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# Measuring Economic Performance in the Private Sector

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## 1. INTRODUCTION

The problem of measuring economic performance involves comparisons. The output of an economic system is greater or less than its output at some previous point in time. The input of factors of production is greater or less in one industry than another. The standard of living in one region is greater or less than in another. Systems of economic accounts have provided a useful framework for organizing the information required for comparisons of this type.

Comparisons between the performance of two economies or the performance of an economic system at two points of time are of great interest from the scientific point of view. They are also of interest for the evaluation of economic policies. Evaluation of alternative policies involves comparison of the present state of affairs and possible alternative states associated with changes in policy.

The description of alternative states of an economic system involves the value of accounting magnitudes associated with each state. Changes from one state to another must be separated into price and quantity components. For example, the measurement of inflation involves an analysis of price changes, while the measurement of real output involves changes in quantity.

In view of the importance of the separation of changes in accounting magnitudes into price and quantity components, it is not surprising that much attention has been given to the measurement of real product. The scope of the product measure—whether and how to include activities internal to households, institutions, and governments, or services of the external environment—has been discussed in great detail.

Denison has recently drawn attention to the limitations inherent in a one-dimensional view of economic performance [17]. In comparing economic systems or alternative states of an economy it is impossible to summarize all the relevant information in a single measure of economic welfare. Real output is important, but the composition of output —by end use, industry of origin, and so on—is equally important in interpreting economic events and evaluating performance.

A complete economic system includes a production account, incorporating data on output and factor input, an income and expenditure account—giving data on factor incomes, expenditures, and saving—and an accumulation account, allocating saving to its uses in various types of capital formation.[1] In addition, a complete system contains data on national wealth from both asset and liability points of view. All of these accounting magnitudes are of interest in evaluating economic performance.

Although the separation of changes in accounting magnitudes into price and quantity components is of fundamental importance for the evaluation of economic performance, only the measurement of real product and real assets is well-established in accounting practice. For the evaluation of economic performance, measures of factor input, income, expenditures, saving, and capital formation in both current and constant prices are essential.

In this paper we present a complete accounting system in constant prices that comprehends all the aspects of economic performance we have listed above. This system is implemented in detail for the private sector of the U.S. economy. Although it would be desirable to implement the system for a detailed breakdown of the economy by sectors, our presentation is limited to national aggregates.

In measuring economic performance, our basic framework consists of a production account for the U.S. private domestic economy and a consolidated income and expenditure account for the U.S. private national economy. The income and expenditure account is consolidated with the accumulation account to provide a complete summary of the income of the private sector and its disposition in the form of consumer expenditures and capital formation.

[1] For a description of a complete accounting system, see [55].

For the production account, the fundamental accounting identity is that the value of output is equal to the value of factor input. Changes in the values of product and factor input are separated into price and quantity components. A summary measure of performance is based on the level of productivity, defined as the ratio of real product to real factor input or the ratio of the price of input to the price of output.

For the consolidated income and expenditure account the fundamental accounting identity is that the value of consumer receipts is equal to consumer outlays plus capital formation. Consumer receipts, consumer outlays, and capital formation can be separated into price and quantity components. A summary measure of performance is based on the standard of living, defined as the ratio of real expenditures to real receipts or the ratio of the price of factor services to the price of expenditures.

The interpretation of real product, real factor input, and total factor productivity requires the notion of a social production possibility frontier.[2] In each period the inputs of factors of production are transformed into outputs. In an extended description of the production possibilities, the inputs may include durable goods of various ages, inventories and financial claims, as well as the services of labor and natural agents.[3] The outputs would include used durable goods, unspent inventories, and goods and services for private or public consumption.

The interpretation of real consumer receipts and outlays and the standard of living requires the notion of a social welfare function.[4] An extended description of the determinants of social welfare must include all "goods" and "bads" relevant to social choice. Within the conventional framework the "goods" would include deliveries to final consumption in every future period and the "bads" would include deliveries of labor services in every future period. Evaluation would involve comparisons of "wealth-like" magnitudes.[5]

In this paper, we concentrate on the development of a complete accounting system in both current and constant prices. We limit the transactions included to those that can be measured or imputed from presently available primary data sources—income tax returns, population and production censuses and surveys, and so on. We present data

[2] This interpretation is developed by Solow [54], Richter [50], and Jorgenson and Griliches [40].

[3] An extended description is presented by Malinvaud [46] and Hicks [36].

[4] This interpretation is developed by Samuelson [51], [52], and many others; detailed references to the literature are given in [52, pp. 44–52].

[5] See [52, pp. 53–56].

for the period 1929–69 for each of the accounting magnitudes we discuss.

The first step in constructing an accounting system for the measurement of economic performance is to develop accounts in current prices. We present income and wealth accounts, including production, income and expenditure, accumulation and wealth accounts for the U.S. private economy for 1929–69 in Section 2 below.

In Section 3, we introduce the problem of constructing accounts in constant prices with a description of our system of index numbers for prices and quantities. In Section 4, we present an extension of the perpetual inventory method, familiar from national wealth accounting, to incorporate prices as well as quantities of capital goods. The price counterpart of the perpetual inventory method involves the estimation of prices of capital goods of every vintage at each point of time.

The presentation of a system of accounts in constant prices begins in Section 5 with the production account. The product side of the account includes consumption and investment goods output in constant prices. The factor outlay side includes labor and capital input. The ratio of output in constant prices to factor input is equal to total factor productivity. We present estimates of product, factor input, and total factor productivity for the U.S. private domestic economy for 1929–69.

In Section 6, we present income and expenditure, accumulation, and wealth accounts in constant prices for the U.S. private national economy, 1929–69. Consolidating the income and expenditure accounts we obtain a single account giving income and its disposition in constant prices.

We conclude with a discussion of possible extensions of the accounting framework in Section 7. The educational sector of the U.S. economy, which is largely governmental rather than private, could be incorporated into our accounting system by compiling data on educational investment, capital and labor input used in the educational sector, and the stock of human capital. Research and development expenditures in the private sector are treated on current account; expenditures on research should be capitalized.

Many other extensions of our accounting framework can be suggested. Activities internal to the household and government sectors could be incorporated into the accounting system by making appropriate imputations for nonmarket activities. Accounts for the educational sector could serve as a prototype for complete accounts for the household and government sectors.

A different range of extensions, not discussed in Section 7, would involve the compilation of accounts in constant prices for individual sectors of the economy. The production account could be disaggregated and complete interindustry accounts in constant prices could be incorporated into the system. The wealth account could be extended to include both assets and liabilities. Accumulation and wealth accounts could be disaggregated to incorporate complete flow of funds accounts.

As a basis for comparison we contrast our approach with two alternative accounting systems. The first is the U.S. national accounts, augmented by Denison's *Sources of Economic Growth,* which extends the framework of the U.S. accounts considerably.[6] The second is the United Nations System of National Accounts, as revised in 1968 [55]. In both systems, efforts have been made to develop accounts in both current and constant prices.

Despite the severe self-imposed limitations of our accounting system, concentrating on national aggregates of transactions that are already included in present accounting systems, our accounts differ very substantially from current practice. In comparing our system with available alternatives we focus attention on these differences. The basic similarities between our approach and eurrent accounting practice can be recognized through the heavy reliance we have placed on data derived from the U.S. national accounts.

## 2. INCOME AND WEALTH

### *2.1. Introduction*

The first problem in accounting for economic performance is the measurement of income and wealth in current prices. The solution of this problem requires a system of four accounts. First, the production account includes data on the output of the producing sector and the outlay of that sector on factor services, both expressed in current prices. Second, the income and expenditure account contains data on transfer payments and income from factor services, consumer outlays, and saving. Third, the accumulation account includes data on saving, capital formation, revaluation of existing assets, and the change in wealth from period to period. Finally, successive values of wealth are contained in the wealth account.

[6] All references to data from the U.S. national income and product accounts are to [49a] and [14].

*2.2. Production Account*

The production account contains data on the value of output and the value of input. As an accounting identity, the value of output is equal to the value of input. The two sides of the production account are linked through production of investment goods and compensation for the services of capital. Investment goods output enters the change in wealth from period to period through capital formation. Accumulated wealth generates factor incomes that arise as compensation for the services of capital. Investment goods output and property compensation must be defined in a consistent manner.

In the U.S. national income and product accounts, total output is divided among services, nondurable goods, durable goods, and structures.[7] The output of services includes the services of owner-occupied dwellings; the output of structures includes the production of new residential housing. Capital formation in the form of residential housing is a component of the change in wealth from period to period; property compensation includes the imputed value of compensation for the use of owner-occupied dwellings. The output of durables includes consumer durables and producer durables used by nonprofit institutions. However, property compensation, as defined in the U.S. national accounts, does not include the imputed value of the services of these durables.

In the U.S. national accounts, the value of the services of owner-occupied residential real estate, including structures and land, is imputed from market rental prices of renter-occupied residential real estate. The value of these services is allocated among net rent, interest, taxes, and capital consumption allowances. A similiar imputation is made for the services of real estate used by nonprofit institutions, but the imputed value excludes net rent.

To preserve consistency between the accounts for investment goods production and for property compensation we introduce imputations for the value of the services of consumer durables and durables used by nonprofit institutions and the net rent of real estate used by institutions. The value of the services of these assets is included in the output of services, together with the services of owner-occupied dwellings. Property compensation also includes the value of these services. This imputation preserves the accounting identity between the value of output and the value of input.

[7] See [49a, Tables 1.4 and 1.5].

We implement the production account for the U.S. private domestic economy, including the production activities of U.S. business and household sectors.[8] In principle, similar accounts could be constructed for government and rest of the world sectors of the economy. Wealth accounts for the government sector would be required for construction of a production account for government comparable to our production account for the private domestic sector.

We define revenue as proceeds to the sector from the sale of output, and outlay as gross outlays by the sector on purchases of input. Our concept of output is intermediate between gross output at market prices and gross output at factor cost, as these terms are usually employed. Output at market prices includes all indirect taxes in the value of output; output at factor cost excludes all indirect taxes. We distinguish between taxes charged against revenue, such as excise or sales taxes, and taxes that are part of the outlay on factor services, such as property taxes. We exclude taxes on output from the value of gross output since these taxes are not included in the proceeds to the sector. We include taxes on input since these taxes are included in the outlay of the sector.

Taxes on output reduce the proceeds of the sector and subsidies increase these proceeds; accordingly, the value of output includes production subsidies. To be more specific, we exclude excise and sales taxes, business nontax payments, and customs duties from the value of output and include other indirect business taxes plus subsidies and less current surplus of federal and state and local government enterprises. The resulting production account is given for 1958 in Table 1.

As an accounting identity, the value of gross private domestic factor outlay is equal to the value of gross private domestic product. Factor outlay is the sum of income originating in private enterprises and private households and institutions, plus the imputed value of consumer durables, producer durables utilized by institutions, and the net rent on institutional real estate, plus indirect taxes included in factor outlay. Factor outlay includes capital consumption allowances, business transfer payments, and the statistical discrepancy. Capital consumption allowances are part of the rental value of capital services. We include business transfer payments and the statistical discrepancy in factor outlay on capital. The value of gross private domestic factor outlay for the year 1958 is presented in Table 1.

[8] Our estimates are based on those of Christensen and Jorgenson [8].

TABLE 1

Production Account, Gross Private Domestic Product and Factor Outlay, United States, 1958 (billions of current dollars)

| | | |
|---|---|---|
| | *Product* | |
| 1. | Private gross national product (Table 1.7) | 405.2 |
| 2. | − Income originating in government enterprises (Table 1.13) | 4.8 |
| 3. | − Rest of the world gross national product (Table 1.7) | 2.0 |
| 4. | + Services of consumers' durables (our imputation) | 40.3 |
| 5. | + Services of durables held by institutions (our imputation) | 0.3 |
| 6. | + Net rent on institutional real estate (our imputation) | 0.8 |
| 7. | − Federal indirect business tax and nontax accruals (Table 3.1) | 11.5 |
| 8. | + Capital stock tax (Table 3.1, note 2) | – |
| 9. | − State and local indirect business tax and nontax accruals (Table 3.3) | 27.0 |
| 10. | + Business motor vehicle licenses (Table 3.3) | 0.8 |
| 11. | + Business property taxes (Table 3.3) | 13.8 |
| 12. | + Business other taxes (Table 3.3) | 2.9 |
| 13. | + Subsidies less current surplus of federal government enterprises (Table 3.1) | 2.7 |
| 14. | − Current surplus of state and local government enterprises (Table 3.3) | 1.8 |
| 15. | = Gross private domestic product | 419.7 |
| | *Factor Outlay* | |
| 1. | Capital consumption allowances (Table 1.9) | 38.9 |
| 2. | + Business transfer payments (Table 1.9) | 1.6 |
| 3. | + Statistical discrepancy (Table 1.9) | 1.6 |
| 4. | + Services of consumers' durables (our imputation) | 40.3 |
| 5. | + Services of durables held by institutions (our imputation) | 0.3 |
| 6. | + Net rent on institutional real estate (our imputation) | 0.8 |
| 7. | + Certain indirect business taxes (product account above, lines 8 + 10 + 11 + 12) | 17.4 |
| 8. | + Income originating in business (Table 1.13) | 312.2 |
| 9. | − Income originating in government enterprises (Table 1.13) | 4.8 |
| 10. | + Income originating in households and institutions (Table 1.13) | 11.4 |
| 11. | = Gross private domestic factor outlay | 419.7 |

NOTE: All table references are to [49].

Product and income accounts are linked through capital formation and the corresponding compensation of property. To make this link explicit we must divide the total product between consumption and investment goods and total factor outlay between labor and property compensation. Investment goods production in the private domestic sector is equal to the total output of durable goods and structures included in the gross national product. Consumption goods production in the private domestic sector is equal to the output of nondurable goods and services in the gross national product, less the output of the foreign and government sectors, plus our imputation for the services of consumer durables and institutional durables and the net rent of institutional real estate. The output of the foreign and government sectors consists entirely of services.

The imputed value of the services of consumer and institutional durables and the net rent on institutional real estate is included in the value of output and the value of capital input. The value of outlay on capital services also includes the property income of self-employed persons; profits, rentals, and interest; capital consumption allowances; business transfer payments; the statistical discrepancy; and indirect taxes included in outlay on capital services, such as motor vehicle licenses, property taxes, and other taxes. The value of labor input includes the compensation of employees in private enterprises and in private households and nonprofit institutions, plus the labor compensation of the self-employed.

We estimate labor compensation of the self-employed by assuming that the compensation per full-time equivalent employee is equal to the labor compensation of proprietors and unpaid family workers.[9] This method is only one of many that have been proposed. Denison has suggested that the results are biased in the direction of allocating too large a proportion of the income of the self-employed to labor compensation.[10] However, Christensen has shown that the method produces results consistent with the assumption that rates of return to property used by the self-employed are comparable to rates of return in the corporate business sector when appropriate corrections are made for taxation and accrued capital gains or losses.[11] Gross private domestic

[9] Self-employed persons include proprietors and unpaid family workers. The method for imputation of labor compensation of the self-employed that underlies our estimates is discussed in detail by Christensen [6]. Alternative methods for imputation are reviewed in [44].

[10] See [16, p. 4].

[11] See [6].

product and factor outlay in current prices for 1929–69 are given in Table 2. Total product is divided between investment and consumption goods output. Total factor outlay is divided between labor and property compensation.

### 2.3. *Income and Expenditure Account*

The income and expenditure account includes data on transfer payments and the value of income from factor services, the value of consumer outlays, and saving. As an accounting identity, the value of consumer receipts is equal to the value of consumer outlays plus saving. The two sides of the income and expenditure account are linked through property compensation and saving. Saving results in the accumulation of tangible assets and financial claims; the accumulated wealth generates future property income. Saving must be defined in a way that is consistent with accounts for property income. Income must include all payments for factor services that result in consumption expenditures or in the accumulation of assets that result in future income.

We implement the income and expenditure account for the U.S. private national economy.[12] For this purpose we consolidate the accounts of private business with those of private households and institutions. Financial claims on the business sector by households and institutions are liabilities of the business sector; in the consolidated accounts these assets and liabilities cancel out. The assets of the private national economy include the tangible assets of the business sector. We treat social insurance funds as part of the private national economy. The claims of these funds on other governmental bodies are treated as assets of the private sector.

In the U.S. national accounts the income and expenditure account of the government sector does not include income from tangible assets owned by governmental bodies. If capital accounts were available for the government sector, we could construct income and expenditure accounts for that sector analogous to our accounts for the private sector. The income and expenditure account of the rest of the world sector of the U.S. national accounts is comparable to our account for the private sector.

We define income of the private national economy as proceeds from the sale of factor services. We define expenditure of the sector as consumer outlays plus saving. Our concept of income is closer to that underlying the concept of gross private saving in the U.S. national accounts

[12] Our estimates are based on those of Christensen and Jorgenson [9].

TABLE 2

Gross Private Domestic Product and Factor Outlay, 1929–69
(billions of current dollars)

| Year | Gross Private Domestic Product | Invest-ment Goods Product | Consump-tion Goods Product | Labor Outlay | Property Outlay |
|---|---|---|---|---|---|
| 1929 | 104.2 | 28.5 | 75.7 | 60.5 | 43.7 |
| 1930 | 91.0 | 20.3 | 70.8 | 55.6 | 35.5 |
| 1931 | 76.6 | 14.1 | 62.5 | 47.3 | 29.3 |
| 1932 | 55.7 | 7.2 | 48.5 | 37.2 | 18.6 |
| 1933 | 54.5 | 7.5 | 47.0 | 34.8 | 19.7 |
| 1934 | 62.3 | 10.4 | 51.9 | 39.3 | 23.0 |
| 1935 | 67.9 | 12.7 | 55.2 | 42.6 | 25.3 |
| 1936 | 77.6 | 17.1 | 60.5 | 47.4 | 30.2 |
| 1937 | 85.3 | 19.6 | 65.7 | 53.7 | 31.6 |
| 1938 | 79.1 | 15.3 | 63.8 | 49.6 | 29.6 |
| 1939 | 85.4 | 19.3 | 66.1 | 53.0 | 32.4 |
| 1940 | 94.0 | 23.9 | 70.1 | 57.1 | 36.9 |
| 1941 | 115.9 | 36.9 | 79.0 | 69.6 | 46.3 |
| 1942 | 140.9 | 47.6 | 93.3 | 86.9 | 54.0 |
| 1943 | 167.8 | 60.5 | 107.3 | 102.3 | 65.5 |
| 1944 | 178.6 | 61.2 | 117.4 | 108.7 | 69.9 |
| 1945 | 177.7 | 52.8 | 124.9 | 108.4 | 69.3 |
| 1946 | 190.3 | 49.7 | 140.6 | 119.5 | 70.8 |
| 1947 | 218.3 | 64.1 | 154.2 | 137.8 | 80.6 |
| 1948 | 244.4 | 72.8 | 171.7 | 151.0 | 93.4 |
| 1949 | 235.1 | 72.1 | 162.9 | 148.9 | 86.1 |
| 1950 | 270.4 | 91.1 | 179.3 | 162.6 | 107.8 |
| 1951 | 305.8 | 106.0 | 199.8 | 183.8 | 122.0 |
| 1952 | 320.7 | 108.2 | 212.5 | 196.0 | 124.7 |
| 1953 | 340.6 | 115.1 | 225.5 | 210.5 | 130.1 |
| 1954 | 341.6 | 111.0 | 230.6 | 209.2 | 132.4 |
| 1955 | 377.7 | 128.5 | 249.2 | 225.0 | 152.7 |
| 1956 | 395.5 | 135.3 | 260.2 | 242.6 | 152.9 |
| 1957 | 413.4 | 140.0 | 273.4 | 254.1 | 159.3 |
| 1958 | 419.7 | 130.6 | 289.2 | 252.8 | 166.9 |
| 1959 | 458.1 | 146.8 | 311.3 | 273.6 | 184.5 |

(continued)

TABLE 2 (concluded)

| Year | Gross Private Domestic Product | Investment Goods Product | Consumption Goods Product | Labor Outlay | Property Outlay |
|---|---|---|---|---|---|
| 1960 | 475.5 | 148.8 | 326.7 | 286.5 | 189.1 |
| 1961 | 488.1 | 147.5 | 340.6 | 292.3 | 195.8 |
| 1962 | 524.6 | 163.4 | 361.2 | 310.7 | 213.9 |
| 1963 | 553.4 | 173.2 | 380.2 | 325.0 | 228.4 |
| 1964 | 592.7 | 186.6 | 406.1 | 346.8 | 245.9 |
| 1965 | 640.5 | 204.5 | 436.0 | 371.4 | 269.1 |
| 1966 | 703.5 | 224.1 | 479.4 | 405.8 | 297.7 |
| 1967 | 739.8 | 229.3 | 510.4 | 429.9 | 309.8 |
| 1968 | 798.3 | 251.3 | 546.9 | 469.8 | 328.5 |
| 1969 | 863.7 | 270.7 | 593.0 | 514.8 | 348.9 |

than to the more commonly employed concept of personal disposable income. Accordingly, we refer to our income concept as gross private national income. Outlay on factor services by the production sector includes indirect taxes such as property taxes and motor vehicle licenses. This outlay also includes direct taxes such as corporate and personal income taxes. Our concept of gross private national income excludes both indirect and direct taxes.

To be specific, gross private national income includes labor and property income originating in the private domestic economy and the rest of the world sectors, labor income originating in the government sector, net interest paid by government, and the statistical discrepancy. Income is net of indirect taxes on factor outlay and all direct taxes on incomes. Gross private national income excludes interest paid by consumers and personal transfer payments to foreigners. Income also includes the investment income of social insurance funds, less transfers to general government by these funds. Contributions to social insurance are included and transfers from social insurance funds are excluded from income. The value of gross private national income and expenditures for the year 1958 are presented in Table 3.

Consumption is equal to personal consumption expenditures on services and nondurable goods plus our imputation for the services of consumer and institutional durables and the net rent of institutional real

## TABLE 3

Gross Private National Receipts and Expenditures, 1958
(billions of current dollars)

| | |
|---|---|
| *Receipts* | |
| 1. Gross private domestic factor outlay [a] | 419.7 |
| 2. + Income originating in general government (Table 1.13) [b] | 42.1 |
| 3. + Income originating in government enterprises (Table 1.13) | 4.8 |
| 4. + Income originating in rest of world (Table 1.13) | 2.0 |
| 5. + Investment income of social insurance funds (Table 3.7) | 1.8 |
| 6. − Transfers to general government from social insurance funds (Table 3.7) | 0.6 |
| 7. + Net interest paid by government (Tables 3.1 and 3.3) | 6.2 |
| 8. − Corporate profits tax liability (Table 1.10) | 19.0 |
| 9. − Business property taxes [c] | 17.4 |
| 10. − Personal tax and nontax payments (Table 2.1) | 42.3 |
| 11. + Personal nontax payments (Tables 3.1 and 3.3) | 2.3 |
| 12. = Gross private national income | 399.5 |
| 13. + Government transfer payments to persons other than benefits from social insurance funds | 8.1 |
| 14. = Gross private national consumer receipts | 407.7 |
| *Expenditures* | |
| 1. Personal consumption expenditures (Table 1.1) | 290.1 |
| 2. − Personal consumption expenditures, durable goods (Table 1.1) | 37.9 |
| 3. + Services of consumer durables (our imputation) [d] | 40.3 |
| 4. + Services of institutional durables (our imputation) [d] | 0.3 |
| 5. + Opportunity cost of equity capital, institutional real estate (our imputation) [e] | 0.8 |
| 6. = Private national consumption expenditure | 293.6 |
| 7. + Personal transfer payments to foreigners (Table 2.1) | 0.6 |
| 8. + Personal nontax payments (Tables 3.1 and 3.3) | 2.3 |
| 9. = Private national consumer outlays | 296.5 |
| 10. + Gross private national saving [f] | 111.2 |
| 11. = Private national expenditures | 407.7 |

[a] [8, Table 1, p. 23]. This series has been revised to include a net rent imputation to institutional structures. Our other imputations have also been slightly modified. See expenditure items 3, 4, and 5 below.

[b] All table references are to [49].

[c] [8, Table 1, p. 23, line 6, in factor outlay].

[d] [8, Section 5].

[e] We have computed an implicit rental value for institutional structures and land based on our estimate of the rate of return to owner-occupied real estate. The opportunity cost of equity capital is the difference between the implicit rental value and the net rent figure [49, Table 7.3]. This imputation was suggested to us by Edward F. Denison.

[f] See Table 5, line 10, below.

estate. Purchases of consumer durables, included in personal consumption expenditures in the U.S. national accounts, are treated as part of saving in our income and expenditure account. The value of consumption includes taxes and excludes subsidies on output; these taxes are excluded from the value of consumption goods output in the production account. Our concept of saving differs from gross private saving as defined in the U.S. national accounts in the treatment of social insurance and the statistical discrepancy. The expenditure account for the consuming sector for the year 1958 is presented in Table 3.

Our definition of income is similar to the concept of income underlying the U.S. national accounts concept of gross private saving. Our concept of income differs from the national accounts concept in the treatment of social insurance and transfer payments, the inclusion of the services of consumer and institutional durables, the net rent on institutional real estate, and the statistical discrepancy. Transfer payments are treated as a nonincome receipt of the consumer sector. The services of durables, net rent, and the statistical discrepancy are treated as part of outlays on capital services. The services of durables are included in output and capital input in order to preserve consistency between the definition of investment goods in the production account and the definition of property compensation in the factor outlay account. Net rent is included in output and factor outlay to preserve consistency between the treatment of owner-occupied residential real estate and institutional real estate. The statistical discrepancy is assigned to factor outlay so that the accounting identity between the value of output and the value of factor outlay is preserved.

Our treatment of social insurance can be compared with the treatment that underlies the U.S. national accounting concepts of personal disposable income and gross private saving. In these income concepts the social insurance funds are treated as part of the government sector rather than the private sector. Contributions to social insurance are treated as a tax, benefits paid by these funds are treated as a transfer payment, and the claims of these sectors on other governmental bodies are treated as claims on the government by itself that cancel out in a consolidated government wealth account. Our concept of income focuses on the separation of contributions to social insurance from other taxes and on the effects of a future stream of benefits on saving decisions by individuals. The national accounts treatment focuses on the involuntary nature of contributions to social insurance.

The differences between our concept of income and the national accounts concept of personal disposable income are very substantial.

In addition to the differences we have already outlined, our concept of income includes undistributed corporate profits, the corporate inventory valuation adjustment, corporate and noncorporate capital consumption allowances, and wage accruals less disbursements. All of these components of factor outlay are excluded from personal disposable income. We also exclude government transfer payments and net interest paid by consumers, which are included in personal disposable income. These differences between gross private national income, as we have defined it, and personal disposable income are primarily attributable to our consolidation of the accounts of the private business sector with those of private households and institutions. The income of the private sector includes all property compensation whether paid out in the form of dividends and interest or retained by the business sector.

Income and expenditure accounts are linked through saving and the resulting income from the services of property. To make this link explicit we must divide income between labor and property compensation and expenditure between saving and consumption. The measurement of labor and property compensation gross of taxes is straightforward. We have already described the allocation of private domestic factor outlay between the value of capital input and the value of labor input. Corresponding allocations for government and rest of the world sectors are available from the U.S. national accounts. The problem is to allocate taxes on factor services between labor services and capital services. We allocate indirect business taxes on factor services and the corporate income tax to income from capital. The problem that remains is to allocate personal income tax payments between income from labor and income from capital.

To allocate personal income tax payments between labor and property compensation we employ a method developed by Frane and Klein [21] and applied by Ando and Brown [1] to U.S. data on the personal income tax for 1929 to 1958. Personal income taxes on income from labor services are a remarkably stable proportion of total personal income tax receipts. The data for 1929 to 1958 show that the proportion of taxes on labor income in total personal taxes for the latter part of the period is .755 with negligible variation. We have extended the estimates of personal income taxes on labor income by Ando and Brown to 1969 by assuming that the proportion of these taxes in total personal income taxes is constant at 0.755. Personal income taxes not allocated to labor income are allocated to property income. Gross private national receipts and expenditures in current prices for 1929–69 are given in Table 4. Income is divided between labor and property

TABLE 4

Gross Private National Receipts and Expenditures, 1929–69
(billions of current dollars)

| Year | Gross Private National Income | Labor Compensation | Property Compensation | Gross Private National Receipts and Expenditures | Consumption Expenditures | Consumer Outlays | Gross Private National Saving |
|---|---|---|---|---|---|---|---|
| 1929 | 102.7 | 65.5 | 37.1 | 103.5 | 77.0 | 78.1 | 25.4 |
| 1930 | 90.1 | 60.8 | 29.3 | 91.0 | 71.4 | 72.4 | 18.6 |
| 1931 | 76.9 | 52.7 | 24.1 | 78.8 | 63.5 | 64.4 | 14.3 |
| 1932 | 56.2 | 42.3 | 13.9 | 57.4 | 50.0 | 50.8 | 6.7 |
| 1933 | 55.3 | 39.9 | 15.3 | 56.5 | 49.3 | 50.0 | 6.5 |
| 1934 | 63.8 | 45.4 | 18.4 | 65.1 | 53.9 | 54.6 | 10.5 |
| 1935 | 69.2 | 49.1 | 20.1 | 70.7 | 56.4 | 57.1 | 13.7 |
| 1936 | 79.0 | 55.2 | 23.8 | 81.7 | 62.8 | 63.5 | 18.2 |
| 1937 | 85.6 | 61.0 | 24.6 | 87.2 | 66.5 | 67.2 | 20.0 |
| 1938 | 80.6 | 57.6 | 23.1 | 82.3 | 65.4 | 66.0 | 16.3 |
| 1939 | 86.8 | 61.1 | 25.8 | 88.5 | 67.6 | 68.2 | 20.3 |
| 1940 | 94.1 | 65.3 | 28.8 | 95.7 | 70.6 | 71.3 | 24.4 |
| 1941 | 112.1 | 79.3 | 32.8 | 113.8 | 78.7 | 79.4 | 34.4 |
| 1942 | 136.7 | 100.7 | 35.9 | 138.4 | 87.1 | 87.8 | 50.6 |
| 1943 | 160.3 | 118.3 | 41.9 | 162.0 | 103.0 | 103.9 | 58.1 |
| 1944 | 178.5 | 130.1 | 48.4 | 180.7 | 111.6 | 112.6 | 68.1 |
| 1945 | 181.8 | 132.0 | 49.8 | 185.9 | 123.1 | 124.3 | 61.6 |
| 1946 | 185.2 | 130.7 | 54.5 | 193.4 | 142.8 | 144.1 | 49.4 |
| 1947 | 204.1 | 142.1 | 62.0 | 212.7 | 158.6 | 159.9 | 52.8 |
| 1948 | 230.0 | 156.9 | 73.1 | 238.0 | 171.0 | 172.6 | 65.4 |
| 1949 | 227.0 | 159.3 | 67.6 | 234.7 | 166.6 | 168.1 | 66.6 |
| 1950 | 254.0 | 174.1 | 79.9 | 261.8 | 184.8 | 186.3 | 75.5 |
| 1951 | 282.9 | 194.6 | 88.3 | 289.6 | 200.7 | 202.2 | 87.4 |
| 1952 | 299.4 | 206.9 | 92.4 | 306.0 | 215.3 | 216.9 | 89.0 |
| 1953 | 317.1 | 221.2 | 96.0 | 323.6 | 227.6 | 229.4 | 94.2 |
| 1954 | 324.3 | 223.6 | 100.7 | 331.0 | 235.2 | 237.2 | 93.8 |

(continued)

TABLE 4 (concluded)

| Year | Gross Private National Income | Labor Compensation | Property Compensation | Gross Private National Receipts and Expenditures | Consumption Expenditures | Consumer Outlays | Gross Private National Saving |
|---|---|---|---|---|---|---|---|
| 1955 | 354.7 | 239.3 | 115.4 | 361.9 | 252.8 | 254.8 | 107.1 |
| 1956 | 370.5 | 256.7 | 113.8 | 377.7 | 265.9 | 268.3 | 109.4 |
| 1957 | 388.6 | 269.2 | 119.5 | 396.2 | 278.7 | 281.4 | 114.9 |
| 1958 | 399.5 | 271.5 | 128.1 | 407.7 | 293.6 | 296.5 | 111.2 |
| 1959 | 431.9 | 292.3 | 139.7 | 440.3 | 315.6 | 318.7 | 121.6 |
| 1960 | 448.5 | 305.8 | 142.7 | 456.9 | 331.8 | 334.9 | 122.0 |
| 1961 | 462.0 | 314.7 | 147.3 | 471.0 | 343.0 | 346.4 | 124.6 |
| 1962 | 496.2 | 333.9 | 162.3 | 505.4 | 360.4 | 364.2 | 141.2 |
| 1963 | 522.8 | 350.0 | 172.8 | 532.5 | 380.8 | 385.0 | 147.5 |
| 1964 | 567.2 | 379.2 | 188.0 | 577.4 | 406.5 | 411.3 | 166.1 |
| 1965 | 609.9 | 404.9 | 205.0 | 621.1 | 433.9 | 439.1 | 181.9 |
| 1966 | 669.4 | 442.2 | 227.2 | 681.3 | 470.8 | 476.6 | 204.8 |
| 1967 | 708.0 | 470.4 | 237.6 | 721.8 | 499.0 | 505.7 | 216.1 |
| 1968 | 756.0 | 511.0 | 245.0 | 771.5 | 536.2 | 543.7 | 227.8 |
| 1969 | 807.8 | 551.8 | 256.0 | 825.7 | 583.5 | 591.9 | 233.8 |

compensation, net of taxes. Expenditure is divided between consumer outlays and saving.

### *2.4. Accumulation Account*

The accumulation account includes data on saving, capital formation, revaluation of existing assets, and the change in wealth from period to period. Gross private national saving is reduced by depreciation to obtain saving as it enters the accumulation account. As an accounting identity, the value of saving is equal to the value of capital formation. The change in wealth from period to period is equal to saving plus the revaluation of existing assets. Although revaluations are part of the change in wealth, they are excluded from income and from saving. In measuring the return from investment in different types of assets, both returns in the form of income and returns from revaluations must be considered.

TABLE 5

Gross Private National Capital Formation, Saving, and Revaluation, 1958
(billions of current dollars)

| | | |
|---|---|---|
| | *Saving* | |
| 1. | Personal saving (Table 2.1) | 22.3 |
| 2. | + Undistributed corporate profits (Table 5.1) | 10.8 |
| 3. | + Corporate inventory valuation adjustment (Table 5.1) | −0.3 |
| 4. | + Corporate capital consumption allowances (Table 5.1) | 22.0 |
| 5. | + Noncorporate capital consumption allowances (Table 5.1) | 16.9 |
| 6. | + Wage accruals less disbursements (Table 5.1) | 0.0 |
| 7. | + Personal consumption expenditures, durable goods (Table 1.1) | 37.9 |
| 8. | + Surplus, social insurance funds (Table 3.7) | 0.0 |
| 9. | + Statistical discrepancy (Table 1.9) | 1.6 |
| 10. | = Gross private national saving | 111.2 |
| 11. | − Depreciation (our imputation) | 80.8 |
| 12. | = Net private national saving | 30.4 |
| 13. | + Revaluation (our imputation) | 31.6 |
| 14. | = Change in private national wealth | 62.1 |
| | *Capital Formation* | |
| 1. | Gross private domestic investment (Table 1.2) | 60.9 |
| 2. | + Personal consumption expenditures, durable goods (Table 1.1) | 37.9 |
| 3. | + Deficit of federal government (Table 3.1) | 10.2 |
| 4. | + Deficit of state and local governments (Table 3.3) | 2.3 |
| 5. | − Deficit, federal social insurance funds (Table 3.7) | −1.6 |
| 6. | − Deficit, state and local social insurance funds (Table 3.7) | 1.7 |
| 7. | + Net foreign investment (Table 5.1) | −0.2 |
| 8. | = Gross private national capital formation | 111.2 |

NOTE: Table references are to [49].

We implement the accumulation account for the U.S. private national economy.[13] Sources of saving include gross private saving, as defined in the U.S. national accounts, the surplus of federal and state and local social insurance funds, personal consumption expenditures on durable goods, and the statistical discrepancy. Capital formation includes gross private domestic investment, personal consumption expenditures on durable goods, deficits of the federal, state, and local governments excluding social insurance funds, and net foreign invest-

[13] Our estimates are based on [9].

ment. Private national saving and capital formation are given for 1958 in Table 5.

In the U.S. national accounts depreciation on tangible assets in the business sector is set equal to depreciation claimed for tax purposes. We replace this estimate of depreciation by our own imputation, described in detail in Section 4 below. No depreciation for consumer durables and durables used by institutions is included in the U.S. national accounts. Our imputed value of depreciation includes depreciation for both these classes of assets.

To estimate the change in wealth from period to period we require estimates of saving net of depreciation and estimates of the revaluation of existing assets due to price changes. Revaluations are not included in the U.S. national accounts, so that an essential link between income and expenditure accounts and wealth accounts is missing. We have estimated the revaluations for private domestic tangible assets as part of our perpetual inventory of capital goods, described in Section 4 below. Our estimates of revaluations for financial claims are based on accounts for stocks of these claims in current prices. We estimate revaluations as the difference between the period to period changes in these stocks and the deficits of the government and rest of the world sectors. Private national saving and capital formation in current prices for 1929–69 are given in Table 6.

### *2.5. Wealth Account*

All of the accounts we have considered up to this point contain data on flows. The production account includes flows of output and input; the income and expenditure account includes the corresponding flows; the flow of saving and changes in wealth from period to period are included in the accumulation account. The wealth account contains data on the stock of wealth in successive periods. The wealth account can be presented in balance sheet form with the value of assets equal to the value of liabilities as an accounting identity. We present only the asset side of the wealth account.

We implement the wealth account for the U.S. private national economy.[14] The wealth accounts of private business are consolidated with those of private households and institutions. Our wealth account includes data on assets in the consolidated account. These assets include the tangible assets of private households and institutions and the tangible assets of private business. In addition, they include net

[14] Our estimates are based on [9].

TABLE 6

Gross Private National Capital Formation, Saving, and Revaluation, 1929–69
(billions of current dollars)

| Year | Gross Private National Saving and Capital Formation | Replacement and Depreciation | Net Private National Saving and Capital Formation | Revaluation | Change in Wealth |
|---|---|---|---|---|---|
| 1929 | 25.4 | 19.2 | 6.2 | 3.4 | 9.7 |
| 1930 | 18.6 | 19.0 | −0.4 | −22.0 | −22.4 |
| 1931 | 14.3 | 17.2 | −2.8 | −39.9 | −42.7 |
| 1932 | 6.7 | 14.7 | −8.0 | −38.7 | −46.7 |
| 1933 | 6.5 | 13.3 | −6.7 | 1.7 | −5.0 |
| 1934 | 10.5 | 13.5 | −2.9 | 19.8 | 16.9 |
| 1935 | 13.7 | 12.9 | 0.7 | 3.4 | 4.2 |
| 1936 | 18.2 | 12.9 | 5.3 | 8.0 | 13.3 |
| 1937 | 20.0 | 14.1 | 5.9 | 17.4 | 23.2 |
| 1938 | 16.3 | 14.8 | 1.4 | −2.1 | −0.7 |
| 1939 | 20.3 | 14.5 | 5.8 | −1.5 | 4.3 |
| 1940 | 24.4 | 15.0 | 9.4 | 6.2 | 15.7 |
| 1941 | 34.4 | 16.8 | 17.6 | 30.2 | 47.8 |
| 1942 | 50.6 | 20.3 | 30.3 | 41.7 | 72.0 |
| 1943 | 58.1 | 20.9 | 37.2 | 27.6 | 64.7 |
| 1944 | 68.1 | 21.7 | 46.5 | 21.8 | 68.3 |
| 1945 | 61.6 | 21.7 | 39.9 | 12.9 | 52.9 |
| 1946 | 49.4 | 22.9 | 26.4 | 52.5 | 79.0 |
| 1947 | 52.8 | 28.4 | 24.4 | 83.2 | 107.6 |
| 1948 | 65.4 | 33.9 | 31.5 | 46.4 | 77.9 |
| 1949 | 66.6 | 37.9 | 28.8 | −10.8 | 17.9 |
| 1950 | 75.5 | 41.8 | 33.7 | 45.4 | 79.0 |
| 1951 | 87.4 | 49.6 | 37.8 | 64.9 | 102.7 |
| 1952 | 89.0 | 53.8 | 35.2 | 15.7 | 50.9 |
| 1953 | 94.2 | 56.5 | 37.7 | 5.2 | 42.9 |
| 1954 | 93.8 | 59.2 | 34.6 | 5.3 | 39.9 |

(continued)

TABLE 6 (concluded)

| Year | Gross Private National Saving and Capital Formation | Replacement and Depreciation | Net Private National Saving and Capital Formation | Revaluation | Change in Wealth |
|---|---|---|---|---|---|
| 1955 | 107.1 | 62.3 | 44.8 | 21.4 | 66.2 |
| 1956 | 109.4 | 69.8 | 39.6 | 58.1 | 97.8 |
| 1957 | 114.9 | 76.5 | 38.4 | 51.9 | 90.3 |
| 1958 | 111.2 | 80.8 | 30.4 | 32.4 | 62.8 |
| 1959 | 121.6 | 83.7 | 37.9 | 39.3 | 77.2 |
| 1960 | 122.0 | 86.7 | 35.3 | 32.1 | 67.4 |
| 1961 | 124.6 | 89.7 | 34.9 | 29.5 | 64.4 |
| 1962 | 141.2 | 92.4 | 48.8 | 40.8 | 89.6 |
| 1963 | 147.5 | 96.3 | 51.2 | 37.6 | 88.7 |
| 1964 | 166.1 | 101.5 | 64.6 | 45.7 | 110.4 |
| 1965 | 181.9 | 107.5 | 74.5 | 52.4 | 126.8 |
| 1966 | 204.8 | 115.7 | 89.0 | 64.1 | 153.1 |
| 1967 | 216.1 | 127.2 | 88.9 | 79.1 | 168.1 |
| 1968 | 227.8 | 138.5 | 89.3 | 98.6 | 187.9 |
| 1969 | 233.8 | 152.0 | 81.8 | 120.7 | 202.5 |

claims on the foreign and government sectors by the private sector. Social insurance funds are treated as part of the private sector rather than as part of government.

Our estimate of the stock of private domestic tangible assets is based on a perpetual inventory of capital goods, as described in Section 4. Our estimate of net claims on foreigners and governments is based on the flow of funds accounts of the Board of Governors of the Federal Reserve System and on *Studies in the National Balance Sheet of the United States* [26] and *The National Wealth of the United States in the Postwar Period* [24].[15] We distinguish between monetary and nonmonetary claims on the federal government by the private sector. Monetary claims include vault cash of commercial banks, member bank reserves, and currency outside banks. Nonmonetary claims on the fed-

[15] Data on flow of funds are based on estimates of [3b], [23], [24], [25], and [26].

eral government include U.S. government total liabilities, less U.S. government financial assets, plus net liabilities of federally sponsored credit agencies and financial assets of included social insurance funds, less U.S. government liabilities to rest of world, plus U.S. government credits and claims abroad, less monetary liabilities. Private sector claims on state and local governments include state and local government total liabilities, less state and local government financial assets, plus assets of cash sickness compensation funds. Net private claims on the rest of the world include private U.S. assets and investments abroad less private U.S. liabilities to foreigners. Private national wealth in 1958 is presented in Table 7. Annual data on the components of private national wealth are presented in Table 8.

### *2.6. The Accounting System*

The production and income and expenditure accounts are related through markets for commodities and factor services. Factor outlay by the producing sector is the most important component of income from the supply of factor services by the consuming sector. Income also includes the value of factor services supplied to the government and rest of the world sectors. The expenditure account is linked to the production account through the market for consumption goods and services. The production of consumption goods also includes goods consumed by the government and the rest of the world sectors. Expenditure on consumption goods includes goods supplied by the rest of the world sector. The expenditure account is also linked to the production indirectly through saving.

The accumulation account allocates saving among its sources and uses. The uses of saving include capital formation through investment in reproducible tangible assets. Expenditure on investment in these assets is linked to the production account through the market for investment goods output. The production of investment goods is partly consumed by government and rest of the world sectors; part of the supply of these goods originates in the rest of the world sector. The accumulation account is linked to the wealth account through the accounting identity between period to period changes in wealth and the sum of saving and revaluations of existing assets.

The structure of this accounting system can be compared with that of the U.S. national accounts. The production account is for gross national product and includes income generated in the government and rest of the world sectors. Our production account is for gross private

TABLE 7

Private National Wealth, 1958
(billions of current dollars)

| | | | |
|---|---|---|---|
| 1. Private domestic tangible assets [a] | | | 1,300.1 |
| 2. + Net claims on the federal, state, and local governments | | | 280.9 |
| a. Federal, monetary [b] | | 50.6 | |
| (i) + Vault cash of commercial banks | 3.2 | | |
| (ii) + Member bank reserves | 18.5 | | |
| (iii) + Currency outside banks | 28.9 | | |
| b. Federal, nonmonetary | | 195.2 | |
| (i) U.S. government total liabilities [b] | 256.4 | | |
| (ii) − U.S. government financial assets [b] | 50.0 | | |
| (iii) + Net liabilities, federally sponsored credit agencies [b] | 0.5 | | |
| (iv) + Assets of included social insurance funds [c] | 30.4 | | |
| (v) − U.S. government liabilities to rest of world [d] | 8.8 | | |
| (vi) + U.S. government credits and claims abroad [d] | 18.3 | | |
| (vii) − Monetary liabilities [b] | 50.6 | | |
| c. State and local | | 35.1 | |
| (i) State and local government total liabilities [b] | 62.6 | | |
| (ii) − State and local government financial assets [b] | 27.7 | | |
| (iii) + Assets of cash sickness compensation fund | 0.2 | | |
| 3. + Net claims on rest of world [d] | | | 13.8 |
| a. Private U.S. assets and investments abroad | | 41.1 | |
| b. − Private U.S. liabilities to foreigners | | 27.3 | |
| 4. = Private national wealth | | | 1,594.9 |

[a] [8]; see [8, pp. 294–301] for a discussion.
[b] [31].
[c] [59, February issues].
[d] [49b, "The International Investment Pattern of the United States," in October issues].

TABLE 8

Private National Wealth, 1929–69
(billions of current dollars)

| Year | Corporate Tangible Assets | Noncorporate Tangible Assets | Household and Institutional Tangible Assets | Net Claims on Governments and Rest of World | Private National Wealth |
|---|---|---|---|---|---|
| 1929 | 116.7 | 106.7 | 158.1 | 33.0 | 414.6 |
| 1930 | 110.3 | 97.9 | 150.0 | 34.0 | 392.2 |
| 1931 | 97.7 | 85.1 | 131.5 | 35.2 | 349.5 |
| 1932 | 84.1 | 73.3 | 108.1 | 37.3 | 302.7 |
| 1933 | 80.4 | 73.4 | 104.4 | 39.4 | 297.6 |
| 1934 | 83.1 | 76.3 | 109.5 | 45.2 | 314.2 |
| 1935 | 83.7 | 79.4 | 107.9 | 47.4 | 318.3 |
| 1936 | 86.6 | 82.7 | 112.5 | 50.0 | 331.8 |
| 1937 | 95.0 | 88.1 | 120.5 | 51.7 | 355.2 |
| 1938 | 92.2 | 85.6 | 122.2 | 54.4 | 354.3 |
| 1939 | 91.9 | 85.3 | 123.6 | 57.8 | 358.6 |
| 1940 | 96.6 | 88.7 | 129.2 | 59.9 | 374.4 |
| 1941 | 109.7 | 98.6 | 143.7 | 70.5 | 422.5 |
| 1942 | 121.1 | 108.6 | 155.1 | 110.3 | 495.0 |
| 1943 | 126.4 | 115.1 | 163.6 | 155.2 | 560.2 |
| 1944 | 130.4 | 121.2 | 173.9 | 202.9 | 628.5 |
| 1945 | 133.1 | 127.2 | 181.2 | 239.0 | 680.6 |
| 1946 | 159.6 | 148.5 | 206.7 | 244.6 | 759.4 |
| 1947 | 199.1 | 175.0 | 252.6 | 240.3 | 867.0 |
| 1948 | 224.2 | 192.2 | 291.8 | 236.0 | 944.2 |
| 1949 | 226.6 | 190.4 | 302.1 | 242.8 | 962.0 |
| 1950 | 248.2 | 212.7 | 341.8 | 238.8 | 1,041.5 |
| 1951 | 286.9 | 234.4 | 384.9 | 238.1 | 1,144.3 |
| 1952 | 303.0 | 237.5 | 408.7 | 245.9 | 1,195.0 |
| 1953 | 315.5 | 238.6 | 427.8 | 256.5 | 1,238.4 |
| 1954 | 323.1 | 244.4 | 442.9 | 267.6 | 1,278.0 |

(continued)

TABLE 8 (concluded)

| Year | Corporate Tangible Assets | Noncorporate Tangible Assets | Household and Institutional Tangible Assets | Net Claims on Governments and Rest of World | Private National Wealth |
|---|---|---|---|---|---|
| 1955 | 344.3 | 253.5 | 477.6 | 268.6 | 1,344.1 |
| 1956 | 381.8 | 269.7 | 518.8 | 271.6 | 1,441.9 |
| 1957 | 411.9 | 287.8 | 553.8 | 278.5 | 1,532.0 |
| 1958 | 422.2 | 303.9 | 574.0 | 294.8 | 1,594.9 |
| 1959 | 443.7 | 315.4 | 612.8 | 300.5 | 1,672.4 |
| 1960 | 461.9 | 330.6 | 640.7 | 306.4 | 1,739.7 |
| 1961 | 476.6 | 347.3 | 663.8 | 316.7 | 1,804.3 |
| 1962 | 500.1 | 367.0 | 698.6 | 328.3 | 1,893.9 |
| 1963 | 524.4 | 384.7 | 737.9 | 335.5 | 1,982.6 |
| 1964 | 556.4 | 404.8 | 785.1 | 346.9 | 2,093.2 |
| 1965 | 598.9 | 433.9 | 831.1 | 355.8 | 2,219.7 |
| 1966 | 660.0 | 464.2 | 880.6 | 367.9 | 2,372.7 |
| 1967 | 714.8 | 494.0 | 943.0 | 389.1 | 2,540.8 |
| 1968 | 771.4 | 529.5 | 1,022.9 | 404.5 | 2,728.3 |
| 1969 | 839.8 | 570.8 | 1,109.5 | 410.5 | 2,930.6 |

domestic product and excludes these two sectors. The income and expenditure account in the U.S. national accounts is for personal income and outlay. Factor outlay in the producing sector taking the form of undistributed corporate profits is excluded from personal income. Our concept of gross private national income is more closely related to the concept of income underlying the U.S. national accounts concept of gross private saving than to the concept of personal disposable income.

The accumulation account of the U.S. national accounts is based on national saving and investment rather than private saving and investment. However, the most serious problem with the accumulation account is the absence of two types of data that are essential in linking income and wealth accounts. The first is an estimate of economic depreciation. Estimates of capital consumption allowances in the U.S. national accounts are based on depreciation reported for tax purposes. As tax laws have evolved over time, these estimates have come to reflect widely varying depreciation formulas and lifetimes of assets for tax purposes.[16] No attempt has been made to replace estimates of depreciation for tax purposes with estimates based on an economic concept of depreciation.[17] We have attempted to remedy this deficiency. The second important omission in the accumulation account is an estimate of the revaluation of assets. Data on revaluations are essential for the construction of an integrated system of national income and wealth accounts.

The structure of our accounting system can also be compared with the United Nations *System of National Accounts* [55]. The principal difference between our system and the U.N. system is that we confine the accounts to the private sector. In the U.N. system the production account is based on the domestic economy rather than the private domestic sector; the income and expenditure account and the accumulation account are based on the national economy rather than the private sector. We have combined the accumulation and revaluation accounts of the U.N. system into a single accumulation account, which also includes period to period changes in national wealth. We have presented only the asset side of the national wealth accounts, while the U.N. system includes a balance sheet with data on both assets and liabilities.

## 3. INDEX NUMBERS

### *3.1. Introduction*

The second problem in accounting for economic performance is the measurement of income and wealth in constant prices. Preliminary to the solution of this problem we must consider the selection of an appropriate system of index numbers. To express any accounting magnitude in constant prices we must separate the change in value from period to period into components associated with change in price and change in quantity. As an illustration, the change in the value of output entering the production account can be separated into a change in the quantity of output and a change in the price of output. Changes in other flows—factor outlay, income, expenditure on consumer goods, and investment—can be decomposed into price and quantity changes in the same way. As a second illustration, the change in the value of

[16] A detailed discussion of tax provisions affecting depreciation and amortization for tax purposes is given in [62].

[17] Estimates of replacement based on the straight-line method and estimates of depreciation and replacement based on the declining balance method for producer durables and nonresidential structures are contained in the Office of Business Economics *Capital Goods Study*. See [30].

wealth entering the wealth account can be separated into a change in the quantity of assets and a change in the price of assets. We identify the change in quantity with saving and the change in price with revaluation of assets.

### *3.2. Divisia Index Numbers*

Our system of index numbers is based on a discrete approximation to continuous index numbers. To illustrate the construction of index numbers of prices and quantities we consider the value of output as it enters the production account. Suppose that $m$ components of output are distinguished in the accounts; the value of output, say $qY$, may be written:

$$qY = q_1Y_1 + q_2Y_2 + \cdots + q_mY_m.$$

Our system of index numbers consists of an index for the price of output $q$ and the quantity of output $Y$, defined in terms of the prices $(q_i)$ and quantities $(Y_i)$ of the $m$ components. The first step in defining these indexes is to differentiate the value of output with respect to time, obtaining:

$$\dot{q}Y + q\dot{Y} = \Sigma\dot{q}_iY_i + \Sigma q_i\dot{Y}_i.$$

We may define the relative shares of the value of the $i$th output in the value of total output, say $w_i$, as follows:

$$w_i = \frac{q_iY_i}{\Sigma q_iY_i}.$$

Dividing both sides of the total derivative of the value of output with respect to time by the value of output, we obtain:

$$\frac{\dot{q}}{q} + \frac{\dot{Y}}{Y} = \Sigma w_i\left(\frac{\dot{q}_i}{q_i} + \frac{\dot{Y}_i}{Y_i}\right).$$

We define the price and quantity indexes for output in terms of the prices and quantities of individual components; the rates of growth of the price index $q$ and the quantity index $Y$ are:

$$\frac{\dot{q}}{q} = \Sigma w_i\frac{\dot{q}_i}{q_i}, \quad \frac{\dot{Y}}{Y} = \Sigma w_i\frac{\dot{Y}_i}{Y_i},$$

respectively. These index numbers are Divisia price and quantity indexes.[18] The indexes are defined in terms of rates of growth of price and

[18] The economic interpretation of Divisia indexes of total factor productivity has been discussed in [54], [50], and [40].

quantity components of the rate of growth of the value of output. To obtain the price and quantity indexes themselves we choose a base for the indexes and integrate the rates of growth with respect to time. For the index numbers given below we choose the base for all price indexes as 1.000 in 1958. The base for the quantity indexes is equal to the value of the corresponding accounting magnitude in 1958.

The principal advantages of Divisia index numbers for social accounting purposes are, first, that rates of growth of these indexes of prices and quantity are symmetrical and add up to the rate of growth of the value of output (factor reversal test). Second, Divisia indexes are unaffected by a change in the direction of time (time reversal test). Finally, these indexes have the important reproductive property that a Divisia index of Divisia indexes is a Divisia index of the components. As an illustration, if the quantity index of total product is a Divisia index of quantity indexes of consumption and investment goods output and if the consumption and investment goods indexes are each Divisia indexes of individual consumption and investment goods, then the total product index is a Divisia index of the individual consumption and investment goods. The Divisia index numbers provide a convenient framework for national accounting since the principles of aggregation for data from subsectors of the economy are the same as those for construction of data for the subsectors. The results for the economy as a whole are independent of the structuring of the subaggregates.

For application to data for discrete points of time an approximation to the Divisia indexes for continuous time is required. Price and quantity index numbers originally discussed by Fisher may be employed for this purpose [19]. Approximating rates of growth by the period-to-period changes in logarithms, we obtain:

$$\log q_t - \log q_{t-1} = \Sigma \bar{w}_{it}(\log q_{it} - \log q_{i,\,t-1}),$$

$$\log Y_t - \log Y_{t-1} = \Sigma \bar{w}_{it}(\log Y_{it} - \log Y_{i,\,t-1}),$$

where the weights ($\bar{w}_{it}$) are arithmetic averages of the relative shares in the two periods,

$$\bar{w}_{it} = \frac{1}{2} w_{it} + \frac{1}{2} w_{i,\,t-1}.$$

These index numbers have been suggested as a discrete approximation to the Divisia index by Tornquist [58]. Obviously, the discrete and continuous index numbers are equal if and only if relative shares are constant. If shares are not constant, the discrete approximation involves an error that depends on the variability of the relative shares and the length of the time period.

Divisia index numbers for discrete time are symmetric in data of different time periods (time reversal). They also have the basic reproductive property that a discrete Divisia index of discrete Divisia indexes is a discrete Divisia index of the components. This property implies that the indexes for the economy as a whole are independent of the structuring of subsectors from which the aggregate data are constructed. The discrete Divisia price and quantity indexes are symmetrical. Theil has demonstrated that the sum of changes in the logarithms of discrete Divisia indexes of price and quantity is approximately equal to the change in the logarithm of the corresponding value (factor reversal) [57]. The factor reversal test is satisfied exactly if relative shares are constant; the accuracy of the approximation depends on the change in relative shares.

As a practical matter the approximation of changes in value by the sum of changes in discrete Divisia price and quantity indexes is extremely accurate. For the annual rate of growth in value of personal consumption expenditures in the Netherlands for the period 1921–63, Theil shows that the error averages only 0.01 per cent of the annual growth rate. It is convenient to have the product of price and quantity indexes equal to the value of transactions so that standard accounting identities hold for variables defined as price and quantity index numbers. Accordingly, we construct discrete Divisia price indexes as the value of the corresponding accounting magnitude divided by the discrete Divisia quantity index. The resulting price indexes are approximately equal to Divisia price indexes and have the reproductive property of Divisia indexes. They also satisfy, approximately, the time reversal and factor reversal tests for index numbers.

### *3.3. Taxes*

At a number of points in our accounting system transactions data are presented net and gross of taxes. As one illustration, consumer purchases of goods and services in the income and expenditure accounts include sales and excise taxes. Sales of the same goods and services in the production account exclude these taxes. As a second illustration, outlay on factor services in the production account includes direct taxes and certain indirect taxes such as property taxes. Income from factor services in the income and expenditure accounts excludes these taxes. We treat sales and excise taxes as part of the price paid by consumers.

We treat property taxes and income taxes as part of the price paid by producers. We can separate the change in the value of transactions into three components—change in price, change in quantity, and change in tax. The tax change is a component of the change in the price paid by the sector making an expenditure; the tax change is excluded from the change in the price received by the sector receiving income.

To illustrate the construction of price, quantity, and tax indexes we consider the value of consumer expenditure as it enters the income and expenditure account. Again, suppose that $m$ components of consumer expenditure are distinguished in the accounts; the value of output, gross of tax, say $q^+Y$, may be written:

$$q^+Y = q_1{}^+Y_1 + q_2{}^+Y_2 + \cdots + q_m{}^+Y_m.$$

The prices $(q_i{}^+)$ include sales and excise taxes; the quantities $(Y_i)$ are measured in the same way as in the production accounts. Price and quantity indexes based on these prices and quantities may be defined in the same way as before.

To introduce taxes into the system of index numbers we let the market price of output $q^+$ be equal to the price received by the producer, say $q$, multiplied by unity plus the effective tax rate, $t$; the value of output at market prices is:

$$q^+Y = (1 + t)qY.$$

The value of output at market prices may be expressed in terms of prices received by producers, each multiplied by unity plus the corresponding tax rate:

$$(1 + t)qY = \Sigma(1 + t_i)q_iY_i,$$

where the prices paid by the consumers $(q_i{}^+)$ are expressed in terms of prices received by producers $(q_i)$ and tax rates $(t_i)$.

Proceeding as before, we express the rate of growth of the value of consumer expenditure as the sum of rates of growth of taxes, prices, and quantities:

$$\frac{(1 + \dot{t})}{1 + t} + \frac{\dot{q}}{q} + \frac{\dot{Y}}{Y} = \Sigma w_i \left[\frac{(1 + \dot{t}_i)}{1 + t_i} + \frac{\dot{q}_i}{q_i} + \frac{\dot{Y}_i}{Y_i}\right].$$

The rate of growth of the tax index, $1 + t$, is:

$$\frac{(1 + \dot{t})}{1 + t} = \Sigma w_i \frac{(1 + \dot{t}_i)}{1 + t_i};$$

rates of growth of price and quantity indexes are analogous to those for the production account described above. To construct a tax index from the rate of growth we choose an appropriate base and integrate the rates of growth with respect to time. For the index numbers given below we choose the base for all tax indexes as the ratio of the corresponding accounting magnitude before taxes to this magnitude after taxes for 1958. To obtain the effective tax rate, we subtract unity from the resulting tax index.

For application to data for discrete points of time we approximate Divisia indexes for continuous time as before. It is convenient to preserve accounting identities for variables defined as price, quantity, and tax index numbers. Accordingly, we construct an index of taxes $1 + t$ by dividing the value of transactions at market prices by the value of transactions at producer prices. The resulting tax index is approximately equal to the Divisia tax index. It should be noted that Divisia price and quantity indexes at market prices differ from the corresponding indexes at producer prices since taxes enter the weights $(w_i)$ employed in constructing the indexes.

### *3.4. Index Number Systems*

In the U.S. national accounts only the output side of the production account is measured in current and constant prices. The index number system employed for the measurement of output in constant prices is based on a Laspeyres index number for the quantity of output and a Paasche index number for the price of output. In the Laspeyres index of output, prices of a base year are employed as weights for quantities of output. The Laspeyres index of the quantity of output, say $Y^L$, is defined by:

$$Y_1{}^L = \frac{\Sigma q_{i0} Y_{i1}}{\Sigma q_{i0} Y_{i0}},$$

where the base period prices $(q_{i0})$ are prices of 1958. Dividing the ratio of the values of transactions in period 1 to those in period 0 by the Laspeyres quantity index, we obtain the Paasche index of the price of output, $q^P$:

$$q_1{}^P = \frac{\Sigma q_{i1} Y_{i1}}{\Sigma q_{i0} Y_{i1}},$$

where the quantities $(Y_{i1})$ are quantities of the current year.

To compare the Divisia index numbers with the system of index numbers used in the U.S. national accounts we consider the rate of growth of the Laspeyres index of real product:

$$\frac{Y_1^L - Y_0^L}{Y_0^L} = \frac{\Sigma q_{i0} Y_{i1}}{\Sigma q_{i0} Y_{i0}} - \frac{\Sigma q_{i0} Y_{i0}}{\Sigma q_{i0} Y_{i0}} = \frac{\Sigma q_{i0} Y_{i1}}{\Sigma q_{i0} Y_{i0}} - 1.$$

Next we consider the Laspeyres approximation to the rate of growth of the Divisia quantity index:

$$\frac{Y_1^D - Y_0^D}{Y_0^D} = \Sigma \frac{q_{i0} Y_{i0}}{\Sigma q_{i0} Y_{i0}} \frac{Y_{i1} - Y_{i0}}{Y_{i0}} = \frac{\Sigma q_{i0} Y_{i1}}{\Sigma q_{i0} Y_{i0}} - 1.$$

The rate of growth of the Laspeyres approximation to the Divisia index is identical with the rate of growth of the usual Laspeyres quantity index.

The first difference between our system of index numbers and the system employed in the U.S. national accounts is that we approximate the underlying continuous index numbers by price and quantity indexes that satisfy the time reversal and factor reversal tests for index numbers. The Laspeyres approximation given above satisfies neither test since the corresponding price index number is a Paasche approximation to the underlying continuous price index number and since the Laspeyres and Paasche formulas are not symmetric in data of different time periods. These differences do not produce large variations in the price and quantity index numbers.

The second difference between our system of index numbers and the system of the U.S. national accounts is that our indexes are chain linked. For each year, current prices are used as weights in estimating the rate of growth of quantity to the following year and current quantities are used as weights in estimating the rate of growth of price. This process is followed for each pair of years, and the resulting indexes are chain linked. In effect the base of the index numbers is moved continually. The main advantage of a continually changing base is in the reduction of errors of approximation as the economy moves from one production or expenditure configuration to another. Chain-linked index numbers reduce the errors of approximation to a minimum. The use of a chain-linked index alters price and quantity indexes substantially for periods in which relative prices and relative quantities are shifting.

Denison has augmented the quantity and price indexes of the production account of the U.S. national accounts to provide quantity and price indexes of factor input [14]. Although he uses the quantity and price indexes of output based on the national accounts, he employs chain-linked indexes of input with weights changing every five years. The

Laspeyres approximation to the Divisia indexes of input and output is employed by Jorgenson and Griliches [40], while Christensen and Jorgenson [8] have used the approximation described above, satisfying factor and time reversal tests for index numbers. The main differences between the price and quantity indexes for these alternative systems of index numbers result from the use of chain-linked indexes. Alternative approximations to continuous indexes produce substantially similar results.

In the United Nations *System of National Accounts,* systems of index numbers like that employed in the U.S. national accounts are recommended as a basis for constructing price and quantity indexes for the output side of the production account [55]. As the base period is changed from time to time, chain linking of the resulting price and quantity indexes is recommended. Continual chain linking is not recommended for general adoption "mainly because the amount of data it requires is altogether greater than the amount required by the alternative." [19] The index numbers we employ in constructing accounts for output in constant prices are chain-linked indexes of component indexes obtained from the U.S. national accounts. They represent a mixture of chain-linked and fixed weight indexes. The index numbers we employ in constructing accounts for input, income, and expenditure are chain-linked indexes based on price and quantity data.

## 4. PERPETUAL INVENTORY

### *4.1. Introduction*

Measurement of the output side of the production account and the asset side of the wealth account in constant prices is well-established in social accounting practice. Index numbers of the price and quantity of output are constructed from data on prices and quantities of individual outputs. Index numbers of the price and quantity of capital assets are constructed from data on prices and quantities of individual assets. Quantities of individual assets are estimated from data on past levels of investment, and investment goods prices by the perpetual inventory method.[20]

Our objective is to develop a complete system of accounts in constant

[19] See [55, p. 58].

[20] The perpetual inventory method is discussed by Goldsmith [22], and employed extensively in his *Study of Saving* [23], and more recent studies of U.S. national wealth [24], [25], [26]. This method is also used in the OBE *Capital Goods Study* and in the study of capital stock for the United States by Tice [57].

prices, linking output in constant prices to assets in constant prices. The most important obstacle to development of a complete accounting system is the lack of appropriate data on capital. To estimate the necessary data we extend the perpetual inventory method to encompass data on prices as well as quantities of capital goods by vintage. An accounting system of this type can be implemented only in a highly simplified form. However, even a simplified accounting system makes it possible to avoid inconsistencies in the treatment of capital that frequently occur in studies of total factor productivity.

### *4.2. Relative Efficiency*

We begin the construction of a complete system of income and wealth accounts in constant prices with a description of the price and quantity data required for a single capital good. As in the perpetual inventory method, our characterization of a capital good is based on the relative efficiency of capital goods of different ages.[21] In the perpetual inventory method, the relative efficiency of a capital good depends on the age of the good and not on the time it is acquired. Replacement requirements are determined by losses in efficiency of existing capital goods as well as actual physical disappearance or retirement of capital goods. When a capital good is retired its relative efficiency drops to zero. The relative efficiency of capital goods of different ages can be described by a sequence of nonnegative numbers, $d_0, d_1, \ldots$.

We normalize the relative efficiency of a new capital good at unity and assume that relative efficiency is nonincreasing so that:

$$d_0 = 1; d_\tau - d_{\tau-1} \leqq 0; \tau = 0, 1, \ldots.$$

We also assume that every capital good is eventually retired or scrapped so that relative efficiency eventually drops to zero:

$$\lim_{\tau \to \infty} d_\tau = 0.$$

Subject to these restrictions, a wide variety of patterns of decline in efficiency may be employed in the perpetual inventory method.

For illustration we consider three patterns of decline in efficiency, "one-hoss shay," straight-line, and declining balance. In the "one-hoss shay" pattern, efficiency is constant over the lifetime of the capital good. Where $T$ is the lifetime, relative efficiency is:

$$d_\tau = 1; \tau = 0, 1, \ldots, T - 1.$$

[21] A more detailed discussion of the economic theory of replacement and depreciation is given by Jorgenson [39].

In the straight-line pattern, efficiency declines linearly over the lifetime of the capital good:

$$d_\tau = 1 - \frac{1}{T}\tau;\ \tau = 0, 1, \ldots, T - 1.$$

In the declining balance pattern, efficiency declines geometrically:

$$d_\tau = (1 - \delta)^\tau;\ \tau = 0, 1, \ldots.$$

These patterns of decline in efficiency and many others may be treated as special cases within the framework of our extension of the perpetual inventory method.

Capital goods decline in efficiency at each point of time, giving rise to needs for replacement to maintain productive capacity. The proportion of an investment to be replaced during the $\tau$th period after its acquisition is equal to the decline in efficiency during that period. We refer to the decline in relative efficiency as the mortality distribution of a capital good, say $m_\tau$, where:

$$m_\tau = -(d_\tau - d_{\tau-1});\ \tau = 1, 2, \ldots.$$

By our assumption that relative efficiency is nonincreasing, the mortality distribution may be represented by a sequence of nonnegative numbers, $m_1, m_2, \ldots$, where:

$$\sum_{\tau=1}^{\infty} m_\tau = \sum_{\tau=1}^{\infty} (d_{\tau-1} - d_\tau) = d_0 = 1.$$

For the patterns of decline in efficiency considered above, we can derive the corresponding mortality distributions. If efficiency is constant over the lifetime of the capital good, the mortality distribution is zero except for period $T$: $m_T = 1$. For linear decline in efficiency, the mortality distribution is constant throughout the lifetime of the capital good:

$$m_\tau = \frac{1}{T};\ \tau = 1, 2, \ldots, T.$$

For geometric decline in efficiency, the mortality distribution declines geometrically:

$$m_\tau = \delta(1 - \delta)^{\tau-1};\ \tau = 0, 1, \ldots.$$

Replacement requirements can be expressed in terms of the mortality distribution for capital goods. Requirements can also be expressed in terms of the proportion of an initial investment replaced $\tau$ periods after the initial acquisition. This proportion includes replacement of the initial investment and subsequent replacements of each succeeding replacement. We refer to the sequence of these proportions as the replacement distribution of a capital good; each coefficient, say $\delta_\tau$, is the rate of replacement of an investment replaced $\tau$ periods after initial acquisition. The sequence of replacement rates $(\delta_\tau)$ can be computed recursively for the sequence of mortality rates $(m_\tau)$. The proportion of an initial investment replaced at time $\nu$ and again at time $\tau > \nu$ is $m_\nu \delta_{\tau-\nu}$. The proportion of the stock replaced in the $\tau$th period is the sum of proportions replaced first in periods $1, 2, \ldots,$ and later at period $\tau$; hence,

$$\delta_\tau = m_1\delta_{\tau-1} + m_2\delta_{\tau-2} + \cdots + m_\tau\delta_0; \; \tau = 1, 2, \ldots .$$

This equation is referred to as the renewal equation.[22]

For constant relative efficiency over the lifetime of a capital good, the replacement distribution is periodic with the period equal to the lifetime of the capital good:

$$\delta_\tau = 1; \; \tau = T, 2T, \ldots .$$

For linear decline in efficiency, the replacement distribution may be represented in the form:

$$\delta_1 = \frac{1}{T};$$

$$\delta_2 = \frac{1}{T}\left(1 + \frac{1}{T}\right);$$

etc.

For geometric decline in efficiency, the replacement distribution is constant:

$$\delta_\tau = \delta; \; \tau = 1, 2, \ldots .$$

*4.3. Quantities and Prices*

The relative efficiency of capital goods of different ages and the derived mortality and replacement distributions are useful in estimating the data required for income and wealth accounts in constant prices. We begin our description of the required capital data with quantities estimated by the perpetual inventory method. First, capital stock at the end of each period, say $K_t$, is the sum of past investments, say $A_{t-\tau}$, each weighted by its relative efficiency:

[22] See [18].

$$K_t = \sum_{\tau=0}^{\infty} d_\tau A_{t-\tau}.$$

For a complete system of accounts, both capital stock and investments in every preceding period are required. For this purpose a system of vintage accounts containing data on investments of every age in every period is essential.

Taking the first difference of the expression for capital stock in terms of past investments, we obtain:

$$\begin{aligned} K_t - K_{t-1} &= A_t + \sum_{\tau=1}^{\infty} (d_\tau - d_{\tau-1}) A_{t-\tau}; \\ &= A_t - \sum_{\tau=1}^{\infty} m_\tau A_{t-\tau}; \\ &= A_t - R_t; \end{aligned}$$

where:

$$R_t = \sum_{\tau=1}^{\infty} m_\tau A_{t-\tau}$$

is the level of replacement requirements in period $t$. The change in capital stock from period to period is equal to the acquisition of investment goods less replacement requirements.

Replacement requirements may also be expressed in terms of present and past changes in capital stock, using the replacement distribution:

$$R_t = \sum_{\tau=1}^{\infty} \delta_\tau (K_{t-\tau} - K_{t-\tau-1}).$$

The average replacement rate for capital stock at the beginning of the period,

$$\hat{\delta}_t = \frac{R_t}{K_{t-1}} = \sum_{\tau=1}^{\infty} \delta_\tau \frac{(K_{t-\tau} - K_{t-\tau-1})}{K_{t-1}},$$

is a weighted average of replacement rates with weights given by the relative proportions of changes in capital stock of each vintage in beginning-of-period capital stock.

We turn next to a description of the price data required for construction of income and wealth accounts in constant prices. These accounts require an extension of the perpetual inventory method to incorporate data on prices of capital goods of each vintage. Our extension of the perpetual inventory method is dual to the usual method in the sense that there is a one-to-one correspondence between the quantities that appear in the perpetual inventory method and the prices that appear in our extension of it.[23] To bring out this correspondence and to simplify the notation we use a system of present or discounted prices. Taking the present as time zero, the discounted price of a commodity, say $p_t$, is the discounted value of the future price, say $q_t$:

$$p_t = \prod_{s=1}^{t} \frac{1}{1 + r_s} q_t.$$

The notational convenience of present or discounted prices results from dispensing with explicit discount factors in expressing prices for different time periods.

In the correspondence between the perpetual inventory method and its dual or price counterpart, the price of acquisition of a capital good is analogous to capital stock. The price of acquisition, say $p_{A,t}$, is the sum of future rental prices of capital services, say $p_{K,t}$, weighted by the relative efficiency of the capital good in each future period:

$$p_{A,t} = \sum_{\tau=0}^{\infty} d_\tau p_{K,t+\tau+1}.$$

This expression may be compared with the corresponding expression giving capital stock as a weighted sum of past investments. The acquisition price of capital goods enters the production account through the price of investment goods output. This price also appears as the price component of capital formation in the accumulation account. Vintage accounts, containing data on the acquisition prices of capital goods of every age at every point of time, are required for a complete system of accounts.

Taking the first difference of the expression for the acquisition price of capital goods in terms of future rentals, we obtain:

$$\begin{aligned} p_{A,t} - p_{A,t-1} &= -p_{K,t} - \sum_{\tau=1}^{\infty} (d_\tau - d_{\tau-1}) p_{K,t+\tau} \\ &= -p_{K,t} + \sum_{\tau=1}^{\infty} m_\tau p_{K,t+\tau} \\ &= -p_{K,t} + p_{D,t}; \end{aligned}$$

where:

$$p_{D,t} = \sum_{\tau=1}^{\infty} m_\tau p_{K,t+\tau}$$

[23] The dual to the durable goods model was developed by Arrow [2], and Hall [32], on the basis of earlier work by Hotelling [38].

is depreciation on a capital good in period $t$. The period to period change in the price of acquisition of a capital good is equal to depreciation less the rental price of capital. In the correspondence between the perpetual inventory method and its price counterpart, investment corresponds to the rental price of capital and replacement corresponds to depreciation.

We can rewrite the expression for the first difference of the acquisition price of capital goods in terms of undiscounted prices:

$$q_{K,t} = q_{A,t-1}r_t + q_{D,t} - (q_{A,t} - q_{A,t-1}),$$

where $q_{A,t}$ is the undiscounted price of acquisition of capital goods, $q_{K,t}$ the price of capital services, $q_{D,t}$ depreciation, and $r_t$ the rate of return, all in period $t$. The price of capital services $q_{K,t}$ is the sum of return per unit of capital $q_{A,t-1}r_t$, depreciation $q_{D,t}$, and the negative of revaluation $-(q_{A,t} - q_{A,t-1})$. The service price enters the production and the income and expenditure accounts through the price component of capital input and property compensation. Depreciation enters the accumulation account as the price component of depreciation on existing capital assets. Revaluation enters the accumulation account as the price component of revaluation of existing assets.

Depreciation may also be expressed in terms of present and future changes in the price of acquisition of investment goods, using the replacement distribution:

$$p_{D,t} = -\sum_{\tau=1}^{\infty} \delta_\tau (p_{A,t+\tau} - p_{A,t+\tau-1}).$$

The average depreciation rate on the acquisition price of a capital good,

$$\bar{\delta}_t = \frac{p_{D,t}}{p_{A,t}} = -\sum_{\tau=1}^{\infty} \delta_\tau \frac{(p_{A,t+\tau} - p_{A,t+\tau-1})}{p_{A,t}},$$

is a weighted average of replacement rates with weights given by the relative proportions of changes in futures prices in the acquisition price of investment goods in the current period. This expression may be compared with that for the average replacement rate, $\hat{\delta}_t$, given above. For a complete system of accounts, vintage data on the depreciation of capital goods of every age at every point of time are required.

In the perpetual inventory method, data on the quantity of investment goods of every vintage are used to estimate capital formation, replacement requirements, and capital stock. In the price counterpart of the perpetual inventory method, data on the acquisition prices of investment goods of every vintage are required. The price of acquisition of an investment good of age $v$ at time $t$, say $p_{A,t,v}$, is the weighted sum of future rental prices of capital prices. The weights are relative efficiencies of the capital good in each future period, beginning with age $v$:

$$p_{A,t,v} = \sum_{\tau=0}^{\infty} d_{\tau+v}\, p_{K,t+\tau+1}.$$

A new investment good has age zero so that:

$$p_{A,t,0} = p_{A,t}.$$

Given the acquisition prices, we require estimates of depreciation and the rental price for goods of each vintage.

To calculate depreciation on capital goods of each vintage we take the first difference of the acquisition prices across vintages at a given point in time:

$$\begin{aligned} p_{A,t,v} - p_{A,t,v+1} &= -\sum_{\tau=1}^{\infty} (d_{\tau+v} - d_{\tau+v-1}) p_{K,t+v+\tau} \\ &= \sum_{\tau=1}^{\infty} m_{\tau+v}\, p_{K,t+v+\tau} \\ &= p_{D,t,v}; \end{aligned}$$

where $p_{D,t,v}$ is depreciation on a capital good of age $v$ at time $t$. Again, a new investment good has age zero so that:

$$p_{D,t,0} = p_{D,t}.$$

To obtain depreciation in terms of futures prices or undiscounted prices, we observe that acquisition prices across vintages at a given point in time and the corresponding depreciation are associated with the same discount factor, so that:

$$q_{A,t,v} - q_{A,t,v+1} = q_{D,t,v}.$$

To calculate the capital service price for goods of each vintage, we first observe that the rental of a capital good of age $v$ at time $t$, say $q_{K,t,v}$, is proportional to the rental of a new capital good,

$$q_{K,t,v} = d_v q_{K,t},$$

with the constant of proportionality given by the efficiency of a capital good of age $v$ relative to that of a new capital good. New and used capital goods are perfect substitutes in production. To calculate the service price for new capital goods, we use the formula derived above:

$$q_{K,t} = q_{A,t-1} r_t + q_{D,t} - (q_{A,t} - q_{A,t-1}).$$

To apply this formula we require a series of undiscounted acquisition prices for capital goods ($q_{A,t}$), rates of return ($r_t$), depreciation on new capital goods ($q_{D,t}$), and revaluation of existing capital goods ($q_{A,t} - q_{A,t-1}$).

To calculate the rate of return in each period, we set the formula for the rental price $q_{K,t}$ times the quantity of capital $K_{t-1}$ equal to property compensation. All of the variables entering this equation—current and past acquisition prices for capital goods, depreciation, revaluation, capital stock, and property compensation—except for the rate of return, are known. Replacing these variables by the corresponding data we solve this equation for the rate of return. To obtain the capital service price itself we substitute the rate of return into the original formula along with the other data. This completes the calculation of the service price.

We conclude that acquisition prices for capital goods of each vintage at each point of time provide sufficient information to enable us to calculate depreciation and rental value for capital goods of each vintage. These data together with current investment, capital stock, replacement, and investments of all vintages at each point of time constitute the basic data on quantities and prices required for an extended perpetual inventory system. The problem that remains is to describe the role of each set of data in a complete accounting system. From this point we consider an accounting system for any number of investment goods. Price and quantity data that we have described above for a single investment good are required for each investment good in the system. The data for all investment goods are used to derive price and quantity indexes that play the role of the price and quantity data for a single investment good outlined above.

### *4.4. Accounting System*

The quantities of investment goods ($A_t$) enter the production account in the period the investment is made through the quantity of investment goods output. An analogous quantity appears as part of capital formation in the accumulation account. The prices associated with investment in the production and accumulation accounts are prices of acquisition of new investment goods ($q_{A,t}$). The value of investment goods output is price times quantity, say $q_{A,t}A_t$. The value of capital formation is also equal to price times quantity; the price includes taxes on investment goods output. For several investment goods the values of investment goods output and capital formation are sums of prices times quantities for the individual investment goods. The price and quantity components of these accounts are derived by application of the Divisia index number formulas to the underlying price and quantity data for the individual investment goods.

Capital stock enters the production account through the quantities of capital service input ($K_{t-1}$); the quantity of capital service input also appears in the income and expenditure account as the quantity component of property compensation. The prices associated with capital services in the production and the income and expenditure accounts are rental prices ($q_{K,t}$). The value of capital input and property compensation is price times quantity, say $q_{K,t}K_{t-1}$. The service prices entering the production account are gross of taxes while the prices entering the property compensation account are net of taxes; these service prices will be discussed in more detail in sections 5 and 6 below. For several capital goods the values of capital services input and property compensation are sums of prices times quantities for each capital good. The price and quantity components of these accounts are derived by application of the Divisia index number formulas to the rental price and service quantity data for the individual capital goods.

Capital stock enters the accumulation account as the quantity component of depreciation. In the accumulation account capital stock must be distinguished by vintage so that vintage accounts containing data on investment of every age ($A_{t-v-1}$) may be regarded as part of the accumulation account in constant prices. The prices associated with capital stock in the accumulation account are the levels of depreciation ($q_{D,t,v}$). The value of depreciation for capital goods of age $v$ is price times quantity, say $q_{D,t,v}A_{t-v-1}$; to obtain the total value of depreciation we sum over vintages, obtaining

$$\sum_{v=0}^{\infty} q_{D,t,v}A_{t-v-1}.$$

Even for a single capital good the separation of prices and quantities of depreciation requires application of an index number formula to the underlying vintage data. For several capital goods, the appropriate price and quantity index numbers can be constructed by applying the Divisia index number formulas to prices and quantities for each capital good derived from vintage data.

Capital stock also enters the accumulation account as the quantity component of revaluation. The prices associated with capital stock in measuring revaluation are the price changes $q_{A,t,v} - q_{A,t-1,v}$. Revaluation for capital goods of age $v$ is price times quantity, say $(q_{A,t,v} - q_{A,t-1,v})A_{t-v-1}$; to obtain total revaluation we sum over vintages, obtaining

$$\sum_{v=0}^{\infty} (q_{A,t,v} - q_{A,t-1,v})A_{t-v-1}.$$

Separation of price and quantity components of revaluation for a single capital good or for several goods requires the application of Divisia index number formulas to prices and quantities for each vintage of each capital good, just as in the depreciation account. The prices used for depreciation and revaluation in the accumulation account must be consistent with those used for capital service prices in the production and the income and expenditure accounts.

Replacement appears in the accumulation account as part of capital formation. Gross capital formation is equal to investment. Net capital formation is equal to gross capital formation less replacement. Net capital formation is equal to the period to period change in capital stock. Replacement represents the change in the quantity of existing capital goods due to a decline in relative efficiency. Depreciation represents the change in the price of existing capital goods due to present and all future declines in efficiency. We have already described the separation of price and quantity components of gross capital formation. The methods for separation of these components of net capital formation and replacement are strictly analogous; quantities of gross capital formation or investment are replaced by quantities of net capital formation and replacement in index number formulas that also depend on prices of acquisition of investment goods.

Finally, capital stock appears in the wealth account as the quantity component of capital assets. In the wealth account, capital stock must be distinguished by vintage so that vintage accounts containing investment of every age in every time period may be regarded as part of both accumulation and wealth accounts. The prices associated with capital stock in the wealth account are the acquisition prices $(q_{A,t,v})$. The value of wealth for capital goods of age $v$ is price times quantity, say $q_{A,t,v}A_{t-v}$; to obtain the total value of wealth we sum over vintages, obtaining

$$\sum_{v=0}^{\infty} q_{A,t,v}A_{t-v}.$$

For a single capital good or for several capital goods, price and quantity index numbers of wealth can be constructed by applying the Divisia index number formulas to prices and quantities of capital assets of each vintage at each point of time.

For capital goods with a full set of data for every time period, including investment of every vintage and the price of acquisition for every vintage, accounts can be compiled for capital input, property compensation, depreciation, capital formation, replacement, and wealth in current and constant prices. Price data corresponding to each of the accounts in constant prices can also be compiled. For capital goods with a less complete set of data, a simplified system of accounts can be constructed on the basis of the assumption that decline in efficiency is geometric. Under this assumption the rate of replacement and the rate of depreciation are constant and equal to the rate of decline in efficiency:

$$\hat{\delta}_t = \bar{\delta}_t = \delta.$$

Constant rates of replacement and depreciation lead to substantial simplifications in our system of income and wealth accounts in constant prices. Vintage accounts can be dispensed with since replacement is proportional to capital stock and depreciation is proportional to the current acquisition price of investment goods.

As a first step in construction of a simplified accounting system for income and wealth in constant prices we estimate capital stock at the end of each period as a weighted sum of past investments:

$$K_t = \sum_{\tau=0}^{\infty} (1 - \delta)^\tau A_{t-\tau}.$$

With a constant rate of replacement, replacement becomes:

$$R_t = \delta K_{t-1}.$$

The price of acquisition of new investment goods is a weighted sum of future rentals:

$$p_{A,t} = \sum_{\tau=0}^{\infty} (1 - \delta)^\tau p_{K,t+\tau+1}.$$

With a constant rate of depreciation, depreciation becomes:

$$q_{D,t} = \delta q_{A,t}.$$

The acquisition price of investment goods of age $v$ at time $t$ is:

$$q_{A,t,v} = (1 - \delta)^v q_{A,t}.$$

The service price for new capital goods becomes:

$$q_{K,t} = q_{A,t-1}r_t + \delta q_{A,t} - (q_{A,t} - q_{A,t-1}).$$

In the complete accounting system for income and wealth in constant prices outlined above, vintage accounts for capital are required for calculating replacement, depreciation, capital formation, revaluation, and wealth. With constant replacement rates $(\delta_\tau)$ the values of replacement and depreciation are equal and depend only on the price of acquisition of new capital goods and the stock of capital:

$$q_{A,t}R_t = \delta q_{A,t}K_{t-1} = q_{D,t}K_{t-1}.$$

Similarly, the value of wealth is the product of the price of acquisition and the stock of capital, $q_{A,t}K_t$. The change in wealth from period to period,

$$q_{A,t}K_t - q_{A,t-1}K_{t-1} = q_{A,t}(K_t - K_{t-1}) + (q_{A,t} - q_{A,t-1})K_{t-1},$$

is the sum of capital formation and revaluation. No vintage accounts for capital goods are required under the assumption of constant replacement rates. For several capital goods the Divisia index number formulas must be employed to separate replacement, depreciation, capital formation, revaluation, and wealth into price and quantity components.

Geometric decline in efficiency is among the patterns most commonly employed in estimating capital stock by the perpetual inventory method.[24] For geometric decline in efficiency, depreciation is proportional to the acquisition price of new capital goods and replacement is proportional to capital stock. These properties result from the constancy of the sequence of replacement rates $(\delta_\tau)$. Neither property holds for any other representation of the relative efficiency of capital goods of different ages. A fundamental result of renewal theory is that $\delta_\tau$ tends to a constant value for almost any pattern of decline in efficiency.[25] Geometric decline in efficiency, resulting in a constant rate of replacement $\delta$, may provide a useful approximation to replacement requirements and depreciation for a wide variety of patterns of decline in efficiency. Where this approxima-

[24] A representative study is the OBE *Capital Goods Study;* in this research straight-line and double-declining balance methods are employed. See [30].

[25] For detailed discussion of the application of renewal theory to replacement and depreciation, see [39].

tion is unsatisfactory, a complete accounting system for income and wealth in constant prices requires vintage accounts for capital goods quantities and prices.

Many different retirement distributions have been found useful in describing the retirement or physical disappearance of capital goods.[26] Considerably less evidence is available on the decline in efficiency of existing capital goods.[27] The available evidence arises from two sources—studies of replacement investment and studies of depreciation on capital goods. Geometric decline in efficiency has been employed by Hickman [37] and by Hall and Jorgenson [34], [35] in studies of investment. This assumption is tested by Meyer and Kuh, who find no effect of the age distribution of capital stock in the determination of replacement investment.[28] Geometric decline in efficiency has been employed in the study of depreciation on capital goods by Cagan, Griliches, and Wykoff.[29] This assumption has been tested by Hall, who finds no effect of the age of a capital good in the determination of the rate of depreciation as measured from prices of capital goods of different vintages.[30] The available empirical evidence supports the use of geometric decline in efficiency as a useful approximation to replacement requirements and depreciation.

### *4.5. Alternative Accounting Systems*

We have outlined the development of a complete system of income and wealth accounts in constant prices. Only the measurement of the output side of the production account and the asset side of the wealth account in constant prices are well-established in social accounting practice. In the study of total factor productivity, attempts have been made to measure the input side of the production account in constant prices. Christensen and Jorgenson [7], [8] have applied the methods we have described for a simplified accounting system to the measurement of factor input in constant prices and the measurement of total factor productivity.

It is very useful to compare our accounting system with an alternative approach developed by Denison in his path-breaking monograph, *Sources of Economic Growth* [14]. Denison's monograph deals with output and input sides of the production account for the United States. Similar

[26] A relatively recent work on capital equipment lifetimes is Marston, Winfrey, and Hempstead [47]. The classic work in the field is E. B. Kurtz [45], which provides other references.

[27] When a capital good is retired, relative efficiency drops to zero.

[28] See [48, pp. 91–100].

[29] See [5, pp. 222–226], [27, pp. 197–200], and [60, pp. 171–172].

[30] See [33, pp. 19–20].

methods have been applied to data for a number of other countries in his book, *Why Growth Rates Differ* [15]. Denison takes gross national product in constant prices from the U.S. national accounts as a point of departure. He measures labor input along lines similar to those we outline below, weighting rates of growth of each type of labor input by relative shares in the values of total labor input to obtain the rate of growth of an index of labor input.[31] In comparing Denison's approach with our own, we concentrate on the measurement of capital input.

Denison points out that the construction of a capital input measure depends on the relative efficiency of capital goods of different ages:

> In principle, the selection of a capital input measure should depend on the changes that occur in the ability of a capital good to contribute to net production as the good grows older (within the span of its economic life). Use of net stock, with depreciation computed by the straight line formula, would imply that this ability drops very rapidly—that it is reduced by one-fourth when one-fourth of the service life has passed, and by nine-tenths when nine-tenths of the service life has passed. Use of gross stock would imply that this ability is constant throughout the service life of a capital good.[32]

Denison adds: "I believe that net value typically declines more rapidly than does the ability of a capital good to contribute to production. . . . On the other hand, the gross stock assumption of constant services throughout the life of an asset is extreme." [33]

Under Denison's gross stock assumption, relative efficiency is constant over the economic lifetime of the equipment:

$$d_\tau = 1;\ \tau = 0, 1, \ldots, T - 1;$$

where $T$ is the economic lifetime of the capital good. Under Denison's net stock assumption, efficiency declines linearly:

$$d_\tau = 1 - \frac{1}{T}\,\tau;\ \tau = 0, 1, \ldots, T - 1.$$

In Denison's *Sources of Economic Growth* gross stock is employed as a measure of the quantity of capital input. In *Why Growth Rates Differ* an arithmetic average of gross stock and net stock is employed; [34] the

[31] A detailed comparison of our estimates of labor input and those of Denison is given by Jorgenson and Griliches [41]; see Section 5.2 below for further discussion.

[32] [15, p. 140].

[33] [15, p. 140].

[34] [15, p. 141].

implied relative efficiency of capital goods is an average of constant and linearly declining relative efficiency:

$$d_\tau = 1 - \frac{1}{2T}\,\tau;\ \tau = 0, 1, \ldots, T - 1.$$

Since Denison does not assume that the relative efficiency of capital goods declines geometrically, depreciation and replacement must be carefully distinguished in order to preserve consistency among production, income and expenditure, accumulation, and wealth accounts in constant prices. Depreciation is a component of the price of capital services. The value of capital services is equal to property income including depreciation. Replacement is the consequence of a decline in the efficiency of capital assets or, in Denison's language, the ability of a capital good to contribute to production. Unfortunately, a confusion between depreciation and replacement pervades Denison's treatment of the output and input sides of the production account and the measurement of capital stock. This confusion leads to a series of inconsistencies, making it impossible to incorporate Denison's measures of product and factor input in constant prices into a complete accounting system.

The first indication of confusion between depreciation and replacement is Denison's definition of net product: "Net product measures the amount a nation consumes plus the addition it makes to its capital stock. Stated another way, it is the amount of its output a nation could consume without changing its stock of capital." [35] The correct definition of net product is gross product less depreciation; this is the definition suggested by the second statement quoted above. The first statement defines net product as gross product less replacement, since the addition to capital stock or net capital formation is equal to investment less replacement. The two definitions are consistent if and only if depreciation is equal to replacement. Under any of Denison's assumptions about decline in relative efficiency, depreciation and replacement are not equal, so that his definition of net product is self-contradictory.

In *Why Growth Rates Differ* Denison measures capital consumption allowances on the basis of Bulletin F lives and the straight-line method.[36] Even under the assumption that relative efficiency or Denison's "ability to contribute to production" declines linearly, this estimate corresponds to replacement rather than depreciation. Denison reduces gross product by his estimate of capital consumption allowances to obtain his measure

[35] [15, p. 14].
[36] [15, p. 351].

of net product.[37] This procedure employs the incorrect definition of net product as gross product less replacement. A similar procedure for calculating capital consumption allowances is employed in *Sources of Economic Growth.* Denison's confusion between depreciation and replacement carries over to the input side of the production account. His measure of net product is reduced by labor compensation to obtain property compensation net of capital consumption allowances. Thus, Denison's measure of property compensation is also calculated net of replacement rather than net of depreciation. This erroneous measure is allocated among capital inputs to obtain weights employed in measuring capital input as a component of factor input in constant prices. Denison's weights for different components of capital input are measured incorrectly; these weights should reflect property compensation less depreciation rather than property compensation less replacement.

A further difficulty with Denison's estimate of capital consumption allowances in *Why Growth Rates Differ* is that in estimating capital stock Denison assumes that decline in efficiency is linear, but at half the straight-line rate. He uses the straight-line method to estimate capital consumption allowances; the resulting estimate is equal to neither depreciation nor replacement for the pattern of decline in efficiency he uses in estimating capital. In *Sources of Economic Growth* Denison assumes that relative efficiency is constant over the lifetime of a capital good.[38] Again, the straight-line estimates of capital consumption allowances are equal to neither depreciation nor replacement for the pattern of decline in efficiency underlying his estimate of capital. In both *Sources of Economic Growth* and *Why Growth Rates Differ* the price and quantity components of the input side of the production account are mutually contradictory.

In our accounting system for capital input and property compensation, the price component of the flow of capital services is the sum of return per unit of capital, depreciation, and revaluation. In estimating the rate of return Denison omits revaluations of existing capital goods and fails to measure depreciation correctly.[39] His implied estimate of return per unit of capital is erroneous. Denison omits capital gains and losses from the revaluation of assets in allocating property income among capital assets; so the weights for different components of capital input are measured incorrectly. The revaluations are required as part of the ac-

[37] [15, p. 14].
[38] [14, pp. 112–113].
[39] [16, p. 8].

cumulation account for an accounting system that includes accumulation and wealth accounts. If Denison's measure of capital input were to be incorporated into a complete accounting system, the omission of revaluations from the price component of capital services would introduce an inconsistency between the production and the income and expenditure accounts on the one hand and the accumulation and wealth accounts on the other.

Denison's assumptions about the decline in relative efficiency of capital goods can be incorporated into a complete accounting system along the lines we have suggested. Since he does not assume that efficiency declines geometrically, vintage accounts for quantities and prices of capital goods of every age at every point of time are required. Vintage data are essential even for the relatively limited objective of measuring net product; net product measurement requires an estimate of depreciation and estimation of depreciation requires vintage prices. The first step in implementing Denison's assumptions would be to assemble data on the acquisition prices of capital goods of every age at every point of time. The second step would be to estimate depreciation for goods of every vintage at every point of time from the vintage data on prices. This estimate of depreciation would replace Denison's estimate of capital consumption allowances in measuring net product and property compensation net of depreciation. The third step would be to estimate capital service prices by combining estimates of the return per unit of capital, depreciation, and revaluation of assets. These prices could be combined with Denison's estimates of capital stock to construct index numbers of the price and quantity of capital input.

We conclude that Denison's assumptions about the relative efficiency of capital goods of different ages can be incorporated into a complete accounting system for income and wealth in constant prices. A broader data base than that Denison has employed would be required. Denison's estimates of both the output and input sides of the production account would have to be revised substantially. To employ an approach that dispenses with vintage accounts for capital goods prices and quantities, like the approach Denison actually uses, it is necessary to assume that the decline in efficiency of capital goods is geometric. In the absence of vintage data the use of Denison's assumptions about relative efficiency leads to a series of inconsistencies in the construction of even a single account, the production account, in current and constant prices. If Denison's estimates of the production accounts were to be incorporated into a complete accounting system, these inconsistencies would ramify throughout the system.

In the United Nations *System of National Accounts,*[40] the construction of a production account in constant prices is discussed at some length. In the United Nations system capital stock is measured as gross stock, following Denison's practice in *Sources of Economic Growth.* Capital consumption allowances are measured by the straight-line method, again following Denison's practice. We conclude that the United Nations system of accounts in constant prices incorporates a production account similar to Denison's. We have already outlined the internal contradictions in Denison's production account; an accounting system incorporating a production account like Denison's would give rise to inconsistencies between the production and the income and expenditure accounts on the one hand and the accumulation and wealth accounts on the other. We conclude that the United Nations system provides a satisfactory solution to the problem of constructing accounts in constant prices only for the output side of the production account. Measurement of the other accounting magnitudes of the system in constant prices requires an extension of the perpetual inventory method like that we have outlined above.

## 5. PRODUCTION ACCOUNT

### *5.1. Introduction*

In sections 3 and 4 our objective has been to develop methods for measuring income and wealth in constant prices. The task that remains is to present production, income and expenditure, accumulation, and wealth accounts in constant prices. To complete this task we must separate the values included in the accounts presented in Section 2 into price and quantity components. For this purpose we employ the system of price and quantity index numbers discussed in Section 3. This system is based on a discrete approximation to continuous Divisia index numbers of prices and quantities.

To construct a complete system of accounts in constant prices we must account for investment goods output, capital input, property compensation, capital formation, and wealth in a way that is internally consistent. For this purpose we have extended the perpetual inventory method to incorporate data on prices as well as quantities of capital goods by vintage. We have also presented a simplified version of the

[40] [55, pp. 52–70].

perpetual inventory method and its price counterpart, based on approximation of replacement rates for individual capital goods by a constant rate of replacement for each good. Our extension of the perpetual inventory method is presented in Section 4.

In this section we present the production account for the U.S. private domestic sector in constant prices. In the following section we present income and expenditure, accumulation, and wealth accounts for the U.S. private national economy in constant prices. In Section 7 we discuss possible extensions of our accounting system.

In constructing the production account in constant prices changes in the value of product and the value of factor outlay must be separated into price and quantity components. The ratio of the quantity of total product to the quantity of total factor input or, alternatively, the ratio of the price of total factor input to the price of total product is equal to total factor productivity. In addition to data on output and input the production account in constant prices includes data on total factor productivity.

### *5.2. Output and Labor Input*

To construct a quantity index for gross product we first allocate the value of output between consumption and investment goods. Investment goods include durable goods and structures. Consumption goods include nondurable goods and services. Data for prices and quantities of both consumption and investment goods are included in the U.S. national accounts as part of gross national product. The product of the rest of the world and government sectors consists entirely of services. Price and quantity index numbers for the services of consumer and institutional durables are constructed as part of our imputation for the value of these services, described below.

The value of output from the point of view of the producing sector excludes certain indirect taxes and includes subsidies. Sales and excise taxes must be allocated between consumption and investment goods output. Since a portion of each of these taxes is levied on intermediate goods, a completely satisfactory allocation would require a detailed interindustry analysis. We have allocated these taxes in proportion to the value of consumption and investment goods output. The price index for each type of output is implicit in the value and quantity of output included in gross national product. We construct price and quantity indexes of gross output by applying Divisia index number formulas to price and quantity data for consumption and investment goods product. The results are given in Table 9.

To construct a quantity index for gross factor input we allocate the value of factor outlay between labor and capital input. The construction of a quantity index of labor input begins with data on the number of persons engaged in the private domestic sector. Persons engaged include full-time equivalent employees and proprietors. Our estimates for the nonfarm business sector are identical to those of the Office of Business Economics for full-time equivalent employees and proprietors. We add Kendrick's estimates of employment in agriculture to obtain total persons engaged.[41] To obtain a measure of labor input our next step is to estimate the number of man-hours worked. For this purpose we employ Kendrick's estimates of man-hours for the private domestic sector.[42]

Denoting the index of man-hours by $L$ and the wage index by $p_L$, we first represent the value of labor input as the sum of the values of labor input for each category of labor:

$$p_L L = \Sigma p_{L,j} L_j,$$

where $p_{L,j}$ is the price of the $j$th type of labor, and $L_j$ is the number of man-hours worked by workers of this type. Divisia indexes of the wage rate and man-hours worked are:

$$\frac{\dot{p}_L}{p_L} = \Sigma v_j \frac{\dot{p}_{L,j}}{p_{L,j}}, \qquad \frac{\dot{L}}{L} = \Sigma v_j \frac{\dot{L}_j}{L_j},$$

where the weights $(v_j)$ are the relative shares of each type of labor in the value of total labor input.

For each category of labor, total man-hours is the product of persons engaged, say $n_j$, and hours per person, say $h_j$. Where $N$ is the total number of persons engaged and $H$ is the number of hours per man, the quantity index of labor input may be rewritten in the form:

$$\frac{\dot{L}}{L} = \Sigma v_j \left(\frac{\dot{n}_j}{n_j} - \frac{\dot{N}}{N}\right) + \Sigma v_j \left(\frac{\dot{h}_j}{h_j} - \frac{\dot{H}}{H}\right) + \left(\frac{\dot{N}}{N} + \frac{\dot{H}}{H}\right).$$

The first term in this expression represents the change in labor input per person engaged due to changes in the composition of the labor force.

[41] These data have been compiled for John W. Kendrick's forthcoming study [43]. We are indebted to Kendrick for providing us with these data in advance of publication. The conceptual basis for compilation of the data is the same as in Kendrick's [42]. The Office of Business Economics data on nonfarm proprietors and employees are from [49a, Tables 6.4 and 6.6].

[42] See note 41, above.

TABLE 9

Gross Private Domestic Product, 1929–69
(constant prices of 1958)

| | Gross Private Domestic Product | | Consumption Goods Product | | Investment Goods Product | | |
|---|---|---|---|---|---|---|---|
| Year | Price Index | Quantity Index | Price Index | Quantity Index | Price Index | Quantity Index | Relative Share |
| 1929 | 0.556 | 187.5 | 0.566 | 133.7 | 0.508 | 56.1 | .278 |
| 1930 | 0.536 | 169.8 | 0.547 | 129.5 | 0.489 | 41.5 | .227 |
| 1931 | 0.489 | 156.6 | 0.497 | 125.8 | 0.453 | 31.2 | .188 |
| 1932 | 0.420 | 132.5 | 0.423 | 114.6 | 0.407 | 17.8 | .134 |
| 1933 | 0.418 | 130.3 | 0.421 | 111.6 | 0.403 | 18.6 | .143 |
| 1934 | 0.440 | 141.7 | 0.445 | 116.6 | 0.414 | 25.2 | .176 |
| 1935 | 0.447 | 151.8 | 0.453 | 121.8 | 0.418 | 30.5 | .197 |
| 1936 | 0.455 | 170.3 | 0.465 | 130.2 | 0.414 | 41.3 | .231 |
| 1937 | 0.471 | 181.1 | 0.476 | 137.8 | 0.441 | 44.5 | .241 |
| 1938 | 0.460 | 171.8 | 0.461 | 138.4 | 0.448 | 34.2 | .203 |
| 1939 | 0.458 | 186.4 | 0.460 | 143.7 | 0.441 | 43.8 | .236 |
| 1940 | 0.463 | 202.8 | 0.465 | 150.7 | 0.446 | 53.5 | .266 |
| 1941 | 0.503 | 230.6 | 0.497 | 159.1 | 0.504 | 73.2 | .333 |
| 1942 | 0.563 | 250.3 | 0.545 | 171.2 | 0.588 | 80.9 | .351 |
| 1943 | 0.626 | 268.3 | 0.625 | 171.7 | 0.617 | 98.1 | .375 |
| 1944 | 0.631 | 283.0 | 0.647 | 181.4 | 0.594 | 103.1 | .358 |
| 1945 | 0.636 | 279.5 | 0.667 | 187.4 | 0.569 | 92.7 | .312 |
| 1946 | 0.701 | 271.5 | 0.728 | 193.0 | 0.643 | 77.3 | .276 |
| 1947 | 0.787 | 277.5 | 0.807 | 191.2 | 0.748 | 85.8 | .309 |
| 1948 | 0.826 | 295.8 | 0.851 | 201.8 | 0.778 | 93.6 | .312 |
| 1949 | 0.796 | 295.3 | 0.800 | 203.6 | 0.791 | 91.2 | .323 |
| 1950 | 0.827 | 326.9 | 0.843 | 212.8 | 0.800 | 114.0 | .354 |
| 1951 | 0.876 | 349.1 | 0.884 | 226.0 | 0.862 | 123.0 | .364 |
| 1952 | 0.897 | 357.7 | 0.907 | 234.4 | 0.879 | 123.0 | .354 |
| 1953 | 0.906 | 375.9 | 0.922 | 244.6 | 0.878 | 131.1 | .355 |
| 1954 | 0.915 | 373.2 | 0.932 | 247.5 | 0.886 | 125.3 | .341 |

(continued)

TABLE 9 (concluded)

| Year | Gross Private Domestic Product Price Index | Gross Private Domestic Product Quantity Index | Consumption Goods Product Price Index | Consumption Goods Product Quantity Index | Investment Goods Product Price Index | Investment Goods Product Quantity Index | Investment Goods Product Relative Share |
|---|---|---|---|---|---|---|---|
| 1955 | 0.932 | 405.4 | 0.953 | 261.5 | 0.893 | 144.0 | .357 |
| 1956 | 0.951 | 415.8 | 0.956 | 272.3 | 0.943 | 143.4 | .359 |
| 1957 | 0.978 | 422.6 | 0.974 | 280.8 | 0.988 | 141.7 | .355 |
| 1958 | 1.000 | 419.7 | 1.000 | 289.2 | 1.000 | 130.6 | .326 |
| 1959 | 1.023 | 447.8 | 1.028 | 302.8 | 1.012 | 145.1 | .336 |
| 1960 | 1.034 | 459.8 | 1.046 | 312.5 | 1.009 | 147.4 | .329 |
| 1961 | 1.039 | 469.6 | 1.052 | 323.6 | 1.011 | 145.8 | .317 |
| 1962 | 1.051 | 499.3 | 1.066 | 338.9 | 1.018 | 160.5 | .327 |
| 1963 | 1.062 | 521.0 | 1.081 | 351.6 | 1.021 | 169.5 | .329 |
| 1964 | 1.074 | 551.6 | 1.096 | 370.6 | 1.029 | 181.3 | .330 |
| 1965 | 1.090 | 587.5 | 1.113 | 391.5 | 1.041 | 196.5 | .335 |
| 1966 | 1.121 | 627.4 | 1.149 | 417.1 | 1.062 | 210.9 | .333 |
| 1967 | 1.146 | 645.6 | 1.170 | 436.2 | 1.095 | 209.5 | .324 |
| 1968 | 1.177 | 678.2 | 1.202 | 455.1 | 1.125 | 223.4 | .330 |
| 1969 | 1.228 | 703.4 | 1.258 | 471.3 | 1.164 | 232.5 | .329 |

The second term represents the change in labor input per hour due to changes in the relative number of hours worked per man among components of the labor force. The last term is the change in total man-hours. Adjustments for changes in the composition of the labor force and the relative number of hours worked per man are required to convert an index of man-hours into an index of the quantity of labor input.

Price and quantity indexes of output require data on the prices and quantities of individual outputs. Similarly, price and quantity indexes of labor input require data on the wages and hours worked for different types of workers. It would be desirable to distinguish among hours worked by workers classified by sex, race, years of schooling, occupation, age, and so on. Price and quantity indexes of labor input would be obtained by applying Divisia index number formulas to price and quantity data for different types of workers. The data available for construction of price and quantity indexes of labor input are very limited. We distinguish among different categories of labor by years of schooling completed. We employ the data compiled by Jorgenson and Griliches and extended by Griliches to estimate the change in labor input due to changes in the educational composition of the labor force.[43]

Kendrick distinguishes among different categories of labor by industry of employment [42]. Jorgenson and Griliches distinguish among different categories by years of schooling completed [40]. Our adjustment of the index of man-hours is limited to changes in the quality of labor input due to changes in the educational composition of the labor force. Adjustments for changes in the distribution of the labor force by age and sex would require more detailed data. We have made no adjustment for changes in the relative number of hours worked by different types of workers. Estimates of the likely effect of additional adjustments of each type are given by Jorgenson and Griliches [41].

Denison has observed that the intensity of effort may vary with the number of hours worked per week [13], [14]. Correction of the quantity index of labor input to reflect changes in intensity of effort would require estimates of wages and man-hours, classified by the number of hours worked per week. Denison suggests that the stock of labor input provides an upper bound for labor input corrected for variations in intensity, while the number of man-hours provides a lower bound. He estimates effective labor input by correcting man-hours for variations in labor intensity. We have employed Denison's adjustment for the intensity of effort applied to actual hours per man rather than potential hours per man. The number of persons engaged and hours per worker, together with price and quantity indexes of labor input for 1929–69, are given for the private domestic economy in Table 10.

### *5.3. Capital Input*

Our estimates of capital input, property compensation, depreciation, replacement, and capital assets are based on an extension of the perpetual inventory method to incorporate data on prices as well as quantities of investment goods by vintage. We estimate capital service prices, depreciation, and acquisition prices for capital goods of different vintages on the basis of the assumption that the decline in efficiency of capital goods is geometric in form. We estimate capital stock, replacement, and quantities of capital goods of different vintages on the basis of the same

[43] See [40] and [28]. We have extended Griliches' estimates back to 1929, using relative earnings for 1939 and estimates of the educational attainment of the labor force for 1930 and 1940 by Folger and Nam [20].

TABLE 10

Private Domestic Labor Input, 1929–69
(constant prices of 1958)

| Year | Private Domestic Persons Engaged (millions) | Private Domestic Hours per Person (thousands per year) | Private Domestic Labor Input | |
|---|---|---|---|---|
| | | | Price Index | Quantity Index |
| 1929 | 43.0 | 2.645 | 0.338 | 178.8 |
| 1930 | 40.8 | 2.600 | 0.326 | 170.6 |
| 1931 | 37.6 | 2.579 | 0.290 | 163.2 |
| 1932 | 34.2 | 2.512 | 0.254 | 146.1 |
| 1933 | 34.2 | 2.488 | 0.238 | 146.0 |
| 1934 | 36.7 | 2.281 | 0.257 | 152.6 |
| 1935 | 37.9 | 2.327 | 0.267 | 159.3 |
| 1936 | 39.8 | 2.380 | 0.281 | 168.7 |
| 1937 | 41.6 | 2.420 | 0.303 | 177.4 |
| 1938 | 39.1 | 2.350 | 0.298 | 166.6 |
| 1939 | 40.5 | 2.389 | 0.305 | 174.0 |
| 1940 | 42.2 | 2.391 | 0.314 | 182.1 |
| 1941 | 45.8 | 2.402 | 0.351 | 198.6 |
| 1942 | 48.1 | 2.458 | 0.411 | 211.6 |
| 1943 | 48.7 | 2.517 | 0.472 | 216.7 |
| 1944 | 47.5 | 2.549 | 0.505 | 215.5 |
| 1945 | 46.0 | 2.487 | 0.520 | 208.5 |
| 1946 | 48.6 | 2.372 | 0.543 | 220.1 |
| 1947 | 50.9 | 2.314 | 0.597 | 230.7 |
| 1948 | 52.0 | 2.287 | 0.640 | 236.0 |
| 1949 | 50.2 | 2.279 | 0.651 | 228.9 |
| 1950 | 51.7 | 2.250 | 0.689 | 236.0 |
| 1951 | 53.7 | 2.242 | 0.746 | 246.5 |
| 1952 | 54.1 | 2.239 | 0.786 | 249.3 |
| 1953 | 54.9 | 2.208 | 0.832 | 252.9 |
| 1954 | 53.2 | 2.185 | 0.854 | 244.9 |
| 1955 | 54.5 | 2.210 | 0.887 | 253.7 |
| 1956 | 55.6 | 2.197 | 0.935 | 259.5 |

(continued)

TABLE 10 (concluded)

| Year | Private Domestic Persons Engaged (millions) | Private Domestic Hours per Person (thousands per year) | Private Domestic Labor Input | |
|---|---|---|---|---|
| | | | Price Index | Quantity Index |
| 1957 | 55.5 | 2.170 | 0.979 | 259.4 |
| 1958 | 53.7 | 2.150 | 1.000 | 252.8 |
| 1959 | 54.8 | 2.175 | 1.040 | 263.0 |
| 1960 | 55.4 | 2.177 | 1.070 | 267.8 |
| 1961 | 54.9 | 2.160 | 1.098 | 266.3 |
| 1962 | 55.8 | 2.163 | 1.138 | 272.9 |
| 1963 | 56.3 | 2.160 | 1.173 | 277.0 |
| 1964 | 57.4 | 2.163 | 1.221 | 284.1 |
| 1965 | 59.2 | 2.166 | 1.262 | 294.4 |
| 1966 | 61.3 | 2.152 | 1.323 | 306.8 |
| 1967 | 62.3 | 2.154 | 1.366 | 314.7 |
| 1968 | 63.8 | 2.151 | 1.447 | 324.6 |
| 1969 | 65.6 | 2.139 | 1.537 | 334.9 |

assumption. Estimates of capital input, property compensation, depreciation, and capital assets in constant prices require data on both prices and quantities of capital goods by vintage. We continue our discussion of the production account for the U.S. private domestic economy in constant prices by describing the construction of prices and quantities of capital input.[44]

The starting point for a quantity index of capital input is a perpetual inventory estimate of the stock of each type of capital, based on past investments in constant prices. At each point of time the stock of each type of capital is the sum of stocks remaining from past investments of each vintage. Under the assumption that efficiency of capital goods declines geometrically, the rate of replacement, say $\delta$, is a constant. Capital stock at the end of every period may be estimated from investment and capital stock at the beginning of the period:

$$K_t = A_t + (1 - \delta)K_{t-1},$$

where $K_t$ is end of period capital stock, $A_t$ the quantity of investment, and $K_{t-1}$ the capital stock at the beginning of the period.

[44] Our estimates are based on those of [8].

For each type of capital included in our accounts we prepare perpetual inventory estimates of the stock as follows: First, we obtain a benchmark estimate of capital stock from data on national wealth in constant prices. Second, we deflate the investment series from the U.S. national accounts to obtain investment in constant prices. Third, we choose an estimate of the rate of replacement from data on the lifetimes of capital goods. Finally, we estimate capital stock in every period by applying the perpetual inventory method described above. We have prepared estimates for the stocks of consumer durables, nonresidential structures, producer durables, residential structures, nonfarm inventories, farm inventories, and land. Benchmark estimates of capital stocks in 1929, expressed in constant prices of 1958, rates of replacement, and price indexes for each type of capital are presented in Table 11.

Our price indexes for consumer and producer durables and for farm and nonfarm inventories are taken directly from the U.S. national accounts. These indexes are the implicit deflators for investment in each category from estimates of gross private domestic investment in current and constant prices. We replace the deflators from the national accounts for residential and nonresidential structures by the "constant cost 2" construction price index employed in the *Capital Stock Study* of the Office of Business Economics.[45] This index results from an attempt to correct implicit deflators for structures for changes in the quality of structures produced. In the *Capital Stock Study* the "constant cost 2" price index is employed to deflate data on investment in nonresidential structures. We employ Goldsmith's price index for land through 1958, extrapolating this index from 1958 to 1969 by assuming a constant rate of growth of the price of land at 6.9 per cent per year.[46] Our price indexes for farm and nonfarm inventory stocks [47] are based on unpublished estimates of the Office of Business Economics.[48]

Rates of replacement for inventories and land are zero by definition. To estimate rates of replacement for structures and durables we employ double declining balance replacement rates from the *Capital Stock Study*.

[45] The Office of Business Economics *Capital Stock Study* is reported in a series of articles. See [30], and the references given there. We are indebted to Robert Wasson for permission to use the underlying data on investment in current and constant prices.

[46] See [24, Tables A-40 and A-41, pp. 186–189].

[47] Asset deflators are weighted by the relative proportion of assets of each type in total assets; investment deflators are weighted by the relative proportion of investment goods of each type in total investment. See [16, p. 12]. Asset deflators are appropriate for deflating asset values and for estimating rental values of capital services.

[48] We are indebted to Shirley Loftus for providing us with these estimates.

TABLE 11

Benchmarks, Rates of Replacement, and Price Indexes Employed in Estimating Capital

| Asset Class | 1929 Benchmark (billions of 1958 dollars) | Replacement Rate | Deflator |
|---|---|---|---|
| 1. Consumer durables | 74.9 | .200 | Implicit deflator, national product accounts [a] |
| 2. Nonresidential structures | 148.2 | .056 | Constant cost 2 deflator [b] |
| 3. Producer durables | 77.5 | .138 | Implicit deflator, national product accounts [a] |
| 4. Residential structures | 214.0 | .039 | Implicit deflator, national product accounts [a] |
| 5. Nonfarm inventories | 57.1 | - | Investment: Implicit deflator, national product accounts [c]<br>Assets: Implicit deflator, OBE [d] |
| 6. Farm inventories | 21.9 | - | Investment: Implicit deflator, national product accounts [c]<br>Assets: Implicit deflator, OBE [d] |
| 7. Land | 321.6 | - | Goldsmith [e] |

[a] [49a, Table 8.1].
[b] [31].
[c] [49a. Tables 1.1 and 1.2].
[d] Unpublished OBE sources.
[e] [25, Tables A-5 and A-6].

For each asset the rate of replacement is $\delta = 2/T$, where $T$ is the mean service life for the asset given in the *Capital Stock Study*.[49] Our estimates of replacement rates incorporate both retirements of capital goods and the decline in efficiency of existing capital goods. In the *Capital Stock*

[49] These lifetimes have been compiled for the Office of Business Economics *Capital Stock Study;* we are indebted to Robert Wasson for providing us with data on service lives.

*Study* investment in nonresidential structures is divided into fifty-two categories. Although it would be possible to compile data on capital input for each of these categories separately, we have limited our estimates to total producer durables and total nonresidential structures. The replacement rate for each group is estimated as a weighted average of replacement rates for the individual components, using relative shares of the value of each category in the total value of capital stock as weights.

Residential structures may be divided into farm and nonfarm components. We estimate service lives for each component on the basis of Bulletin F lifetimes; the replacement rate for residential structures is a weighted average of double declining balance replacement rates with weights based on the relative shares of farm and nonfarm residential structures in the total.[50] We assume that the rate of replacement for consumer durables is 0.200; this estimate was developed by deLeeuw in estimating stocks of consumer durables [12].

We have described the measurement of capital stocks for each category of capital goods by the perpetual inventory method. Our next step is to describe the measurement of capital service prices by the price counterpart of the perpetual inventory method. For property with an active rental market the price of capital services may be observed directly as the rental price of the corresponding asset. A substantial portion of the range of capital goods employed in the U.S. private domestic sector has an active rental market; most classes of structures can be rented and a rental market exists for many types of equipment, especially large pieces of equipment such as aircraft, trucks, construction equipment, computers, and so on. Unfortunately, very little effort has been devoted to compiling data on rental rates for either structures or equipment. Data on the flow of rent payments among industrial sectors have been compiled by Creamer [11]. However, both current price and constant price flows are required for direct measurement of the price and quantity of capital services by class of asset.

Given market rental prices by class of asset, the implicit rental values paid by owners for the use of their property may be imputed by applying rental rates to capital stocks employed by owner-users. This method for imputation is used to estimate the price and quantity of capital services from owner-occupied dwellings in the U.S. national accounts. Data on rental prices of dwellings occupied by renters are employed to impute

[50] Bulletin F [4] lives have been compared with alternative lifetimes by Wasson [60].

the rental value of dwellings occupied by owners. The total rental value of owner-occupied dwellings is divided among taxes, capital consumption allowances, interest payments, and net rent. A somewhat similar but not identical method of imputation is used for the space rental value of institutional buildings. Capital consumption allowances and interest payments by institutions are estimated as components of imputed space rental value. Net rent is omitted from the imputation, but this component of space rental value could be estimated from the market rental prices of space comparable to that used by institutions. The main obstacle to broader application of this method of imputation is the lack of appropriate data on market rental prices.

An alternative method for imputation of the rental value of owner-utilized assets is included in our extension of the perpetual inventory method to incorporate data on prices of capital goods by vintage. For each type of capital included in our accounts we prepare perpetual inventory estimates of acquisition prices, service prices, depreciation, and revaluation by vintage. Under our assumption of geometrically declining relative efficiency of capital goods, perpetual inventory estimates of prices can be simplified considerably. First, beginning with acquisition prices for new capital goods of each type, the acquisition prices for goods of each vintage decline geometrically with vintage. The formula for the value of capital stock,

$$q_{A,t}K_t = \Sigma q_{A,t}(1 - \delta)^{\tau} A_{t-\tau} = \Sigma q_{A,t,\tau} A_{t-\tau},$$

may be regarded as the sum of past investments weighted by relative efficiency and evaluated at the acquisition price for new capital goods or, equivalently, as the sum of past investments evaluated at the acquisition price for the corresponding vintage of capital.

Second, under our assumption that replacement rates are constant, depreciation is proportional to the value of beginning of period capital stock:

$$q_{D,t}K_{t-1} = \delta q_{A,t}K_{t-1}.$$

This measure of depreciation can also be obtained by estimating depreciation separately for each vintage and summing over vintages:

$$\Sigma q_{D,t,\tau} A_{t-\tau-1} = \Sigma \delta q_{A,t,\tau} A_{t-\tau-1} = \delta q_{A,t} K_{t-1}.$$

Similarly, revaluation is equal to the change in the acquisition price of new capital goods multiplied by beginning of period capital stock. This measure can also be obtained by estimating revaluation separately for each vintage and summing over vintages:

$$(q_{A,t} - q_{A,t-1})K_{t-1} = \Sigma(q_{A,t,\tau} - q_{A,t-1,\tau-1})A_{t-\tau-1}.$$

In the absence of taxation, the value of capital services is the sum of the cost of capital and depreciation, less revaluation:

$$q_{K,t}K_{t-1} = [q_{A,t-1}r_t + q_{A,t}\delta - (q_{A,t} - q_{A,t-1})]K_{t-1}.$$

We can obtain this expression by estimating the capital service price for capital goods of each vintage and summing over vintages:

$$q_{K,t}K_{t-1} = \Sigma(1 - \delta)^{\tau}q_{K,t}A_{t-\tau-1} = \Sigma q_{K,t,\tau}A_{t-\tau-1}.$$

Given the quantity of each type of asset held, the acquisition price, and the rate of replacement, only the rate of return remains to be determined in compiling data on the price and quantity of capital services. In measuring the rate of return, differences in the tax treatment of property compensation from different sectors must be taken into account.

For tax purposes the private domestic sector of the U.S. economy can be divided into corporate business, noncorporate business, and households and nonprofit institutions. Households and institutions are not subject to direct taxes on the flow of capital services they utilize. Noncorporate business is subject to personal income taxes on income generated from capital services, while corporate business is subject to both corporate and personal income taxes. Households and corporate and noncorporate business are subject to indirect taxes on property income through taxes levied on the value of property. In order to take these differences in taxation into account we first allocate each class of assets among the four sectors of the U.S. private domestic economy—corporations, noncorporate business, households, and institutions. The relative proportions of capital stock by asset class for each sector for 1958 are given in Table 12.

For a sector not subject to either direct or indirect taxes on property income, the value of property compensation is equal to the value of capital services, i.e., property compensation = $q_{K,t}K_{t-1}$. This formula is appropriate for a single class of assets. For several classes of assets, property compensation is the sum of price times quantity of capital services for all classes of assets. We assume that the rate of return is the same for all assets held by a given sector; rates of return can be estimated for each flow of property compensation that can be measured separately. Flows of property compensation can be separately measured for industry groups or even for individual firms.

Given property compensation, the acquisition prices of new capital

TABLE 12

Relative Proportions of Capital Stock by Asset Class and Sector, 1958

| Asset Class | Sector: 1. Corporate Business | 2. Noncorporate Business | 3. Households and Institutions | Total |
|---|---|---|---|---|
| 1. Consumer durables | - | - | .138 | 0.138 |
| 2. Nonresidential structures | .104 | .027 | .014 | 0.145 |
| 3. Producer durables | .09 | .041 | .002 | 0.132 |
| 4. Residential structures | .019 | .009 | .211 | 0.238 |
| 5. Nonfarm inventories | .065 | .013 | - | 0.078 |
| 6. Farm inventories | - | .021 | - | 0.021 |
| 7. Land | .047 | .124 | .077 | 0.247 |
| Total | .325 | .234 | .442 | 1.000 |

goods ($q_{A,t}$), the rate of replacement ($\delta$), and capital stocks estimated by the perpetual inventory method ($K_{t-1}$), we can solve for the rate of return by substituting the capital service price,

$$q_{K,t} = q_{A,t-1}r_t + q_{A,t}\delta - (q_{A,t} - q_{A,t-1}),$$

into the expression for property compensation. In this expression only the rate of return is unknown and we may solve for the rate of return in terms of the observed data, obtaining:

$$r_t = \frac{\text{Property compensation} - q_{A,t}\delta K_{t-1} + (q_{A,t} - q_{A,t-1})K_{t-1}}{q_{A,t-1}K_{t-1}}.$$

The rate of return is the ratio of property compensation less depreciation plus revaluation of capital assets to the value of capital stock at the beginning of the period. For more than one capital good we estimate depreciation, revaluation, and the value of capital stock by summing over all capital goods.

The formula for the rate of return given above is appropriate only with no direct or indirect taxes on property compensation. For the U.S. private domestic economy, this formula can be applied only to nonprofit institutions. We discuss the imputation of the value of the capital services utilized by these institutions below. Households hold consumer durables and owner-occupied dwellings. The property compensation associated with these assets is not taxed directly; however, part of the income is taxed indirectly through property taxes. To incorporate property taxes into our estimates of the price and quantity of capital services we add taxes to the cost of capital, depreciation, and revaluation, obtaining the capital service price:

$$q_{K,t} = q_{A,t-1}r_t + q_{A,t}\delta - (q_{A,t} - q_{A,t-1}) + q_{A,t}\tau_t,$$

where $\tau_t$ is the rate of property taxation. To estimate the rate of return we proceed as before, substituting the capital service price including property taxes into the expression for property compensation. The rate of return is the ratio of property compensation less depreciation plus revaluation of capital assets less taxes to the value of capital stock at the beginning of the period.

In measuring the capital service flow utilized by households and institutions we first estimate the value of the services of owner-occupied residential real estate, including both land and structures. This value is obtained directly from the U.S. national accounts. Using prices of acquisition for land and residential structures, the corresponding stocks in constant prices, the rate of replacement for structures, and the value of owner-occupied housing services, we estimate the implicit rate of return for the household sector. We assume that rates of return for consumer durables and for producer durables, nonresidential structures, and land utilized by institutions are the same as for owner-occupied residential real estate. This assumption results in a single rate of return for households and institutions. Adding the cost of capital and depreciation, subtracting revaluation for assets held by households and institutions, and adding property taxes for the household sector, we obtain the imputed value of property compensation, gross of taxes, for households and institutions. The imputed value of the services of owner-occupied dwellings is identical to the value of the flow of services from these dwellings from the U.S. national accounts.

Given the rate of return for households and institutions, we can construct estimates of capital service prices for each class of assets held by households and institutions—land held by households and institutions, residential structures, nonresidential structures, producer durables, and consumer durables. These estimates require acquisition prices for each capital good, rates of replacement, rates of taxation for assets held by households, and the rate of return for the sector as a whole. We employ separate effective tax rates for owner-occupied residential property, both land and structures, and for consumer durables. Corresponding to these price data we can construct estimates of capital service quantities for each class of assets. Price and quantity measures of capital input by class of asset can be combined into price and quantity index numbers of capital input by households and institutions, utilizing the Divisia index number formulas presented in Section 3 above.

Our measure of the gross output of the private domestic sector of the U.S. economy differs from that of the U.S. national accounts in the treatment of consumer and institutional durables and institutional real estate. We assign personal consumption expenditures on durables to gross investment rather than consumption. This change leaves the product of the private domestic sector unchanged. We add the service flow from consumer and institutional durables to the value of output and the value of capital input. We also add the net rent component of the services of institutional real estate to values of both output and input. The values of these service flows enter the product and factor outlay accounts given in Table 1 above and represent net additions to the value of gross product of the private domestic sector from the U.S. national accounts.

Our method for estimating the prices and quantities of capital services in the noncorporate sector is similar to the method we have described for households and institutions. For the noncorporate sector we estimate property compensation directly as the sum of income originating in business, other than income originating in corporate business and government enterprises and net rent of owner-occupied dwellings, less labor compensation in the noncorporate sector, including imputed labor compensation of proprietors and unpaid family workers, plus noncorporate capital consumption allowances, less allowances for owner-occupied dwellings and institutional structures, and plus indirect business taxes allocated to the noncorporate sector. We also allocate the statistical discrepancy to noncorporate property compensation.

To obtain an estimate of the noncorporate rate of return we deduct property taxes from noncorporate property compensation, add revaluation of assets, subtract depreciation, and divide the result by the value of noncorporate assets at the beginning of the period. The noncorporate rate of return is gross of personal income taxes on noncorporate property compensation. Property compensation of households and institutions is not subject to the personal income tax.

The value of property compensation in the noncorporate sector is equal to the value of the flow of capital services from residential and nonresidential structures, producer durable equipment, farm and nonfarm inventories, and land held by the sector. All farm inventories are assigned to the noncorporate sector. Given the noncorporate rate of return, estimated from noncorporate property compensation by the method outlined above, and given data on prices of acquisition, stocks, tax rates, and replacement rates for each class of assets, we can estimate capital service prices for each class of assets held by the noncorporate sector. Quantity data on capital services for each class of assets are constructed by the perpetual inventory method. Price and quantity measures of capital input by class of asset can be combined into price and quantity index numbers of capital input by noncorporate business, using Divisia index number formulas as before.

We next consider the measurement of prices and quantities of capital services for corporate business. We measure corporate property compensation as income originating in corporate business, less compensation of employees, plus corporate capital consumption allowances, plus business transfer payments, plus the indirect business taxes allocated to the corporate sector. To obtain an estimate of the corporate rate of return we must take into account the corporate income tax. The capital service price, modified to incorporate the corporate income tax and indirect business taxes, becomes:

$$q_{K,t} = \left[\frac{1 - u_t z_t - k_t + y_t}{1 - u_t}\right][q_{A,t-1} r_t + q_{A,t}\delta - (q_{A,t} - q_{A,t-1})] + q_{A,t}\tau_t,$$

where indirect business taxes $q_{A,t}\tau_t$ are deducted from corporate property compensation before taxes as an expense, $u_t$ is the corporate tax rate, $z_t$ is the present value of depreciation allowances on one dollar's worth of investment, $k_t$ the investment tax credit, and $y_t = k_t u_t z_t$.[51] The variable $y_t$ is set equal to zero for all years but 1962 and 1963; it is used in accounting for the fact that the investment tax credit was deducted from the value of an asset for depreciation in those years. The tax credit is different from zero only for producer durables. Depreciation allowances are different from zero only for durables and structures.

[51] A detailed derivation of prices of capital services is given by Hall and Jorgenson [34], [35] for continuous time. We have converted their formulation to discrete time, added property taxes, and introduced alternative measurements for the tax parameters. Similar formulas have been developed by Coen [10].

Our method for estimating the corporate rate of return is the same as for the noncorporate rate of return. Property compensation in the corporate sector is the sum of the value of services from residential and nonresidential structures, producer durable equipment, nonfarm inventories, and land held by that sector. To estimate the rate of return in the corporate sector we require estimates of the variables that describe the corporate tax structure—the effective corporate tax rate, the present value of depreciation allowances, and the investment tax credit. We obtain estimates of all the variables—acquisition prices and stocks of assets, rates of replacement, and variables describing the tax structure—that enter the value of capital services except, of course, for the rate of return. We then solve for the rate of return in terms of these variables and total property compensation.

Our estimate of the effective rate of the corporate income tax is obtained as the ratio of federal and state and local corporate profits tax liability plus the investment tax credit to corporate property income less taxes on corporate property and the imputed value of depreciation allowances for tax purposes. Imputed depreciation differs from depreciation for tax purposes in reflecting changes in the present value of future depreciation allowances as well as the current flow of depreciation allowances. The present value of depreciation deductions on new investment depends on depreciation formulas allowed for tax purposes, the lifetimes of assets used in calculating depreciation, and the rate of return. We assume that the rate of return used for discounting future depreciation allowances in the corporate sector is constant at 10 per cent. Our estimate of the effective rate of the investment tax credit is based on estimates of the tax credit claimed by corporations. The effective rate is the investment tax credit divided by investment in producer durable equipment by corporations.

To estimate the rate of return in the corporate sector our first step is to subtract property taxes from total property compensation before taxes. The second step is to subtract federal and state and local corporate profits tax liability. We then add revaluation of assets, subtract depreciation, and divide the result by the value of corporate assets at the beginning of the period. The corporate rate of return is gross of personal income taxes, but net of the corporate income tax. We estimate the price of capital services for each asset employed in the corporate sector by substituting the corporate rate of return into the corresponding formula for the price of capital services. These formulas also depend on acquisition prices of capital assets, rates of replacement, and variables describing the tax structure. Quantity data for each class of assets are constructed by the perpetual inventory method. Price and quantity indexes of capital input by class of asset are combined into price and quantity indexes of capital input for the corporate sector, utilizing Divisia index number formulas.

In separating changes in the value of capital input into price and quantity components we preserve the accounting identity that property compensation for each sector of the U.S. private domestic economy is equal to the value of all capital services utilized in that sector. Denoting the index of capital input by $K$ and the capital service price index by $p_K$, total property compensation is the sum of values of capital input for each category of capital:

$$p_K K = \Sigma p_{K,j} K_j,$$

where $p_{K,j}$ is the price of the $j$th type of capital service and $K_j$ is the quantity of capital of this type. Divisia indexes of the capital service price and capital input are:

$$\frac{\dot{p}_K}{p_K} = \Sigma v_j \frac{\dot{p}_{K,j}}{p_{K,j}}, \quad \frac{\dot{K}}{K} = \Sigma v_j \frac{\dot{K}_j}{K_j},$$

where the weights are the relative shares of each type of capital input in total property compensation.

We assume that the rate of return is the same for all assets within a given sector. This rate of return is inferred from the value of property compensation, acquisition prices and stocks of capital goods, rates of replacement, and variables describing the tax structure. To obtain price and quantity indexes of capital input for the private domestic sector as a whole we apply the Divisia index formulas to Divisia price and quantity indexes for each of the three subsectors—corporations, noncorporate business, and households and institutions. By the reproductive property of Divisia index numbers the resulting price and quantity indexes are equivalent to Divisia indexes computed from data on prices and quantities of capital goods distinguished by class of asset and sector. Price and quantity indexes of capital services for corporations, noncorporate business, households and institutions, and the U.S. private domestic sector as a whole are given for 1929–69 in Table 13.

*5.4. Total Factor Productivity*

We construct price and quantity index numbers for total factor input by combining Divisia indexes of labor and capital input into a Divisia

TABLE 13

Gross Private Domestic Capital Input, 1929–69
(constant prices of 1958)

| Year | Corporate Capital Input | | Noncorporate Capital Input | | Household Capital Input | | Private Domestic Capital Input | |
|---|---|---|---|---|---|---|---|---|
| | Price Index | Quantity Index | Price Index | Quantity Index | Price Index | Quantity Index | Price Index | Quantity Index |
| 1929 | .070 | 261.7 | .052 | 204.6 | .053 | 280.9 | .057 | 765.8 |
| 1930 | .056 | 268.2 | .029 | 210.2 | .050 | 285.1 | .045 | 782.4 |
| 1931 | .039 | 267.9 | .025 | 210.9 | .048 | 279.8 | .038 | 776.2 |
| 1932 | .026 | 260.0 | .011 | 216.9 | .035 | 270.2 | .024 | 758.1 |
| 1933 | .025 | 242.6 | .013 | 211.2 | .043 | 254.1 | .028 | 714.3 |
| 1934 | .040 | 228.0 | .017 | 204.5 | .044 | 240.5 | .034 | 676.6 |
| 1935 | .048 | 220.8 | .025 | 209.6 | .041 | 232.5 | .038 | 661.5 |
| 1936 | .059 | 216.8 | .031 | 210.3 | .047 | 231.6 | .046 | 655.9 |
| 1937 | .063 | 219.3 | .032 | 213.2 | .046 | 238.1 | .047 | 667.6 |
| 1938 | .053 | 224.9 | .029 | 220.6 | .046 | 244.6 | .043 | 686.4 |
| 1939 | .060 | 220.6 | .034 | 220.3 | .048 | 242.6 | .048 | 678.7 |
| 1940 | .076 | 220.7 | .038 | 221.9 | .048 | 247.3 | .054 | 684.5 |
| 1941 | .099 | 227.3 | .051 | 225.0 | .048 | 256.8 | .066 | 704.0 |
| 1942 | .119 | 239.6 | .065 | 230.2 | .040 | 269.5 | .073 | 735.7 |
| 1943 | .137 | 239.0 | .074 | 228.5 | .061 | 261.6 | .090 | 728.3 |
| 1944 | .139 | 234.5 | .094 | 224.6 | .065 | 250.4 | .098 | 710.8 |
| 1945 | .123 | 231.1 | .103 | 223.6 | .075 | 239.0 | .099 | 697.4 |
| 1946 | .110 | 234.5 | .103 | 223.9 | .094 | 232.0 | .102 | 695.8 |
| 1947 | .133 | 252.9 | .095 | 229.3 | .099 | 255.9 | .108 | 744.1 |
| 1948 | .155 | 272.1 | .100 | 235.1 | .098 | 285.4 | .117 | 799.3 |
| 1949 | .143 | 288.1 | .089 | 245.9 | .074 | 313.3 | .101 | 852.4 |
| 1950 | .165 | 295.0 | .098 | 254.0 | .101 | 341.6 | .121 | 891.0 |
| 1951 | .177 | 310.6 | .117 | 266.2 | .093 | 383.5 | .128 | 955.7 |
| 1952 | .163 | 331.0 | .107 | 274.6 | .101 | 408.2 | .123 | 1,010.0 |
| 1953 | .162 | 344.3 | .101 | 279.4 | .108 | 426.8 | .124 | 1,047.1 |
| 1954 | .157 | 357.0 | .098 | 284.1 | .108 | 452.8 | .121 | 1,090.2 |

(continued)

TABLE 13 (concluded)

| | Corporate Capital Input | | Noncorporate Capital Input | | Household Capital Input | | Private Domestic Capital Input | |
|---|---|---|---|---|---|---|---|---|
| Year | Price Index | Quantity Index | Price Index | Quantity Index | Price Index | Quantity Index | Price Index | Quantity Index |
| 1955 | .184 | 365.3 | .098 | 288.2 | .119 | 476.6 | .136 | 1,125.4 |
| 1956 | .178 | 382.5 | .089 | 294.7 | .114 | 514.5 | .129 | 1,187.1 |
| 1957 | .175 | 402.5 | .096 | 298.3 | .112 | 540.2 | .129 | 1,239.5 |
| 1958 | .162 | 418.1 | .111 | 302.0 | .117 | 562.0 | .130 | 1,282.0 |
| 1959 | .187 | 423.7 | .100 | 304.6 | .130 | 574.0 | .142 | 1,302.2 |
| 1960 | .182 | 437.2 | .096 | 309.1 | .133 | 598.7 | .140 | 1,346.1 |
| 1961 | .180 | 452.6 | .104 | 313.4 | .132 | 620.9 | .141 | 1,389.8 |
| 1962 | .197 | 463.1 | .114 | 316.9 | .136 | 637.5 | .150 | 1,421.3 |
| 1963 | .202 | 479.7 | .115 | 323.5 | .142 | 663.5 | .155 | 1,471.5 |
| 1964 | .214 | 497.4 | .115 | 330.8 | .146 | 695.0 | .161 | 1,528.8 |
| 1965 | .231 | 520.1 | .125 | 338.8 | .146 | 731.2 | .168 | 1,597.5 |
| 1966 | .239 | 549.5 | .140 | 349.7 | .152 | 775.7 | .177 | 1,684.1 |
| 1967 | .224 | 590.0 | .145 | 359.6 | .153 | 819.7 | .174 | 1,783.9 |
| 1968 | .232 | 620.9 | .140 | 368.8 | .155 | 856.4 | .176 | 1,864.0 |
| 1969 | .230 | 650.2 | .136 | 379.3 | .164 | 903.2 | .179 | 1,952.4 |

index of total factor input. The weights for labor and capital are the relative shares of labor and property compensation in the value of total factor outlay. Price and quantity index numbers for gross private domestic product may be represented in the form:

$$\frac{\dot{p}}{p} = v_L \frac{\dot{p_L}}{p_L} + v_K \frac{\dot{p_K}}{p_K},$$

$$\frac{\dot{X}}{X} = v_L \frac{\dot{L}}{L} + v_K \frac{\dot{K}}{K},$$

where $p$ is the price index for total factor input, $X$ is the quantity index, $v_L$ is the relative share of labor, and $v_K$ the relative share of capital. Discrete approximations to these continuous Divisia indexes for the price and quantity of total factor input for the U.S. private domestic economy are given for 1929–69 in Table 14.

TABLE 14

Gross Private Domestic Factor Input, 1929–69
(constant prices of 1958)

| Year | Gross Private Domestic Factor Input: Price Index | Gross Private Domestic Factor Input: Quantity Index | Property Outlay, Relative Share |
|---|---|---|---|
| 1929 | 0.376 | 277.5 | .419 |
| 1930 | 0.335 | 272.1 | .389 |
| 1931 | 0.290 | 264.0 | .382 |
| 1932 | 0.228 | 243.8 | .333 |
| 1933 | 0.228 | 238.7 | .362 |
| 1934 | 0.259 | 240.7 | .370 |
| 1935 | 0.277 | 245.2 | .373 |
| 1936 | 0.306 | 253.2 | .389 |
| 1937 | 0.324 | 263.1 | .370 |
| 1938 | 0.310 | 255.5 | .374 |
| 1939 | 0.327 | 261.4 | .379 |
| 1940 | 0.348 | 269.7 | .392 |
| 1941 | 0.403 | 287.3 | .399 |
| 1942 | 0.464 | 303.8 | .383 |
| 1943 | 0.546 | 307.2 | .390 |
| 1944 | 0.589 | 303.2 | .391 |
| 1945 | 0.602 | 295.0 | .390 |
| 1946 | 0.624 | 304.8 | .372 |
| 1947 | 0.678 | 321.8 | .369 |
| 1948 | 0.729 | 335.3 | .382 |
| 1949 | 0.698 | 337.0 | .366 |
| 1950 | 0.774 | 349.3 | .399 |
| 1951 | 0.829 | 368.7 | .399 |
| 1952 | 0.845 | 379.4 | .389 |
| 1953 | 0.878 | 388.1 | .382 |
| 1954 | 0.884 | 386.4 | .388 |
| 1955 | 0.945 | 399.8 | .404 |
| 1956 | 0.955 | 413.9 | .387 |
| 1957 | 0.982 | 420.8 | .385 |
| 1958 | 1.000 | 419.7 | .398 |
| 1959 | 1.059 | 432.5 | .403 |

(continued)

TABLE 14 (concluded)

| Year | Gross Private Domestic Factor Input: Price Index | Gross Private Domestic Factor Input: Quantity Index | Property Outlay, Relative Share |
|---|---|---|---|
| 1960 | 1.073 | 443.0 | .398 |
| 1961 | 1.091 | 447.2 | .401 |
| 1962 | 1.145 | 458.0 | .408 |
| 1963 | 1.181 | 468.6 | .413 |
| 1964 | 1.226 | 483.2 | .415 |
| 1965 | 1.275 | 502.4 | .420 |
| 1966 | 1.337 | 526.2 | .423 |
| 1967 | 1.352 | 547.1 | .419 |
| 1968 | 1.407 | 567.3 | .412 |
| 1969 | 1.467 | 589.0 | .404 |

Total factor productivity is defined as the ratio of real product to real factor input or, equivalently, as the ratio of the price of factor input to the product price.[52] Growth in total factor productivity may be regarded as an increase in the efficiency of the use of input to produce output or as a decline in the cost of input required to produce a given value of output. We may define a Divisia index of total factor productivity, say $P$, as:

$$\frac{\dot{P}}{P} = \frac{\dot{Y}}{Y} - \frac{\dot{X}}{X},$$

where $Y$ is the quantity index of total output and $X$ is the quantity index of total factor input. Equivalently, the index of total factor productivity may be defined as:

$$\frac{\dot{P}}{P} = \frac{\dot{p}}{p} - \frac{\dot{q}}{q},$$

where $p$ is the price index of total factor input and $q$ is the price index of output. A discrete approximation to the Divisia index of total factor productivity is given in Table 15. For comparison, indexes of total factor productivity for a number of alternative conventions for the measurement of total factor input are also included in this table.

[52] For further discussion of this index of total factor productivity, see [40], especially pp. 250–254. The Divisia index of total factor productivity described in the text is a discrete approximation to the continuous Divisia index discussed by Jorgenson and Griliches.

TABLE 15

Total Factor Productivity, 1929–69
(1958 = 1.000)

| Year | Labor Services and Capital Services | Labor Services and Capital Stock | Unweighted Man-hours and Capital Stock |
|---|---|---|---|
| 1929 | 0.674 | 0.637 | 0.519 |
| 1930 | 0.623 | 0.590 | 0.487 |
| 1931 | 0.592 | 0.560 | 0.475 |
| 1932 | 0.543 | 0.512 | 0.438 |
| 1933 | 0.545 | 0.511 | 0.439 |
| 1934 | 0.589 | 0.547 | 0.489 |
| 1935 | 0.619 | 0.575 | 0.511 |
| 1936 | 0.673 | 0.624 | 0.549 |
| 1937 | 0.688 | 0.641 | 0.560 |
| 1938 | 0.673 | 0.629 | 0.560 |
| 1939 | 0.713 | 0.667 | 0.591 |
| 1940 | 0.752 | 0.704 | 0.625 |
| 1941 | 0.802 | 0.754 | 0.669 |
| 1942 | 0.823 | 0.777 | 0.686 |
| 1943 | 0.873 | 0.823 | 0.722 |
| 1944 | 0.933 | 0.878 | 0.773 |
| 1945 | 0.948 | 0.892 | 0.796 |
| 1946 | 0.891 | 0.841 | 0.773 |
| 1947 | 0.863 | 0.819 | 0.765 |
| 1948 | 0.882 | 0.847 | 0.798 |
| 1949 | 0.877 | 0.847 | 0.802 |
| 1950 | 0.936 | 0.909 | 0.867 |
| 1951 | 0.947 | 0.924 | 0.887 |
| 1952 | 0.943 | 0.924 | 0.890 |
| 1953 | 0.969 | 0.952 | 0.924 |
| 1954 | 0.966 | 0.953 | 0.930 |
| 1955 | 1.014 | 1.003 | 0.980 |
| 1956 | 1.005 | 0.998 | 0.980 |
| 1957 | 1.004 | 1.001 | 0.991 |
| 1958 | 1.000 | 1.000 | 1.000 |
| 1959 | 1.036 | 1.036 | 1.040 |

(continued)

TABLE 15 (concluded)

| Year | Labor Services and Capital Services | Labor Services and Capital Stock | Unweighted Man-hours and Capital Stock |
|---|---|---|---|
| 1960 | 1.038 | 1.042 | 1.050 |
| 1961 | 1.050 | 1.057 | 1.073 |
| 1962 | 1.091 | 1.100 | 1.120 |
| 1963 | 1.112 | 1.125 | 1.151 |
| 1964 | 1.142 | 1.160 | 1.191 |
| 1965 | 1.169 | 1.195 | 1.230 |
| 1966 | 1.192 | 1.227 | 1.270 |
| 1967 | 1.180 | 1.224 | 1.274 |
| 1968 | 1.196 | 1.247 | 1.305 |
| 1969 | 1.195 | 1.254 | 1.319 |

Solow uses a stock concept of capital input, omitting changes in the quantity of capital due to changes in the composition of capital input [54]. Denison distinguishes among residential real estate, farm capital, and all other capital input [14]. Since this breakdown of capital input does not coincide with sectors distinguished by a legal form of organization, Denison's measure fails to take account of differences in rates of return due to differences in the tax structure. Denison omits revaluation of assets in estimating rates of return and fails to account for the quantity of capital and depreciation in an internally consistent way. Kendrick adjusts capital input for changes in the industrial composition of capital stock [42]. This breakdown of capital input also fails to capture differences in rates of return due to the tax structure.

Solow employs unweighted man-hours as a measure of labor input, omitting the effects of changes in the composition of the labor force on the quantity of labor input. Denison weights persons engaged by an index of labor quality that incorporates the effects of growth in educational attainment, but differs in a number of details from the index we have used. Kendrick adjusts labor input for changes in the industrial composition of man-hours worked. For comparison with our index of total factor productivity we present indexes based on man-hours and capital stock and based on our index of labor input and capital stock. The first of these indexes provides an approximation to the conventions for measuring total factor productivity used by Solow. The second provides an ap-

TABLE 16

Relative Importance of Productivity Change, 1929–69
(average annual rates of growth)

| | 1929–49 | 1949–69 | 1929–69 |
|---|---|---|---|
| Gross private domestic product | | | |
| Real product | 2.28 | 4.34 | 3.31 |
| Real factor input | 0.97 | 2.79 | 1.88 |
| Total factor productivity | 1.31 | 1.55 | 1.43 |
| Relative proportion of productivity change | 0.57 | 0.36 | 0.43 |

proximation to the conventions employed by Denison. It is obvious from a comparison of the alternative estimates of total factor productivity given in Table 15 that the results are very sensitive to the choice of methods for measuring real factor input.

Finally, to evaluate the relative importance of growth in real factor input and growth in total factor productivity as sources of economic growth, we present the relative proportion of growth in real factor input. Geometric average annual rates of growth are given for real product and real factor input for 1929–49 and 1949–69 in Table 16. The relative proportion of growth in total factor productivity in the growth of real product is also given.

## 6. INCOME AND EXPENDITURE, ACCUMULATION, AND WEALTH ACCOUNTS

### *6.1. Introduction*

In Section 5 we presented the production account for the U.S. private domestic economy in constant prices. We gave data in constant prices for both product and factor input sides of the production account. In this section we present income and expenditure, accumulation, and wealth accounts for the U.S. private national economy in constant prices. In constructing these accounts in constant prices we must separate changes in income, consumer outlays, and capital formation into price and quantity components.

The fundamental accounting identity for the income and expenditure account is that consumer receipts are equal to consumer outlays plus saving. The corresponding identity for the accumulation account is that saving is equal to capital formation. The income and expenditure account is linked directly to the production account through factor income and consumer outlays. The income and expenditure and production accounts are linked indirectly through the accumulation account. The accumulation account is linked to the production account through capital formation. Capital formation includes expenditures on investment goods. Through the accumulation account, production and income and expenditure are linked to wealth. The change in wealth from period to period is equal to capital formation less depreciation plus revaluation of assets.

The accumulation account is also linked to production through net capital formation, defined as capital formation less replacement. If the decline in efficiency of capital goods is geometric, replacement is equal to depreciation and net capital formation is equal to the change in wealth from period to period less the revaluation of assets. If decline in efficiency is not geometric, a perpetual inventory of prices and quantities of capital goods is required. Net capital formation is linked to changes in capital input, while net saving is linked to changes in wealth.

Consumption expenditures in the income and expenditure account include sales and excise taxes and customs duties on consumption goods. Taxes are excluded from the value of consumption goods output in the production account. Factor outlay in the production account includes both direct taxes on factor income and indirect taxes that form a part of outlay on factors of production. In the income and expenditure account factor incomes exclude both direct and indirect taxes. Similarly, capital formation in the accumulation account includes sales and excise taxes and customs duties on investment goods. Taxes are excluded from the value of investment goods output in the production account.

### *6.2. Labor Income and Consumer Outlays*

We begin by presenting estimates of labor income and consumer outlays in constant prices for the U.S. private national economy. To construct price and quantity indexes of consumer outlays, we obtain data for consumption expenditures on nondurable goods and services, excluding the services of institutional real estate, in constant prices from the U.S. national accounts. We combine these data with imputed values of the services of consumer and institutional durables, and the services of institutional real estate in constant prices. Prices of services and nondurable goods are implicit in the data on personal consumption expenditures in current prices from the U.S. national accounts. Price indexes for the services of consumer and institutional durables, and institutional real estate are the capital service prices described in Section 5 above.

TABLE 17

Private National Consumption Expenditures, Consumer Outlays, and National Labor Compensation, 1929–69 (constant prices of 1958)

| Year | Consumption Expenditures and Consumer Outlays (price index) | Consumption Expenditures (quantity index) | Consumer Outlays (quantity index) | Private National Labor Compensation | | |
|---|---|---|---|---|---|---|
| | | | | Price Index | Quantity Index | Effective Tax Rate |
| 1929 | 0.546 | 141.0 | 142.9 | 0.278 | 235.8 | .001 |
| 1930 | 0.527 | 135.5 | 137.5 | 0.260 | 234.0 | .001 |
| 1931 | 0.481 | 132.0 | 133.9 | 0.226 | 233.8 | .001 |
| 1932 | 0.411 | 121.5 | 123.4 | 0.185 | 229.0 | .001 |
| 1933 | 0.414 | 119.1 | 120.8 | 0.171 | 233.1 | .004 |
| 1934 | 0.437 | 123.3 | 124.8 | 0.203 | 224.0 | .004 |
| 1935 | 0.447 | 126.3 | 127.7 | 0.212 | 231.6 | .005 |
| 1936 | 0.458 | 137.2 | 138.7 | 0.227 | 243.5 | .005 |
| 1937 | 0.469 | 141.9 | 143.4 | 0.248 | 246.3 | .007 |
| 1938 | 0.457 | 143.0 | 144.4 | 0.237 | 242.7 | .009 |
| 1939 | 0.457 | 147.9 | 149.3 | 0.246 | 248.1 | .008 |
| 1940 | 0.460 | 153.6 | 155.1 | 0.261 | 250.5 | .009 |
| 1941 | 0.487 | 161.5 | 162.9 | 0.304 | 260.8 | .011 |
| 1942 | 0.526 | 165.6 | 166.9 | 0.367 | 274.6 | .024 |
| 1943 | 0.602 | 171.0 | 172.4 | 0.392 | 301.7 | .086 |
| 1944 | 0.630 | 177.1 | 178.6 | 0.424 | 306.8 | .086 |
| 1945 | 0.660 | 186.7 | 188.4 | 0.456 | 289.3 | .090 |
| 1946 | 0.730 | 195.5 | 197.3 | 0.505 | 259.2 | .081 |
| 1947 | 0.804 | 197.2 | 198.9 | 0.553 | 257.1 | .092 |
| 1948 | 0.840 | 203.7 | 205.5 | 0.606 | 258.9 | .081 |
| 1949 | 0.799 | 208.6 | 210.5 | 0.620 | 256.9 | .068 |
| 1950 | 0.845 | 218.7 | 220.5 | 0.670 | 260.0 | .065 |
| 1951 | 0.881 | 227.7 | 229.4 | 0.723 | 269.1 | .092 |
| 1952 | 0.909 | 236.9 | 238.7 | 0.759 | 272.7 | .103 |
| 1953 | 0.926 | 245.7 | 247.7 | 0.811 | 272.7 | .101 |

(continued)

TABLE 17 (concluded)

| Year | Consumption Expenditures and Consumer Outlays (price index) | Consumption Expenditures (quantity index) | Consumer Outlays (quantity index) | Private National Labor Compensation | | |
|---|---|---|---|---|---|---|
| | | | | Price Index | Quantity Index | Effective Tax Rate |
| 1954 | 0.932 | 252.3 | 254.5 | 0.831 | 269.0 | .089 |
| 1955 | 0.953 | 265.4 | 267.5 | 0.874 | 273.7 | .090 |
| 1956 | 0.957 | 277.8 | 280.2 | 0.928 | 276.7 | .094 |
| 1957 | 0.974 | 286.0 | 288.7 | 0.981 | 274.3 | .095 |
| 1958 | 1.000 | 293.6 | 296.5 | 1.000 | 271.5 | .094 |
| 1959 | 1.031 | 306.0 | 309.0 | 1.059 | 275.9 | .095 |
| 1960 | 1.052 | 315.4 | 318.4 | 1.093 | 279.7 | .099 |
| 1961 | 1.057 | 324.3 | 327.6 | 1.123 | 280.2 | .098 |
| 1962 | 1.071 | 336.4 | 340.0 | 1.180 | 283.0 | .101 |
| 1963 | 1.090 | 349.2 | 353.1 | 1.224 | 286.0 | .102 |
| 1964 | 1.106 | 367.5 | 371.8 | 1.306 | 290.4 | .090 |
| 1965 | 1.121 | 386.9 | 391.6 | 1.360 | 297.7 | .094 |
| 1966 | 1.157 | 407.0 | 412.0 | 1.441 | 306.8 | .099 |
| 1967 | 1.178 | 423.7 | 429.3 | 1.496 | 314.4 | .102 |
| 1968 | 1.214 | 441.9 | 448.0 | 1.591 | 321.2 | .111 |
| 1969 | 1.271 | 459.1 | 465.7 | 1.678 | 328.8 | .123 |

The value of consumption expenditures includes customs duties, excise and sales taxes, and excludes subsidies. In Section 5 we have outlined the method for allocating excise and sales taxes between investment and consumption goods output. We construct a quantity index of consumption expenditures as a Divisia index of the quantity indexes of nondurables, services and our estimate of imputed capital services. The price index is then computed as the ratio of consumption expenditures to the quantity index. We deflate consumer outlays by the price index of consumption expenditures. We present price and quantity indexes for consumption expenditures and consumer outlays in Table 17.

Labor services offered are not identified with hours actually worked. Unemployment is a measure of the number of persons willing to offer labor at the current wage rate who do not have a demand for their labor.

We include a "normal workday" for the unemployed in working time. All nonworking time is considered to be leisure. A case could be made for including even more in working time offered on the grounds that there is an interaction between labor force participation and unemployment rates. As unemployment is reduced, people previously discouraged from entering the labor force by high unemployment are induced to enter. We include in working time offered only the time of the unemployed, assuming that the average workweek is the same as for the employed.

Our data for man-hours are from Kendrick.[53] Kendrick provides total man-hours for the farm sector, the general government sector, and the total private domestic sector. Hours for proprietors and unpaid family workers are included in his estimates. We provide our own hours estimate only for the rest of world sector. We assume that hours per man employed are equal to hours per man for the private domestic nonfarm economy. We adjust the total time endowment and the quantity of working time offered for quality change as measured by educational attainment. Both work and leisure are composed of quantities of labor services of varying qualities. Quantities of the different categories of labor services offered are combined into a Divisia quantity index of labor offered. In principle, a quantity index of labor supply could be built up from man-hours worked, classified by sex, race, years of schooling, occupation, age, and so on. Wage rates net of tax could be estimated for each class of worker. Our adjustment of the quantity of man-hours for changes in the educational composition of the labor force fails to take into account differences in taxes paid by workers at different levels of income.

Our concept of labor income is net of personal income taxes. The effective tax rate on labor income is computed as the ratio of taxes on labor income to labor income including taxes. Price, quantity, and tax indexes for labor income are presented in Table 17.

### *6.3. Property Income*

The starting point for estimating price and quantity components of property income is a set of perpetual inventory estimates of stocks of each type of capital employed in measuring capital input in constant prices in the production account. We assume that the flow of capital services from each type of tangible asset is proportional to the stock. Real property compensation for each asset is equal to the real service

[53] See footnote 41, above.

flow. Similarly real property compensation from the government and rest of world sectors is proportional to the quantity of net claims on governments and foreigners.

Prices of capital input from the point of view of the producer include both direct and indirect taxes. To obtain prices for capital input from the point of view of the owner of the asset we exclude all taxes. Excluding both direct and indirect taxes, the price of capital services becomes:

$$q_{K,t} = q_{A,t-1}r_t + q_{A,t}\delta - (q_{A,t} - q_{A,t-1}),$$

where $r_t$ is the after-tax rate of return. The depreciation rate $\delta$ is different from zero only for structures, equipment, and consumer durables employed in the private domestic sector. For inventories, land, and financial claims on the government and rest of world sectors the capital service price reduces to the cost of capital $q_{A,t-1}r_t$ less revaluation of assets $q_{A,t} - q_{A,t-1}$. For a financial asset the value of capital services is equal to earnings on the asset, for example, interest payments on a bond.

To construct price and quantity indexes of property compensation for the income and expenditure account our procedure is analogous to the methods we have used for the production account, except for the treatment of taxes. Property compensation before taxes includes the property share of gross private domestic factor outlay, corporate profits and net interest originating in the foreign sector, net interest paid by government, and investment income of social insurance funds net of transfers to general government. We have described effective rates of business property taxation and corporate income taxation in our presentation of the production account. We compute an effective rate of personal income taxation on property compensation net of business property taxes and the corporate income tax, and an effective rate of estate, death, and gift taxation on wealth.

We allocate federal estate and gift taxes and state and local death and gift taxes proportionally to all the components of private national wealth. Property income from assets in the household sector is not subject to personal income taxation; thus we must allocate personal income taxes attributed to property compensation among the corporate, noncorporate, government, and foreign sectors. A detailed allocation of personal income taxes to the various types of property compensation would be desirable; we simply allocate the taxes proportionately to all nonhousehold property compensation after corporate and property taxes but before personal taxes. The effective rate of personal income taxation on property compensation is estimated as the ratio of personal income taxes to property compensation before personal taxes other than household and institutional property compensation.

The after-tax return to capital in each sector includes property compensation, net of all taxes; it also includes capital gains and excludes economic depreciation. Our estimates of capital gains and economic depreciation for corporate and noncorporate tangible assets are discussed in detail in Christensen and Jorgenson [7]. Depreciation is zero for the financial assets which constitute net claims on governments and foreigners. Capital gains on net claims on foreigners are computed as the yearly increase in net claims less net private foreign investment. Capital gains on net claims on governments are computed as the yearly increase in net claims on governments less the current government deficit. These items are discussed in greater detail below.

The after-tax rate of return in each sector is computed by dividing the after-tax return to capital by the value of assets. These rates of return are nominal or money rates. We can also compute the real or own rates of return by excluding capital gains from the return to capital. Nominal and own rates of return for each sector and for the private national economy are presented in Table 18, together with effective tax rates on property compensation. We can now estimate the price of capital services for each asset from the formula above as a function of the rate of return, the depreciation rate, and the current and lagged acquisition price. Real property income for each sector and the private national economy is obtained as a Divisia quantity index of real property income from each asset. The price indexes for property income are computed as the ratios of property income to the quantity indexes. The price and quantity indexes of property income are presented in Table 19.

### *6.4. Accumulation Account*

The fundamental accounting identity for the accumulation account is that gross private national saving, taken from the income and expenditure account, is equal to gross private national capital formation. Gross private national saving may be expressed as the sum of depreciation and net private national saving. Net private national saving is equal to the change in wealth from period to period less revaluation of assets. Gross private national capital formation can be expressed as the sum of replacement and net private national capital formation. We present data in con-

TABLE 18

Gross Private National Property Compensation, Rates of Return, and Effective Rates of Taxation, 1929–69

| Year | Corporate Sector | Non-corporate Sector | House-holds and Institutions | Net Claims on Gov-ernments and Rest of World | Private National Economy |
|---|---|---|---|---|---|
| | | a. Nominal Rates of Return | | | |
| 1929 | .076 | .056 | .029 | .078 | .053 |
| 1930 | –.008 | –.067 | –.031 | .042 | –.028 |
| 1931 | –.065 | –.117 | –.092 | –.017 | –.084 |
| 1932 | –.091 | –.141 | –.151 | .036 | –.113 |
| 1933 | –.005 | .010 | .017 | .043 | .012 |
| 1934 | .082 | .062 | .090 | .108 | .083 |
| 1935 | .062 | .049 | .002 | .032 | .034 |
| 1936 | .078 | .071 | .059 | .009 | .060 |
| 1937 | .131 | .073 | .078 | .034 | .084 |
| 1938 | .029 | –.005 | .032 | –.001 | .017 |
| 1939 | .052 | .027 | .019 | .007 | .027 |
| 1940 | .096 | .070 | .041 | .002 | .056 |
| 1941 | .154 | .159 | .092 | .088 | .123 |
| 1942 | .181 | .187 | .079 | .109 | .136 |
| 1943 | .129 | .156 | .096 | .010 | .098 |
| 1944 | .124 | .172 | .103 | –.025 | .087 |
| 1945 | .077 | .170 | .080 | –.017 | .066 |
| 1946 | .158 | .265 | .125 | .027 | .123 |
| 1947 | .243 | .253 | .186 | .008 | .154 |
| 1948 | .140 | .141 | .114 | .018 | .099 |
| 1949 | .055 | .036 | –.021 | .024 | .020 |
| 1950 | .096 | .152 | .074 | .042 | .087 |
| 1951 | .136 | .161 | .085 | .028 | .100 |
| 1952 | .062 | .065 | .040 | .024 | .047 |
| 1953 | .048 | .048 | .027 | .034 | .038 |
| 1954 | .048 | .067 | .017 | .030 | .037 |

(continued)

TABLE 18 (continued)

| Year | Corporate Sector | Non-corporate Sector | House-holds and Institutions | Net Claims on Gov-ernments and Rest of World | Private National Economy |
|---|---|---|---|---|---|
| 1955 | .076 | .073 | .055 | .029 | .058 |
| 1956 | .103 | .092 | .071 | .034 | .076 |
| 1957 | .086 | .096 | .054 | .032 | .066 |
| 1958 | .049 | .097 | .037 | .037 | .051 |
| 1959 | .065 | .064 | .064 | .041 | .060 |
| 1960 | .050 | .069 | .051 | .042 | .053 |
| 1961 | .050 | .076 | .046 | .031 | .050 |
| 1962 | .068 | .082 | .056 | .040 | .061 |
| 1963 | .067 | .073 | .061 | .034 | .060 |
| 1964 | .080 | .075 | .069 | .030 | .067 |
| 1965 | .094 | .092 | .060 | .039 | .072 |
| 1966 | .104 | .100 | .069 | .035 | .079 |
| 1967 | .095 | .094 | .085 | .023 | .080 |
| 1968 | .089 | .092 | .088 | .034 | .081 |
| 1969 | .087 | .087 | .093 | .041 | .082 |
| | | b. Own Rates of Return | | | |
| 1929 | .074 | .058 | .012 | .052 | .044 |
| 1930 | .050 | .016 | .008 | .047 | .025 |
| 1931 | .022 | .010 | .014 | .044 | .018 |
| 1932 | −.002 | −.019 | −.001 | .035 | −.002 |
| 1933 | −.004 | −.012 | .019 | .033 | .007 |
| 1934 | .026 | −.002 | .015 | .036 | .017 |
| 1935 | .042 | .017 | .009 | .031 | .023 |
| 1936 | .060 | .031 | .020 | .027 | .034 |
| 1937 | .063 | .027 | .013 | .026 | .031 |
| 1938 | .040 | .020 | .011 | .026 | .023 |
| 1939 | .056 | .033 | .015 | .025 | .032 |
| 1940 | .075 | .042 | .014 | .027 | .038 |
| 1941 | .076 | .069 | .008 | .025 | .043 |
| 1942 | .074 | .089 | −.020 | .024 | .037 |
| 1943 | .067 | .085 | .011 | .018 | .042 |
| 1944 | .075 | .115 | .008 | .018 | .048 |

(continued)

TABLE 18 (continued)

| Year | Corporate Sector | Non-corporate Sector | House-holds and Institutions | Net Claims on Gov-ernments and Rest of World | Private National Economy |
|---|---|---|---|---|---|
| 1945 | .057 | .119 | .016 | .016 | .045 |
| 1946 | .046 | .115 | .037 | .018 | .046 |
| 1947 | .057 | .086 | .034 | .019 | .044 |
| 1948 | .070 | .079 | .025 | .021 | .045 |
| 1949 | .060 | .061 | −.003 | .023 | .032 |
| 1950 | .054 | .068 | .025 | .022 | .040 |
| 1951 | .049 | .081 | .010 | .024 | .037 |
| 1952 | .042 | .062 | .017 | .024 | .034 |
| 1953 | .037 | .054 | .024 | .023 | .033 |
| 1954 | .040 | .051 | .024 | .025 | .033 |
| 1955 | .056 | .051 | .036 | .024 | .042 |
| 1956 | .045 | .036 | .026 | .025 | .033 |
| 1957 | .040 | .039 | .019 | .027 | .030 |
| 1958 | .034 | .048 | .022 | .026 | .031 |
| 1959 | .043 | .035 | .033 | .027 | .035 |
| 1960 | .039 | .029 | .034 | .028 | .033 |
| 1961 | .039 | .033 | .032 | .028 | .033 |
| 1962 | .050 | .038 | .035 | .030 | .039 |
| 1963 | .051 | .036 | .039 | .031 | .040 |
| 1964 | .059 | .035 | .041 | .035 | .044 |
| 1965 | .068 | .039 | .040 | .035 | .047 |
| 1966 | .070 | .046 | .045 | .036 | .050 |
| 1967 | .058 | .044 | .043 | .037 | .047 |
| 1968 | .052 | .036 | .039 | .038 | .042 |
| 1969 | .042 | .027 | .042 | .036 | .038 |

(continued)

TABLE 18 (continued)

| Year | Effective Corporate Income Tax Rate | Effective Personal Income Tax Rate on Property Compensation | Effective Rate of Wealth Taxation |
|---|---|---|---|
| | c. Effective Tax Rates | | |
| 1929 | .108 | .070 | .000 |
| 1930 | .083 | .106 | .000 |
| 1931 | .074 | .098 | .000 |
| 1932 | .113 | [a] | .000 |
| 1933 | .207 | [a] | .000 |
| 1934 | .136 | .092 | .000 |
| 1935 | .139 | .065 | .001 |
| 1936 | .172 | .063 | .001 |
| 1937 | .156 | .101 | .001 |
| 1938 | .133 | .112 | .001 |
| 1939 | .167 | .054 | .001 |
| 1940 | .243 | .051 | .001 |
| 1941 | .440 | .061 | .001 |
| 1942 | .492 | .089 | .001 |
| 1943 | .531 | .208 | .001 |
| 1944 | .495 | .157 | .001 |
| 1945 | .492 | .183 | .001 |
| 1946 | .470 | .170 | .001 |
| 1947 | .443 | .150 | .001 |
| 1948 | .391 | .123 | .001 |
| 1949 | .331 | .123 | .001 |
| 1950 | .486 | .163 | .000 |
| 1951 | .520 | .155 | .000 |
| 1952 | .463 | .182 | .000 |
| 1953 | .477 | .195 | .001 |
| 1954 | .481 | .187 | .001 |
| 1955 | .481 | .173 | .001 |
| 1956 | .476 | .212 | .001 |
| 1957 | .469 | .212 | .001 |
| 1958 | .472 | .198 | .001 |
| 1959 | .497 | .206 | .001 |

(continued)

TABLE 18 (concluded)

| Year | Effective Corporate Income Tax Rate | Effective Personal Income Tax Rate on Property Compensation | Effective Rate of Wealth Taxation |
|---|---|---|---|
| 1960 | .495 | .230 | .001 |
| 1961 | .488 | .222 | .001 |
| 1962 | .467 | .200 | .001 |
| 1963 | .477 | .201 | .001 |
| 1964 | .479 | .173 | .001 |
| 1965 | .470 | .165 | .001 |
| 1966 | .463 | .169 | .001 |
| 1967 | .449 | .186 | .001 |
| 1968 | .504 | .222 | .001 |
| 1969 | .511 | .282 | .001 |

[a] Income base is zero or negligible.

stant prices for saving and capital formation, both gross and net, and for depreciation, replacement, and revaluation. Gross private national capital formation is equal to gross private domestic investment, as defined in the U.S. national accounts, plus personal consumption expenditures on durable goods, plus the current deficits of the federal and state and local social insurance funds, plus the current surpluses of federal and state and local social insurance funds, plus net foreign investment.

We divide the components of gross private national capital formation into prices and quantities using the following deflators: The implicit deflators from the U.S. National Income and Product Accounts are used for investment in producer and consumer durables, and for farm and nonfarm inventories. For residential and nonresidential structures we use the "constant cost 2" price index for structures from the Bureau of Economic Analysis (formerly the Office of Business Economics) *Capital Stock Study* for both capital formation and replacement.[54] We have constructed price indexes for claims on the government and rest of world sectors from data on changes in the value of claims from period to period and data on the corresponding components of capital formation from the U.S. national accounts. We set the price of claims of each type equal to 1.000 in 1958 and the quantity in 1958 equal to the value of

[54] See footnote 45, above.

TABLE 19

Gross Private National Property Compensation, 1929–69
(constant prices of 1958)

| Year | Corporate Property Compensation | | Noncorporate Property Compensation | | Households and Institutions, Property Compensation | | Government and Rest of World, Property Compensation | | Private National Property Compensation | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Price Index | Quantity Index | Price Index | Quantity Index | Price Index | Quantity Index | Price Index | Quantity Index | Price Index | Quantity Index |
| 1929 | .056 | 257.9 | .039 | 200.9 | .048 | 273.9 | .048 | 35.1 | .045 | 834.0 |
| 1930 | .044 | 264.0 | .017 | 212.2 | .045 | 278.2 | .044 | 35.0 | .034 | 854.7 |
| 1931 | .029 | 263.4 | .013 | 216.0 | .044 | 272.8 | .041 | 36.3 | .028 | 849.8 |
| 1932 | .018 | 255.5 | .000 | 214.9 | .031 | 263.0 | .031 | 40.0 | .017 | 830.7 |
| 1933 | .016 | 237.2 | .003 | 208.6 | .039 | 246.3 | .029 | 42.3 | .020 | 784.7 |
| 1934 | .029 | 221.5 | .007 | 202.6 | .039 | 232.5 | .032 | 44.2 | .025 | 745.3 |
| 1935 | .036 | 213.8 | .015 | 197.6 | .036 | 224.4 | .030 | 47.3 | .028 | 725.8 |
| 1936 | .043 | 209.5 | .020 | 197.5 | .042 | 223.6 | .025 | 49.5 | .033 | 721.3 |
| 1937 | .046 | 211.6 | .020 | 201.8 | .041 | 230.6 | .025 | 53.2 | .033 | 738.2 |
| 1938 | .038 | 216.9 | .017 | 211.4 | .041 | 237.5 | .024 | 54.6 | .030 | 760.9 |
| 1939 | .045 | 212.8 | .022 | 211.3 | .043 | 235.5 | .023 | 59.0 | .034 | 756.2 |
| 1940 | .053 | 212.8 | .026 | 213.4 | .043 | 240.5 | .024 | 63.8 | .038 | 766.8 |
| 1941 | .056 | 219.2 | .038 | 216.7 | .043 | 250.2 | .022 | 67.8 | .041 | 791.3 |
| 1942 | .060 | 230.9 | .051 | 221.9 | .034 | 263.4 | .023 | 75.1 | .043 | 828.7 |
| 1943 | .061 | 229.7 | .053 | 219.5 | .055 | 255.3 | .018 | 108.4 | .050 | 831.1 |
| 1944 | .069 | 225.0 | .073 | 215.3 | .059 | 243.9 | .018 | 153.7 | .059 | 823.6 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1945 | .061 | 222.2 | .079 | 214.2 | .069 | 232.4 | .016 | 209.8 | .061 | 822.7 |
| 1946 | .057 | 227.0 | .081 | 214.8 | .088 | 225.4 | .017 | 255.8 | .066 | 831.1 |
| 1947 | .073 | 245.2 | .073 | 220.6 | .093 | 249.7 | .018 | 259.0 | .070 | 888.5 |
| 1948 | .092 | 266.1 | .079 | 227.3 | .092 | 279.5 | .020 | 257.2 | .076 | 956.5 |
| 1949 | .089 | 282.8 | .069 | 238.6 | .067 | 307.9 | .021 | 253.7 | .066 | 1,019.7 |
| 1950 | .085 | 290.6 | .074 | 248.0 | .094 | 337.0 | .020 | 260.5 | .074 | 1,074.0 |
| 1951 | .086 | 306.0 | .090 | 261.4 | .086 | 379.7 | .022 | 250.9 | .077 | 1,154.3 |
| 1952 | .084 | 326.0 | .079 | 270.6 | .094 | 404.9 | .023 | 249.2 | .076 | 1,216.0 |
| 1953 | .082 | 339.8 | .072 | 276.1 | .100 | 423.7 | .022 | 257.3 | .076 | 1,262.9 |
| 1954 | .085 | 353.3 | .070 | 281.4 | .099 | 450.2 | .024 | 265.9 | .076 | 1,320.0 |
| 1955 | .100 | 363.1 | .071 | 286.2 | .110 | 474.3 | .024 | 275.5 | .084 | 1,369.8 |
| 1956 | .095 | 380.4 | .061 | 293.6 | .104 | 513.2 | .024 | 275.5 | .079 | 1,447.5 |
| 1957 | .095 | 401.0 | .066 | 297.9 | .101 | 539.5 | .026 | 276.0 | .079 | 1,510.0 |
| 1958 | .091 | 418.1 | .078 | 302.0 | .105 | 562.0 | .026 | 282.3 | .082 | 1,564.2 |
| 1959 | .102 | 424.7 | .068 | 305.0 | .118 | 574.0 | .027 | 294.7 | .088 | 1,594.1 |
| 1960 | .098 | 438.1 | .062 | 310.3 | .120 | 599.3 | .028 | 296.6 | .087 | 1,548.7 |
| 1961 | .098 | 454.3 | .067 | 315.7 | .117 | 622.3 | .029 | 298.4 | .086 | 1,703.1 |
| 1962 | .111 | 465.2 | .074 | 319.9 | .121 | 639.4 | .031 | 307.0 | .093 | 1,744.9 |
| 1963 | .113 | 482.2 | .073 | 328.1 | .126 | 666.5 | .032 | 315.3 | .095 | 1,809.8 |
| 1964 | .122 | 500.6 | .073 | 337.4 | .130 | 699.5 | .036 | 321.4 | .100 | 1,884.3 |
| 1965 | .133 | 525.0 | .080 | 347.8 | .129 | 737.9 | .037 | 333.8 | .104 | 1,975.9 |
| 1966 | .138 | 556.8 | .090 | 361.5 | .134 | 785.4 | .037 | 340.9 | .109 | 2,088.6 |
| 1967 | .129 | 599.7 | .092 | 374.3 | .134 | 832.8 | .039 | 353.0 | .107 | 2,215.5 |
| 1968 | .125 | 634.3 | .084 | 386.3 | .135 | 872.6 | .039 | 378.6 | .105 | 2,326.9 |
| 1969 | .119 | 667.8 | .076 | 400.9 | .143 | 923.3 | .037 | 395.4 | .104 | 2,450.2 |

outstanding claims in that year. These price indexes are then used to deflate the government deficit and net foreign investment.

To construct an index of the quantity of gross private national capital formation we first construct a Divisia index of the quantities of investment in producer and consumer durables, residential and nonresidential structures, and the quantity indexes of net foreign investment and government deficits. Real investment in inventories of durable and nondurable goods is added to the Divisia index to obtain the quantity index of gross private national capital formation. The price index of replacement is computed as the ratio of the value of replacement to the Divisia index of replacement. A quantity index of net private national capital formation is computed as the quantity index of gross private national capital formation less the quantity index of replacement. The price of net private national capital formation is computed as the ratio of the value in current prices to the quantity index. The price and quantity indexes of gross private national capital formation, replacement, and net private national capital formation are presented in Table 20.

Net private national capital formation in constant prices is equal to the change in the quantity of capital for each type of capital utilized in the U.S. private domestic economy. Capital input and net capital formation in a given period are combined in the perpetual inventory formula to obtain capital input from each capital good in the following period. Changes in the value of capital input can be decomposed into price and quantity components. The quantity component must be carefully distinguished from the quantity of net capital formation. The quantity of capital input is weighted by capital service prices, while the quantities of gross and net capital formation are weighted by capital asset prices.

The value of gross private national saving is taken from the income and expenditure account. To construct the saving side of the accumulation account in constant prices we begin with gross private national capital formation in constant prices. The capital formation and saving sides of the accumulation account are equal in both current and constant prices. To complete the saving side of the accumulation account in constant prices we must construct accounts for depreciation and revaluation of assets in constant prices. We outline methods for constructing these accounts from a perpetual inventory of prices and quantities of capital goods; we then specialize to the case of geometric decline in efficiency of capital goods.

For a single capital good the value of wealth is the sum of values of investment goods of each vintage, summed over all vintages:

$$W_t = \sum_{v=0}^{\infty} q_{A,t,v} \, A_{t-v}.$$

The change in wealth from period to period may be written:

$$\begin{aligned} W_t - W_{t-1} &= \sum_{v=0}^{\infty} q_{A,t,v} \, A_{t-v} - \sum_{v=0}^{\infty} q_{A,t-1,v} \, A_{t-v-1} \\ &= q_{A,t,0} \, A_t + \sum_{v=0}^{\infty} q_{A,t,v-1} \, A_{t-v-1} - \sum_{v=0}^{\infty} q_{A,t-1,v} \, A_{t-v-1} \\ &= q_{A,t} \, A_t + \sum_{v=0}^{\infty} (q_{A,t,v-1} - q_{A,t,v}) A_{t-v-1} \\ &\quad + \sum_{v=0}^{\infty} (q_{A,t,v} - q_{A,t-1,v}) A_{t-v-1}. \end{aligned}$$

In this expression for change in the value of wealth, the first term is the value of gross capital formation, the second is the negative of depreciation on capital goods of all vintages, and the third is the revaluation of assets of all vintages.

We have already described the construction of price and quantity index numbers for gross capital formation. Treating the change in prices across vintages, $q_{A,t,v} - q_{A,t,v-1}$, as the price component of depreciation and $A_{t-v-1}$ as the quantity component, we may apply Divisia index number formulas to perpetual inventory data on prices and quantities of each vintage of a capital good to obtain price and quantity index numbers for depreciation on a single capital good. To obtain index numbers for several capital goods we again apply Divisia index number formulas, this time to the price and quantity indexes for each capital good. Similarly, treating the change in prices across time periods, $q_{A,t,v} - q_{A,t-1,v}$, as the price component of revaluation, we may obtain price and quantity index numbers of revaluation for any number of capital goods.

The value of gross saving is equal to change in wealth plus depreciation less revaluation of assets. We may define the quantity of gross saving as the sum of quantities of change in wealth and depreciation less the quantity of revaluation. The quantity of change in wealth itself is the sum of quantities of gross capital formation and revaluation less the

TABLE 20

Gross Private National Capital Formation, 1929–69
(constant prices of 1958)

| Year | Gross Private National Capital Formation | | Replacement | | Effective Sales Tax Rate of Investment Goods |
|---|---|---|---|---|---|
| | Price Index | Quantity Index | Price Index | Quantity Index | |
| 1929 | 0.474 | 53.6 | 0.463 | 41.3 | .017 |
| 1930 | 0.473 | 39.3 | 0.449 | 42.3 | .019 |
| 1931 | 0.471 | 30.5 | 0.411 | 41.8 | .021 |
| 1932 | 0.441 | 15.2 | 0.365 | 40.2 | .029 |
| 1933 | 0.423 | 15.5 | 0.352 | 37.7 | .042 |
| 1934 | 0.483 | 21.8 | 0.379 | 35.5 | .048 |
| 1935 | 0.429 | 31.8 | 0.379 | 34.1 | .047 |
| 1936 | 0.436 | 41.8 | 0.381 | 33.8 | .045 |
| 1937 | 0.434 | 46.1 | 0.408 | 34.6 | .044 |
| 1938 | 0.490 | 33.2 | 0.416 | 35.7 | .045 |
| 1939 | 0.467 | 43.4 | 0.410 | 35.3 | .044 |
| 1940 | 0.460 | 53.2 | 0.418 | 35.8 | .044 |
| 1941 | 0.510 | 67.5 | 0.453 | 37.2 | .044 |
| 1942 | 0.765 | 66.1 | 0.516 | 39.2 | .039 |
| 1943 | 0.848 | 68.5 | 0.551 | 38.0 | .037 |
| 1944 | 0.861 | 79.1 | 0.595 | 36.4 | .042 |
| 1945 | 0.822 | 75.0 | 0.617 | 35.2 | .048 |
| 1946 | 0.661 | 74.6 | 0.655 | 35.0 | .053 |
| 1947 | 0.728 | 72.5 | 0.741 | 38.3 | .049 |
| 1948 | 0.798 | 82.0 | 0.792 | 42.9 | .047 |
| 1949 | 0.809 | 82.4 | 0.799 | 47.4 | .050 |
| 1950 | 0.803 | 94.1 | 0.817 | 51.2 | .048 |
| 1951 | 0.885 | 98.8 | 0.880 | 56.3 | .046 |
| 1952 | 0.906 | 98.3 | 0.898 | 59.9 | .048 |
| 1953 | 0.903 | 104.2 | 0.901 | 62.7 | .048 |
| 1954 | 0.906 | 103.6 | 0.895 | 66.2 | .046 |

(continued)

TABLE 20 (concluded)

| Year | Gross Private National Capital Formation | | Replacement | | Effective Sales Tax Rate of Investment Goods |
|---|---|---|---|---|---|
| | Price Index | Quantity Index | Price Index | Quantity Index | |
| 1955 | 0.904 | 118.4 | 0.901 | 69.2 | .045 |
| 1956 | 0.949 | 115.3 | 0.945 | 73.9 | .046 |
| 1957 | 0.989 | 116.2 | 0.986 | 77.6 | .046 |
| 1958 | 1.000 | 111.2 | 1.000 | 80.8 | .045 |
| 1959 | 1.017 | 119.5 | 1.017 | 82.2 | .046 |
| 1960 | 1.020 | 119.5 | 1.018 | 85.2 | .048 |
| 1961 | 1.018 | 122.4 | 1.017 | 88.2 | .047 |
| 1962 | 1.027 | 137.5 | 1.023 | 90.3 | .047 |
| 1963 | 1.030 | 143.2 | 1.026 | 93.8 | .048 |
| 1964 | 1.040 | 159.8 | 1.035 | 98.1 | .047 |
| 1965 | 1.048 | 173.6 | 1.040 | 103.4 | .046 |
| 1966 | 1.059 | 193.4 | 1.051 | 110.2 | .043 |
| 1967 | 1.081 | 199.9 | 1.080 | 117.7 | .043 |
| 1968 | 1.117 | 204.0 | 1.115 | 124.2 | .046 |
| 1969 | 1.164 | 200.9 | 1.155 | 131.6 | .047 |

quantity of depreciation. The quantity of net saving is equal to the quantity of gross saving less the quantity of depreciation. Quantities of gross saving and gross capital formation are, of course, identical.

If the decline in efficiency of capital goods is geometric the change in wealth from period to period for a single capital good may be written:

$$\begin{aligned} W_t - W_{t-1} &= q_{A,t}K_t - q_{A,t-1}K_{t-1} \\ &= q_{A,t}(K_t - K_{t-1}) + (q_{A,t} - q_{A,t-1})K_{t-1} \\ &= q_{A,t}A_t - q_{A,t}\delta K_{t-1} + (q_{A,t} - q_{A,t-1})K_{t-1}. \end{aligned}$$

Gross saving is represented by $q_{A,t}A_t$, which is equal to gross capital formation and has the same price and quantity components. Depreciation is represented by $q_{A,t}\delta K_{t-1}$ and is equal to replacement; the price and quantity components of depreciation differ from the price and quantity components of replacement. We construct the quantity index of depreciation as a Divisia index of the various lagged stocks, $K_{t-1}$, with depreciation shares as weights. The quantity index of replacement is a Divisia index of the $\delta K_{t-1}$ with replacement shares as weights. The weights are, of course, the same for replacement and depreciation under geometric decline in efficiency; so the quantity indexes for depreciation and replacement are proportional. The price index of depreciation is computed as the ratio of depreciation to the quantity index of depreciation.

Revaluation is represented by $(q_{A,t} - q_{A,t-1})K_{t-1}$. We construct a quantity index of revaluation as a Divisia index of the various lagged capital stocks with revaluation shares as weights. The price index of revaluation is computed as the ratio of revaluation to the quantity index of revaluation. Price and quantity index numbers of private national saving, depreciation, and revaluation are presented in Table 21.

### *6.5. Standard of Living*

At this point we can consolidate the receipt and expenditure account with the accumulation account to obtain a consolidated receipt and expenditure account. In the consolidated account consumer receipts are equal to the sum of consumer outlays and gross capital formation. Price and quantity index numbers for factor income can be constructed by combining Divisia index numbers of labor and property income into a Divisia index of factor income. The weights for labor and property are the relative shares of labor and property compensation in the value of total factor income. We use the price index of factor income to deflate government transfer payments to persons, except for social insurance benefits. Adding deflated transfer payments to the quantity index of factor income provides an index of total real consumer receipts. The construction of an index of total real consumer receipts is analogous to the construction of an index of total factor input in the production account; the scope of transactions covered by the two indexes is different and consumer receipts are net of both direct and indirect taxes in the consolidated consumer receipts and expenditures account.

Price and quantity index numbers for total expenditures can be constructed by combining Divisia index numbers of consumer outlays and capital formation into a Divisia index of total expenditures. The weights for consumer outlays and capital formation are the relative shares of these components of expenditure in the value of total expenditure. The price and quantity indexes of expenditures are analogous to indexes for total product in the production account; the scope of transactions is different and expenditures include sales and excise taxes, while the value of total product excludes such taxes.

TABLE 21

Gross Private National Saving, Depreciation, and Revaluation, 1929–69 (constant prices of 1958)

| | Gross Private National Saving | | Depreciation | | Revaluation | |
|---|---|---|---|---|---|---|
| Year | Price Index | Quantity Index | Price Index | Quantity Index | Price Index | Quantity Index |
| 1929 | 0.474 | 53.6 | .046 | 418.3 | .003 | 1,200.3 |
| 1930 | 0.473 | 39.3 | .044 | 428.3 | –.018 | 1,214.8 |
| 1931 | 0.471 | 30.5 | .041 | 422.8 | –.033 | 1,212.2 |
| 1932 | 0.441 | 15.2 | .036 | 407.1 | –.032 | 1,200.5 |
| 1933 | 0.423 | 15.5 | .035 | 381.3 | .001 | 1,293.4 |
| 1934 | 0.483 | 21.8 | .037 | 359.2 | .015 | 1,361.8 |
| 1935 | 0.429 | 31.8 | .037 | 345.4 | .003 | 1,298.2 |
| 1936 | 0.436 | 41.8 | .038 | 342.1 | .008 | 1,301.3 |
| 1937 | 0.434 | 46.1 | .040 | 350.3 | .013 | 1,299.7 |
| 1938 | 0.490 | 33.2 | .041 | 361.3 | –.002 | 1,384.7 |
| 1939 | 0.467 | 43.4 | .041 | 357.6 | –.001 | 1,445.9 |
| 1940 | 0.460 | 53.2 | .041 | 362.6 | .004 | 1,491.5 |
| 1941 | 0.510 | 67.5 | .045 | 376.3 | .020 | 1,514.1 |
| 1942 | 0.765 | 66.1 | .051 | 396.8 | .026 | 1,581.3 |
| 1943 | 0.848 | 68.5 | .054 | 384.6 | .017 | 1,598.6 |
| 1944 | 0.861 | 79.1 | .059 | 368.4 | .015 | 1,470.6 |
| 1945 | 0.822 | 75.0 | .061 | 355.7 | .010 | 1,260.3 |
| 1946 | 0.661 | 74.6 | .065 | 354.4 | .044 | 1,188.0 |
| 1947 | 0.728 | 72.5 | .073 | 387.8 | .067 | 1,235.3 |
| 1948 | 0.798 | 82.0 | .078 | 433.9 | .036 | 1,287.3 |
| 1949 | 0.809 | 82.4 | .079 | 479.4 | –.008 | 1,317.4 |
| 1950 | 0.803 | 94.1 | .081 | 518.1 | .035 | 1,314.2 |
| 1951 | 0.885 | 98.8 | .087 | 570.0 | .048 | 1,367.2 |
| 1952 | 0.906 | 98.3 | .089 | 606.5 | .011 | 1,411.0 |
| 1953 | 0.903 | 104.2 | .089 | 634.2 | .004 | 1,456.7 |
| 1954 | 0.906 | 103.6 | .088 | 670.0 | .003 | 1,469.6 |

(continued)

TABLE 21 (concluded)

| | Gross Private National Saving | | Depreciation | | Revaluation | |
|---|---|---|---|---|---|---|
| Year | Price Index | Quantity Index | Price Index | Quantity Index | Price Index | Quantity Index |
| 1955 | 0.904 | 118.4 | .089 | 699.7 | .015 | 1,446.6 |
| 1956 | 0.949 | 115.3 | .093 | 747.6 | .039 | 1,489.3 |
| 1957 | 0.989 | 116.2 | .097 | 785.1 | .034 | 1,534.5 |
| 1958 | 1.000 | 111.2 | .099 | 817.7 | .020 | 1,564.2 |
| 1959 | 1.017 | 119.5 | .101 | 832.1 | .025 | 1,581.2 |
| 1960 | 1.020 | 119.5 | .101 | 862.1 | .020 | 1,599.1 |
| 1961 | 1.018 | 122.4 | .101 | 892.3 | .018 | 1,603.7 |
| 1962 | 1.027 | 137.5 | .101 | 913.9 | .025 | 1,613.3 |
| 1963 | 1.030 | 143.2 | .101 | 949.4 | .023 | 1,627.5 |
| 1964 | 1.040 | 159.8 | .102 | 992.2 | .028 | 1,641.8 |
| 1965 | 1.048 | 173.6 | .103 | 1,045.7 | .032 | 1,659.1 |
| 1966 | 1.059 | 193.4 | .104 | 1,114.7 | .038 | 1,684.7 |
| 1967 | 1.081 | 199.9 | .107 | 1,190.9 | .046 | 1,721.1 |
| 1968 | 1.117 | 204.0 | .110 | 1,256.4 | .056 | 1,751.6 |
| 1969 | 1.164 | 200.9 | .114 | 1,331.9 | .067 | 1,791.3 |

The standard of living may be defined as the ratio of real expenditures to real receipts or, equivalently, the ratio of the price of factor income to the price of expenditures. A Divisia index of the standard of living may be defined as the ratio of Divisia indexes of the quantity of expenditures to the quantity of consumer receipts or, equivalently, the ratio of Divisia indexes of the price of factor income to the price of consumer expenditures. Divisia price and quantity indexes of consumer receipts and total expenditures and the standard of living for the U.S. private national economy are given in Table 22 for 1929–69.

### *6.6. Wealth Account*

In Section 2 we described the asset side of the wealth account for the U.S. private national economy in current prices. Changes in the value of wealth from period to period may be separated into price and quantity components. The price component is equal to gross saving less depreciation or net saving. Capital formation is related to the change in capital input, but not to the change in capital assets, except where the decline in efficiency of capital goods is geometric. Under this assumption deprecia-

TABLE 22

Gross Private National Expenditures, Receipts, and Standard of Living, 1929–69 (constant prices of 1958)

| | Gross Private National Expenditures | | Gross Private National Consumer Receipts | | |
|---|---|---|---|---|---|
| Year | Price Index | Quantity Index | Price Index | Quantity Index | Standard of Living |
| 1929 | 0.531 | 194.8 | 0.346 | 298.8 | 0.652 |
| 1930 | 0.516 | 176.3 | 0.303 | 300.4 | 0.587 |
| 1931 | 0.479 | 164.3 | 0.259 | 304.3 | 0.540 |
| 1932 | 0.416 | 138.1 | 0.193 | 297.1 | 0.465 |
| 1933 | 0.416 | 135.8 | 0.191 | 296.7 | 0.458 |
| 1934 | 0.444 | 146.5 | 0.230 | 283.6 | 0.517 |
| 1935 | 0.443 | 159.6 | 0.245 | 288.8 | 0.553 |
| 1936 | 0.453 | 180.4 | 0.271 | 301.9 | 0.598 |
| 1937 | 0.461 | 189.2 | 0.289 | 301.7 | 0.627 |
| 1938 | 0.464 | 177.4 | 0.273 | 301.8 | 0.588 |
| 1939 | 0.459 | 192.9 | 0.290 | 305.6 | 0.631 |
| 1940 | 0.459 | 208.5 | 0.310 | 308.6 | 0.676 |
| 1941 | 0.493 | 230.9 | 0.356 | 319.7 | 0.722 |
| 1942 | 0.594 | 233.0 | 0.413 | 335.1 | 0.695 |
| 1943 | 0.672 | 241.0 | 0.451 | 358.8 | 0.672 |
| 1944 | 0.695 | 259.9 | 0.498 | 363.0 | 0.716 |
| 1945 | 0.704 | 263.9 | 0.529 | 351.1 | 0.752 |
| 1946 | 0.710 | 272.3 | 0.582 | 332.5 | 0.819 |
| 1947 | 0.782 | 272.0 | 0.632 | 336.6 | 0.808 |
| 1948 | 0.827 | 287.7 | 0.693 | 343.5 | 0.837 |
| 1949 | 0.801 | 293.1 | 0.674 | 348.4 | 0.841 |
| 1950 | 0.832 | 314.6 | 0.736 | 355.6 | 0.885 |
| 1951 | 0.882 | 328.3 | 0.783 | 369.9 | 0.887 |
| 1952 | 0.908 | 337.1 | 0.808 | 378.9 | 0.890 |
| 1953 | 0.919 | 352.0 | 0.846 | 382.6 | 0.920 |
| 1954 | 0.924 | 358.2 | 0.861 | 384.3 | 0.932 |

(continued)

TABLE 22 (concluded)

| | Gross Private National Expenditures | | Gross Private National Consumer Receipts | | |
|---|---|---|---|---|---|
| Year | Price Index | Quantity Index | Price Index | Quantity Index | Standard of Living |
| 1955 | 0.938 | 385.8 | 0.920 | 393.3 | 0.981 |
| 1956 | 0.955 | 395.6 | 0.937 | 402.9 | 0.982 |
| 1957 | 0.978 | 405.0 | 0.977 | 405.7 | 0.998 |
| 1958 | 1.000 | 407.7 | 1.000 | 407.7 | 1.000 |
| 1959 | 1.027 | 428.6 | 1.063 | 414.2 | 1.035 |
| 1960 | 1.043 | 438.0 | 1.081 | 422.5 | 1.037 |
| 1961 | 1.047 | 450.0 | 1.101 | 427.6 | 1.052 |
| 1962 | 1.059 | 477.3 | 1.166 | 433.5 | 1.101 |
| 1963 | 1.074 | 496.0 | 1.205 | 441.9 | 1.123 |
| 1964 | 1.088 | 530.9 | 1.277 | 452.2 | 1.174 |
| 1965 | 1.101 | 564.3 | 1.329 | 467.2 | 1.208 |
| 1966 | 1.128 | 603.9 | 1.404 | 485.4 | 1.244 |
| 1967 | 1.150 | 627.7 | 1.432 | 504.1 | 1.245 |
| 1968 | 1.186 | 650.7 | 1.483 | 520.2 | 1.251 |
| 1969 | 1.240 | 665.9 | 1.534 | 538.2 | 1.237 |

tion is equal to replacement so that net saving is equal to net capital formation. Net capital formation, like net saving, may be interpreted as the quantity component of the change in the value of wealth, but only under the assumption of geometric decline in efficiency of capital goods.

To construct price and quantity indexes of wealth we require a perpetual inventory of prices and quantities of capital goods. We first outline methods for constructing these indexes from perpetual inventory data; we then specialize to the case of geometric decline in efficiency of capital goods. For a single capital good, the value of wealth, as given above, is the sum of values of investment goods of all vintages:

$$W_t = \sum_{v=0}^{\infty} q_{A,t,v} A_{t-v}.$$

Price and quantity indexes of wealth may be constructed from price and quantity data for each vintage, treating $q_{A,t,v}$ as the price and $A_{t-v}$ as the quantity. Price and quantity indexes for several capital goods may be constructed by applying the Divisia index numbers to price and quantity indexes of wealth for each capital good.

With geometric decline in efficiency the expression for the value of wealth reduces to:

$$W_t = q_{A,t}K_t.$$

For several capital goods the acquisition price $q_{A,t}$ and quantity of capital $K_t$ for each capital good can be combined into price and quantity indexes for wealth. Our wealth account for the U.S. private national economy includes tangible assets held by private households and institutions, and by corporate and noncorporate business, and net claims on the government and foreign sectors, including the claims of social insurance funds. We estimate the price and quantity of assets for each of the five sectors by applying Divisia index number formulas to price and quantity data for each class of capital assets held by the sector. We construct price and quantity index numbers for the U.S. private national economy by applying these index number formulas to Divisia price and quantity indexes for the five sectors. Price and quantity indexes of wealth for 1929–69 are given in Table 23.

## 7. EXTENDING THE ACCOUNTING FRAMEWORK

### *7.1. Introduction*

As a long-term objective the basic accounting framework must be expanded to incorporate investment in human capital. Investment in human capital is primarily a product of the educational sector, which is not included in the private domestic sector of the economy. In addition to data on education already incorporated into the national accounts, data on physical investment and capital stock in the educational sector would be required for incorporation of investment in human capital into a complete accounting system.[55] We outline methods for incorporation of the educational sector into the basic accounting framework below.

A second objective for long-term research is the incorporation of research and development into a complete system of accounts.[56] At present research and development expenditures are treated as a current expenditure. Labor and capital employed in research and development activities are commingled with labor and capital used to produce marketable output. The first step in accounting for research and development is to

[55] Estimates of the stock of educational capital have been compiled by Schultz [53]; see especially pp. 123–131.

[56] The incorporation of research and development into a complete system of accounts has been discussed by Griliches [29].

## TABLE 23

Private National Wealth, 1929–69
(constant prices of 1958)

| | Corporate Tangible Assets | | Noncorporate Tangible Assets | | Household and Institutional Tangible Assets | | Net Claims on Governments and Rest of World | | Private National Wealth | |
|---|---|---|---|---|---|---|---|---|---|---|
| Year | Price Index | Quantity Index | Price Index | Quantity Index | Price Index | Quantity Index | Price Index | Quantity Index | Price Index | Quantity Index |
| 1929 | 0.424 | 275.3 | 0.417 | 256.2 | 0.427 | 370.7 | 0.943 | 35.0 | 0.572 | 725.3 |
| 1930 | 0.399 | 276.4 | 0.382 | 256.0 | 0.410 | 366.1 | 0.938 | 36.3 | 0.532 | 736.7 |
| 1931 | 0.364 | 268.2 | 0.333 | 255.1 | 0.366 | 358.8 | 0.881 | 40.0 | 0.475 | 736.3 |
| 1932 | 0.332 | 253.5 | 0.293 | 250.4 | 0.312 | 346.7 | 0.882 | 42.3 | 0.414 | 732.0 |
| 1933 | 0.331 | 242.6 | 0.299 | 245.6 | 0.311 | 335.3 | 0.891 | 44.2 | 0.417 | 714.0 |
| 1934 | 0.350 | 237.5 | 0.318 | 240.3 | 0.335 | 327.5 | 0.955 | 47.3 | 0.450 | 698.2 |
| 1935 | 0.357 | 234.6 | 0.328 | 242.2 | 0.332 | 324.5 | 0.956 | 49.5 | 0.460 | 691.9 |
| 1936 | 0.363 | 238.6 | 0.341 | 242.5 | 0.345 | 325.9 | 0.940 | 53.2 | 0.478 | 693.6 |
| 1937 | 0.388 | 244.8 | 0.356 | 247.0 | 0.368 | 327.9 | 0.947 | 54.6 | 0.504 | 705.4 |
| 1938 | 0.384 | 240.2 | 0.348 | 246.2 | 0.375 | 325.6 | 0.922 | 59.0 | 0.494 | 717.6 |
| 1939 | 0.382 | 240.6 | 0.345 | 247.1 | 0.376 | 328.4 | 0.905 | 63.8 | 0.498 | 720.6 |
| 1940 | 0.390 | 247.7 | 0.355 | 249.7 | 0.387 | 334.3 | 0.883 | 67.8 | 0.511 | 732.8 |
| 1941 | 0.420 | 260.9 | 0.387 | 254.7 | 0.419 | 342.9 | 0.938 | 75.1 | 0.562 | 752.2 |
| 1942 | 0.465 | 260.3 | 0.425 | 255.3 | 0.461 | 336.5 | 1.018 | 108.4 | 0.629 | 787.3 |
| 1943 | 0.494 | 255.8 | 0.456 | 252.5 | 0.500 | 327.2 | 1.010 | 153.7 | 0.653 | 857.8 |
| 1944 | 0.518 | 251.6 | 0.482 | 251.6 | 0.547 | 317.7 | 0.967 | 209.8 | 0.669 | 939.4 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1945 | 0.528 | 251.8 | 0.506 | 251.2 | 0.582 | 311.4 | 0.935 | 255.8 | 0.660 | 1,031.0 |
| 1946 | 0.587 | 271.7 | 0.582 | 255.0 | 0.633 | 326.7 | 0.944 | 259.0 | 0.693 | 1,096.0 |
| 1947 | 0.696 | 286.0 | 0.680 | 257.5 | 0.728 | 346.9 | 0.934 | 257.2 | 0.763 | 1,136.8 |
| 1948 | 0.746 | 300.8 | 0.722 | 266.2 | 0.793 | 368.1 | 0.931 | 253.7 | 0.806 | 1,171.4 |
| 1949 | 0.742 | 305.5 | 0.705 | 270.2 | 0.778 | 388.3 | 0.932 | 260.5 | 0.794 | 1,211.1 |
| 1950 | 0.773 | 321.2 | 0.764 | 278.6 | 0.816 | 418.9 | 0.951 | 250.9 | 0.835 | 1,247.9 |
| 1951 | 0.840 | 341.5 | 0.824 | 284.4 | 0.877 | 439.0 | 0.956 | 249.2 | 0.885 | 1,293.0 |
| 1952 | 0.857 | 353.4 | 0.827 | 287.2 | 0.897 | 455.4 | 0.956 | 257.3 | 0.893 | 1,337.6 |
| 1953 | 0.866 | 364.2 | 0.822 | 290.3 | 0.900 | 475.4 | 0.966 | 265.9 | 0.899 | 1,377.9 |
| 1954 | 0.873 | 370.0 | 0.835 | 292.7 | 0.894 | 495.4 | 0.971 | 275.5 | 0.899 | 1,421.6 |
| 1955 | 0.891 | 386.5 | 0.853 | 297.1 | 0.911 | 524.5 | 0.976 | 275.5 | 0.920 | 1,460.9 |
| 1956 | 0.942 | 405.2 | 0.901 | 299.2 | 0.952 | 545.0 | 0.984 | 276.0 | 0.954 | 1,512.0 |
| 1957 | 0.985 | 418.1 | 0.953 | 302.0 | 0.985 | 562.0 | 0.989 | 282.3 | 0.986 | 1,555.0 |
| 1958 | 1.000 | 422.2 | 1.000 | 303.9 | 1.000 | 574.0 | 1.000 | 294.7 | 1.000 | 1,594.7 |
| 1959 | 1.021 | 434.5 | 1.028 | 306.7 | 1.031 | 594.2 | 1.014 | 296.6 | 1.029 | 1,625.8 |
| 1960 | 1.033 | 447.4 | 1.069 | 309.2 | 1.049 | 611.0 | 1.028 | 298.4 | 1.046 | 1,664.0 |
| 1961 | 1.044 | 456.5 | 1.115 | 311.3 | 1.063 | 624.5 | 1.032 | 307.0 | 1.062 | 1,698.6 |
| 1962 | 1.062 | 470.8 | 1.164 | 315.2 | 1.086 | 643.6 | 1.042 | 315.3 | 1.093 | 1,732.5 |
| 1963 | 1.079 | 486.0 | 1.207 | 318.7 | 1.109 | 665.4 | 1.045 | 321.4 | 1.115 | 1,778.5 |
| 1964 | 1.101 | 505.3 | 1.255 | 322.4 | 1.140 | 688.7 | 1.039 | 333.8 | 1.146 | 1,825.9 |
| 1965 | 1.129 | 530.5 | 1.322 | 328.3 | 1.162 | 715.3 | 1.044 | 340.9 | 1.178 | 1,884.7 |
| 1966 | 1.167 | 565.4 | 1.393 | 333.2 | 1.190 | 739.9 | 1.043 | 353.0 | 1.217 | 1,950.4 |
| 1967 | 1.210 | 590.7 | 1.462 | 337.8 | 1.240 | 760.4 | 1.028 | 378.6 | 1.254 | 2,026.8 |
| 1968 | 1.255 | 614.5 | 1.545 | 342.7 | 1.300 | 786.9 | 1.023 | 395.4 | 1.299 | 2,100.7 |
| 1969 | 1.312 | 640.3 | 1.638 | 348.5 | 1.366 | 812.2 | 1.028 | 399.2 | 1.350 | 2,171.5 |

develop data on factors of production devoted to research. The second step is to develop measures of investment in research and development. The final step is to develop data on the stock of accumulated research. A similar accounting problem arises for advertising expenditures, also treated as a current expenditure.

Both education and investment in research and development are heavily subsidized in the United States, so that private costs and returns are not equal to social costs and returns. The effects of these subsidies would have to be taken into account in measuring the effects of human capital and accumulated research on productivity in the private sector. If the output of research activities is associated with external benefits in use, these externalities would not be reflected in the private cost of investment in research.

### *7.2. Investment in Human Capital*

To illustrate the design of a system of accounts incorporating the educational sector, we suppose that the stock of human capital at any point of time, say $E$, can be imputed from past investment in education, say $I_E$:

$$I_E = \dot{E} + \delta E,$$

where $\delta$ is the rate of required replacement of human capital. Total labor compensation in the private domestic economy, $q_L L$, may be divided between the value of services of human capital, say $q_E E$, and the value of labor services, $q_{N \cdot H} N \cdot H$, where $N$ is number of persons engaged, $H$ is effective man-hours per person engaged, and $N \cdot H$ is the number of effective man-hours:

$$q_L L = q_E E + q_{N \cdot H} N \cdot H.$$

Our present measure of real labor input, corrected for quality change, is an estimate of the services of both labor, $N \cdot H$, and human capital, $E$.

Next, we suppose that the value of the product of the private domestic sector is equal to the value of factor outlay, as before:

$$p_I I + p_C C = q_K K + q_E E + q_{N \cdot H} N \cdot H.$$

The product of the educational sector consists entirely of investment in human capital, produced with physical capital, human capital, and labor in the educational sector: [57]

$$p_E I_E = q_K K_E + q_E E_E + q_{N \cdot H} N_E H_E,$$

[57] Labor may include the imputed value of the time of students as well as the market value of the time of teachers.

where $p_E$ is the unit value of investment in human capital, $K_E$, $E_E$, $N_E$, and $H_E$ are physical capital, human capital, persons engaged, and effective man-hours per person, all in the educational sector.[58]

An important obstacle to implementation of a consolidated system of accounts is the need to compile data on the stock of physical capital in the educational sector. In compiling data on the stock of human capital and its service flow the procedure we have followed for physical capital would be reversed. Data on the flow of services is readily available; from these data we would infer an appropriate implied rate of return on educational investment.

*7.3. Research and Development*

To incorporate investment in the form of research into our accounting framework, we may suppose, as in our analysis of investment in education, that accumulated research and development can be treated as a stock, say $R$, with a corresponding investment flow, $I_R$. The value of output, including research and development investment, is equal to the value of factor outlay, including the services of accumulated research:

$$p_R I_R + p_I I + p_C C = q_K K + q_R R + q_L L,$$

where $p_R$ is the unit value of investment in research and development and $q_R$ is the service price of accumulated research. The value of labor and capital employed in producing research are, of course, included in the value of factor outlay. The absolute contribution of productivity change is the sum of productivity changes in research and in ordinary production activities.

Now, suppose that research and development are treated, erroneously, as a current expenditure so that no investment is recorded as an output. The value of output may then be written:

$$p_I I + p_C C = q_K K + q_R R + q_L L - p_R I_R.$$

If factor outlay on capital is computed as a residual equal to the value of output less the value of outlay on labor, the service price of capital is estimated, erroneously, as:

$$q^*_K = \frac{q_K K + q_R R - p_R I_R}{K}.$$

[58] Educational expenditures, including student time, are not equal to private outlays, since a substantial part of total expenditures is publicly funded. Subsidies to the educational sector, like subsidies to the output of the private domestic sector, are included in the value of the output of the educational sector.

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## COMMENT

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The basic vision behind Christensen and Jorgenson's opus is laudable. Their system comprises consistent sets of accounts—production, income and expenditure, accumulation, and revaluation—which link flows with stocks as of the beginning and end of each period. An integrated system of economic accounts, of course, has been one of the ultimate goals of this conference, which was wisely named, by Simon Kuznets and other founders, the Conference on Research in Income and Wealth. We have been pushed in the direction of developing a consistent system ending up with balance sheets and wealth statements by the National Accounts Review Committee, by the work of the Ruggleses, and others, and by the U.N. revised System of National Accounts which accommodates an ultimate integration of the various types of accounts.[1] The work of Christensen and Jorgenson moves us further in that direction.

I would like first to make a few comments on the basic structure of their system. It is a partial system, combined for the private domestic economy. It will be nice if ultimately they can prepare accounts for governments and the rest of the world, and include the financial flows in their capital accounts and the financial assets and liabilities for complete combined balance sheets. That is what we ultimately want, but they confined themselves to the private domestic economy because of their particular interest in productivity analysis. But even from their point of view, I think it would have been better if they could have separated their accounts into households and institutions, and business separately, because as soon as you start adding imputations in the households and institutional sector you reduce the usefulness of the productivity estimates which are a major objective of their *tour de force*. That is, they quite rightly included consumer durables in the capital account, but in imputing the rental value of those durables in real terms, I presume that the services of the durables parallel the movement of the stocks of the durables; so that their own estimates of productivity may be biased downward.

Also, one wonders why they stop with rental values of consumer durables, because one might easily add imputations for the value of the work of housewives and other unpaid household persons. At the National Bureau, Robert Eisner and I are working on these and additional imputations in the national accounts which will make them more useful for the kinds of social analysis that we are discussing at this session. What we do is to impute a value to inputs—the opportunity costs of students in school work, of housewives and others in household work, and so on. So our measures are really input measures, and later we will be comparing

[1] A recent presentation of economic accounts as a comprehensive, integrated system is contained in John W. Kendrick, *Economic Accounts and Their Uses*, New York, McGraw-Hill, 1972.

these nonmarket inputs with the market inputs, not with output, to see what has happened in the structure of the economy. Maybe in some far distant happy day we will have measures of outputs of households in terms of numbers of meals prepared, square feet of floor space cleaned, children brought up, and so on. The same thing goes for government output, and the Office of Management and Budget is pushing forward to find yet more measures of output for government agencies. For the time being, however, productivity analysis is better confined to the business economy. To the extent that Christensen and Jorgenson have imputations in their combined sector, this gives a downward bias to the productivity estimates.

In regard to structure, I approve heartily of their concept of the production account in real terms, in which they deflate final expenditure by product price indexes and factor costs by input price indexes, so that the ratio of real product to real factor costs yields a measure of changes in total factor productivity. This concept, of course, underlies my own book on productivity trends. As I indicated in the introduction to Volume 25 of the Income and Wealth series on output, input, and productivity measurement, this conception goes back at least to Morris Copeland at the first of these income and wealth conferences. He pointed out the productivity implications of real income and product accounts, although he did not follow up along these lines since his primary interest was in money flows.

Regarding Christensen and Jorgenson's execution of this useful concept, I have a few comments and exceptions that I would take to the methodology. In the case of their real labor cost, or the real value of labor services, I would prefer to weight by industry, and if possible, by occupational categories within industries instead of adjusting man-hours of labor input by a factor that reflects the effect of increasing levels of education per worker and per man-hour. If we follow the market-oriented approach of the accounts, I think we find that labor is marketed in terms of occupational categories, not in terms of years of schooling, with a few exceptions. The universities usually require Ph.D's, but that is in order to maintain the market for their output. In general, employers are not buying services according to years of education but according to skills, which of course, may be related to education. Although this is really partly a matter of taste, I also prefer to try to measure the inputs net of changes in quality or productive efficiency so that the productivity relationship brackets the whole change in productive efficiency. Now it is true that the interindustry, interoccupational shifts in part reflect an increase in quality, but only as a result of shifts, not as a result of changes in quality within the categories. This remark applies to Denison's work, too. Yet, my point is not crucial, because what we include in input or include in the residual is not so important, as long as we make clear what we are doing, and keep the effects separate so that we can identify the forces at work influencing economic growth, whether or not we include them in input or in the productivity residual.

As far as capital services are concerned, I think that Christensen and Jorgenson's method is very ingenious for obtaining service prices of capital as rental rate deflators. Yet I believe that their real net capital input would not differ much from mine because, in effect, my implicit deflators for net capital costs reflect changes in rates of return, as well as in the prices of the underlying assets. But with regard to gross capital costs, I think they are correct in taking account in their deflators of changes in depreciation or replacement rates, revaluation effects, and indirect taxes.

I have inferred that Christensen and Jorgenson have dropped their adjustment for changing rates of utilization of capital stock. I think this is good, partly because Denison has demonstrated that we do not have decent estimates of rates of utilization of capital in the economy as a whole. Also, I would argue conceptually that the real capital cost is related to the capital owned regardless of rates of utilization of capacity, and changes in those rates should show up in the change in productivity, because the capital charge goes on regardless of the rate of utilization. Many rental contracts, of course, are on an annual, weekly, or monthly basis, and the rental does not take account of hours within that period that the capital is utilized.

I think it is interesting that, as estimated using geometric depreciation rates, Christensen and Jorgenson's depreciation equals replacement, again on a somewhat peculiar definition that retirements occur piecemeal as capital goods lose their efficiency. However, there is also the rather peculiar result that their gross and net capital stocks are equal, as I see it. I would think that the traditional concept of gross capital stock which keeps the goods in until they actually disappear, defining retirement as the disappearance, or scrapping, at the end of the life, is a useful concept, particularly in looking at productivity of capital. In this connection, I think that Denison is justified in defining his stocks for purposes of productivity measurement in terms of their capacity to produce net output. Christensen and Jorgenson used the term "declining efficiency" with regard to maintaining production capacity, but without defining these terms. After working through their paper, one sees that they are really talking about "declining efficiency" in capital goods with respect to producing real net property income, not maintaining capacity intact in respect to producing either gross or net output. Presumably, with a slight decline of the capacity of capital goods for producing gross output, over their lives, there is a somewhat greater decline for producing net output, since you have to take into account increasing intermediate expenses for repair and maintenance. I think this is what Denison was trying to get at—the decline in capacity of producing net output, not in producing real net income for the capital factor. However, I do think that from a purely economic viewpoint Christensen and Jorgenson's concept is correct that depreciation is the decline in the value of a capital good as it ages, and this reflects the shortening as well as the declining stream of net income that it produces over its lifetime.

Just a few more general comments. On their income and expenditure account I am dubious about their deflating income (including net transfers), as distinguished from factor costs, by a factor price index. By this deflation procedure the identity of disposable income less outlay equaling personal saving (investment) does not obtain when the variables are expressed in constant dollars. Personally, I would confine price deflation to production accounts; but if I had to deflate income-expenditure accounts I would use consumption-price indexes for that portion of income, and investment-price indexes for the saving residual in order to maintain the basic saving-investment identities. Also, I do not find useful Christensen and Jorgenson's concept of the "standard of living" as a ratio of real expenditure to real income deflated by factor prices.

The use of Divisia indexes for factoring value changes into price and quantity components has its attractions. While not representing an ultimate solution of the index number problem,[2] Divisia index numbers have the advantage of not requiring periodic reweighting of aggregative price and/or quantity measures. But the fact that cyclical as well as secular changes in the structure of the economy affect annually chang-

[2] In addition to the approximation error, other problems have been discussed by Richard R. Nelson, "Recent Experiences in Growth Accounting: New Understanding or Dead End?" Yale University Economic Growth Center, Discussion Paper 18 (processed), October 1971, pp. 6–9.

ing weights introduces new problems. In particular, the relative weights of labor and property inputs can show major changes over the cycle. Nevertheless, I believe federal government statistical agencies should give serious thought to the possible use of Divisia indexes, where applicable and feasible.

Many of the details of Christensen and Jorgenson's concepts and estimating methodology deserve closer scrutiny than I have had time to give them. For example, I do not consider the use of the implicit price index for services to be the most appropriate deflator for income originating in general governments and the rest of the world. Also, I would question their method of deflating inventory stocks, when they could readily cumulate Commerce estimates of the real net change in inventories on a single stock estimate as of the end of the base period.

Finally, I am in agreement with Christensen and Jorgenson's discussion of the utility of expanding the capital accounts and balance sheets to include estimates of investments in research and development, and education, and the resulting stocks of intangible capital. I have already developed such estimates, and recently reported preliminary findings which will be of interest to economic growth accountants.[3]

In conclusion, I should like to reiterate my basically positive appraisal of Christensen and Jorgenson's contribution. We look forward to further refinements and extensions of their work in the economic accounts and growth analysis in the future.

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The handling of depreciation, replacement, and "relative efficiency" by Laurits Christensen and Dale Jorgenson merits some comment lest the reader miss critical implications of several special simplifying assumptions.

Christensen and Jorgenson distinguish usefully between changes in the current services available from a capital good, or its pattern of "efficiency" over time, and depreciation or capital consumption which, for constant prices of capital services, involves the reduction in the present value of future services as they are used up. Christensen and Jorgenson

[3] See John W. Kendrick, "The Treatment of Intangible Resources as Capital," *Review of Income and Wealth*, March 1972. The paper was presented in early September 1971, at the meetings of the International Association for Research in Income and Wealth at Ronneby, Sweden.

point out correctly the inconsistency in certain sets of assumptions regarding efficiencies and depreciation, in particular some by Edward Denison. For assumptions about the sequence of capital services or rentals or efficiencies determine, given discount rates, the value and depreciation of capital over time. Thus, both constant relative efficiency and a linear decline in efficiency, variously assumed by Denison, are inconsistent with straight-line depreciation of the value of capital. Nevertheless, Denison may be correct in his belief "that net value typically declines more rapidly than does the ability of a capital good to contribute to production" (as quoted by Christensen and Jorgenson, in Section 4.5). This would relate to a relative efficiency pattern quite different from that assumed by them, indeed a pattern which might even result in a change in value less than the change in efficiency because both are *rising* over time!

Christensen and Jorgenson's critical assumption is that of a geometric decline in efficiency over the (infinite) lifetime of capital goods. "Under this assumption," they declare, "the rate of replacement and the rate of depreciation are constant and equal to the rate of decline in efficiency." And on this basis they assume the geometric decline in value or "declining balance" depreciation which they use in their "simplified system of accounts."

In fact, however, a geometric decline in efficiency is not a sufficient condition for a geometric (and equal) decline in the value of assets. The necessary further assumption is that of a constant rate of discount. This may be demonstrated in Christensen and Jorgenson's terms as follows.

First, with $p_{A,t,v}$ the "acquisition price" or value of a capital good $v$ years old at the beginning of the year $t$, $d_v$ the efficiency or service of a capital good $v$ years old, $r_t$ the one-year rate of discount in the year $t$, $r_{t+s}$ the one-year rate of discount for the year $t+s$ anticipated in the year $t$, and with the price of capital services assumed constant and equal to unity, we have for that value or acquisition price at the beginning of the year $t$,

$$p_{A,t,v} = \sum_{\tau=0}^{\infty} d_{v+\tau} / \prod_{s=0}^{\tau} (1 + r_{t+s}). \tag{1}$$

Then, if we assume a geometric decline in efficiency at the (constant) rate $\delta$, we have

$$d_{v+\tau} = d_v(1 - \delta)^{\tau}. \tag{2}$$

If we add the further assumption that

$$r_{t+s} = r = \text{a constant for all } s \geq 0, \tag{3}$$

we can simplify (1) to

$$p_{A,t,v} = d_v \sum_{\tau=0}^{\infty} \left(\frac{1-\delta}{1+r}\right)^{\tau} = d_v \left(\frac{1+r}{r+\delta}\right). \tag{4}$$

But then this same capital good one year later and one year older is expected to have a value

$$\begin{aligned} p_{A,t+1,v+1} &= d_v(1-\delta) \sum_{\tau=0}^{\infty} \left(\frac{1-\delta}{1+r}\right)^{\tau} \\ &= d_v(1-\delta)\left(\frac{1+r}{r+\delta}\right) = (1-\delta)p_{A,t,v} \end{aligned} \tag{5}$$

and the value of depreciation in the year $t$ may be written

$$p_{D,t,v} = p_{A,t,v} - p_{A,t+1,v+1} = d_v\delta\left(\frac{1+r}{r+\delta}\right) = \delta p_{A,t,v}, \tag{6}$$

which is Christensen and Jorgenson's geometric or "declining balance" depreciation.

However, where we have a geometric decline in efficiency (2) but no constant geometric increase in the rate of discount (3), that is, where $r_{t+s}$ does not equal a constant for all $s$, we can only reduce the expression for the value at the beginning of the year $t$ of a capital good $v$ years old to

$$p_{A,t,v} = d_v \sum_{\tau=0}^{\infty} (1-\delta)^{\tau} / \prod_{s=0}^{\tau} (1 + r_{t+s}) \tag{7}$$

and the value anticipated for that good one year later is

$$p_{A,t+1,v+1} = d_v(1-\delta) \sum_{\tau=0}^{\infty} (1-\delta)^{\tau} / \prod_{s=0}^{\tau} (1 + r_{t+1+s}). \tag{8}$$

Then the value of depreciation in the year $t$ is (7) minus (8) or

$$\begin{aligned} p_{D,t,v} = d_v \sum_{\tau=0}^{\infty} &\left[(1-\delta)^{\tau} / \prod_{s=0}^{\tau} (1 + r_{t+s})\right] \\ &- \left[(1-\delta)^{\tau+1} / \prod_{s=0}^{\tau} (1 + r_{t+1+s})\right], \end{aligned} \tag{9}$$

which, for $r_{t+1+s} = r_{t+s}$ for all $s$, implies

$$r_{t+s} = r, \text{ a constant for all } s \geq 0, \tag{3}$$

and, hence,

$$p_{D,t,v} = d_v \delta \left( \frac{1 + r}{r + \delta} \right) = \delta p_{A,t,v}, \tag{6}$$

*but not otherwise.*

That this may be more than a technical curiosity may be suggested by the casual empirical judgment that, whether because of imperfection of information, increasing relative risks over time, or the finitude of life, annual rates of interest or discount tend to be greater for longer term commitments. This implies that $r_{t+1+s}$ tends to be greater than $r_{t+s}$, which would in turn tend to make depreciation (or $p_{D,t,v}$) less in earlier years (for lesser $v$) and more in later years (for greater $v$) than would be the case if $r_{t+s} = r_{t+1+s}$ for all $s$. In nonalgebraic terms what this comes down to is that the exhaustion of early capital services does not reduce the value of a capital good as much because the component of present value due to later capital services is rising relatively sharply as these prospective services move closer in time.[1]

There are other, probably more substantial, factors that may lead one to question the correctness of geometric decline in value and declining balance depreciation. Some of these relate to the basic assumption of geometric decline in efficiency. Many capital goods, particularly plant, show little or no decline in efficiency with age. One need only compare rents by "vintage" on houses or office space, after adjustment for quality differences, to realize that in these cases declines in efficiency are minimal or nonexistent over significant portions of economic lives. Where efficiency is constant, equal say to unity, the traditional "one-hoss-shay" case, we may write the acquisition price as

$$p_{A,t,v} = \sum_{\tau=0}^{n-v} 1 / \prod_{s=0}^{\tau} (1 + r_{t+s}), \tag{10}$$

or

$$p_{A,t,v} = \sum_{\tau=0}^{n-v} (1 + r)^{-\tau} = \frac{1 + r - (1 + r)^{v-n}}{r}, \tag{11}$$

[1] Throughout we are dealing, as Christensen and Jorgenson must implicitly, with *expected* future rates of discount. To the extent that there are changes in expectations of future rates of discount, or of prices, we would recognize capital gains or losses, or "revaluations" in Christensen and Jorgenson's terminology.

for $r_{t+s} = r$ for all $s$ where, in both cases $n$ = the expected years of life of the asset and hence $v \leq n$. Then, for $r_{t+s} = r$, the asset is expected one year later to have the value

$$p_{A,t+1,v+1} = \sum_{\tau=0}^{n-v-1} (1 + r)^{-\tau} = \frac{1 + r - (1 + r)^{v+1-n}}{r}, \tag{12}$$

and depreciation is (11) minus (12) or,

$$p_{D,t,v} = \frac{(1 + r)^{v+1-n} - (1 + r)^{v-n}}{r} = (1 + r)^{v-n}. \tag{13}$$

Far from approximating declining balance depreciation, which implies much more rapid depreciation in early years of the asset's life, (13), derived from the one-hoss-shay or constant efficiency assumption, implies slow depreciation at first and more rapid depreciation later, as $v$ approaches $r$. This may be confirmed readily by taking the derivative of (13) with respect to $v$ whence,

$$\frac{\partial p_{D,t,v}}{\partial v} = (1 + r)^{v-n} \ln (1 + r) > 0 \text{ for } r > 0. \tag{14}$$

We may further question Christensen and Jorgenson's seemingly innocuous and modest assumption "that relative efficiency is nonincreasing so that:

$$d_0 = 1;\ d_\tau - d_{\tau-1} \leqq 0;\ \tau = 0,\ 1,\ \ldots \text{"}$$

It seems probable that for many assets built to "lead" expected increases in demand or requiring substantial "break-in time," efficiency or the rate of production of capital services is initially low and rises for a significant early period. In the case of some capital goods, particularly plant and equipment, which is an integral part of new plant, there may be major capital expenditures over several years which have zero productivity or "efficiency" until construction is completed and the new plant is in operation. Thus, $d_0 = d_1 = d_2 = 0$ would prove common for substantial portions of capital additions that take three years to complete.

Some evidence that this is so can be found in work by Allan Mendelowitz [1], who estimated the time path of revenues attributable to capital expenditure by using McGraw-Hill data in regressions of profits on a distributed lag function of capital expenditures. His estimates suggest an initially rising curve of earnings. For reasonable positive rates of discounts his age profiles of earnings or efficiency actually imply initially negative rates of depreciation. This is indeed not as implausible as it may appear; as with a fruit tree, as the fruit-bearing years approach, the value of the tree rises.

And finally, further evidence that depreciation is at least less rapid than implied by a geometric decline in value is to be seen in the work of Wolfhard Ramm [2]. Ramm estimated the effect of age on value of automobiles in regressions that adjusted for quality changes. With a sample of some 8,980 observations of "Red Book" prices for specific types and models of used cars, Ramm was able to estimate year-by-year depreciation up to age six for each of the years 1961 through 1968. From his results [2, Table III, p. 156] we may calculate the mean estimates for the years 1961 to 1968 of depreciation as a ratio of value at the beginning of the years as a function of age—what might be called $\delta_s$. These turn out to be a monotonically rising sequence, 0.209, 0.237, 0.239, 0.254, 0.294, 0.306, and 0.389, for ages 0 to 6, respectively.

This is not a marked departure from geometric depreciation and it is indeed more rapid than straight-line; the corresponding sequence of depreciation rates in terms of original cost is 0.209, 0.187, 0.145, 0.117, 0.101, 0.074, and 0.065. But it must be recalled that this kind of estimate, relating to values of used capital goods for which there are market transactions, is likely to be biased in the direction of faster rates of depreciation, since used capital goods put on the market will tend to be those whose values have declined more than the values of goods which are retained. If goods are valuable to their owners they will not be offered for sale. In the case of automobiles there is as well a substantial element of "moral hazard." A disproportionate number of cars put on the market may be offered for sale because they have proved to be "lemons." Prospective purchasers will be fearful of this and will not be willing to pay as much for "second-hand" cars as unoffered vehicles of the same vintage and specifications will be worth to their original owners. Observations of this sort, we may add, are relevant to arguments over the years by George Terborgh and others that prices of used assets indicate faster depreciation than what was (previously) accepted for tax purposes.

This note does not presume to indicate just what rates and patterns of depreciation are appropriate for our old or new income, product, and capital accounts. It should be seen clearly though that the assumption of geometric decline in value or "declining balance" depreciation, however convenient, has profound effects on our measures of income, input, output, and capital. Particularly with the *double-rate* declining balance of our tax laws, which Christensen and Jorgenson employ, we may well derive estimates of depreciation which are very much too rapid and, in an economy where capital values are generally growing, substantially high, year after year. Major empirical work to illuminate the actual path of depreciation is very much in order for all of us concerned with improving and extending our national accounts. At this point, however, if a choice has to be made, I for one would opt for the old straight-line depreciation employed by Denison as coming closest to reflecting the complex of slowly declining efficiencies and increasing rates of discounts which characterize the returns from many of our capital goods.

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## REPLY BY CHRISTENSEN AND JORGENSON

### EISNER

Eisner's comment is based on an unfortunate confusion. We state that the value of assets declines geometrically if efficiency declines geometrically (middle of Section 4.4). This statement refers to a decline in value across vintages at a given point of time. Eisner misinterprets our statement as referring to a decline in value over time. This error in interpretation leads Eisner to lengthy but irrelevant deductions about conditions for geometric decline in value over time.

Geometric decline in the value of assets across vintages is the essence of the "simplified accounting system" described in our paper. Under this condition depreciation is equal to replacement, and both are proportional to the product of the price of acquisition of capital goods and the stock of capital. The geometric decline in the value of assets over time analyzed by Eisner is irrelevant to the "simplified accounting system." In general, assets will not decline in value geometrically over time in this system.

Eisner correctly points out that Denison employs inconsistent assumptions about efficiency and depreciation. He then endorses Denison's errors by stating that: "I for one would opt for the old straight-line depreciation employed by Denison . . . ." Here Eisner exhibits a personal predilection for logical inconsistency over logical consistency; this is a taste that few economic statisticians can be expected to share.

## KENDRICK

Turning to the comments of John W. Kendrick, our first observation is that "real net capital input" as defined by Kendrick ignores differences in rates of return at a given point in time due to differences in direct taxes. For example, corporate returns are taxed at a rate of nearly 50 per cent while noncorporate returns are not subject to the corporate income tax. If the rate of return after taxes is the same for the two sectors, as seems likely, the before-tax rate of return should be almost twice as high in the corporate sector. Before-tax rates of return are the appropriate rates for weighting capital input. Kendrick's measure of capital input based only on capital stock fails to take account of this difference.

Deterioration of capital goods is associated with decline in efficiency as well as retirement of these goods. The concept of capital appropriate for productivity measurement is an efficiency weighted sum of past investments. Under the assumption that efficiency declines geometrically, the appropriate measure of capital is net capital stock calculated by the declining balance method. The traditional concept of gross capital stock, an unweighted sum of past investments, is not relevant to productivity measurement unless efficiency does not decline until the asset is retired. As Kendrick points out, this appears to be implausible. The accounting system developed in our paper is not limited to geometric decline in efficiency. In actual implementation other patterns are much more costly to employ consistently; it is not yet clear whether the additional expense is justified by greater realism.

The essential novelty of the complete accounting system proposed in our paper is the development of accounts in constant prices throughout. For this purpose both production accounts and income-expenditure accounts are essential. Kendrick agrees at least in principle with the methods we have developed for implementing the production account. For the income and expenditure account he prefers to deflate expenditure by price indexes for expenditure categories and to deflate income by the same price indexes. The effect of this procedure is identical to that of deflating both sides of the production account by output price indexes, which would obviously give a distorted picture of the development of real factor input.

Kendrick proposes to deflate the income side of the income and expenditure account by an expenditure price index. The correct procedure is to deflate income by an income price index, weighting the prices of individual components of factor income by their relative weight in total factor income. These factor prices differ from those that appear in the production account by direct taxes. It would not suffice to carry over the price of real factor input from the production account for deflation of the income side of the income and expenditure account. Correct deflation of the income and expenditure account gives rise to a measure of the standard of living, integrating this useful concept into a national accounting system. It is difficult for us to understand Kendrick's reservations about the usefulness of incorporating the standard of living into social accounts.

Finally, the basic justification of Divisia index numbers is that they solve the index number problem where that problem has a solution. The index number problem has a solution if and only if there exists a corresponding economic aggregate. For example, in the production function underlying total factor productivity measurement, an invariant and path-independent index number of capital exists if and only if there exists a capital aggregate; the Divisia real capital index captures the variations in the capital aggregate. No other index number formula has this property. Of course, where no aggregate exists, the index number problem has no solution, and the choice of an index number formula is completely arbitrary.