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# III

# THE PURE CONSUMPTION MODEL

In the previous chapter, I explored the effects of variations in the shadow price of health in the context of the pure investment model. These variations were traced in turn to shifts in the rate of depreciation, market efficiency, and nonmarket efficiency. In this chapter, I utilize the pure consumption model to examine the effects of age, the wage rate, and education.[1] The purpose of the analysis is to indicate the major differences between the investment and consumption models rather than to develop the latter in detail. Consequently, a formal presentation of the consumption model, including derivations of all formulas, has been relegated to Appendix C.

## 1. LIFE CYCLE PATTERNS

If the cost of capital were large relative to the monetary rate of return on an investment in health and if $\tilde{\pi}_{i-1} = 0$, all $i$, then equation (1-13) could be approximated by

$$\frac{Uh_iG_i}{\lambda} = \frac{UH_i}{\lambda} = \frac{\pi(r + \delta_i)}{(1 + r)^i}. \tag{3-1}$$

Equation (3-1) indicates that the monetary equivalent of the marginal utility of health capital in period $i$ must equal the discounted user cost of $H_i$.[2] Division of the equilibrium condition for $H_{i+1}$ by the equilibrium

[1] The model of life cycle behavior presented in this chapter is similar to Gilbert R. Ghez's analysis of life cycle demand for durable consumer goods. See "A Theory of Life Cycle Consumption," unpublished Ph.D. dissertation, Columbia University, 1970, Chapter I.

[2] Solving equation (3-1) for the monetary equivalent of the marginal utility of healthy time, one has

$$\frac{Uh_i}{\lambda} = \frac{\pi(r + \delta_i)/G_i}{(I + r)^i},$$

where $\pi(r + \delta_i)/G_i$ is the undiscounted price of a healthy day. Given diminishing marginal productivity of health capital, this price would be positively correlated with $H$ or $h$ even if $\pi$ were constant. Therefore, the consumption demand curve would be influenced by scale effects. To emphasize the main issues at stake in the consumption model, I ignore these scale effects essentially by assuming that $\phi_i$ and hence $G_i$ are constant. The analysis would not be greatly altered if they were introduced.

condition for $H_i$ generates the basic equation for the analysis of life cycle demand:

$$\frac{UH_{i+1}}{UH_i} = (1 + r)^{-1}\left(\frac{r + \delta_{i+1}}{r + \delta_i}\right). \tag{3-2}$$

Condition (3-2) simply states that the marginal rate of substitution between $H_i$ and $H_{i+1}$ must equal the ratio of the discounted user cost of $H_{i+1}$ to the discounted user cost of $H_i$.

To characterize the life cycle path of health capital in a precise manner, certain restrictions must be placed on the utility function. In particular, it must be assumed that this function is weakly separable in $H_i$ and $H_{i+1}$. That is, the marginal rate of substitution between $H_i$ and $H_{i+1}$ depends only on these two stocks and is independent of the other $H$'s and all other commodities. It is also assumed for the present that there is no "health time preference." This means that $UH_{i+1}/UH_i = m = 1$ when $H_{i+1} = H_i$. Of course, since indifference curves are convex to the origin, a reduction in $H_{i+1}$ relative to $H_i$ along a given indifference curve would increase $UH_{i+1}$ relative to $UH_i$.

Suppose the rate of interest were zero and the rate of depreciation were independent of age. Then the discounted user cost ratio and, therefore, the marginal rate of substitution between $H_i$ and $H_{i+1}$ would equal unity. Given no time preference, this implies $H_i = H_{i+1}$. If the rate of interest were positive, the discounted user cost ratio and, hence, $UH_{i+1}/UH_i$ would be less than unity. Convexity of indifference curves implies $H_{i+1} > H_i$ in this situation. Therefore, under the stated conditions of no time preference and constant depreciation rates, $H_i$ would rise over time if $r > 0$ and would be stationary if $r = 0$. Either of these two life cycle patterns suggests that individuals would choose to live forever.

As in the investment model, a positive correlation between the rate of depreciation and age generates a stock of health pattern that is consistent with a finite life. To see this, first let the rate of interest equal zero. Then the discounted user cost ratio would equal $\delta_{i+1}/\delta_i$, which is clearly greater than one. It follows that $UH_{i+1}/UH_i$ must exceed one, and this implies $H_{i+1} < H_i$. Thus, the stock of health would fall throughout the life cycle because the price of the next period's health in terms of its present period price is always greater than one.

Now let the rate of interest be positive. In this case, $H_{i+1}$ might exceed $H_i$ even if $\delta_{i+1} > \delta_i$. But if $\delta_i$ grew at a constant rate, the discounted user cost ratio would rise over time.[3] Since this price ratio increases over time, so must the marginal rate of substitution between $H_{i+1}$ and $H_i$. Convexity of indifference curves dictates that $H_{i+1}/H_i$ must fall with age. Health capital might increase for a while but would peak when the depreciation effect began to outweigh the interest rate effect. After the peak is reached, $H_i$ would decline until death is "chosen."

A formula for the percentage rate of change in health capital over the life cycle is given by[4]

$$\tilde{H}_i = \sigma[\ln m + \ln(1 + r) - s_i\tilde{\delta}]. \tag{3-3}$$

In this formula, $\sigma$ is the elasticity of substitution in consumption between $H_i$ and $H_{i+1}$:

$$\sigma = \frac{\partial(\ln H_i/H_{i+1})}{\partial(\ln UH_{i+1}/UH_i)}.$$

Equation (3-3) includes a time preference effect as well as interest and depreciation effects. If there were time preference for the present, the marginal utility of $H_i$ would exceed the marginal utility of $H_{i+1}$ when $H_i = H_{i+1}$. Hence $\ln m < 0$, and $H_i$ would fall faster and death would occur sooner given preference for the present. On the other hand, preference for the future makes $\ln m > 0$ and prolongs the time interval during which $H$ increases. Equation (3-3) also indicates that $H_i$ reaches its maximum quantity when $\ln m + \ln(1 + r) = s_i\tilde{\delta}$.[5]

Although the demand for health capital declines after some point in the life cycle, gross investment would tend to be positively correlated with age if the elasticity of substitution between present and future health were less than unity.[6] Put differently, if present and future health were relatively poor substitutes, individuals would have an incentive to

[3] If $\tilde{\delta}$ were constant, then the derivative of the natural logarithm of the discounted user cost ratio would be

$$\tilde{\delta}\left[\frac{r(\delta_{i+1} - \delta_i)}{(r + \delta_i)(r + \delta_{i+1})}\right] > 0,$$

since $\delta_{i+1} > \delta_i$.

[4] For a derivation of equation (3-3), see Appendix C, Section 1.

[5] When $\ln m + \ln(1 + r) = s_i\tilde{\delta}$, $\tilde{H}_i = 0$. This stationary point gives a maximum since $\tilde{H}_{ii} < 0$:

$$\tilde{H}_{ii} = -s_i(1 - s_i)\sigma\tilde{\delta}^2.$$

The formula for $\tilde{H}_{ii}$ assumes $\sigma$ and $\tilde{\delta}$ are constant.

[6] For a proof, see Appendix C, Section 1.

offset part of the reduction in health caused by an increase in the rate of depreciation by increasing their gross investments. In fact, there is reason to believe that the elasticity of substitution is relatively small, at least in the vicinity of the death stock. To see why this is the case, redefine $\sigma$ as

$$\sigma = \frac{\partial(\ln H_i'/H_{i+1}')}{\partial(\ln UH_{i+1}'/UH_i')},$$

where $H_i' = H_i - H_{\min}$.[7] I have shown that $H_i'/H_{i+1}'$ rises with age, which increases $UH_{i+1}'/UH_i'$. Since $UH_{i+1}'/UH_i' \to \infty$ as $H_{i+1}' \to 0$ and since this condition must be satisfied at death, small increases in $H_i'/H_{i+1}'$ must have large effects on the ratio of marginal utilities around the death age.[8]

In both the consumption and investment models, biological factors associated with aging cause individuals to substitute away from future health until death is chosen. There are two major differences between the two models. First, even if the depreciation rate rises continuously with age, the existence of time preference for the future or a positive rate of interest might cause health capital to increase for a while in the consumption model. Second, the elasticity of substitution between present and future health, rather than the elasticity of the MEC schedule, determines (1) the responsiveness of health to a change in the rate of depreciation and (2) the life cycle behavior of gross investment.

## 2. MARKET AND NONMARKET EFFICIENCY

To study the effects of variations in the shadow price of health among individuals of the same age, a cross-sectional consumption demand curve must be specified. The simplest specification is

$$H = H(R^*, Q^*), \tag{3-4}$$

where $R^* = R/Q$, real full wealth; $Q^* = \pi(r + \delta)/Q$, the relative user cost or shadow price of health; $\ln Q = w \ln \pi(r + \delta) + (1 - w) \ln q$, the natural logarithm of a weighted geometric price level of health and the

[7] This definition implies the death time utility function is

$$U = U(\phi_i H_i', Z_i).$$

If both consumption and investment aspects of the demand for health were considered, $H_i'$ and $Z_i$ would equal zero at death. Hence, total utility would be driven to zero provided $U(0, 0) = 0$.

[8] If the elasticity of substitution were constant, it would have to be small at all stages of the life cycle and not just in the vicinity of the death stock.

aggregate commodity $Z$, where the weights $w$ and $(1 - w)$ are shares of these commodities in full wealth, and where $q$ is the price of $Z$. A reduction in the relative shadow price of health would lead consumers to substitute $H$ for the aggregate commodity $Z$. Moreover, if health were not an inferior commodity, an increase in real full wealth would increase demand. The magnitudes of responses in $H$ to changes in relative price and real wealth are summarized by $e_H$, the own price elasticity of demand, and $\eta_H$, the wealth elasticity.

### Wage Effects

Differentiation of equation (3-4) with respect to the wage rate yields[9]

$$e_{H,W} = -e_H(K - \bar{K}). \tag{3-5}$$

In this equation, $K$ is the fraction of the total cost of gross investment accounted for by time and $\bar{K}$ is the average time intensity of nonmarket production. The derivation of (3-5) holds real wealth constant so that the equation shows the pure substitution effect of a change in the wage rate. Since $e_H$ is positive by definition, $e_{H,W} \lesseqgtr 0$ as $K \gtreqless \bar{K}$.

The sign of the wage elasticity is ambiguous because an increase in the wage raises the marginal cost of gross investment in health and the marginal cost of $Z$. Hence, both $\pi$ and the price level are positively correlated with $W$. If time costs were relatively more important in the production of health than in the production of a typical nonmarket commodity, the relative price of health would rise with the wage rate, which would reduce the quantity demanded. The ambiguity of the wage effect here is in sharp contrast to the situation in the investment model. In that model, the wage rate would be positively correlated with health as long as $K$ were less than one.

### The Role of Human Capital

To study the effects of variations in nonmarket productivity associated with education, note that since $E$ influences productivity in all nonmarket activities, it alters the marginal costs of all home-produced commodities. Given factor-neutrality, the percentage reduction in the

[9] For a proof, see Appendix C, Section 2. The corresponding equation for the wage elasticitv of medical care is

$$e_{M,W} = K\sigma_p - (K - \bar{K})e_H.$$

marginal cost of the aggregate commodity $Z$ would be $-r_Z$, where $r_Z$ is the percentage increase in either the marginal product of $Z$'s goods input or time input as $E$ increases. Therefore, human capital's effect on the weighted geometric price level is given by

$$\hat{Q} = -r_E = -[wr_H + (1 - w)r_Z].$$

With money full wealth fixed, the term $r_E$ can be viewed as the percentage change in real full wealth due to the change in nonmarket productivity associated with education. It indicates the nonmonetary return to an investment in education.[10]

If education improved productivity, then the last equation suggests that an increase in this variable would reduce the absolute shadow prices of all home-produced commodities, increase real wealth, and also alter relative prices provided the improvements in productivity were not the same for all commodities. Therefore, a shift in $E$ would set in motion wealth and substitution effects that would alter the demand for health. Differentiating the demand function (3-4) with respect to $E$, holding money full wealth and the wage rate constant, one obtains[11]

$$\hat{H} = r_E\eta_H + e_H(r_H - r_E). \tag{3-6}$$

The first term on the right-hand side of equation (3-6) reflects the wealth effect, and the second term reflects the substitution effect. If $E$'s productivity effect on the gross investment function were the same as its average productivity effect, then $r_H = r_E$, and $\hat{H}$ would reflect the wealth effect alone. In this situation, a shift in education would be "commodity-neutral." If $r_H > r_E$, $E$ would be "biased" toward health, its relative price would fall, and the wealth and substitution effects would both operate in the same direction. Consequently, an increase in $E$ would definitely increase the demand for health. If $r_H < r_E$, $E$ would be biased away from health, its relative price would rise, and the wealth and substitution effects would operate in opposite directions.

The human capital parameter in the demand curve for medical care is given by

$$\hat{M} = r_E(\eta_H - 1) + (r_H - r_E)(e_H - 1). \tag{3-7}$$

[10] For an exhaustive discussion of the above method of specifying the nonmonetary benefits of education, see Robert T. Michael, *The Effect of Education on Efficiency in Consumption*, New York, NBER, Occasional Paper 116, 1972, Chap. 1.

[11] For derivations of the human capital parameters given by equations (3-6) and (3-7), see Appendix C, Section 2.

If shifts in $E$ were "commodity-neutral," then medical care and education would be negatively correlated unless $\eta_H \geq 1$. If, on the other hand, there were a bias in favor of health, these two variables would still tend to be negatively correlated unless the wealth and price elasticities both exceeded one.

The preceding discussion reveals that the analysis of variations in nonmarket productivity in the consumption model differs in two important respects from the corresponding analysis in the investment model. In the first place, wealth effects are not relevant in the pure investment model. This follows because an increase in wealth with no change in the interest rate and the rate of depreciation would not alter the equality between the cost of capital and the rate of return on an investment in health. Note that health would have a positive wealth elasticity in the investment model if wealthier people faced lower rates of interest.[12] But the analysis of shifts in education assumes money wealth is fixed. Thus, one could not rationalize the positive relationship between education and health in terms of an association between wealth and the interest rate.

In the second place, if the investment framework were utilized, then whether or not a shift in human capital is commodity-neutral would be irrelevant in assessing its effect on the demand for health. As long as the rate of interest were independent of education, $H$ and $E$ would be positively correlated. Put differently, if individuals could always receive, say, a 5 percent rate of return on savings deposited in a savings bank, then a shift in education would create a gap between the cost of capital and the marginal efficiency of a given stock.

## 3. GLOSSARY

| | |
|---|---|
| $UH_i$ | Marginal utility of $H_i$ |
| $m$ | Index of time preference |
| $\sigma$ | Elasticity of substitution between $H_{i+1}$ and $H_i$ |
| $R^*$ | Real full wealth |
| $Q^*$ | Relative user cost or shadow price of health |
| $Q$ | Weighted geometric price level |

[12] If the rate of interest depends on full wealth and if health does not enter the utility function, then

$$\eta_H = -(1 - s)\varepsilon\eta_r,$$

where $1 - s$ is the share of interest in the cost of health capital and $\eta_r$ is the wealth elasticity of the interest rate.

$e_H$ Own price elasticity of demand for health
$\eta_H$ Wealth elasticity of demand for health
$\overline{K}$ Average time intensity of nonmarket production
$r_Z$ Percentage change in either the marginal product of goods or time in the $Z$ production function for a one unit change in $E$
$r_E$ Percentage change in real full wealth for a one unit change in $E$