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A Model of Monetary Effects on Interest Rates

This chapter presents a model of the portfolio effect described in Chapter 1. The purpose is to explain the empirical results of Chapters 4 and 5 and to show how variations in monetary growth that are not due to bank lending can still affect interest rates. New money is channeled into financial markets, not only initially when lent by banks but also subsequently by the recipients of the additional money as it circulates through the economy. The model gives results broadly consistent with the evidence.

FORMULATION OF THE MODEL

The following assumptions of the model fall within the usual simplifications found in the literature. The first-round effects of money creation are ignored, so that the means of issue need not be specified. In addition to money there is one uniform security which is the vehicle for all borrowing and lending in the economy. To simplify matters further, it is assumed that all investment expenditures are financed by current borrowing (selling new securities) and encompass the total demand for credit or loanable funds. The total supply comprises lending by households, which is derived from two sources: from saving part of current income to add to wealth and from transferring money into securities to change the form in which wealth already accumulated is held. The reverse transfer—from securities into money—reduces the supply of credit or loanable funds.[1]

Such portfolio transfers are induced by the emergence of a discrepancy between desired and actual holdings of securities and money in the sense that current holdings do not correspond with the long-run desired disposition of wealth. People are willing to hold undesired amounts in the short run until at their convenience they make adjustments. Emphasis on such discrepancies as the origin of monetary effects on interest rates in this dynamic model is the point of departure from the usual static analysis.

Portfolio transfers of money represent a flow supply of funds. In the model the net flow is made proportional—as a plausible first approximation—to the percentage discrepancy between actual and desired holdings,

$$c \ln (M^s/M^d) \qquad (1)$$

where M^s is actual money balances (the outstanding stock) and M^d the desired level. The constant positive parameter c indicates the size of the resulting transfer; it converts a percentage discrepancy into the units of a rate of change of expenditure. If c equaled, say, 0.5 per year, an excess of actual over desired balances of 10 per cent would produce transfers at the rate of 5 per cent per year. If, instead, people acted on the average to remove a discrepancy entirely within, say, three months, c would be 4 per year. (The time dimension of c has to be the same as that for expenditures in the model.)

In the monetary literature the two principal variables found to affect desired money balances are nominal income (Y) and interest rates, represented here by i, the market rate on the one uniform security. Aggregate expenditure and income are assumed to be identical. The re-

[1] Some transactions cut across this subdivision and must be classified arbitrarily. Borrowing to increase money holdings does not add to investment expenditures and so may be treated as a deduction from the supply of loanable funds; it should be netted against transfers from money to securities. Saving to increase money holdings does not add to the supply of loanable funds but is canceled by an implicit transfer from securities to money. Investment financed by selling old securities from portfolio is equivalent to a transfer of wealth from securities to money combined with borrowing from oneself; the source of the funds is a reduction in someone's desired money balances. By such interpretations of the definitions of borrowing, saving, and transfers, the assumption that investment expenditures equal total current borrowing can be preserved.

lation may be written

$$\ln M^d = \ln Y - bi, \tag{2}$$

where the income elasticity of demand is, for simplicity, assumed to be unity.[2] The constant parameter b is positive. A rise in the interest rate reduces desired balances according to the demand function for money balances. The use of current income rather than permanent income or wealth in the equation is a departure from the practice of the latest empirical studies. (The use of permanent income is discussed in the appendix to this chapter.) The rate of change of prices, which appears to affect desired money balances importantly only in times of very rapid inflation, is also ignored.

How a discrepancy between actual and desired balances arises and how it affects the interest rate is best described by an example.

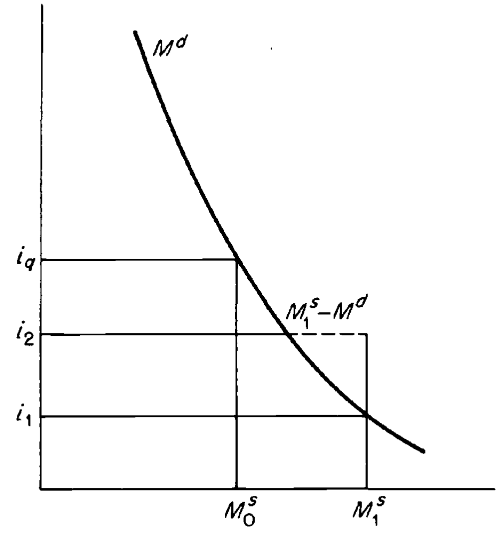
Suppose there is a change in M^s starting from a position of equilibrium in which actual and desired balances are equal. Figure 6-1 shows M^d in relation to i for a given Y, the initial interest rate i_q for M_0^s and the new stock M_1^s. The increase in the money stock to M_1^s would, if the rate fell to i_1, make $M^s = M^d$. But the attempts of money holders to exchange their excess balances for securities will not produce a rate as low as i_1. The rate declines as moneyholders bid for securities, but with the result that investment (borrowing) demand increases and saving (lending) declines, which thus raises the supply of securities and reduces the demand. Equilibrium in the loan market requires, for the moment, some intermediate rate i_2 between i_q and i_1 which equates the *total* demand and supply of securities. The flow of funds supplied out of the discrepancy between actual and desired money balances is part of the demand for securities. It makes up the difference between investment and saving ($I - S$). The size of the discrepancy at any time therefore helps determine the values of the two variables in the system, i and Y. We may consider the determination of each in turn.

[2] A parameter other than unity could be inserted without difficulty as a factor in the first term. So long as this parameter is unity, the logarithm of the price index can be subtracted from both sides of the equation to put the money stock and income in real terms, a more conventional form of the demand equation. This change makes no essential difference, and for convenience it has not been done here.

If the coefficient of the income term is not unity, however, the adjustment to real terms does make a difference and must be done explicitly.

FIGURE 6-1

Effect of Desired and Actual Balances on Interest Rate



The Determination of i

Investment and saving are functions of income and the interest rate. If the functions are linear and invariant to growth in dollar amounts, we may write

$$(I - S)/Y = -n(i - i_q) \tag{3}$$

where i_q is the equilibrium rate and n is a positive constant. A difference between investment and saving as a percentage of income is produced at any given interest rate by a flow of funds originating in the discrepancy between actual and desired balances. Then, from (1),

$$i - i_q = -a \ln (M^s/M^d) \tag{4}$$

determines the interest rate, where $a = c/n$. Equation 4 is the heart of the model and is a simple way to rationalize the empirical results presented above.

As noted above, the model disregards any first-round effect of issuing money. Such effects involve a change in the saving of the issuers of money and could be introduced by adding, for example, some fraction of the growth in the money stock to the supply of loanable funds. The model also makes no allowance for the effect of changing prices on nominal interest rates: an expansion of the money supply increases prices and eventually the expectation of price changes, which tends to *raise* nominal interest rates and to counteract the effect of changing prices on the value of fixed-dollar securities. This effect of inflation on nominal interest rates is ignored here.

Equation 4 could be written, instead, as

$$\frac{di}{dt} = -a \ln (M^s/M^d) \tag{4'}$$

if it is reasoned that the interest rate would change until the discrepancy was erased. But (4′) ignores the effect of the interest rate on investment and saving and their role in the loanable funds market, since the equation says that the interest rate keeps changing so long as actual and desired money balances are unequal. Certain day-to-day adjustments in portfolios may be explained by equation 4′, but it is inappropriate to the intermediate-run movements examined in this study. Moreover, equation 4′ leads to a relation between the level of the interest rate and of current and past *stocks* of money, whereas the model is designed to explain a dependence of the level of the interest rate on the current and past *rates of change* of the money stock.[3]

The Determination of Y[4]

An excess of investment over saving is financed by the lending of undesired money holdings. The increased flow of lending supplied by the discrepancy raises investment expenditures and produces a rising level of income. The growth of income continues so long as the discrepancy does. Income also grows, even if the discrepancy is zero, by the expected trend of income, which is anticipated and allowed for ahead of time. The expected growth of income equals the average rate of growth of real income (g_0, assumed constant) plus the expected rate of change of prices, $d \ln P^e/dt$. If current income moves along the expected trend, the variables maintain an equilibrium relationship; in particular, de-

[3] For an analysis of the kind implied by equation 4′ see William E. Gibson, "Interest Rates and Monetary Policy," *Journal of Political Economy,* May/June 1970.

[4] It is necessary to specify how Y is determined in the model, because Y affects M^d.

sired and actual money balances grow at the trend rate and remain equal. Income growth departs from the expected trend when actual and desired balances diverge.

The equation for income growth can be written

$$\frac{d \ln Y}{dt} = c \ln (M^s/M^d) + \frac{d \ln P^e}{dt} + g_0. \tag{5}$$

The use of the same coefficient c here as in (1) implies that all adjustments of money balances involve the purchase and sale of securities. This is not necessary, however, and the discrepancy in (5) can be interpreted as including some expenditures directly on goods and services as well as some indirectly on capital goods through the net purchase of new securities.

Undesired money balances lead to increases in income, which in turn tend to remove the discrepancy by raising M^d. Hence the equality between the total demand and supply of loanable funds implied by (4) describes a moving equilibrium which determines i at each moment in time; it is continually changing to produce new values of i and Y. Equations 4 and 5 can be regarded as a simple extension of the usual static equilibrium analysis. In the static analysis with full employment and flexible prices, monetary growth raises nominal income and prices without affecting the equilibrium interest rate, which remains at i_q. The present model describes the path to that equilibrium when the system is disturbed. The use here of liquidity preference and loanable funds is quite conventional; together the two provide a stock-flow analysis of the movement to equilibrium.[5] All that has been added here is the idea of a temporary discrepancy between desired and actual balances which allows a simplified description of the movement in terms of a differential equation.

This formulation eliminates the crucial importance of the money demand function which it has in static models. In equation 2, b could be quite small with no important alteration in behavior of the variables. Presumably b would not be zero, for that would imply that money balances and securities are not substitutes in portfolios. The only consequence of a large b is that the full effect on income of an increase

[5] Credit for first combining liquidity preference and loanable funds in a stock-flow analysis of monetary changes belongs to George Horwich, *Money, Capital, and Prices*, Homewood, Ill., R. D. Irwin, 1964 (see also his earlier work cited therein).

in monetary growth is quite prolonged. The lending of excess balances reduces interest rates, which raises the desired balances. The desired increase in balances absorbs part of the excess, to that extent temporarily closing part of the discrepancy in money balances and delaying the full rise in expenditures and income. In the new long-run equilibrium the full effect on income of an increase in monetary growth is the same as it would be if desired balances did not depend upon interest rates.

To complete the model we need to specify how price expectations are formed. They are assumed to depend upon the discrepancy between actual and expected price changes; such a relation is commonly used in economic models. Since the adjustment is by all indications slow, we may simplify the mathematics by using the trend of the actual price change, $(d \ln P/dt)_q$, rather than the concurrent change, as follows:

$$\frac{d^2 \ln P^e}{dt^2} = f\left[\left(\frac{d \ln P}{dt}\right)_q - \frac{d \ln P^e}{dt}\right] \tag{6}$$

where the constant parameter f is positive. The trend of price changes can be approximated by the average rate of monetary growth less the average rate of growth in real income,

$$\left(\frac{d \ln P}{dt}\right)_q = \left(\frac{d \ln M^s}{dt}\right)_q - g_0. \tag{7}$$

We may also specify that $(d \ln M^s/dt)_q$ is an exponentially weighted average (with slope h) of the actual past rates of monetary growth. The approximation (7) will be reasonably accurate and consistent with the other equations in the model, provided that no major long-run changes in M^d occur due to changes in i. This supposition is plausible, since changes in i in this model are small and temporary, and the value of b is relatively low (about 0.5, according to most studies).

If we treat the money stock as exogenous, the above equations form a complete system and can be combined to eliminate all but one endogenous variable. The reduced form is a second-order differential equation. To derive it in terms of the interest rate, we differentiate (5) with respect to time and eliminate the price variables by successive substitution of (6), (7), and (5). Income, desired money balances, and their derivatives can be eliminated by means of (2), (4), and their derivatives.

(Also, $d^2 \ln M^s/dt^2$ is assumed to be zero, because only discrete changes in monetary growth will be analyzed.) The final result is

$$[(1/a) + b](d^2i/dt^2) + \{[(c+f)/a] - bf\}(di/dt) + (cf/a)i$$
$$= (cf/a)i_q + f[(d \ln M^s/dt)_q - (d \ln M^s/dt)]. \quad (8)$$

SOLUTION OF THE MODEL

To solve the differential equation 8 for i we must specify the time path of monetary growth. We may illustrate the behavior of the model by analyzing the simple case in which the money stock has grown at the constant rate g_0 up to time zero and then begins to grow at the higher constant rate g_1. Up to $t = 0$, i will have been constant at i_q.

By the definition of $(d \ln M^s/dt)_q$ we have, for $t \geqslant 0$,

$$\left(\frac{d \ln M^s}{dt}\right)_q - \frac{d \ln M^s}{dt} = h \int_{-\infty}^{0} g_0 e^{h(T-t)}\, dT + h \int_0^t g_1 e^{h(T-t)}\, dT - g_1$$
$$= -(g_1 - g_0)e^{h(T-t)} \quad (9)$$

where $he^{h(T-t)}$ is the weighting pattern (which sums to unity when T goes from $-\infty$ to t). Consequently, a particular solution of (8) is simply

$$i_t = i_q - J(g_1 - g_0)e^{-ht} \quad (10)$$

where J is a constant, and substitution of (10) in (8) shows that[6]

$$J = \frac{-af}{(h-f)[c - h(1+ab)]}. \quad (11)$$

In the long run, therefore, i will approach i_q; there is no permanent effect of monetary growth on the interest rate (ignoring as we do any changes in the marginal productivity of capital i_q).

A general solution is given by the homogeneous equation, setting the right side of (8) equal to zero. Its discriminant[7] is

$$\left(\frac{c+f}{a} + bf\right)^2 - 4\left(\frac{1}{a} + b\right)\frac{cf}{a} = \left[\frac{c - f(1+ab)}{a}\right]^2. \quad (12)$$

The quadratic equation associated with the homogeneous equation, therefore, has the roots

[6] My thanks to Carl Christ for pointing out an algebraic error at this point in an earlier version of this chapter.

[7] A homogeneous second-order differential equation with constant coefficients may be represented by $p(d^2i/dt^2) + q(di/dt) + ri = 0$. Its discriminant is $q^2 - 4pr$.

$$\frac{\frac{c+f}{a}+bf\pm\left(\frac{c-f(1+ab)}{a}\right)}{2\left(\frac{1}{a}+b\right)}=-f \text{ and } \frac{-c}{1+ab}. \tag{13}$$

The complete general solution of (8) is the sum of (10) and two exponential terms with powers given by (13), that is,

$$i_t = i_q - J(g_1 - g_0)e^{-ht} + Ke^{-ft} + Le^{-[c/(1+ab)]t} \tag{14}$$

where K and L are arbitrary constants of integration.

K and L are limited by certain specified conditions of the solution. Before $t = 0$, $i = i_q$; hence, K and L contain the factor $(d \ln M^s/dt) - g_0$ which is zero up to $t = 0$ and equals $g_1 - g_0$ thereafter. At the moment $t = 0$ when monetary growth jumps to g_1, no discrepancy in money balances has had time to develop and i still equals i_q. Hence $i_0 = i_q$ and

$$K + L = J(g_1 - g_0). \tag{15}$$

Further restrictions on K and L can be derived from the specification that i initially declines after monetary growth increases. It is also necessary, however, to rank the exponents by size. From the derivation of the model it seems likely that $c/(1 + ab)$ is considerably larger than h, and the latter somewhat larger than f. It then follows that J as given by (11) and K are negative.[8] L may be positive or negative by these initial conditions, but it is most likely positive.[9]

[8] That K is negative follows from the assumption that $(di/dt)_0$ is negative. To show this, first differentiate (14) and set $t = 0$; then

$$(di/dt)_0 = -fK - [c/(1 + ab)]L + hJ(g_1 - g_0) < 0. \tag{16}$$

Since K and L contain the factor $(g_1 - g_0)$, define $K' = K/(g_1 - g_0)$ and $L' = L/(g_1 - g_0)$, where K' and K have the same sign and likewise L' and L. Hence, from (15), $K' + L' = J$. We can then write (16) as

$$K'(h-f) - L'\{[c/(1 + ab)] - h\} < 0. \tag{17}$$

If K' were positive, L' would also have to be positive. But that is impossible, since $K' + L' = J$ is negative. Hence K' and K are negative.

[9] L and L' can be positive or negative. If negative, however, L' must be appreciably smaller in absolute value than K', because of (17) and the assumption that $[c/(1 + ab)] - h$ greatly exceeds $h - f$. But the closer L' is to zero, the more the long-run influences in the model dominate the initial short-run movements (that is, the third term of (14) becomes unimportant). If the model is at all relevant to monetary effects on interest rates, the third term of (14) will be important, which requires that L' be positive and relatively large. The figure has been drawn accordingly. If $(di/dt)_0 = 0$, it can be shown that L is negative, but there is no reason to impose such a condition.

The solution is illustrated by Figure 6-2, assuming L is positive and K is negative but that otherwise their values are arbitrary. The interest rate first declines, then gradually returns to its original level.[10]

A once-and-for-all change in the money stock can be analyzed as a rise in the monetary growth rate which terminates after a short period of time. The solution can be found in the same manner by taking account of the two steps—first up, then down—in the monetary growth rate,[11] as in Figure 6-3. After first declining, the interest rate gradually returns to its original position, but more rapidly, of course, than for a continuing rise in monetary growth. In Figure 6-3, where the parameters have the same values as in Figure 6-2 and the constants of integration the same initial values, there is a slight temporary overshooting of the long-run equilibrium position. (Not all possible values of the constants produce overshooting.)

DESCRIPTION OF THE SOLUTION

Certain characteristics of the solution deserve attention. With an unanticipated increase in monetary growth, actual balances begin to grow faster than the desired amount. The public responds by increasing the flow of spending and lending. The lending reduces interest rates

[10] This can be shown by examining the number of times that di/dt becomes zero. We have

$$di/dt = -fKe^{-ft} - [c/(1+ab)]Le^{-[c/(1+ab)]t} + hJ(g_1 - g_0)e^{-ht} = 0. \qquad (18)$$

Since $K' + L' = J$, equation (18) gives

$$K'(-fe^{-ft} + he^{-ht}) = L'\{[c/(1+ab)]e^{-[c/(1+ab)]t} - he^{-ht}\}. \qquad (19)$$

The functions on each side of (19) have the same general pattern as that for i in Figure 6-2: starting above zero, they decline below zero, reach a mimimum point, and then approach zero asymptotically from below. If L' is positive (and K' negative), the left-hand side is inverted and forms a hill, while the right-hand side forms a valley, and the two then intersect only once; after the first intersection, the right-hand function remains below the left-hand function and approaches the zero line from below. That means there is only one value of t which satisfies (18) and (19).

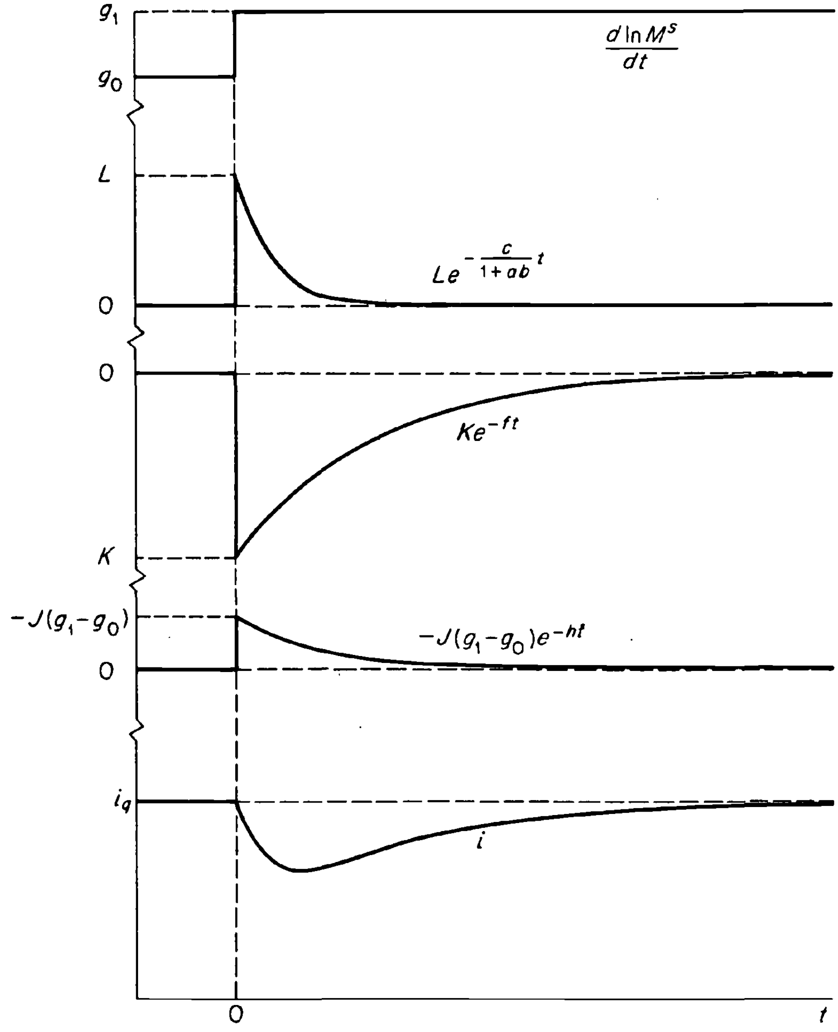
[11] If monetary growth jumps from a rate of g_0 to g_1 during the interval from 0 to θ and then drops back to g_0, we have, instead of (9), for $t \geqslant \theta$

$$(d \ln M^s/dt)_a - (d \ln M^s/dt) = (g_1 - g_0)(e^{h\theta} - 1)e^{-ht}.$$

The values of K and L for $t \geqslant \theta$ can then be found by the condition that i_θ has the same value by the new and the old constants. These values are given in the note to Figure 6-3, where $\theta = .5$.

FIGURE 6-2

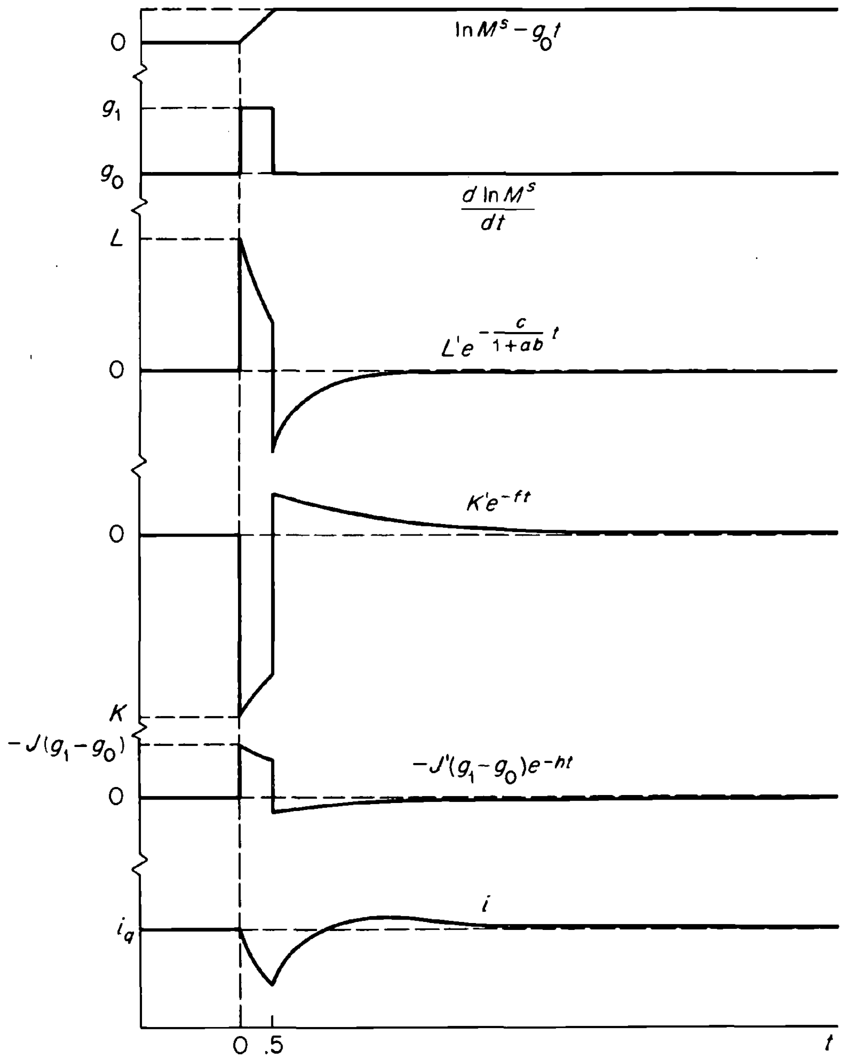
Graphical Solution of Equation 14 with Increase in Monetary Growth Rate



(described by equation 4) and, along with the greater spending (described by equation 5), increases the growth of income and the rate of change of prices. The increase in income growth and fall in interest rates raise the growth rate of desired balances, but not enough at first to prevent the discrepancy between actual and desired money balances

FIGURE 6-3

Graphical Solution of Equation 14 for Temporary Spurt in Monetary Growth Rate



Note: For $t < 0.5$, $L' = L$, $K' = K$, $J' = J$. For $t \geqslant 0.5$, $L' = -L(e^{0.5c/(1+ab)} - 1)$, $K' = -K(e^{0.5f} - 1)$, $J' = -J(e^{0.5h} - 1)$.

from enlarging. Eventually the increase in income growth raises the growth rate of desired balances sufficiently to begin to reduce the discrepancy, and interest rates slowly return to initial levels. (The model disregards the possibility that interest may remain forever lower because of a fall in the marginal productivity of capital resulting from the increase in the capital stock.)

In long-run equilibrium, desired money balances rise at the new rate of monetary growth; the growth in nominal income due to price increases keeps money demand growing at that rate, and no new discrepancy occurs so long as the growth of the money stock remains constant at the new rate.

The path for the growth rate of nominal income will necessarily involve some overshooting. The effect of a sudden increase in monetary growth is to reduce the ratio M^s/Y, whereas in the final equilibrium this ratio returns to its original level.[12] Hence the cumulative percentage changes in money and income are equal. If the percentage change in income is initially below that of the money stock, therefore, the former must for a time exceed the latter.

If desired money balances depend upon permanent rather than current income, there is an additional reason for overshooting of income growth,[13] as shown in the appendix to this chapter.

APPENDIX: MODIFICATION OF THE MODEL USING PERMANENT INCOME

The model of Chapter 6 departs, as noted above, from the practice of empirical studies that use permanent income in the demand-for-money equation. With this modification the basic equations of the model are

[12] The final equilibrium will be different if the income elasticity of the demand for real money balances is not unity and if part of a change in money income is in real terms.

[13] On this point see also A. A. Walters, "Professor Friedman and the Demand for Money" and "The Demand for Money—The Dynamic Properties of the Multiplier," *Journal of Political Economy,* October 1965, pp. 545–51 and June 1967, pp. 293–98, respectively. Walters shows that a dependence of money demand on permanent income produces an overshooting in the adjustment of current income to monetary changes. But his analysis does not allow for a discrepancy between desired and actual balances and so does not have a lag in the initial effect on income of a monetary change, as does the model here.

$$\ln M^d = \ln Y_p - bi \tag{1A}$$

$$\frac{d \ln Y}{dt} = c(\ln M^s - \ln M^d) + \frac{d \ln P^e}{dt} + g_0 \tag{2A}$$

$$i - i_q = -a(\ln M^s - \ln M^d) \tag{3A}$$

$$\frac{d^2 \ln P^e}{dt^2} = f\left[\left(\frac{d \ln P}{dt}\right)_q - \frac{d \ln P^e}{dt}\right] \tag{4A}$$

where Y_p is permanent income. It may be formed by the usual adaptive adjustment as follows: [14]

$$\frac{d \ln Y_p}{dt} = p(\ln Y - \ln Y_p) + \frac{d \ln P^e}{dt} + g_0. \tag{5A}$$

The last two terms of (5A) impart the expected trend to permanent income, which then grows at the trend rate when there is no discrepancy between the actual and the permanent level.

The reduced form of these equations, containing the interest rate, the money stock, and their derivatives, is a third-order differential equation with an overabundance of possible solutions. The model can be simplified by dropping the expected-price-change variable and equation 4A, which determines it, on the assumption that the adjustment of this variable to the current change is slow (f is presumably very small) and that therefore its effects on short-run movements in the interest rate and income are only slight. (Its effect on M^d is ignored here.) The simpler model has an analytic solution and may be legitimately used for an analysis of once-and-for-all monetary changes which do not affect the long-run trend.

With the expected price change omitted, the model is reduced by differentiating (5A) with respect to time, substituting for the derivatives of Y_p and Y by means of (1A) and (2A), and removing the derivatives of M^d by means of (3A). The resulting equation can be written

$$\left(\frac{1}{a} + b\right)\frac{d^2 i}{dt^2} + \left(\frac{p}{a} + pb\right)\frac{di}{dt} + \left(\frac{pc}{a}\right) i = \left(\frac{pc}{a}\right) i_q + p\left(g_0 - \frac{d \ln M^s}{dt}\right). \tag{6A}$$

This equation has a general solution, given a specified change in monetary growth. Suppose that the monetary growth rate rises from

[14] See Milton Friedman, *A Theory of the Consumption Function*, Princeton for NBER, 1957, p. 143.

g_0 to g_1 at $t = 0$ and after one period drops back to g_0 and remains at that rate. A particular solution of (6A) then is

$$\begin{aligned} i &= i_q - (a/c)(g_1 - g_0) \text{ for } 0 \leqslant t < 1 \\ i &= i_q \qquad\qquad\qquad\quad \text{for } t \geqslant 1. \end{aligned} \tag{7A}$$

The solution of the homogeneous equation is the sum of two exponential terms with powers given by the roots of the quadratic equation formed from the coefficients on the left side of (6A):

$$[(1/a) + b]X^2 + [(p/a) + pb]X + (pc/a) = 0.$$

The roots are

$$X = \frac{-p \pm \{p^2 - [4pc/(1 + ab)]\}^{1/2}}{2}. \tag{8A}$$

The expression inside the braces can be written

$$[p/(1 + ab)][p(1 + ab) - 4c]. \tag{9A}$$

This quantity is negative on assumptions like those made in Chapter 6 that c exceeds the coefficient of expectation (here p) and that $1 + ab$ is well below 4. Consequently, the roots are complex and the general solution of (6A) may be written

$$i_t = e^{-pt/2}(A \sin \eta t + B \cos \eta t) + i_q \tag{10A}$$

where $t \geqslant 1$, η is the square root of the negative of (9A) and is therefore a positive real number, and A and B are constants of integration determined by initial conditions.

The effect on the interest rate of a change in the money stock therefore involves damped oscillations, with overshooting of the long-run equilibrium position. The reason for this result is not hard to see in terms of the modified model. The adjustment to an emerging discrepancy between actual and desired money balances is delayed by the lagged response of permanent income to changes in current income. Consequently, current income grows and interest rates decline more than would otherwise be necessary, and the delayed adjustments to these changes then push the variables too far in the opposite direction. As a result, the variables continually overshoot the equilibrium position (though by less and less).