

This PDF is a selection from an out-of-print volume from the National Bureau of Economic Research

Volume Title: The Detroit Prototype of the NBER Urban Simulation Model

Volume Author/Editor: Gregory K. Ingram, John F. Kain, and J. Royce Ginn

Volume Publisher: UMI

Volume ISBN: 0-870-14258-5

Volume URL: <http://www.nber.org/books/ingr72-1>

Publication Date: 1972

Chapter Title: GROSS PRICE EFFECTS AND ESTIMATION OF SUBMARKET DEMAND PARAMETERS

Chapter Author: Gregory K. Ingram, John F. Kain, J. Royce Ginn

Chapter URL: <http://www.nber.org/chapters/c3505>

Chapter pages in book: (p. 129 - 162)

8

GROSS PRICE EFFECTS AND ESTIMATION OF SUBMARKET DEMAND PARAMETERS

THE HYPOTHESIS that workplace-specific gross housing prices systematically influence both the type of housing consumed and its location is central to the design of the NBER Urban Simulation Model. Moreover, the econometrically estimated submarket demand equations incorporate a great deal of the model's systematic information about household behavior. Therefore, it is hardly surprising that the econometric estimation of these central behavioral relationships has motivated much of our empirical research on housing markets.

This chapter contains descriptions of tests of these key hypotheses, using data from Detroit, and of the methods used to estimate the parameters of the demand allocation submodel of the Detroit Prototype. Also included in the chapter is a more limited discussion of findings on the demand for specific housing types by San Francisco households. Appendix B contains a more complete description of the San Francisco analysis.

Other NBER Studies

In addition to the demand estimates summarized in this chapter by Dresch and by Brown and Kain, persuasive evidence about the important role of gross prices was obtained by Straszheim in his study of San Francisco households. Other studies by Ingram, based on Pittsburgh data, and a separate analysis of the Pittsburgh rental housing market by Quigley also confirm the role of gross prices in

the housing choices of urban households. Kain and Quigley's findings for St. Louis households are also generally consistent with the demand analyses presented in this chapter, but their equations do not include gross price variables because the sample employed was too small and was geographically too concentrated.¹ Ingram estimated submarket demand equations for the Pittsburgh housing market to calibrate Pittsburgh I, and these are discussed in Chapter 9.

Straszheim's analysis uses a different sample of households than the Brown-Kain study summarized in this chapter and in Appendix B. Moreover, he formulates his demand equations in a manner that is somewhat different from the one used in the NBER model. First, he analyzes the demand for specific housing attributes such as lot size and number of rooms, rather than the demand for discrete housing types such as those in the NBER model. Second, he does not use the gross price framework of the NBER model to represent housing and travel costs. Instead, he includes both the minimum housing price prevailing at a distance of twenty minutes from the workplace of the household head and the head's actual commuting time as explanatory variables in his demand equations. In spite of these differences, Straszheim's extensive analysis of the demand for San Francisco households is perhaps the strongest evidence in support of the NBER model's theoretical structure.

Straszheim uses a two-step procedure to analyze the housing choices of San Francisco households. First, he obtains estimates of the rent and value surfaces (gradients) for renter- and owner-occupied housing in the San Francisco-Oakland metropolitan area. These surfaces summarize the prices of housing services in different parts of the metropolitan area. His analysis of these price surfaces differs from previous attempts to estimate value (rent) gradients in that his techniques describe a different rent surface for each attribute of the housing bundle. The attributes considered are number of rooms, age of the structure, lot size, and condition.

Straszheim's research reveals rather large variations in housing prices that appear to be related to the differential accessibility of units to employment opportunities. Moreover, his analysis indicates

1. Straszheim, "Demand for Housing Services"; idem, "An Econometric Analysis"; Ingram, "Model of a Housing Market"; Kain and Quigley, "Discrimination and a Heterogeneous Housing Stock"; and Quigley, "Residential Location."

Table 8.1
Suburban-Central-City Differences in Housing Prices in the San Francisco-Oakland SMSA, by Characteristics of the Housing Bundle: Owner-occupied and Rental Units

	Owner Occupant		Rental Occupant	
	CBD ^a	Suburb ^b	CBD ^a	Suburb ^b
Price of standardized unit ^c	\$57,150	\$26,647	\$185.80	\$122.12
Incremental cost per room	4,515	2,723	28.45	10.62
Incremental savings on standardized units				
Age of structure				
1950-65	15,151	3,640	34.16	15.41
1940-50	26,277	3,795	42.05	22.79
Pre-1940	32,485	5,911	51.01	30.15
Lot size				
0.2 acres	5,357	6,100	-	-
0.3-0.5 acres	-8,605	-3,354	-	-
4.5 acres	-30,505	-16,116	-	-
Unsound condition	17,821	14,251	38.38	-

Source: Straszheim, "Demand for Housing Services," tables A-1 and A-2.

a. Average for four zones located in Census tracts B1-B10 and J1-J20 in downtown San Francisco.

b. Average for ten zones located 0.55-0.65 hour's commute from the CBD.

c. Owner-occupied unit of 5.5 rooms in sound condition, built since 1960, on lots of 0.2-0.3 acres or renter-occupied unit of 4 rooms, built since 1960, in sound condition.

that these price surfaces vary substantially among the housing attributes included in his analysis, a finding which directly contradicts the long-run-equilibrium assumption of the competitive model. Table 8.1 illustrates Straszheim's findings about CBD-suburban variations in housing prices. The first row in Table 8.1 gives the estimated price in 1965 of a standardized rental and owner-occupied unit near the CBD and in the suburbs. (The standardized owner-occupied unit has 5.5 rooms, was built after 1960, has a lot of between two-tenths and three-tenths of an acre, and is in sound condition.) Straszheim estimates that this standard unit would cost \$57,150 adjacent to the San Francisco CBD, but only \$26,647 an hour's commuting time from downtown. The standardized rental unit, which has four rooms, was built after 1960, and is in sound condition, costs an estimated \$186 per month adjacent to the CBD and \$122 per month in the suburbs an hour distant from downtown.

The remaining statistics in Table 8.1 indicate the premium or

savings for different attributes in each location. Value and rent estimates for units whose characteristics differ from those of the standard unit can be obtained by adding or subtracting the indicated amounts from the cost of the standardized unit. Thus, for owner-occupied units, an additional room costs \$4,515 near the CBD and \$2,723 in the suburbs. The price gradient for the age of the structure is much steeper. The figures in Table 8.1 indicate that a household can expect to save \$32,485 by purchasing a structure more than thirty years old near the CBD. However, the savings from purchasing a unit more than thirty years old in the suburbs are only \$5,911.

In the second part of Straszheim's analysis, he obtains demand functions for each housing attribute. Household consumption of these different attributes is postulated to depend on household income, the length of the journey to work of the household head, the price of the attribute, and the price of attributes that are close substitutes. The demand equations for lot size, summarized in Table 8.2, provide some flavor of this analysis.

Table 8.2
Demand for Lot Size: Owner-occupied Dwellings
(elasticities at mean)

Life-Cycle Class	Y	T	P_0	Relative Prices	
				P_0/P_{50}	P_L/P_0
Single, alone	.2015		-.2642	-1.1544	-1.0201
Single, with others	.3092	.3666	-.8147		-1.3822
Separated, no children	.1330	.0582	-.3759	-1.4323	-1.8074
Separated, with children			-.2619		-2.1977
Married, 0 children	.3070	.0757	-.2250	-0.3810	-0.4046
Married, 1 child	.3541		-.1789	-0.4525	-1.0684
Married, 2+ children	.4021	.0483	-.0702	-0.5196	-1.4017

Y = family income.

T = travel time to work.

P_0 = price at a commuting distance of 0.4 hour for a 5.5-room house built since 1960 and occupying a lot of less than 0.2 acres.

P_{50} = price at a commuting distance of 0.4 hour of a 5.5-room house built between 1950 and 1960 and occupying a lot of less than 0.2 acres.

P_L = price at a commuting distance of 0.4 hour of a 5.5-room house built since 1960 and occupying a lot of 0.3-0.5 acres.

Source: Straszheim, "Demand for Housing Services," Table 6.

John Quigley's analysis of the demand for rental housing in Pittsburgh resembles Straszheim's research in that Quigley also analyzes the demand for individual housing attributes. However, Quigley uses a different definition of gross prices. He constructs gross price surfaces for individual Pittsburgh renters in precisely the manner described in Chapter 7, except that he uses the wage rather than 0.4 of the wage to value travel time. He then lists the locations of each housing type from the highest to the lowest gross price for each workplace and household class and uses the average price a household would have to pay for the least-cost 5 per cent of the stock as the gross price variable in his demand analyses. This gross price variable differs by workplace and income level, just as in the NBER model.

Tests of Housing Market Hypotheses for the Detroit Prototype

In both the tests of workplace-specific gross price effects and the efforts to estimate gross price parameters, described in this chapter and in Appendix B, it is assumed that the workplace-specific gross price effects, although distributed around zero, are in general significantly different from zero when compared across housing types for a given workplace, or across workplaces for a given housing type. Ideally, the demand model would be estimated directly by using the workplace-specific gross price of each housing type to explain the variation among workplaces in the proportion of each household class choosing each housing type.

Direct estimates of this kind could be obtained in at least two ways. First, the proportion of each household class choosing each housing type could be defined for each workplace. Regressing these proportions on workplace-specific relative gross prices would then provide direct estimates of the price coefficients and measures of their statistical significance. Alternatively, individual households could be identified by workplace by using dichotomous dependent variables to estimate demand equations: 1, the household consumes this housing type; or 0, the household does not consume this housing type. In this application household characteristics and relative gross

prices would be used as explanatory variables. Again, the significance of the price coefficients could be tested directly.

Either of the preceding methods would provide estimates of the gross price coefficients, test their statistical significance, and permit a direct examination of the responsiveness of households to variations in relative gross prices. Unfortunately, the data needed for directly estimating the gross price parameters of submarket equations for Detroit do not exist, or exist only in a highly unsatisfactory form.

The most serious weakness of the Detroit data is the unavailability of information on rent or value for individual sampled properties. Indeed, the most detailed information on housing prices available for Detroit are median rents of renter-occupied units and median values of single-family, owner-occupied structures by Census tract. Because these data are aggregate values by Census tract rather than price observations for specific housing types, they cannot be used to estimate demand equations or to test workplace price effects directly. Therefore, a two-stage procedure is used to evaluate workplace effects on the choice of housing types and to estimate parameters for the submarket demand model. First, submarket demand equations are estimated using only household characteristics. Then an attempt is made to explain the residual variation by workplace in the observed housing consumption of households by using estimates of relative gross prices. The two-stage procedure used provides a test of the underlying workplace-effect hypothesis, even if it proves to be impossible to estimate the relative price coefficients because the price data are inadequate. Appendix C contains a rigorous derivation of the two-stage procedure.

Alternative sets of price coefficients may be obtained by regressing the mean workplace residuals on alternative estimates of relative gross prices. For example, estimates based on different assumptions about the value of travel time, which yield different gross prices, can be compared in this manner. More importantly, the crucial hypothesis on workplace-specific gross price effects can be tested without the use of the prices themselves. The effect of workplace location on residence consumption choices can be examined in reduced form through the distribution of mean workplace residuals. While estimation of the individual price effects is desirable, the ability to examine net workplace effects directly is a major advantage of the two-stage technique employed here.

Definition of Submarkets

Definition of housing submarkets is the first step in empirical implementation of the demand allocation submodel. Unfortunately, few empirical or theoretical guidelines exist. The technical literature on stratification, particularly multidimensional stratification, is limited. Cluster analysis and the theory of grouping might provide a basis for statistical identification, but the data for these methods are not available for Detroit. In addition, existing grouping techniques require that the relative importance of the various dimensions be precisely specified. Since this is the heart of the problem of defining housing submarkets, these techniques are useful only as refinements. Data limitations simply did not support this level of refinement.

The procedure used combines a priori notions about the relative importance of various components of housing bundles, recognition of the real-data constraints, and considerable trial and error. The classifications used are not proposed as the correct or optimal ones, either for the immediate purposes of the simulation model or for a full understanding of housing market processes. They are, however, a beginning.

Structural type is the first and perhaps the single most important variable used in defining housing submarkets. Fortunately, the TALUS survey obtained information on the structural type occupied by each sampled household. The four structural types employed were: (1) single family, (2) duplex and row (any common-wall unit), (3) small apartment structure (two to four units per structure), and (4) large apartment structures (five units or more per structure).

The single-family and apartment categories were further subdivided by lot size or parcel area for each sample unit. The TALUS survey did not obtain the parcel area of sampled units. Therefore, the parcel area variables used to stratify the single-family and multifamily units are based on Census tract averages for each structural type. The divisions between small and large lot size categories are a mean lot size of 0.15 acres for single-family structures, 0.1 acres per dwelling unit for small apartment buildings, and 0.025 acres per dwelling unit for large apartment buildings.

In addition to structural type and lot size (Census tract density), at least three other housing characteristics would seem important in defining housing submarkets: (1) dwelling unit size (number of

rooms), (2) dwelling unit quality, and (3) neighborhood quality. No data were available from the TALUS survey on the quality of individual dwelling units. Therefore, both dwelling unit quality and neighborhood quality were represented by Census tract variables. The proportion of units in sound condition was used as a measure of dwelling unit quality, and median education was used as a measure of neighborhood quality. Dwelling unit size was similarly imputed from Census tract data. Table 8.3 summarizes the twenty-six housing types defined in terms of these four variables.

The validity of using Census tract statistics to characterize the individual housing units depends upon the degree of internal homogeneity of Census tracts. To examine this question, we computed the relative intra- and intertract variations for a number of Census tract variables. A summary of these tests of intratract homogeneity is presented in Table 8.4. The F ratio shown in the third column of the table is a test of the significance of differences between mean tract values. The ratios suggest the differences are highly significant, but the large number of degrees of freedom deprives them of much information concerning the extent of intratract variation. The fourth-column figures—sums of squared differences between observations and their intratract means as ratios to squared differences between the observations and the metropolitan area mean—are somewhat better measures of the extent of intratract homogeneity. They indicate that the intratract variance is smaller than the variance among tracts. The intratract variance in adult educational attainment, used to measure neighborhood quality and prestige; in housing condition, used to measure dwelling unit condition; and in rooms per unit, used to measure size, is quite small (9.6 per cent, 3.2 per cent, and 6.0 per cent, respectively). In short, the “efficiency of tracting,” i.e., the internal homogeneity of the tracts, appears to be quite high for those characteristics most needed to supplement the TALUS survey data.

The distribution of sample households by income class over the twenty-six housing types is shown in Table 8.5. No final claims can be made for these house-type stratifications. No direct tests of statistical significance of the groupings can be made, and only further experimentation will permit refinements in the classification. Appendix B contains the results of experiments with other

Table 8.3
Definition of Housing Types—Detroit Demand Analysis

Single Family

Large lots (less than 0.15 acres per unit)

High quality (90 per cent or more of units sound with all facilities)

1. High rooms (more than 5.4 rooms per unit, median)

High education (11.8 years or more of school completed, adult median)

2. Median rooms (5.1–5.4 rooms per unit, median)
3. Low rooms (5 rooms or less per unit, median)

Low education (less than 11.8 years of school completed, adult median)

4. Median rooms
5. Low rooms

Low quality (less than 90 per cent of units sound with all facilities)

6. High rooms
7. Median rooms
8. Low rooms

Small lots (0.15 acres or less per unit)

High quality

9. High rooms

High education

10. Median rooms
11. Low rooms

Low education

12. Median rooms
13. Low rooms

Low quality

14. High rooms
15. Median rooms
16. Low rooms

Common Wall, Duplex and Row

17. High quality
18. Low quality

Multiple Family

Small apartment structures (2 to 4 units per structure)

High quality

19. High density (0.1 acres or less per unit)
20. Low density (more than 0.1 acres per unit)

Low quality

21. High density
22. Low density

Large apartment structures (5 units or more per structure)

High quality

23. High density (0.025 acres or less per unit)
24. Low density (more than 0.025 acres per unit)

Low quality

25. High density
 26. Low density
-

Table 8.4
 Detroit TALUS Area Analysis of Socioeconomic Variations by Tract:
 Means and Standard Deviations (σ), F Ratios, and Ratios of Within
 to Total Sums of Squares

	Mean	σ	F Ratio ^a	Within ^b Total
Education (yrs.)	9.98	3.54	25,140	.10
Income (dollars)	8,277	8,049	1,894	.39
Housing condition (per cent sound)	93.89	19.30	44,612	.03
Number of baths	1.15	0.46	11,829	.11
Rooms per unit	5.13	1.47	23,234	.06
Units per structure	1.20	1.00	2,863	.30
Structural age (yrs.)	19.96	11.12	11,487	.11
Persons per household (owner)	3.60	1.68	4,732	.17
Persons per household (renter)	2.95	1.80	1,295	.23
Persons per room	0.62	0.39	3,960	.26
Length of tenure (yrs.)	8.43	7.52	2,013	.41
Number of autos	1.47	1.07	3,747	.33
Value (owner) (dollars)	17,037	9,315	5,054	.15
Gross rent (dollars)	93.09	42.10	3,213	.11
Contract rent (dollars)	77.66	29.69	3,860	.09

Source: Dresch, "The Demand Model," tables 1a and 1b.

a. Degrees of freedom approximately 800/300,000.

b. Sums of squared differences between observations and their intratract means as ratios to squared differences between the observations and the metropolitan area means.

definitions of housing submarkets, using San Francisco data. The principal advantage of the San Francisco sample is that data are available on the number of rooms and the lot size of single-family units.

The Effect of Household Characteristics on Housing Choice

Once a discrete set of housing types has been defined, it becomes necessary to consider what household characteristics influence a household's choice of housing type. The NBER Urban Simulation Model uses discrete household types. Therefore, it is desirable to cast the estimation of the submarket demand equations in the same terms. Seventy-two household classes are used in the Detroit Prototype.

The TALUS sample includes information on all household characteristics needed to estimate the submarket demand equations.

Table 8.5
Distribution of TALUS Subsamples over Housing Types

Housing Type ^a	Household Income			
	\$5,000 or Less	\$5,001 to \$10,000	\$10,001 to \$15,000	More than \$15,000
1	2.2%	4.4%	16.2%	49.1%
2	4.0	11.2	21.3	12.1
3	1.1	4.0	15.3	3.5
4	2.9	4.4	4.1	2.2
5	9.3	15.2	10.6	5.3
6	2.0	1.7	2.3	2.3
7	6.6	8.6	8.8	6.3
8	6.9	6.7	4.4	2.2
9	0.4	2.0	3.3	1.7
10	2.0	2.1	2.8	2.0
11	0.9	2.3	1.8	1.2
12	2.2	3.4	1.8	0.4
13	5.3	6.6	2.5	0.8
14	0.4	0.6	0.2	0.3
15	3.5	2.9	1.9	1.0
16	5.8	3.4	2.0	1.2
17	2.7	2.0	1.4	1.6
18	3.1	1.6	0.6	0.4
19	4.4	2.0	0.7	0.3
20	2.2	3.0	1.5	1.4
21	9.7	3.3	0.9	0.3
22	7.1	2.7	0.8	0.3
23	2.0	1.4	1.6	1.8
24	3.5	1.5	1.6	1.6
25	6.9	2.3	1.0	0.5
26	2.9	0.9	0.5	0.4
No. of households	452	4,711	1,626	768

Source: Dresch, "The Demand Model," Table 2b.

a. See Table 8.3 for definitions.

Since we expect that housing choices may have changed significantly over time, only those households which had relocated in the four years preceding the 1965 survey were included in the analysis. In addition, the analysis was restricted to white households with employed heads and only one worker.

The functional forms used for the submarket demand equations were suggested by several considerations. First, nonlinear effects

might be anticipated for a number of household characteristics. For example, initial increases in income from very low levels might increase the probability that a household would consume, say, small-lot, low-quality housing, but further increases in income might reduce that probability, as households shift to higher-quality, lower-density housing. The explanatory variables were defined as dummy variables, e.g., education 9–11 years, etc., to allow for these nonlinear effects. The first-stage demand equations include income, family size, education, and age of head as explanatory variables. The categories used are summarized in Table 8.6. Except for age of head, which the TALUS survey gave as a continuous variable (years), no information was lost, since the original data were classified by these same categories.

Second, significant interactions might be anticipated for some variables. For example, increases in family size might affect housing consumption choices differently at different income levels. The total number of possible interactions is, obviously, quite large. Only interactions between income and the other explanatory variables were incorporated, however. Income is believed to affect the value

Table 8.6
Detroit Household-Characteristic Stratifications

Major Income Subsamples and Subsample Stratifications	Education (head of household)
Income \$5,000 or less	1. 8 years or less (intercept)
0–\$2,000 (intercept)	2. 9–11 years
\$2,001–\$3,000	3. 12 years; high school graduate
\$3,001–\$4,000	4. 13–15 years; some college
\$4,001–\$5,000	5. 16 or more years; college graduate
Income \$5,001–\$10,000	Family Size
\$5,001–\$6,000 (intercept)	1. 1 or 2 persons (almost invariably 2)
\$6,001–\$7,000	(intercept)
\$7,001–\$8,000	2. 3 or 4 persons
\$8,001–\$9,000	3. 5 persons
\$9,001–\$10,000	4. 6 persons or more
Income \$10,001–\$15,000 (intercept)	Age of Head
Income over \$15,000	1. 30 years or less (intercept)
\$15,001–\$20,000 (intercept)	2. 31–45
Over \$20,000	3. 46–60
	4. Over 60

of travel time and to have especially important effects on housing consumption.

Some Preliminary Demand Equation Estimates

To allow full interactions with income, the sample was stratified into four subsamples: income of \$5,000 or less, \$5,001 to \$10,000, \$10,001 to \$15,000, and greater than \$15,000. Within each of these subsamples interactions among other household characteristics were assumed to be negligible. Therefore, separate equations were estimated for each housing type and each income level.

The lowest stratum of each socioeconomic variable is incorporated in the intercept term. The various coefficients then represent the incremental effect of each variable on the household's probability of consuming a particular housing type. The dependent variables are also dichotomous; they take the value 1 if a household consumes the particular type of housing, and the value zero otherwise.

The 104 individual equations estimated for the twenty-six housing types do not lend themselves to easy evaluation or interpretation. The number of possible substitutions consistent with any single set of coefficient values is simply too large. To overcome this difficulty, the equations were aggregated into nine categories based on the three basic structural types: Four single-family types are defined by lot size and quality, four multiple-family types are defined by structural size and quality, and there is one common-wall type. Even so, 36 individual equations must be estimated, since there are four income levels for each aggregate housing type. For illustrative purposes Table 8.7 presents nine of these equations—for households with annual incomes between \$5,001 and \$10,000.

It would have been possible to re-estimate the submarket demand equations using the aggregate categories, but the data processing and re-estimation would have been very expensive. They were obtained instead by simply adding the coefficients of the original 104 equations. Since all equations contain the same explanatory variables, this procedure provides correct estimates of the coefficients of the aggregate equation. It would also have been very expensive to derive test statistics for the aggregate equations; therefore, they are not presented.

A further summary of some of the principal findings of the demand

Table 8.7
 First-Stage Submarket Demand Functions for Nine Aggregate Housing Types for Incomes of \$5,001-\$10,000

	Housing Type								
	LL-HQ	LL-LQ	SL-HQ	SL-LQ	Dup-Row	SA-HQ	SA-LQ	LA-HQ	LA-LQ
Income									
\$6,001-\$7,000	.081	.049	-.003	-.011	-.011	-.007	-.045	-.015	-.122
\$7,001-\$8,000	.140	.037	-.023	-.007	-.022	-.013	-.076	-.007	-.029
\$8,001-\$9,000	.192	.017	-.017	-.016	-.029	-.017	-.087	-.008	-.034
\$9,001-\$10,000	.248	.027	-.056	-.047	-.027	-.019	-.090	-.008	-.028
Education									
9-11 yrs.	.100	-.029	.017	-.033	-.017	-.008	-.026	-.007	-.003
12 yrs.	.141	-.050	.007	-.045	-.012	.001	-.047	.012	-.007
13-15 yrs.	.180	.111	.024	-.061	-.003	.011	-.058	.022	-.005
16 yrs. or more	.128	.095	.029	-.045	.031	-.017	-.052	.030	-.008
Family size									
3-4 persons	.124	.025	.053	.024	-.024	-.038	-.032	-.066	-.068
5 persons	.160	.111	.064	.037	-.041	-.054	-.050	-.085	-.084
6 persons or more	.106	.100	.072	.053	-.027	-.070	-.054	-.083	-.092
Age									
30-45 yrs.	.054	-.002	.023	-.002	-.014	-.027	-.017	-.001	-.004
46-60 yrs.	.011	.008	.063	.016	-.009	-.033	-.039	-.010	-.007
Over 60 yrs.	-.006	-.002	.027	.021	-.040	-.034	-.049	.107	-.024
Intercept	.010	.162	.103	.094	.091	.116	.206	.089	.130

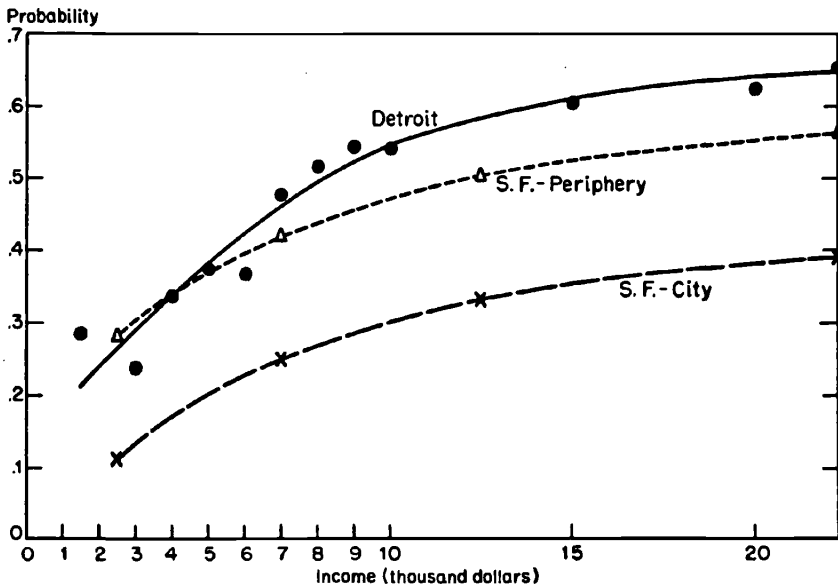
LL = large-lot single-family.
 SL = small-lot single-family.
 SA = small apartment structure.
 LA = large apartment structure.
 HQ = high quality.
 LQ = low quality.
 Dup-Row = duplex-row.

estimation is provided by figures 8.1–8.10, which depict changes in the probability of Detroit households choosing each of several types of housing as a function of several of the most important determinants of demand. Also depicted, in most cases, are similar estimates for San Francisco households.

The San Francisco estimates are based on a somewhat different estimating procedure than that used for Detroit and pertain to all moving households, i.e., nonwhites and households with more than a single wage earner are included. Therefore the Detroit and San Francisco curves are not entirely comparable. Even so they provide rough tests of the over-all consistency of the relationships used in the Detroit Prototype and suggest important differences in the structure of gross prices of different housing types in the San Francisco and Detroit housing markets. A more complete presentation of the San Francisco estimates and a discussion of the methodology employed is presented in Appendix B.

The first graph, Figure 8.1, shows differences in the proportions of Detroit and San Francisco households that choose single-family housing as a function of income. Both the Detroit and San Francisco curves are for small households headed by a young person with less than a high school education. Two curves are shown for San Francisco households. The one labeled "S.F.-Periphery" applies to households in which the primary wage earner is employed at a workplace located at the periphery of the San Francisco–Oakland metropolitan area. The other, labeled "S.F.-City," applies to households in which the primary wage earner is employed in the city of San Francisco. The difference in the level of the two curves reflects, of course, the impact of gross prices on the proportion of households choosing to live in single-family units. For the San Francisco equations, differences in gross prices are represented by an intercept shift. From Figure 8.1 it is apparent that the probability of both Detroit and San Francisco households choosing single-family housing is strongly dependent on income. For Detroit, this probability increases from less than 0.30 at the lowest income levels to greater than 0.65 at the highest income level. For workers employed in the city of San Francisco the probability increases from 0.11 for households whose annual income is less than \$4,000 to 0.39 for households with annual incomes over \$15,000.

Figure 8.1
Proportions of Detroit and San Francisco Households Choosing Single-Family Units, Classified by Income and Workplace

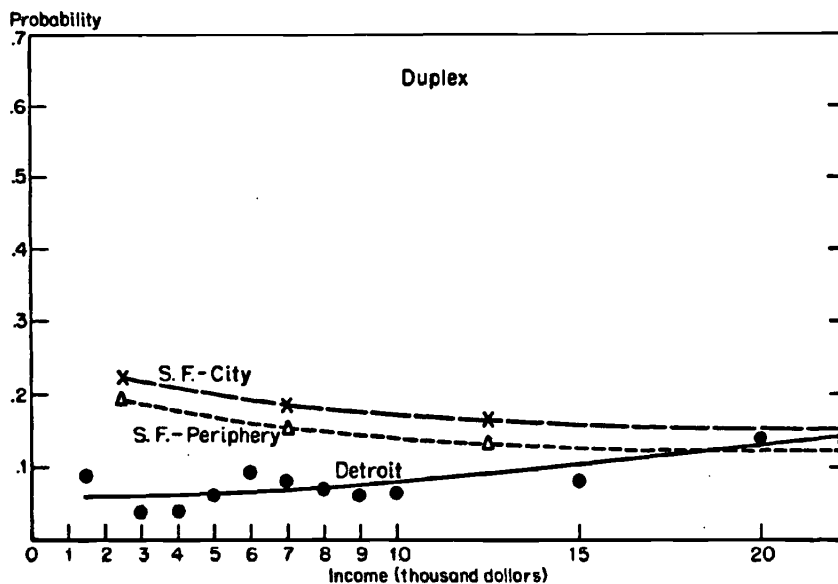


Source: Computed from Detroit and San Francisco equations.

Figure 8.2, for duplexes, illustrates an interesting difference in the Detroit and San Francisco housing markets. In Detroit the proportion of households choosing a duplex unit increases irregularly as income increases. In San Francisco, by comparison, it decreases steadily as income increases.

Figure 8.3 illustrates the proportion of Detroit and San Francisco households choosing small apartment structures, i.e., structures with fewer than five units. In both metropolitan areas, this fraction declines with income, with the decline being more pronounced in Detroit than in San Francisco. The graphs in Figure 8.4 show the proportion of Detroit and San Francisco households choosing large apartment structures (more than five dwelling units) as a function of income. Two categories of large structure are shown for San Francisco: those with between 5 and 19 units and those with 20 units or more. The proportions of households occupying large structures

Figure 8.2
Proportions of Detroit and San Francisco Households Choosing
Two-Family Units, Classified by Income and Workplace



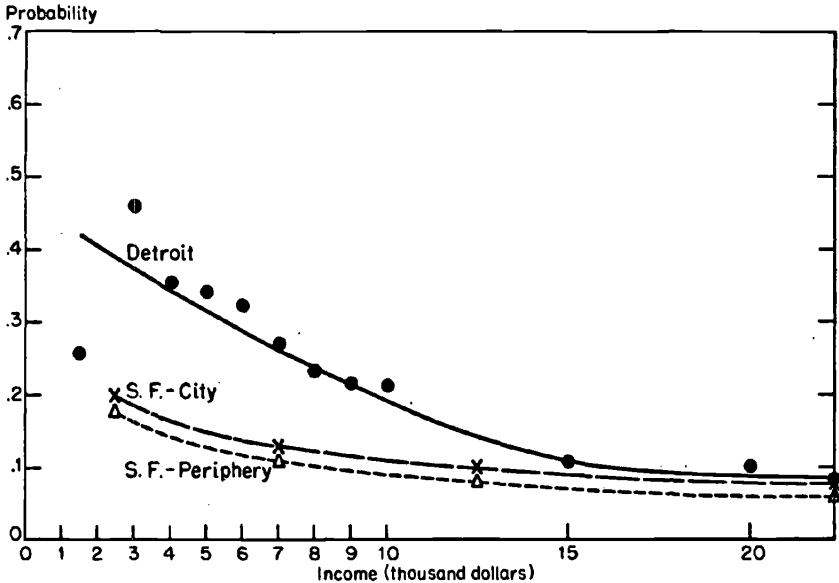
Source: Computed from Detroit and San Francisco equations.

in Detroit and medium-sized structures in San Francisco decline with income. However, the share of San Francisco households living in the largest apartment structures shows no tendency to decline with income.

As the curves in Figure 8.5 indicate, the proportion of households choosing large-lot, single-family units increases with income in both Detroit and San Francisco. The effect is particularly pronounced in Detroit and in outlying workplaces in San Francisco. In contrast, the proportion choosing small-lot, single-family units (Figure 8.6) declines somewhat with increasing income in Detroit and in both central-city and peripheral workplaces in San Francisco.

The effect of income on the consumption of single-family dwellings classified by neighborhood quality is evident in Figure 8.7. The proportion of Detroit households choosing high-quality single-family units increases from approximately zero for households with incomes

Figure 8.3
 Proportions of Detroit and San Francisco Households Choosing Small
 Apartment Structures, Classified by Income and Workplace



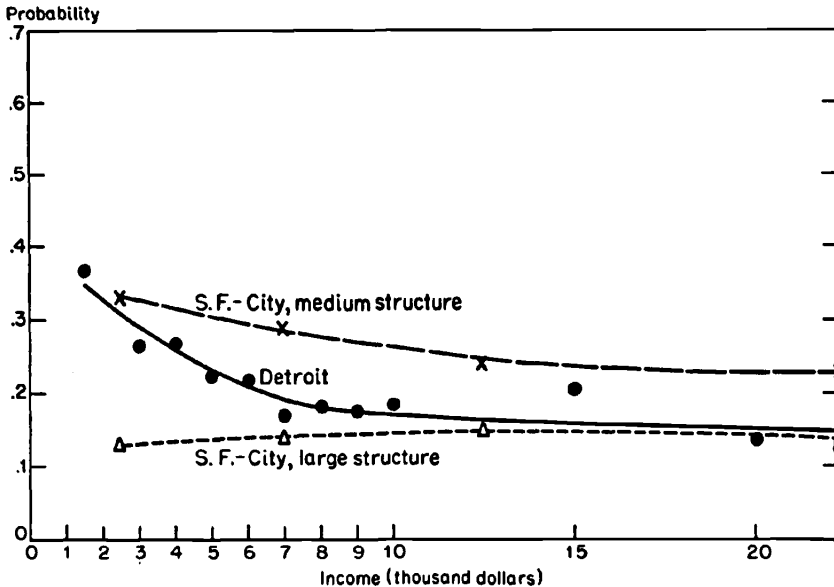
Source: Computed from Detroit and San Francisco equations.

of \$2,000 or less per year to about 0.3 for households with incomes above \$10,000 a year. The proportion living in single-family, low-quality tracts is approximately the same at all income levels.

Figure 8.8 illustrates a somewhat more complex analysis of the relationship between structural type, lot size, neighborhood quality, and income. The curves show the proportions of San Francisco households of various income levels residing in large-lot, single-family units in high-, medium-, and low-income neighborhoods. Unlike the previous diagrams, however, the graph is drawn for large families, whose heads have more than a high school education, and who are between 31 and 60 years old.

A positive relationship between income and tract quality is evident for apartment structures as well (Figure 8.9). For both small and large apartment structures the probability of occupying units in high-quality neighborhoods increases slightly with increases in income. The

Figure 8.4
Proportions of Detroit and San Francisco Households Choosing Large Apartment Structures, Classified by Income and Size of Structure



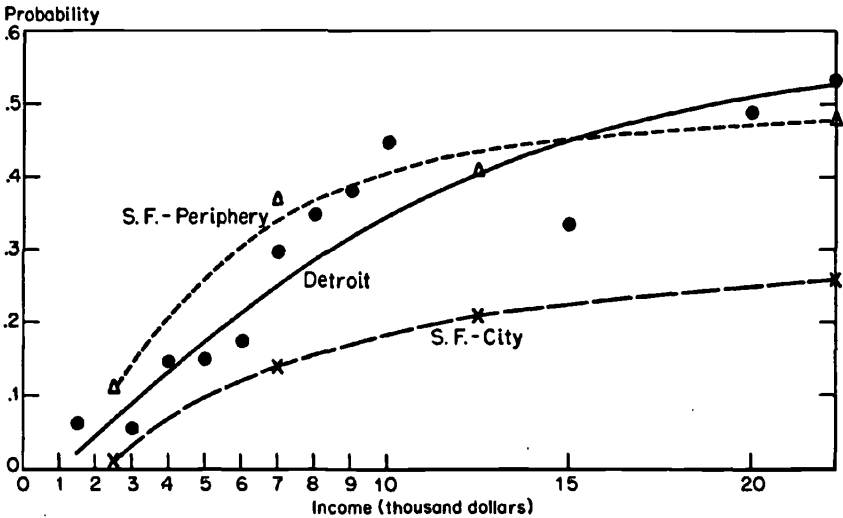
Source: Computed from Detroit and San Francisco equations.

probability of living in apartments in low-quality neighborhoods drops sharply as incomes rise.

One of the principal justifications for the estimation of separate sets of housing demand equations for the four major income groups in Detroit is that family size, age, and other household characteristics are expected to have different effects on the housing consumption patterns of households at different income levels. The importance of these interaction effects is evident in Figure 8.10, which shows the incremental probability of families of varying sizes choosing small, medium, and large single-family units for each of four income classes.

The significance and consistency of the family size-room relationship is, in fact, somewhat surprising. Although the intratract variance in number of rooms per unit is relatively small, there is still significant variation in room size within tracts. Further, it can be

Figure 8.5
 Proportions of Detroit and San Francisco Households Choosing
 Large-Lot, Single-Family Units, Classified by Income and Workplace



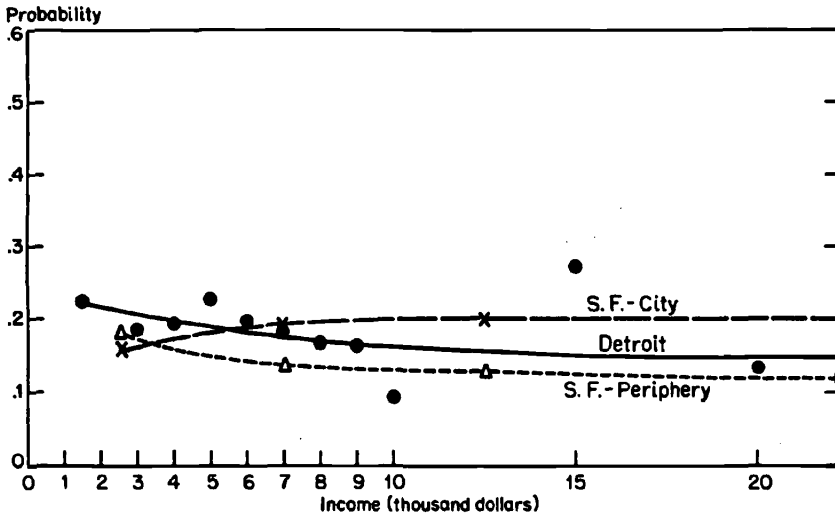
Source: Computed from Detroit and San Francisco equations.

anticipated that the choice of a unit with a specific number of rooms within a tract will be nonrandomly related to family size and other household characteristics. The consistency of the relationships among income, family size, and number of rooms provides strong support for our use of both the room size proxy and the Census summary statistics as proxies for characteristics of individual units.

Patterns of Workplace-Specific Deviations in Housing Consumption Choices

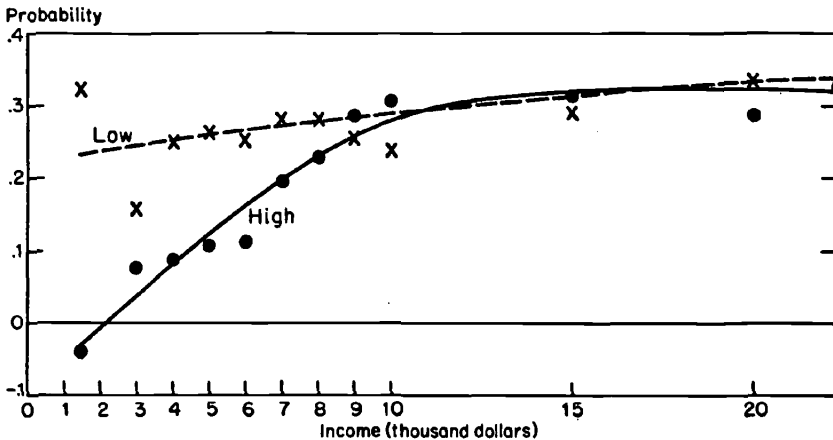
The theoretical discussions of the demand allocation submodel emphasize the role of workplace-specific differences in gross prices on the demand for housing types. The first stage estimates for Detroit, illustrated in Table 8.7, do not incorporate the effects of differences in relative prices among workplaces on the probability of households' consuming each housing type. If the hypothesized workplace effects exist, the proportion of households at each workplace actually consuming a given housing type will differ

Figure 8.6
 Proportions of Detroit and San Francisco Households Choosing Small-Lot, Single-Family Units, Classified by Income and Workplace



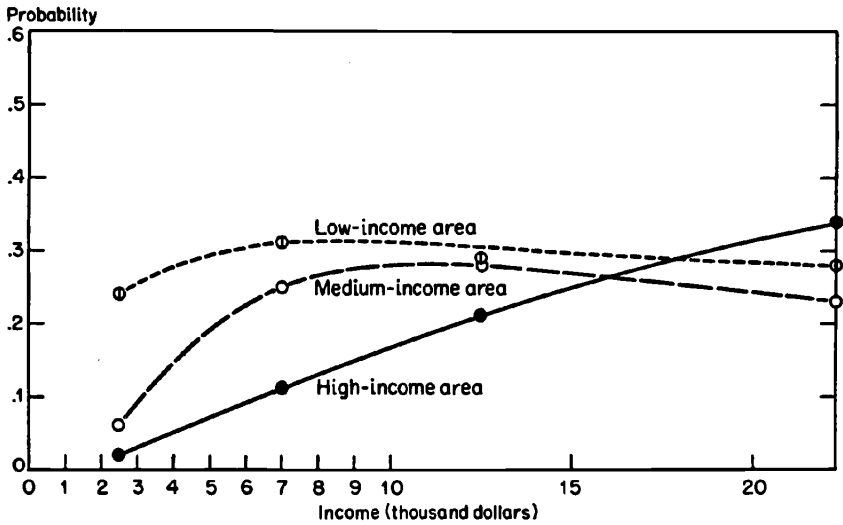
Source: Computed from Detroit and San Francisco equations.

Figure 8.7
 Proportion of Detroit Households Choosing Single-Family Units in High- and Low-Quality Neighborhoods, Classified by Income



Source: Computed from Detroit equations.

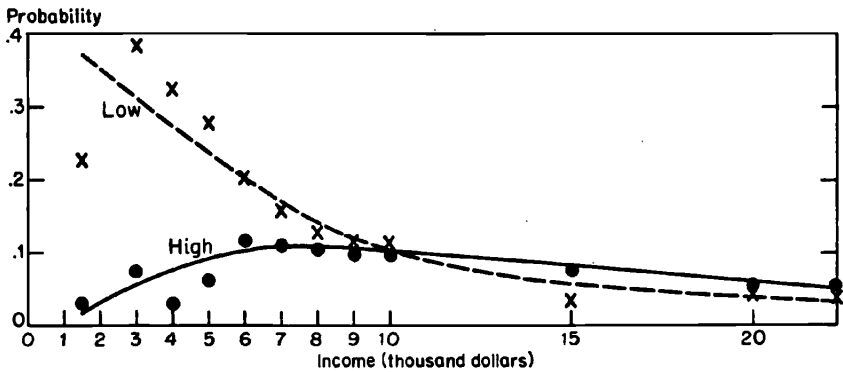
Figure 8.8
 Proportion of San Francisco Households Choosing Large-Lot, Single-Family Units in Neighborhoods of High, Medium, and Low Quality, Classified by Income^a



Source: Computed from San Francisco equations.

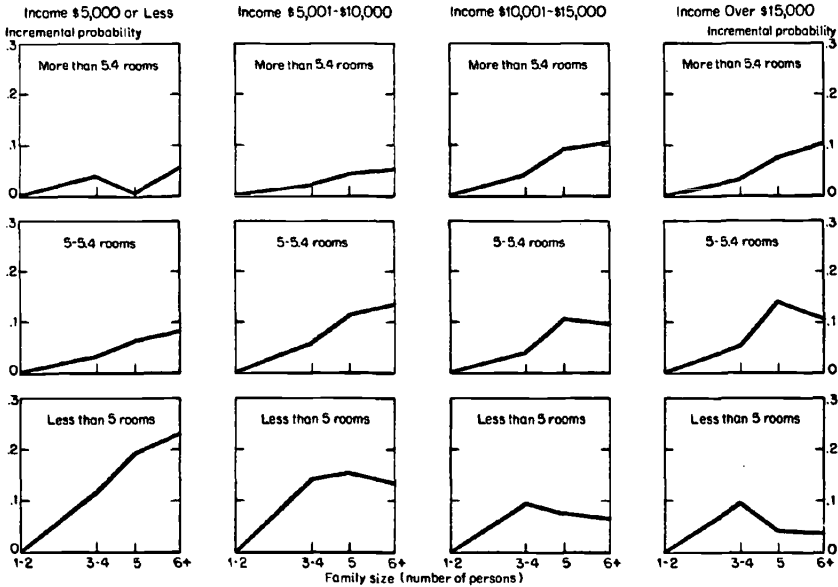
a. Curves are drawn for households that contain more than five persons and in which the head is employed in San Jose, has more than a high school education, and is between 30 and 60 years old.

Figure 8.9
 Effect of Income on the Probability of Choosing Apartments in High- and Low-Quality Neighborhoods



Source: Computed from Detroit equations.

Figure 8.10
Effects of Income and Family Size on the Probability of Choosing Single-Family Units of Various Sizes



systematically from the proportion predicted on the basis of household characteristics alone. The mean difference between the actual and predicted proportions by workplace then reflects workplace-specific differences in the gross price of each type of housing at each workplace. This *mean within-workplace* residual for a given housing submarket demand equation is a weighted sum of the relative gross prices specific to that workplace, where the weights are the relative gross price coefficients from the complete demand equation.²

The across-workplace residuals for a given housing type are distributed around zero. Similarly, for a given workplace, the residuals across housing types are distributed around zero. In the first case, the weighted mean across workplaces is identically zero, while in the second, the algebraic sum is zero. Thus, the magnitude of the workplace price effect is reflected by the *absolute* difference of the residuals from zero.

2. See Appendix A for a derivation of this property.

In the Detroit analyses mean workplace residuals for each of twenty-six housing types were computed for the approximately forty workplaces shown in Figure 8.11. These workplaces are TALUS "superdistricts," the largest subcounty areas in the TALUS zoning system. Instead of examining the residuals of the original 26 equations, however, the residuals were combined to obtain mean residuals for the set of nine aggregate housing types. These residuals are summarized in Table 8.8. Because of the small sample sizes, the residuals are further aggregated to two, rather than four, income

Figure 8.11
Detroit Superdistricts

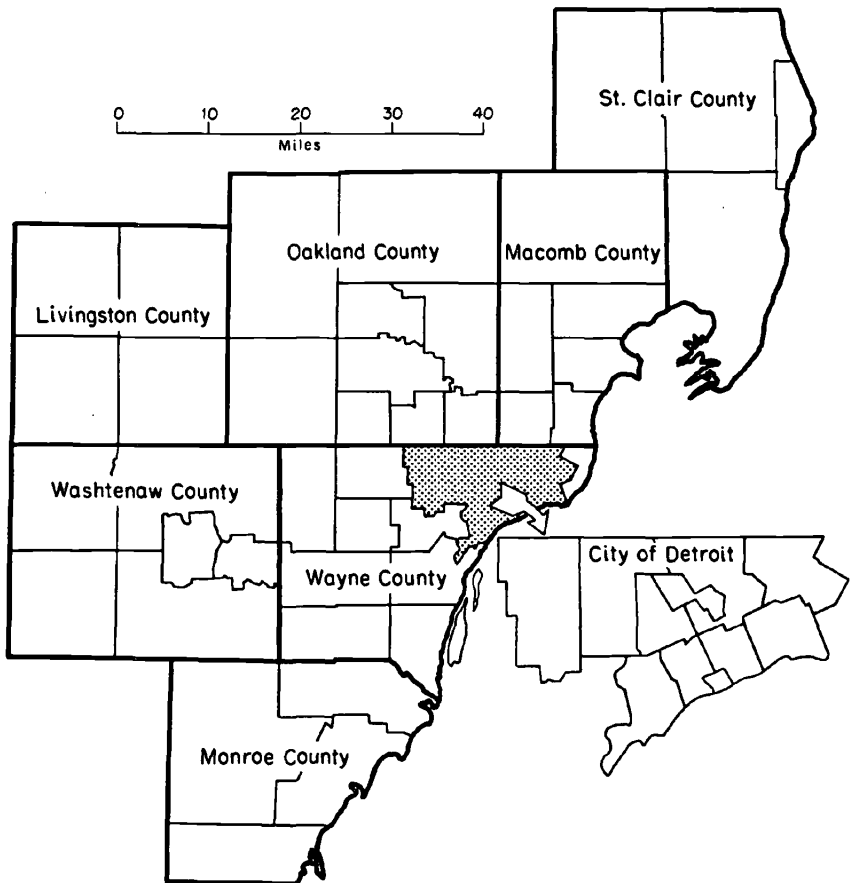


Table 8.8
Mean Absolute Residuals—Superdistricts

Housing Type	Income Above \$10,000			Income \$10,000 or Less		
	Mean Abs. Residual (1)	Ratio: Mean Abs. Residual to Mean Proportion (2)	Mean Proportion (3)	Mean Abs. Residual (4)	Ratio: Mean Abs. Residual to Mean Proportion (5)	Mean Proportion (6)
LL-HQ	.1202	0.1932	.622	.1416	0.3776	.375
LL-LQ	.1224	0.8742	.140	.1875	1.1160	.168
SL-HQ	.0668	0.6485	.103	.0731	0.4568	.160
SL-LQ	.0335	1.0468	.032	.0741	1.1160	.168
Dup-row	.0215	1.0750	.020	.0567	1.4921	.038
SA-HQ	.0210	1.0000	.021	.0750	1.4705	.051
SA-LQ	.0150	1.1538	.013	.1375	1.9927	.069
LA-HQ	.0321	0.9727	.033	.0430	1.3437	.032
LA-LQ	.0202	1.5538	.013	.0625	1.6447	.038

LL = large-lot single-family.

SL = small-lot single-family.

SA = small apartment structure.

LA = large apartment structure.

HQ = high quality.

LQ = low quality.

Dup-row = duplex-row.

classes: high income (more than \$10,000) and low income (\$10,000 or less).

Analysis-of-variance techniques could have been used to test the statistical significance of the workplace effects, but because of their high cost these formal tests were not performed. The general significance of the workplace effects can be evaluated, however, from the relative magnitudes of the mean residuals and the mean proportions consuming each housing type.

Examination of the mean absolute residuals indicates that for high-income households the "mean absolute workplace effect" alters the probability of consuming any of the nine aggregate housing types by from 0.02 to 0.12, depending on the type of housing considered. For low-income households the mean absolute workplace effect varies from 0.06 to 0.19. For every housing type the workplace effect is greater for low-income than for high-income households. This suggests low-income households are more "price-responsive" than

high-income households, a plausible result and one which further supports our decision to estimate separate price coefficients for each income class.

For large-lot, low-quality, single-family units the mean low-income workplace effect is 1.5 times that for high-income households (0.19 versus 0.12). Even more dramatically, the mean workplace residual for low-quality, small apartment structures is approximately 9 times as large for low-income households as for high-income households.

Absolute residuals are one measure of the price responsiveness of households in their housing consumption choices. Since the proportion choosing each housing type varies considerably, however, these should be related to the mean proportion consuming each housing type. These mean proportions are shown in columns 3 and 6 of Table 8.8. The ratio of mean absolute residuals to mean proportions, columns 2 and 5, provides a good summary of the impact of the average workplace effect (in terms of absolute change in housing consumption probabilities) relative to the impact of other characteristics determining housing consumption choices.

From Figure 8.5 it is evident that the probability of consuming large-lot, single-family housing in high-quality neighborhoods is highly sensitive to income. In addition, for high-income families, large-lot, single-family units exhibit low price sensitivities. The ratio of the mean absolute residual to the mean proportion is only 0.19, the lowest value observed for any housing type for either income class. Households with incomes of over \$10,000 are, therefore, very likely to consume low-density housing and are relatively insensitive to price effects. This insensitivity to price is not characteristic of low-income households, who are much less likely to consume large-lot, single-family, detached units and appear to be much more responsive to workplace-specific price variations.

In the case of small-lot, single-family housing, high-income groups appear to be more price-responsive because the average proportion of high-income households choosing this housing type is much smaller. The shift of high-income groups from low- to high-density housing in response to price variations is small relative to the large proportion choosing low-density housing, but large relative to the small proportion choosing high-density housing.

For the several multifamily housing types, relative workplace

effects are much greater for low-income households than for their high-income counterparts, even though the areawide proportions of low-income households choosing these housing types are invariably much higher for the low-income group. These results indicate that low-income households are more easily persuaded to change their housing consumption patterns in response to spatial variations in relative prices than are high-income households.

In summary, analysis of the mean absolute workplace residuals strongly supports the view that workplace location exerts a substantial effect on housing choices of both low- and high-income households.

Estimating the Gross Price Coefficients for the Detroit Prototype

In this section we describe our efforts to estimate the gross price coefficients from Detroit data. The first step in the estimation procedure is to compute gross price surfaces by housing submarket and to summarize this information in a form suitable for estimation.

First, for housing type K and workplace J , mean commuting time is computed for each of two income classes: those with incomes up to \$10,000 per year and those with incomes above \$10,000. Use of more income classes would have been desirable, but the small sample sizes limited the analyses to only these two. In the same manner, the mean monthly housing expense is computed for these same classes of workers, i.e., two income classes for each workplace J and housing type K . Monthly housing expense for each renter or owner household is imputed from the median rent or value of the household's Census tract of residence. Renters are assigned the median rent of their Census tract of residence as their monthly housing expense, while owners are assigned one-hundredth of the median value of single-family homes in their tract.³ As before, only households that recently relocated are included.

To form the travel cost component of gross prices by housing type, however, it is necessary to convert mean travel time into a monthly

3. A monthly gross rent multiplier of 100 is a widely accepted rule of thumb for converting monthly rents to property value.

travel cost. First the cost per work trip is obtained by multiplying travel time by a wage rate multiplier, which incorporates both out-of-pocket costs for travel by auto and the value of travel time. Then, the total monthly commuting cost is estimated by multiplying the estimated cost per trip times 42 (two trips per day for 21 monthly working days). High-income households are assumed to have an average annual wage of \$13,500, and low-income households, an average annual wage of \$7,500. These annual rates are divided by an average wage year of 2,000 hours to obtain an hourly wage of \$6.75 per hour for high-income households and \$3.75 per hour for low-income households.

For reasons discussed in Chapter 4, 0.40 of the wage was used as the value of commuting time. Alternative estimates using 0.2 and 1.0 times the wage as the value-of-time measure were obtained as well. While these estimates are not presented here, it appears the results are not highly sensitive to the precise time-value parameter employed.

Estimation of the Price Effects⁴

Estimation of the relative gross price coefficients also used the set of nine aggregate housing types identified in the preceding section. These include four types of single-family housing, defined by lot size and quality, one duplex-row category, and four multiple-family housing categories, defined by the size and quality of the structure.

The simple correlations between the logarithms of relative gross prices and the residuals are shown in Table 8.9 for the nine housing types. Because the relative prices are collinear, the simple correlation between a residual and its own price need not be negative. Even so the first-order correlations are negative in all cases where they are significant. The largest negative correlation is -0.69 , while the largest positive correlation is 0.11 . Six of the eight own-price correlations are negative for both income groups.

Regressions of house-type residuals on the relative prices are shown in tables 8.10 and 8.11 for the high- and low-income samples, respectively. As the a priori restrictions would dictate for the high-income sample, seven of the own-price coefficients are negative,

4. Appendix C contains a more complete discussion of the procedure used to estimate the relative price coefficients from Detroit data.

Table 8.9
Simple Correlations Between Workplace-Specific Residuals for
Selected Dwelling Units and Their Relative Prices

	Low Income	High Income
Single family		
Large lot, high quality	-	-
Large lot, low quality	-.34	-.09
Small lot, high quality	.11	.03
Small lot, low quality	-.16	.10
Duplex-row	-.20	-.56
Apartment		
Small structure, high quality	-.24	-.41
Small structure, low quality	-.37	-.50
Large structure, high quality	.11	-.43
Large structure, low quality	.00	-.69

Source: Dresch, "The Demand Model," tables 10a and 10b.

and four are highly significant statistically. The results are less encouraging for the low-income sample; only six own-price coefficients are negative, and only two of these are significant. However, even these results are **not** too disheartening when the poor quality of the housing price data is recognized.

The other restriction, that all cross-price coefficients should be positive, is less often satisfied. For the high-income sample, 27 of the 64 cross-price coefficients are negative. However, only 4 of the negative coefficients are statistically significant. For the low-income sample, 30 of the 64 cross-price effects are negative, but once again only 4 are statistically significant. The estimated gross price coefficients are not fully consistent with the prior specification.

On average, mean travel time for a given housing type should be greater for workplaces less accessible to the available stock of a particular type of housing. Workers employed at these less accessible workplaces would be expected both to travel farther and to pay relatively higher housing prices to consume that housing type even in their optimal location. Therefore, relative mean travel costs may be fairly highly correlated with relative gross prices although the relationship need not be exact.

Table 8.10
Gross Price Equations for Households with Income \$10,000 or Less.

	Single Family						Apartment					
	Large Lot		Small Lot		Duplex-Row	Quality	Small Structure		Large Structure		Quality	Quality
	High	Low	High	Low			High	Low	High	Low		
P2	-0.39	-4.65	1.50	0.67	-2.52 ^a	0.95	3.53	-0.45	1.22			
P3	-13.50†	0.28	4.77 ^a	2.88 ^a	-3.55†	2.81 ^a	5.43 ^a	0.16	0.82			
P4	11.54*	0.47	-2.68	-2.76‡	3.01*	-1.57	-4.66 ^a	-1.44 ^a	-1.92 ^a			
P5	-3.54 ^a	0.80	-0.17	0.32	-2.02†	0.37	1.56	0.82	1.80‡			
P6	1.48	1.70	-1.80	-0.32	-0.22	-1.39 ^a	0.94	-0.59	0.16			
P7	3.91‡	-2.27	-0.15	1.24‡	0.11	-0.68	-2.15 ^a	0.82 ^a	-0.81 ^a			
P8	6.86 ^a	2.59	-1.20	-2.52 ^a	3.33†	-2.89 ^a	-4.78 ^a	-0.02	-1.36			
P9	-3.63*	4.75*	-0.28	0.47‡	0.03	-0.98*	-0.36	-0.07	0.06			
Constant	0.67	-18.32	-5.10	-1.80	14.37 ^a	13.45	-5.82	4.62	-1.61			
R ²	0.664	0.648	0.0	0.182	0.228	0.456	0.089	0.00	0.00			

Note: The true coefficients of the transformed relative price variables are expanded by a factor of 10.

Source: Dressch, "The Demand Model," Table 11a.

* Significant at .01 level.

† Significant at .05 level.

‡ Significant at .10 level.

a. Greater than one standard error.

Table 8.11
Gross Price Equations for Households with Income over \$10,000

	Single Family						Apartment					
	Large Lot		Small Lot		Duplex-Row	Quality	Small Structure		Large Structure		High Quality	Low Quality
	High Quality	Low Quality	High Quality	Low Quality			High Quality	Low Quality	High Quality	Low Quality		
P2	6.33†	-2.40	-2.18	-0.43	0.83 ^a	-0.91 ^a	-.66 ^a	0.68	-1.24*			
P3	1.09	-2.92	1.53	-0.68	0.14	0.16	.00	0.80	-0.14			
P4	-1.50	-0.60	-0.80	-0.02	0.29	0.71†	.27	0.95 ^a	0.69†			
P5	-0.89	0.27	0.42	1.26*	-0.78*	0.41 ^a	-.02	-0.79†	0.07			
P6	0.79	1.23	-1.08	-0.04	0.12	-0.96*	.09	-0.01	-0.12			
P7	0.91	-0.04	-0.93	0.29	-0.02	0.64*	-.45†	-0.52†	0.11			
P8	-8.74*	8.01†	1.22	1.07 ^a	-0.03	0.23	.45	-2.13*	0.93†			
P9	0.53	0.10	-0.34	0.76 ^a	-0.15	0.17	.35	-0.07	-1.36			
Constant	8.62	-15.35	11.04	-9.94†	-2.50	-1.48	.15	4.16	5.15			
R ²	0.186	0.046	0.00	0.315	0.248	0.418	0.205	0.446	0.607			

Note: The true coefficients of the transformed relative price variables are expanded by a factor of 10.

Source: Dresch, "The Demand Model," Table 11b.

* Significant at .01 level.

† Significant at .05 level.

‡ Significant at .10 level.

a. Greater than one standard error.

The preceding suggests that one method of evaluating the source of error in the price coefficient equations would be to estimate equations using only travel cost as the measure of gross price. If more consistent results are obtained using travel cost, this would indicate the housing price data are a major source of error. Table 8.12 contains simple correlations between the logarithm of mean workplace residuals and the logarithm of mean monthly travel costs. All eight of the own-price, first-order correlations are negative for both income groups.⁵ Further, the cross-price coefficients in the travel time equations are also more consistent with the a priori restrictions.

A fairly high degree of association between the relative prices of close substitutes would be expected on theoretical grounds. In fact, the multicollinearity in the relative prices across workplaces was severe. Certain of the negative cross-price and positive own-price coefficients obtained for the relative prices of close substitutes may arise from these statistical problems.

Calibrating the Demand Allocation Submodel

The demand equations actually used in the Detroit Prototype are based on the gross price equations shown in tables 8.10 and 8.11. Since the Detroit Prototype employs twenty-seven housing types and four income classes whereas the equations estimated from the Detroit data use only nine housing types and two income classes, the demand equations used in the model represent an interpolation and extrapolation of the estimated results. This interpolation and extrapolation relies upon general patterns of housing demand that were revealed in the analysis of the housing markets in Detroit, San Francisco, St. Louis, and other cities. For example, the Detroit results suggest that as income increases, households become somewhat less price-responsive in their housing demand.

In expanding the Detroit estimates to form the demand equations used in the Detroit Prototype, the two income classes (\$0-\$10,000, over \$10,000) were first split into the four income classes used in the

5. Gross price equations were obtained using travel time as well. The resulting equations were at least as good as those using gross prices. This further suggests inadequate price information is responsible for a large part of our estimation problems.

Table 8.12
Simple Correlation Between Workplace-Specific Residuals
and Mean Relative Travel Time

	Low Income	High Income
Single family		
Large lot, high quality	-	-
Large lot, low quality	-.33	-.09
Small lot, high quality	-.32	-.29
Small lot, low quality	-.09	-.02
Duplex-row	-.07	-.59
Apartment		
Small structure, high quality	-.26	-.47
Small structure, low quality	-.28	-.52
Large structure, high quality	-.01	-.36
Large structure, low quality	-.19	-.75

Source: Dresch, "The Demand Model," tables 12a and 12b.

Detroit Prototype (\$0-\$5,000, \$5,001-\$10,000, \$10,001-\$15,000, more than \$15,000), and price coefficients were formed for each income class and each of the nine aggregate housing types. The nine aggregate housing types were then decomposed into the twenty-seven housing types utilized in the Detroit Prototype. In this decomposition it is assumed that the third quality class employed in the Detroit Prototype is intermediate between the two quality classes used in defining the nine aggregate housing types. In expanding the single-family units to include a size dimension, use was made of evidence such as that shown in Figure 8.10 above, which suggests that low-income households who choose single-family units are more likely to choose small units and high-income households are likely to choose larger units.

Unfortunately, experience with the Detroit Prototype suggests that its demand equations are unsatisfactory. Several peculiarities in the model's performance are directly attributable to the poor calibration of the demand allocation submodel even though the estimates derived from the Detroit data tend to support the behavioral structure of the submodel.

Summary and Conclusions

In this chapter we have discussed the theoretical rationale of the allocation submodel used in the Detroit Prototype of the NBER Urban Simulation Model and described efforts to obtain empirical estimates of the crucial submarket demand equations. These analyses provide considerable support for one of the basic behavioral premises of the NBER model; that workplace-specific variations in the gross price of various types of housing (housing prices plus monthly travel costs) systematically influence the kind of housing selected by urban households.

Because of serious data problems, efforts to estimate gross price coefficients were not fully successful. Still, in spite of their limitations, it was necessary to use these estimates to calibrate the demand allocation submodel of the Detroit Prototype. The parameters of the demand allocation submodel of the Detroit Prototype are a somewhat artful translation of the estimates described in this chapter.

In the end, however, the deficiencies of the submarket demand equations and the lack of data on the price (rent or value) of individual units prevented us from satisfactorily calibrating the Detroit Prototype. Although it might ultimately have been possible to overcome these problems, when more complete data became available for Pittsburgh we shifted model development to the latter city in order to use its richer data set.