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LOCATION, MOVERS,AND VACANCY SUBMODELS

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MODELING DEMOGRAPHIC BEHAVIOR: THE EMPLOYMENT LOCATION, MOVERS, AND VACANCY SUBMODELS

THE DIMENSIONS of the NBER model, which were enumerated in the last chapter, resulted in a very large number of possible combinations of household classes, dwelling unit types, residence zones, and workplaces. Perhaps the major obstacle encountered in transforming the model's design to a workable simulation model was that an array of all possible combinations, i.e., one with 1,625,184 cells, was too large to be accommodated in any generally available computer. And even if such a computer were accessible, the cost of operating the model would have been prohibitive. Thus, the challenge faced in implementing the model was to develop methods of retaining most of the information contained in the large array without actually maintaining the array itself.

In the model this is accomplished by having it carry several smaller arrays which are marginal summaries of the information in the large array. For example, households are indexed only by household class and workplace in the model; so it is impossible to know precisely where a given household lives or in what dwelling unit type. However, by using distributions of household classes over dwelling unit types and distributions of workers by workplace over residence zones, a distribution of households by workplace over dwelling types and residence zones can be calculated. This method of using several marginal distributions to generate more comprehensive probability distributions is used in the supply, demand, and price formation sectors of the model.

In order to give the reader a more detailed understanding of the over-all model and its operation, we describe how the employment location, movers, and vacancy submodels work individually and interact with the other submodels. The submodels are described in the order in which they appear in the over-all model (shown in Figure 3.1, above).

The Employment Location Submodel

The location of employment has a long-standing place of importance in studies of residential land use in urban areas. In most modern land-use theories, residential densities and location rents are functions of employment location and travel cost.[1] Employment location has also been a significant determinant of residence location in many residential land use models.[2] Although the NBER Urban Simulation Model continues in this tradition, all changes in employment location are currently provided as exogenous forecasts to the over-all model. Exigencies of model development have thus far prevented us from making population-serving employment endogenous. In addition, the development of a behavioral model of employment location for basic industries, a major theoretical and empirical challenge, has been avoided in this work. Studies of industry location are, however, a major component of the NBER Urban Studies program.

We have been carrying out extensive analyses of the processes which determine the location of employment within metropolitan areas.[3] The elements of these processes include moves by establishments from one location to another within the region, migration to and from the region, establishment births and deaths, and expansion and contraction of employment in existing establishments.[4] These analyses, based on linked files of individual

1. Alonso, *Location and Land Use;* Wingo, *Transportation and Urban Land.*

2. Lowry, *Model of a Metropolis;* idem, "Seven Models of Urban Development"; Brown et al., *Empirical Models.*

3. Struyk and James, "Intrametropolitan Industrial Location"; and Leone, *Manufacturing in New York.*

4. To the best of our knowledge there have been only two previous significant analyses of the determinants of intrametropolitan manufacturing employment based on micro data for individual establishments. These are: Moses and Williamson, "Economic Activity in Cities"; and Creamer, *Manufacturing Employment.*

manufacturing establishments from Dun and Bradstreet's Dun's Market Identifier file, permit much more detailed analyses of the determinants of industry location than have heretofore been possible.[5] Using these data, Leone has completed a detailed analysis of changes in the level and spatial distribution of employment within the New York metropolitan area between July 1967 and August 1969. Similarly, Raymond J. Struyk and Franklin James have completed comparable analyses of changes in the location of manufacturing employment in the Cleveland, Minneapolis–St. Paul, Boston, and Phoenix metropolitan areas between 1965 and 1968.

One important finding of both the Leone and Struyk-James studies is the high rate of mobility of manufacturing firms. Leone found that almost 10 per cent of all manufacturing jobs in the New York metropolitan area moved in the two-year period he considered. Struyk and James's analyses indicate that similar proportions of manufacturing employment in the four cities studied relocated in the three-year period 1965–68.

These rates of mobility are much higher than we anticipated and suggest the spatial distributions of employment are quite fluid. They also suggest that behavioral analyses of the location decisions of basic industry may be feasible, since firm location decisions occur more frequently than we had believed. For example, in New York, 4,500 establishments representing over 100,000 jobs moved within the region in only two years. In addition, nearly 3,000 manufacturing establishments closed, and over 4,000 manufacturing establishments began operation.[6]

Although these analyses have supplied detailed descriptions of the spatial patterns of growth, relocation, births, and deaths of manufacturing establishments in several cities, our studies of industrial location are still some distance from yielding behavioral models of location for basic employment. Therefore, the employment location submodel presently is designed to transform forecasts of employment changes during each simulated period into spatial distributions of workers by socioeconomic and demographic characteristics, and to simulate a number of demographic changes

5. See Leone, "Workplace Location Studies."
6. Leone, *Manufacturing in New York.*

in the characteristics of existing labor forces. The studies of employment location currently being carried out are of considerable value in formulating meaningful employment change scenarios for use with the NBER Urban Simulation Model, and at some future time they may enable us to develop an endogenous basic employment submodel.

In contrast to basic employment, it is relatively easy to make population-serving employment endogenous, and many other models have done this.[7] Even so, neither the Detroit Prototype nor Pittsburgh I, the second version of the model, incorporate an endogenous population-serving employment submodel. In the Detroit Prototype primary employment is represented by industry and workplace zone. Although nine industries are used in the Detroit Prototype (Table 5.1), this number could easily be enlarged, since industry detail is not costly to maintain in the model in terms of either storage or running time.

Spatial and industry detail is represented in the model in a manner that permits sensitivity analyses to be carried out easily for several different kinds of employment scenarios. First, employment patterns can be altered solely on the basis of work zones. This feature allows the model to be used to evaluate questions about the effects of a revitalization of work opportunities in the central city or the effects of an extensive shift of industry to the suburbs. Second, employment distributions can be altered on an industry level; e.g., the wholesaling and retailing industry can be shifted to suburban locations while durable manufacturing is retained in its present location. And finally, employment can be changed by skill level (the education and income of workers) in industries or at specific workplace zones.

The name "employment location submodel" is somewhat misleading, since this portion of the computer program performs a number of other vital functions as well. For example, during each time period this submodel transforms the new levels of employment

7. See Brown et al., *Empirical Models*, for descriptions of how such employment is handled in several models. Many techniques for forecasting the location of population-serving employment are discussed in the literature. See Niedercorn and Kain, "Food and General Merchandise Stores"; Berry and Pred, *Central Place Studies, Bibliography;* Berry, "Commercial Structure."

by zone and industry into a manpower requirements matrix, *MAN(J, HY, HED)*, by multiplying the number of jobs by industry times the worker characteristics matrix shown in Table 6.1. This matrix multiplication is of the form

$$MAN(J, HY, HED) = JOB(J, IND) * SICMAN(IND, HY, HED); \quad (6.1)$$

where:

MAN(J, HY, HED) =	manpower requirements by workplace, income class, and equation class;
JOB(J, IND) =	number of jobs of primary workers by workplace and industry;
SICMAN(IND, HY, HED) =	worker characteristics matrix by industry, income class, and education class.

The worker characteristics matrix was estimated from the Detroit TALUS data. It represents the average distribution of each industry's employees over their income and education characteristics.

Manpower requirements are grouped only by workers' income and education classes, rather than by the complete set of household characteristics, which includes family size and the age of the household head. Income and education are used because they reflect job and industry requirements, as well as worker productivity, better than the omitted characteristics do. Maintaining information on the family size and age of household head by industry would increase the running time and storage requirements of the submodel without providing enough useful information to make the additional costs worthwhile.

In using the worker characteristics matrix, it is assumed that the manpower composition of industries in terms of income and educational levels is constant over time, i.e., it is a distribution with fixed coefficients. There are two ways of relaxing this assumption within the model. First, the coefficients of the worker characteristics matrix can be modified over time to reflect anticipated technological changes in an industry. Second, when the income and educational

Table 6.1
Worker Characteristics Matrix (*SICMAN* Table)
(proportion of households in each income class)

Industry Group	Income of Workers Having Low Education (0–11 years)				Income of Workers Having High Education (12+ years)			
	$5,000 or Less	$5,001 to $10,000	$10,001 to $15,000	$15,000+	$5,000 or Less	$5,001 to $10,000	$10,001 to $15,000	$15,000+
1	.309	.239	.027	.101	.069	.153	.027	.096
2	.097	.316	.073	.100	.017	.221	.114	.063
3	.105	.314	.045	.065	.030	.257	.098	.085
4	.063	.355	.071	.077	.021	.246	.103	.065
5	.069	.258	.057	.088	.029	.321	.106	.071
6	.186	.215	.043	.092	.078	.209	.080	.097
7	.104	.116	.040	.060	.068	.305	.145	.161
8	.272	.198	.046	.111	.072	.162	.063	.076
9	.129	.115	.039	.058	.083	.305	.145	.126

levels of the jobs introduced in a zone are known, jobs may be introduced directly into the manpower requirements matrix. This capability allows, for example, the characteristics of jobs in the central city to be changed to reflect a disproportionate growth in white collar–professional employment. Programs which would provide jobs for low-skilled workers in specified parts of the city could be simulated in a similar manner. To facilitate the simulation of such work-zone-specific phenomena, the worker characteristics matrix (*SICMAN*) can also be varied by work zone.

The output of the employment location submodel, the manpower requirements matrix, *MAN*(*J, HY, HED*)—a summary cross tabulation of the total number of employees in each workplace zone for this time period—is used as input to the second and third major submodels, the movers and vacancy submodels.

The Movers Submodel

The movers submodel identifies for the larger model those households that will demand housing during the year. The aggregate of all households seeking housing during the time period, termed "demanders," is composed of intrametropolitan movers, new households, and migrants from other regions.

In addition to identifying those households that will demand units during the year, the movers submodel also simulates a number of important aspects of demographic change. Many demographic changes occur in the metropolitan population with the passage of time: workers become older, they marry, they have families, they separate and divorce, they become still older, they retire, they die. These demographic processes, which have significant impacts in both labor and housing markets, are represented in the movers submodel in a rudimentary form.

Identifying Demanders

The NBER model has a strong workplace orientation in that it assumes that all households begin to search for a place to live only after determining their place of work. As we discussed in Chapter 4, this assumption is somewhat unrealistic since for some households, workplace and residence location are almost certainly chosen simultaneously and for others, such as blacks, the opposite causal assumption may be more correct. But the additional information required to model a simultaneous system of residence-workplace determination would increase the size and complexity of the model prohibitively, and there is little doubt that workplace location strongly influences the residential choices of a large fraction of households.

Because of the workplace-dominance assumption, the determination of the number of demanders by socioeconomic characteristics begins with a provisional forecast of the number of movers grouped by household class, at each workplace at the start of each simulated year. This basic mobility forecast represents changes in residences from many potential causes, e.g., life-cycle changes, neighborhood changes, changes in income, and job changes. These provisional estimates of mobility by workplace and household class are made independently of changes in the distribution of employment which occur within the modeled area during the year and are based on average moving rates by household class for the entire region.

The total number of workers in each household class employed at each workplace at the end of the previous simulation year is used as the starting point for the basic mobility forecast. Moving rates for each household class, $RATE(H)$, are multiplied by the number of households to provide a provisional estimate of the number of movers by household type and workplace. Equation 6.2 illustrates this computation.

$$PROVIS(J, H) = RATE(H) * F(J, H); \qquad (6.2)$$

where:

$PROVIS(J, H)$ = basic mobility forecast—provisional estimate of number of movers (no employment change);
$RATE(H)$ = relocating rates for each household class H;
$F(J, H)$ = the number of households at each work zone J in each household class H.

The mobility rates used in equation 6.2, shown in Table 6.2, consist of three components: (1) rates of intrametropolitan moving by

Table 6.2
Basic Mobility Rates for 72 Household Classes

Age (yrs.)	0–11 Years' Education by Income Class (thousands of dollars)				12 Years' Education or More by Income Class (thousands of dollars)			
	0 to 5	5+ to 10	10+ to 15	15+	0 to 5	5+ to 10	10+ to 15	15+
Family Size: 1-2 Persons								
0–30	.254	.282	.281	.199	.290	.363	.310	.299
31–60	.133	.161	.153	.152	.208	.198	.192	.199
60+	.176	.173	.170	.170	.153	.175	.174	.172
Family Size: 3–4								
0–30	.227	.296	.250	.248	.270	.360	.280	.278
31–60	.133	.141	.137	.123	.188	.191	.178	.140
60+	.170	.155	.140	.139	.150	.165	.163	.156
Family Size: 5 Persons or More								
0–30	.191	.273	.242	.193	.240	.338	.275	.235
31–60	.150	.187	.155	.138	.170	.203	.175	.161
60+	.140	.135	.140	.135	.130	.152	.148	.140

Source: Computed from rates shown in Table 6.3, data on migration from the 1960 *Census of Population*, and mortality tables.

household class, (2) out-migration rates estimated from the 1960 Census of Population by age categories, and (3) household dissolution rates estimated from mortality tables. Intrametropolitan moving rates for age, income, education, and family size classes, computed from a sample of 3,000 San Francisco households, are shown in Table 6.3.

In the absence of changes in the level and composition of employment by workplace, it would be reasonable to assume that the socioeconomic composition of the labor force of each workplace would remain relatively constant over time. Under these circumstances the basic mobility forecast by workplace and household class provided by equation 6.2 would provide reasonably satisfactory estimates of the numbers and kinds of households that will seek new residences during the year. However, stability of this kind hardly characterizes cities over the long time spans which the model is intended to simulate. Therefore, the number of housing demanders by workplace is estimated by modifying the basic mobility forecasts to reflect shifts in demand among workplaces resulting from

Table 6.3
Intrametropolitan Moving Rates for 72 Household Classes

Age (yrs.)	0–11 Years' Education by Income Class (thousands of dollars)				12 Years' Education or More by Income Class (thousands of dollars)			
	0 to 5	5+ to 10	10+ to 15	15+	0 to 5	5+ to 10	10+ to 15	15+
			Family Size: 1–2 Persons					
0–30	.348	.363	.363	.288	.200	.431	.335	.367
31–60	.103	.116	.110	.099	.165	.151	.141	.141
60+	.055	.055	.050	.000	.038	.057	.074	.074
			Family Size: 3–4 Persons					
0–30	.194	.350	.291	.400	-	.333	.308	.392
31–60	.112	.099	.089	.076	.182	.141	.117	.090
60+	.056	.035	.027	.042	.000	.037	.024	.020
			Family Size: 5 Persons or More					
0–30	.167	.298	.255	.152	.000	.246	.247	.154
31–60	.148	.113	.097	.081	.182	.112	.084	.103
60+	.071	.045	.000	.000	-	-	.000	.000

Source: Computed from Bay Area Transportation Study supplementary home interview survey.

workplace-specific increases and decreases in employment over time, the birth and death of firms, and the growth of employment and population over time.[8]

The shifts in the socioeconomic characteristics and size of the labor force at each workplace are based on the changes in employment by industry at each workplace provided by the employment location submodel. If there is no change in employment of a given household class at a particular workplace, the final number of demanders is equal to the basic mobility forecast obtained from equation 6.2, i.e., *PROVIS*(*J*, *H*). But if the employment of a particular household class increases or decreases at a given workplace, the number of demanders of that class must be increased or decreased to reflect these employment changes during the simulation year.

The total number of demanders of a particular household class at a particular workplace, *PMOVE*(*H*, *J*), given by equation 6.3, is the

8. The NBER studies of industry location by Leone and by Struyk and James underline the importance of these changes.

basic mobility forecast, *PROVIS(H, J)*, obtained from equation 6.1, plus additions resulting from the growth of employment, *ADDITIONS(H, J)*, minus losses resulting from employment declines, *LOSSES(H, J)*.

$$PMOVE(H, J) = PROVIS(H, J) + ADDITIONS(H, J) - LOSSES(H, J); \tag{6.3}$$

where:

PMOVE(H, J) = number of demanders by household type and workplace location for use in the demand and allocation models;

ADDITIONS(H, J) = additional demanders by household class and workplace resulting from workplace-specific employment increases by income and education classes;

LOSSES(H, J) = reduction in demanders by household class and workplace resulting from workplace declines by income and education classes.

Translation of the employment increases and decreases by workplace and industry into the final estimates of the number of household types and workplaces shown in equation 6.3, is complicated because the employment location submodel produces changes in employment at each workplace only by income and education classes while equation 6.3 also requires a breakdown by age and family size. In adding age and family size to the household descriptions, the model treats employment increases and decreases separately. Employment increases at a workplace are expected to attract workers who are younger than other employees at the workplace. On the other hand, employment declines are assumed to affect the oldest workers the most and be less severe for middle-aged workers than for younger ones.

The number of additional demanders (*ADDITIONS*) by age, family size, income, and education that result from an increase in employment is depicted in equation 6.4. The weights, *ADD RATE(H)*, used in converting increases in employment by income and education class to additional demanders during the period are shown in Table 6.4.

Table 6.4
ADD RATES Used in Estimating Socioeconomic Characteristics of New Workers, 72 Household Classes

Annual Income (000 dollars)	Head 0–30 Years by Family Size (number of persons)			Head 31–60 Years by Family Size (number of persons)			Head 60+ Years by Family Size (number of persons)		
	1 to 2	3 to 4	5+	1 to 2	3 to 4	5+	1 to 2	3 to 4	5+
				Education: 0–11 Years					
0 to 5	.30	.19	.03	.07	.26	.07	.07	.01	.00
5+ to 10	.20	.20	.05	.08	.27	.10	.07	.03	.00
10+ to 15	.12	.12	.11	.08	.46	.14	.05	.02	.00
15+	.03	.06	.01	.09	.52	.17	.06	.06	.00
				Education: 12 Years or More					
0 to 5	.52	.28	.05	.08	.07	.00	.00	.00	.00
5+ to 10	.26	.22	.07	.08	.24	.11	.01	.01	.00
10+ to 15	.20	.14	.04	.08	.33	.16	.02	.03	.00
15+	.11	.10	.01	.19	.38	.14	.03	.04	.00

Source: Estimated from data obtained from Bay Area Transportation Study supplementary home interview survey.

$$ADDITIONS(H, J) = ADD\ RATE(H) * EMPLOY\ INCREASE(HED, HY, J); \quad (6.4)$$

where:

ADD RATE(H) = weights for converting projected increases in employment by income and education class into increases by income, education, family size, and age classes;

EMPLOY INCREASES (HED, HY, J) = projected increases in employment by education, and income classes and workplace.

The *ADD RATES* in Table 6.4 reflect the fact that new workers added to a workplace as a result of an increase in employment will tend to be younger and have smaller families than either all workers or all movers. This tendency is assumed to be more pronounced for workers having less income and education. Thus, only 11 per cent of workers with more than $15,000 income are less than thirty-one years old and belong to families with one or two persons. By comparison 30 per cent of new workers with less than a high school education and earnings of $5,000 or less per year fall into this same age and family size category.

The reduction in housing demand at a particular workplace caused by employment declines at that workplace is simulated by subtracting the number of workers who have lost jobs at that workplace from the basic mobility forecast for that household class and workplace. The implicit assumption of this procedure is that all job losses are effected by not replacing workers involved in normal turnover. Some of these workers will appear in the model as demanders at other workplaces which have counterbalancing employment increases in the same household classes. Others will remain at the same workplace as a result of life cycle, demographic, and other changes not explicitly included in the model. Others will leave the area for jobs in other regions. Still others will leave the labor force.

If the decline in employment in a particular income-education category is unusually large, the basic mobility forecast may provide too few provisional movers in that category. For this reason, the movers submodel must first determine whether the basic mobility forecast for each income-education category is at least as great as the category's projected decline. If it is not, the basic mobility forecast is augmented for all household classes in the requisite income-education category until the total in the category equals the number of movers needed:

Increase *PROVIS(H, J)* until

$$\sum_{HAG,HFS} PROVIS(H, J) \geq EMPLOY\ DECLINE(HED, HY, J); \quad (6.5)$$

where:

HAG, HFS = age and family size categories;

EMPLOY DECLINE(HED, HY, J) = employment declines by income-education category and workplace.

Then, as illustrated by equation 6.6, *LOSS RATES* (shown in the first row of Table 6.5) are multiplied by the group of provisional demanders to determine which are eligible for job loss.

Table 6.5
LOSS RATES: Selected Age for Employment Declines

Iteration	Age Group 0 to 30	31 to 60	60+
First	0.52	0.28	1.00
Second	0.72	0.50	1.00
Third	0.80	0.70	1.00
Fourth	1.00	1.00	1.00

Source: Estimated from data obtained from Bay Area Transportation Study supplemental home interview survey.

$$ELIGIBLE(J, H) = PROVIS(J, H) * LOSS\ RATE(HAG); \quad (6.6)$$

where:

$LOSS\ RATE(HAG)$ = weights shown in Table 6.5;
$ELIGIBLE(J, H)$ = households eligible for losing jobs this period.

If the first set of weights provides too few eligible candidates from among the provisional demanders to satisfy the employment decline, the basic mobility forecasts—*PROVIS(J, H)*—are multiplied by the second or third set of weights shown in Table 6.5. The fourth and final set of weights makes all provisional demanders eligible for job losses (all weights equal unity). Since the model determined, in equation 6.5, that the basic mobility forecast provided sufficient moves to accommodate the projected job loss in the income-education class, the fourth set of weights must provide enough workers in the age and family size categories to satisfy the projected employment decline. When the weighting system provides more households eligible for job losses than are required, the job losses are allocated proportionally among eligible households.

The Vacancy Submodel

The function of the vacancy submodel is to identify dwelling units which will be available for occupancy during the simulation year. These consist of vacant units remaining from the previous model period as well as units vacated this period by intrametropolitan movers, out-migrants, and defunct households.

The level of housing demand and the number of available vacant units are obviously interrelated because intrametropolitan movers represent a large fraction of all households who seek housing during each year, and their former residences constitute a large fraction of each year's available housing supply. Therefore, the vacancy model relies heavily on the basic mobility forecast defined by equation 6.2 and altered by equation 6.5 of the movers submodel. Vacated units are identified by using this information in a process which matches moving households with their previously occupied units. Units which are vacated in the current year or period are stored in an array, *AVAIL(K, I)*, indexed by unit type, *K*, and residence zone, *I*. Vacancies remaining from the previous period's simulation are also identified in this way.

The basic mobility forecast is the starting point for identifying the number of available units by structural type and residence zone because it represents households who are vacating units in the housing stock. These estimates are employed rather than *PMOVE(H, J)*, the estimates of the housing demanders from equation 6.3, because the latter includes new households and in-migrants, neither of which leave vacancies in the stock. In contrast, all households included in the basic demographic forecast, *PROVIS(H, J)*, occupy units at the beginning of the period.

The translation of the basic mobility forecast by household class and workplace, *PROVIS(H, J)*, to available units by housing type and residence location, *AVAIL(K, I)*, is done in two steps, First, as equation 6.7 illustrates, the model obtains estimates of the kinds of structures vacated by movers at each workplace. Then, equation 6.8 is used to distribute the vacancies created by movers among the residence zones. Available units, *PAVAIL(J, K, HY)*, are distributed among residence zones in proportion to weights which reflect work-trip patterns and the shares of each housing type found in each residence zone.

$$\begin{aligned} PAVAIL(J, K, HY) = {} & OCC\ RATE\ 2(J, K, HAG, HFS) \\ & * OCC\ RATE\ 1(J, K, HY) \\ & * PROVIS(J, H); \end{aligned} \qquad (6.7)$$

where:

$OCC\ RATE\ 1(J, K, HY)$ = proportion of each income class HY residing in housing type K at workplace J at the end of the previous period;

$OCC\ RATE\ 2\ (J, K, HAG, HFS)$ = proportion of each family size, HFS, and age class, HAG, residing in housing type K at each workplace J at the end of the previous period.

$$AVAIL(K, I) = \frac{PAVAIL(J, K, HY)}{POTENTIAL(J, K, HY)} * TRIP(I, J, HY) * STOCK(K, I) + VACANT(K, I); \qquad (6.8)$$

where:

$AVAIL(K, I)$ = number of available units of each housing type K in each residence zone I;
$STOCK(K, I)$ = number of occupied units of each type K in each residence zone I;
$TRIP(I, J, HY)$ = number of trips from workplace J to zone I for income class HY;
$VACANT(K, I)$ = number of units available but not occupied in previous period;
$POTENTIAL(J, K, HY) = \sum_I [TRIPS(I, J, HY) * STOCK(K, I)]$.

One important theoretical consideration that influences the method of transforming the estimates of vacant units classified by household characteristics and workplace (equation 6.7) to estimates of vacant units classified by housing type and workplace (equation 6.8) is the knowledge that workers with the same household characteristics will select different kinds of housing depending on where they work. The method of assigning housing types represented by equation 6.7 illustrates one of the many necessary compromises found in the model between what would be desirable theoretically and what is feasible from the viewpoint of over-all model design. Ideally we would maintain information on household class, housing type, and place of residence for each workplace. If such a matrix were feasible to construct, vacancies could be generated easily. For example, movers of a given household type and workplace location could produce vacancies of a specific housing type in proportion to the share of the household-workplace group living in that housing type. Unfortunately, the matrix suggested above has roughly 86,000 cells for each workplace, or nearly 1.7 million for the Detroit Prototype. Since such a matrix is not feasible given present computer technology, some aggregation of the cells in this matrix is necessary.

After considerable experimentation, it became obvious that two transformation matrices, *OCC RATE* 1(*J, K, HY*) and *OCC RATE* 2(*J, K. HAG, HFS*) in equation 6.7 based on income, age, and family size were needed to estimate the kinds of housing vacated by moving households. Including some household characteristics in these matrices helps account for differences in the mix of household classes by work zone. The transformation matrices are calculated from corresponding-state matrices which record the number of households in each class. Since these state matrices are revised by the vacancy and demand allocation submodels each period, the transformation matrices also change over time.

Some Further Demographic Considerations

The over-all simulation model represents the population of the modeled area as a matrix of households indexed by household class and workplace—the *F*(*J, H*) matrix in equation 6.2. We have described the manner in which the model simulates the effect of shifting employment patterns on the composition of the population by workplace. There are, however, other demographic processes which modify the composition of the population over time.

In both the population matrix of equation 6.2 and the *OCC RATE* arrays of equation 6.7, households are not classified by their current household characteristics, but rather by the characteristics they possessed when they chose their current residences. Programming considerations make it most economical to change a household's characteristics only when it moves from its former residence. Therefore, after the number of demanders is determined from equation 6.3, the age and family size of these households are systematically altered by equation 6.9 to reflect demographic changes in household characteristics that are responsible for much intrametropolitan mobility:

$$RMOVE(H, J) = CHAR(H) * PMOVE(H, J); \qquad (6.9)$$

where:

$RMOVE(H, J)$ = housing demanders classified by characteristics at time of move;
$PMOVE(H, J)$ = housing demanders classified by old characteristics;
$CHAR(H)$ = demographic change matrix—modifies age and family size of demanders.

The empirical bases for the life-cycle adjustments in *CHAR*(*H*) are San Francisco data on intrametropolitan mobility. These data were used to estimate a transformation matrix that assigns "new" household characteristics to demanders of each initial household class. These revised estimates of the number of demanders by household class and workplace are then used as an input to the demand allocation submodel.

This method of representing demographic processes in an urban area presents two difficulties. First there is the problem of representing the alteration of the population caused by in- and out-migration; and second, there is a need to account for changes in population characteristics resulting from household deaths and formations. The migration adjustments were dealt with using Census data to increase the moving rates, *RATE*(*H*), of certain household classes to reflect their higher intrametropolitan mobility rates. Representing household deaths and formations requires a somewhat more complicated procedure.

To account for household deaths and formations, household moving rates were increased on the basis of mortality data to include defunct households in the basic mobility forecast. This adjustment allows the model to vacate the appropriate types of dwelling unit formerly occupied by such households. When the array of households that will demand units during the year, *RMOVE*(*H, J*), is formed from *PMOVE*(*H, J*), defunct households are replaced by families whose household characteristics reflect in aggregate the characteristics of newly formed and in-migrating households. The model does not require that a defunct household in a given area be replaced by a newly formed household or one migrating into the region. Instead, a defunct household may well be replaced by a household in which the head is already employed somewhere else in the region. To the extent that intrametropolitan moving reflects this type of "replacement," its effect will already have been picked up in *RATE*(*H*) and subsequently offset by the generation of demanders. All that is assumed in the model is that on a net basis defunct households will be replaced by a given distribution of household types. It does not limit a priori the extent of gross flows generated in this process.

Bookkeeping and Other Tasks

In addition to their principal functions of forming arrays of available dwelling units and housing demanders, the movers and vacancy submodels perform some essential bookkeeping tasks for the over-all simulation model. For example, the model maintains an interzonal travel matrix which gives the number of trips made between each workplace and residence zone by income class. When households change their workplace, they obviously alter their commuting pattern as well. During each simulation year 15 to 25 per cent of households will in fact change their travel patterns. To represent these changes, the trips of households that vacate units must be subtracted from the previous year's interzonal travel matrix as shown in equation 6.10. When households that demand housing units during this simulation period pick new residential locations, the trips resulting from their revised workplace and residence choices are added to the travel matrix:

$$REV\ TRIPS(I, J, HY) = TRIP(I, J, HY) - PROVIS\ TRIPS(I, J, HY); \quad (6.10)$$

where:

$REV\ TRIPS(I, J, HY)$ =	revised trip pattern by residence, workplace, and income class; includes only nonmoving households;
$TRIP(I, J, HY)$ =	trip pattern of all households at beginning of period;
$PROVIS\ TRIPS(I, J, HY)$ =	trips of households that vacate units this period.

The movers and vacancy submodels accomplish the first half of this process by subtracting the number of trips by workplace, residence zone, and income class that is consistent with the processes of mobility simulated by equation 6.8 of the model. The demand allocation and market-clearing submodels subsequently accomplish the second half by augmenting the interzonal travel matrix to reflect the new workplace and residence choices made by households during the simulation year.

The movers and vacancy submodels also perform a variety of bookkeeping functions which update descriptions of the housing stock and population of households from one simulation period to another. Each of these tasks involves subtracting moving households or vacated dwelling units from inventories of households and dwelling units generated at the end of the previous simulation period. Other submodels then add new households and new housing units to reflect the changes occurring during the current simulation period.