Volume Title: The Detroit Prototype of the NBER Urban Simulation Model

Volume Author/Editor: Gregory K. Ingram, John F. Kain, and J. Royce Ginn

Volume Publisher: NBER

Volume ISBN: 0-870-14258-5

Volume URL: http://www.nber.org/books/ingr72-1

Publication Date: 1972

Chapter Title: Appendix D: A Summary of Major Variables and Program Operations

Chapter Author: Gregory K. Ingram, John F. Kain, J. Royce Ginn

Chapter URL: http://www.nber.org/chapters/c3510

Chapter pages in book: (p. 216 - 224)

APPENDIX D

A SUMMARY OF MAJOR VARIABLES AND PROGRAM OPERATIONS

Employment Location Submodel

$$MAN(J, HY, HED) = JOB(J, IND) * SICMAN(IND, HY, HED); \quad (6.1)$$

where:

$MAN(J, HY, HED)$ = manpower requirements by workplace, income class, and equation class;
$JOB(J, IND)$ = number of jobs of primary workers by workplace and industry;
$SICMAN(IND, HY, HED)$ = worker characteristics matrix by industry, income class, and education class.

$$PROVIS(J, H) = RATE(H) * F(J, H); \quad (6.2)$$

where:

$PROVIS(J, H)$ = basic mobility forecast—provisional estimate of number of movers (no employment change);
$RATE(H)$ = relocating rates for each household class H;
$F(J, H)$ = the number of households at each work zone J in each household class H.

$$PMOVE(H, J) = PROVIS(H, J) + ADDITIONS(H, J) - LOSSES(H, J); \quad (6.3)$$

where:

$PMOVE(H, J)$ = number of demanders by household type and workplace location for use in the demand and allocation models;

$ADDITIONS(H, J)$ = additional demanders by household class and workplace resulting from workplace-specific employment increases by income and education classes;

$LOSSES(H, J)$ = reduction in demanders by household class and workplace resulting from workplace declines by income and education classes.

$$ADDITIONS(H, J) = ADD\ RATE(H) * EMPLOY\ INCREASE(HED, HY, J); \quad (6.4)$$

where:

$ADD\ RATE(H)$ = weights for converting projected increases in employment by income and education class into increases by income, education, family size, and age classes;

$EMPLOY\ INCREASES\ (HED, HY, J)$ = projected increases in employment by education, and income classes and workplace.

$$\sum_{HAG,HFS} PROVIS(H, J) \geq EMPLOY\ DECLINE(HED, HY, J); \quad (6.5)$$

where:

HAG, HFS = age and family size categories;

$EMPLOY\ DECLINE(HED, HY, J)$ = employment declines by income-education category and workplace.

$$ELIGIBLE(J, H) = PROVIS(J, H) * LOSS\ RATE(HAG); \quad (6.6)$$

where:

LOSS RATE(HAG) = weights shown in Table 6.5;
ELIGIBLE(J, H) = households eligible for losing jobs this period.

Vacancy Submodel

$$PAVAIL(J, K, HY) = OCC\ RATE\ 2(J, K, HAG, HFS) * OCC\ RATE\ 1(J, K, HY) * PROVIS(J, H); \quad (6.7)$$

where:

OCC RATE 1(J, K, HY) = proportion of each income class *HY* residing in housing type *K* at workplace *J* at the end of the previous period;
OCC RATE 2 (J, K, HAG, HFS) = proportion of each family size, *HFS*, and age class, *HAG*, residing in housing type *K* at each workplace *J* at the end of the previous period.

$$AVAIL(K, I) = \frac{PAVAIL(J, K, HY)}{POTENTIAL(J, K, HY)} * TRIP(I, J, HY) * STOCK(K, I) + VACANT(K, I); \quad (6.8)$$

where:

AVAIL(K, I) = number of available units by housing type and residence zone;
STOCK(K, I) = number of occupied units by each type in each residence zone;
TRIP(I, J, HY) = number of trips from workplace *J* to zone *I* for income class *HY;*
VACANT(K, I) = number of units available but not occupied in previous period;
$POTENTIAL(J, K, HY) = \sum_I [TRIPS(I, J, HY) * STOCK(K, I)]$.

Other Bookkeeping

$$RMOVE(H, J) = CHAR(H) * PMOVE(H, J); \qquad (6.9)$$

where:

$RMOVE(H, J)$ = housing demanders by characteristics at time of move;
$PMOVE(H, J)$ = housing demanders by old characteristics;
$CHAR(H)$ = demographic change matrix, modifies age and family size of demanders.

$$REV\ TRIPS(I, J, HY) = TRIP(I, J, HY) - PROVIS\ TRIPS(I, J, HY); \qquad (6.10)$$

where:

$REV\ TRIPS(I, J, HY)$ = revised trip pattern by residence, workplace, and income class; includes only nonmoving households;
$TRIP(I, J, HY)$ = trip pattern of all households at beginning of period;
$PROVIS\ TRIPS(I, J, HY)$ = trips of households that vacate units this period.

Demand Allocation Submodel

$$TCOST(I, J, HY, M) = OPC(I, J, M) + 0.4 * WAGE(HY) * HRS(I, J, M); \qquad (7.1)$$

where:

$TCOST(I, J, HY, M)$ = the travel cost from residence zone I to workplace J for income class HY and mode M;
$OPC(I, J, M)$ = out-of-pocket costs for mode M;
$WAGE(HY)$ = implicit wage rate of income class HY;
$HRS(I, J, M)$ = interzonal travel time for mode M.

$$RES(I, J, K, HY) = TCOST(I, J, HY, MIN) + P(K, I); \quad (7.2)$$

where:

$RES(I, J, K, HY)$ = array of gross price surfaces over residence zones I, for each workplace J, housing type K, and income class HY;
$TCOST(I, J, HY, MIN)$ = travel cost for the cheapest mode for trips from residence zone to work zone by income class;
$P(K, I)$ = array of expected prices by housing type and residence zone.

$$WT(I, J, K, HY) = \frac{AVAIL(K, I) * TRIP(I, J, HY)}{\sum_I [AVAIL(K, I) * TRIP(I, J, HY)]}; \quad (7.3)$$

where:

$WT(I, J, K, HY)$ = weight applied to gross price surfaces by residence zone I, workplace J, housing type K, and income class HY;
$AVAIL(K, I)$ = number of units available for occupancy by type and location;
$TRIP(I, J, HY)$ = work trips made by income class.

$$R(J, K, HY) = \sum_I [WT(I, J, K, HY) * RES(I, J, K, HY)]; \quad (7.4)$$

where $R(J, K, HY)$ = expected gross housing price by workplace, housing type, and income class.

$$PCT(H, J, K) = A(H, K) + B1 * REL(J, HY, 1) + B2 * REL(J, HY, 2) + \ldots; \quad (7.5)$$

where:

$PCT(H, J, K)$ = the proportion of housing demanders of class H at workplace J that chooses housing type K;
$A, B1, B2, \ldots$ = estimated parameters of the demand equation;
$REL(J, HY, 1)$ = the expected gross price of unit 1 divided by the expected gross price of unit 10, e.g., $R(J, 1, HY)/R(J, 10, HY)$, and so on for each housing type.

$$XMOV(J, K, H) = PCT(H, J, K) * RMOVE(H, J); \qquad (7.6)$$

where:

$XMOV(J, K, H)$ = number of housing demanders by workplace J and household class H who choose housing type K.

$$DEMAND(K) = DMND(K) * VRATE(K); \qquad (7.7)$$

where:

$DEMAND(K)$ = total expected demand for each housing type K;
$DMND(K)$ = demand by households for each housing type $= \Sigma_{J,HY} AMOV(J, K, HY)$;
$VRATE(K)$ = 1 + normal vacancy rate for each housing type.

Filtering Submodel

$$PQ(KS, Q1, Q2, I) = P(KS, Q1, I) - P(KS, Q2, I); \qquad (7.8)$$

where:

$PQ(KS, Q1, Q2, I)$ = the quality premium between quality levels $Q1$ and $Q2$ for structural type KS in zone I;
$P(KS, Q1, I)$ = the expected price for a unit of structural type KS, and quality level $Q1$ in zone I;
$P(KS, Q2, I)$ = expected price of similar units except of quality level $Q2$.

$$RATIO(KS, Q1, Q2, I) = PQ(KS, Q1, Q2, I)/COSTF(KS, Q1, Q2); \qquad (7.9)$$

where:

$RATIO(KS, Q1, Q2, I)$ = a profitability measure of transforming a unit of structural type KS from quality level $Q2$ to quality level $Q1$ in zone I;
$COSTF(KS, Q1, Q2)$ = cost of upgrading a unit of structural type KS from quality level $Q2$ to quality level $Q1$.

$$AVAILF(K, I) = FILTER\,[RATIO(KS, Q1, Q2, I)] * AVAIL(K, I); \tag{7.10}$$

where:

$AVAILF(K, I)$ = the stock of available dwelling units after some units have changed quality level;
$AVAIL(K, I)$ = number of available units supplied by the vacancy submodel;
$FILTER$ = the filtering rate response function.

Supply Submodel

$$XDMND(K) = DEMAND(K) - \sum_I AVAILF(K, I); \tag{7.11}$$

where $XDMND(K)$ = the excess demand including normal vacancies for each housing type K in the current period.

$$PSTRUT(K, I) = P(K, I) * AVGNO(K); \tag{7.12}$$

where:

$PSTRUT(K, I)$ = expected structure price by housing type and zone;
$AVGNO(K)$ = average number of units per structure by housing type K.

$$PINPUT(K, KO) = PSTRUT(K, I) * INPTNO(K, KO) * AGLOM; \tag{7.13}$$

where:

$PINPUT(K, KO)$ = price of input K when housing type KO is output;
$INPTNO(K, KO)$ = number of structures of type K required to produce a structure of type KO;
$AGLOM$ = [1.0 + 0.025 * $INPTNO(K, KO)$] if $INPTNO$ exceeds 1.0, and 1.0 otherwise.

$$PROFIT(I, K, KO) = POUTPUT(I, K, KO) - [PINPUT(I, K, KO) + COST(K, KO)]; \tag{7.14}$$

where:

$PROFIT(I, K, KO) =$ the expected profit of producing structures of type KO from inputs of type K in zone I;
$POUTPUT(I, K, KO) =$ the total expected price of output structures produced by the activity;
$COST(K, KO) =$ exogenously estimated cost of transforming K to KO.

$$RATE(I, K, KO) = PROFIT(I, K, KO)/[PINPUT(I, K, KO) + COST(K, KO)]; \quad (7.15)$$

where $RATE(I, K, KO) =$ gross profit rate of producing output KO from input K.

$$AVAILS(K, I) = SUPPLY[AVAILF(K, I)]; \quad (7.16)$$

subject to:

a. Profit:

$$RATE(I, K, KO) > 0;$$

b. Availability:

$$AVAILS(KO, I) \leq \sum_K [AVAILF(K, I)/INPTNO(K, KO)] + VLAND(I)/INPTNO(28, KO);$$

c. Zoning:

$$AVAILS(K, I) - AVAILF(K, I) \leq ZONE(K, I);$$

d. Forecast demand:

$$\sum_I AVAILS(K, I) - \sum_I AVAILF(K, I) \leq XDMND(K);$$

where:

$AVAILS(K, I) =$ the number of units available for occupancy this period after new construction and transformations;
$SUPPLY =$ algorithm used to assign levels to transformation activities; chooses activities according to profitability;
$VLAND(I) =$ the quantity of vacant land available in each zone during a period;
$ZONE(K, I) =$ the zoning constraint which limits the number of output structures of each type which can be produced in a zone.

Market Clearing Submodel

$$MIN \sum_{I,J,HY} TCOST(I, J, HY) * X(I, J, HY); \qquad (7.17)$$

for each separate K subject to:

$$\sum_{I} X(I, J, HY) = AMOV(J, K, HY);$$

$$\sum_{J,HY} X(I, J, HY) = AVAILS(K, I);$$

where $X(I, J, HY)$ = households of income class HY employed at workplace J who locate in zone I, given they have chosen housing type K.

$$PLAND(I) = \sum_{K} \left[\frac{LRENT(K, I) * STOCK(K, I)}{\sum_{K} STOCK(K, I)} \right]; \qquad (7.18)$$

where:

$PLAND(I)$ = the one-period equilibrium price of land in zone I;
$LRENT(K, I)$ = the location rent of land in zone I under housing type K;
$STOCK(K, I)$ = the number of units of type K in zone I.