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## CHAPTER 3

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# *The Evidence of Seasonality in Interest Rates*

### INTRODUCTION

THIS CHAPTER EVALUATES the evidence of seasonality in postwar interest rates and suggests suitable adjustments where appropriate. The evaluation consists of graphically identifying biases in the SI ratios over or under the 100.0 level. While the one-way analysis of variance test for seasonality is a useful method for identifying systematic deviations of the SI ratios from 100.0, its reference to the entire period makes it ineffective as a means of distinguishing the subperiods with evidence of seasonality from those without it.<sup>1</sup> The consistent deviation of a given month's SI ratio in the same direction from the 100.0 line is strong evidence of seasonality regardless of the variation in the magnitude of these deviations, that is, in the seasonal amplitude. The evidence of seasonality is weak when there are constant reversals of direction or when the relationship among the patterns of SI ratios is generally unstable from year to year. The summary statistics that most computer programs for seasonal adjust-

<sup>1</sup> One can apply the analysis of variance to separate subperiods, but the problem of choosing the limits of the subperiods remains.

ment supply (such as the average month-to-month change in the seasonal component by itself or relative to that of the other components) do not help evaluate the evidence of seasonality since, aside from the analysis of variance test, they only summarize what the program has done. They do not provide independent measures of either the evidence of seasonality or the quality of the adjustment. Once the existence of a seasonal pattern is confirmed and the adjustment decided, then the summary statistics provide a useful summary of the results.

In seasonal analysis, as in regression analysis, one places greater confidence in tests for the existence of a relation than in its actual measurement. In addition to the problem of sampling error common to both analyses, the moving seasonal amplitude implies changing parameters and requires the adjustment method, in effect, to shoot at a moving target. Except for a few experiments this study does not originate any methods of adjustment, nor does it even compare the adjustments obtainable with existing methods.<sup>2</sup> Instead, charts of the seasonal factors obtained with the X-11 are superimposed on charts of the corresponding SI ratios to determine the method's success in capturing what appears to be the systematic movement of the SI ratios. There is no question but that one could fit by eye a curve that is more faithful than is the curve of factors to variation of the SI ratios; in the extreme, one could perfectly fit a curve to the SI ratios by simply connecting the points—that is, by simply treating them as the factors. The art of the adjustment is in identifying the *systematic* movement of the SI ratios. When the pattern of SI ratios is stable from year to year there is no problem; nor is there any when the seasonal amplitude changes gradually or the pattern evolves with an apparent method. But during transition periods, in which the series has strong irregular movements such as 1954 the SI ratios and estimated factors appear to be virtually unrelated. This study recommends ignoring the adjustment when the gap between the two is pervasive.

<sup>2</sup> The potential gain from these experiments is not, in this study's view, commensurate with the effort required.

## SHORT-TERM SECURITIES

## SUMMARY STATISTICS

Table 1 lists some of the summary statistics that are useful in describing the extent and significance of seasonal influence.<sup>3</sup> Columns 1–3 divide the total variance of the series into the parts due to each of the three components: the trend-cycle, seasonal, and irregular. These figures are analogous to readings from the spectral density function, which decomposes the variance of a time series according to the frequency of the recurring variation.<sup>4</sup> Column 4 lists the average month-to-month percentage changes (without regard to sign) of the seasonal component, and column 5 the ratio of column 4 to the corresponding statistics of the cyclical component. These figures, columns 1 through 5, strike averages for the whole study period, averages not of the true seasonal but of the estimated one, including a spill-over into periods without significant seasonality. Columns 7–10 give the dates and amplitudes of the highest and lowest seasonal factors observed during the study period.

But these figures may be only statistical artifacts; hence the need exists to ascertain their statistical significance. To partially satisfy this need, column 6 records the *F*-statistics.

The *F*-statistic may be low for any of several reasons, the enumeration of which will help in evaluating the charts that follow. The most important reason, of course, is the absence of any bias in the monthly SI ratios away from the mean value of 100.0; in other words, no

<sup>3</sup> The statistics are copied directly from the X-11 printout and are described more fully in the X-11 Manual, *op. cit.*

<sup>4</sup> Some recent studies have applied spectral analysis to the problem of identifying seasonal variation. While the principle is the same as that in the moving average method—to simultaneously or sequentially filter, or separate, different frequencies of variation—spectral analysis is a more sophisticated and more rigorous method of doing so. Some of the mathematical advantage is lost, however, in its application to a limited number of observations. This method, moreover, provides no direct adjustment for the seasonal and resembles, in this respect, analysis of variance instead of regression analysis. In his forthcoming study for the National Bureau, Tom Sargent applied spectral analysis to interest rates, among other financial variables, and reached conclusions virtually identical to those in the present study with respect to the extent and evolution of the seasonal in interest rates.

**TABLE 1**  
**Measures of the Relative Importance of the Seasonal Components of the**  
**Four Series of Short-Term Securities, 1948-65**

Series	Percentage of Total Variance of Series Due to Each Component		Average Month-to-Month Percentage Changes Without Regard to Sign of Seasonal Component $\bar{S}$	Ratio of Column 4 to Corresponding Figures for the Cyclical Component $\frac{S}{\bar{C}}$	F-Test for Stable Seasonality <sup>b</sup>	Date and Factor of Seasonal High and Low for Whole Period <sup>c</sup>				
	I	C				High	Factor (percentage)	Low	Factor (percentage)	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<i>Yields on:</i>										
91 day bills	37.72	42.52	19.76	2.54	0.68	9.662	Dec. 1957	113.9	July 1957	87.4
9-12 month securities	35.91	49.77	14.32	1.85	0.54	4.557	Dec. 1956	109.0	June 1957	90.7
Commercial paper <sup>a</sup>	35.38	42.98	21.64	1.81	.71	4.034	Dec. 1958	107.5	May 1959	92.8
Bankers' acceptances	24.57	49.39	26.04	2.07	.72	11.293	Dec. 1957	111.4	July 1956	89.3

<sup>a</sup>The sample period for commercial paper rates ends in June 1965.

<sup>b</sup>All ratios are statistically significant at the 5 per cent level.

<sup>c</sup>A moving seasonal component was estimated for both series. Columns 7 and 9 list the dates when the amplitudes of the seasonal variations were greatest and columns 8 and 10 the values of the estimated seasonal factors for these dates.

seasonality. But a low  $F$ -statistic does not imply the absence of seasonality over the whole sample period. The smaller the subperiod of true seasonality the greater the burden on this period's SI ratios to influence the means for the whole sample period and thereby enlarge the between means variance, the numerator of the  $F$ -ratio. The burden is aggravated by the fact that a moving seasonal component combines with the irregular component to enlarge the within-group variance, the denominator of the  $F$ -ratio. When, for example, the  $F$ -statistic is computed for long-term Treasury securities over the entire period, its value (1.787) signifies the absence of seasonality; whereas, when computed over the period 1955 through 1962 the result (4.726) confirms the presence of seasonality. Similarly, the  $F$ -statistic in Table 1 for nine- to twelve-month Treasury securities is low because the seasonal pattern before 1955 was at best highly irregular. When the seasonal pattern changes over the sample period, even when in each subperiod the pattern is unambiguous, the  $F$ -statistic suffers as the differences among the mean monthly SI ratios are reduced. Combine this problem with the fact that seasonal patterns do not change instantaneously but rather evolve through periods of transition during which a coherent pattern is virtually nonexistent. In the eighteen-year sample period the seasonal pattern of commercial paper rates underwent several changes, and the low  $F$ -statistic shown in Table 1 in part reflects this fact.<sup>5</sup> Finally, the  $F$ -statistic may be low not because there is no seasonality but because of a strong irregular component; the means of the SI ratios are different from 100.0, but the standard errors of the means are high. This condition applies to all the interest rate series and in particular to commercial paper rates and yields on municipal bonds. Here again the diagrams are essential for determining whether the seasonal pattern has sufficient stability to warrant adjustment.

At its highest the seasonal component pushes the Treasury bill rate 14 per cent (rounded to nearest integer) above its trend-cycle value; at its lowest, 13 per cent below. For a bill rate in the neighborhood of 4 per cent (i.e., 400 basis points) these seasonal factors correspond to about 50 basis points.<sup>6</sup> At 11 per cent on

<sup>5</sup> These changes are described later in the section.

<sup>6</sup> These figures are actually underestimates since they embody the dampening effects of the lower peak levels of adjacent years. Later in this chapter an experiment is described that exemplifies this point.

either side of the trend-cycle values, the peak seasonal amplitude for yields on bankers' acceptances is somewhat less. Relative to the total variation of the series, however, the seasonal component of the yields on bankers' acceptances is the most important of the four series, and its  $F$ -statistic is highest. The diagrams, to be discussed presently, support the conclusion that this series evinces the strongest seasonal component. The series for which the summary statistics are least reliable is the series on commercial paper rates, for which, in addition, the  $F$ -statistic is lowest. The diagrams will justify this conclusion as well.

#### TREASURY BILL RATES

Chart 7 plots the seasonal factors for Treasury bill rates superimposed on the corresponding SI ratios. From a relative high in January the Treasury bill rate seasonal pattern typically declines past seasonally neutral February, downward through the spring months to its trough in June or July and then turns sharply upward through seasonally neutral August to September, from which it rises gradually to its peak in December.<sup>7</sup> Surprisingly, this pattern is quite apparent, although the amplitude is small, in 1948, when the Federal Reserve pegged the prices of Treasury bills within narrow limits. This curve is shown in the first panel of Chart 7 together with the unmodified SI ratios. In subsequent panels of Chart 7 the pattern is shown to dissolve until about 1953 and then gradually to emerge again, but with greater amplitude, in the middle fifties, keeping this shape into the sixties as its amplitude virtually disappeared. By 1965 there was little left but a 2 per cent trough in June–July and a 2 per cent peak in December–January.

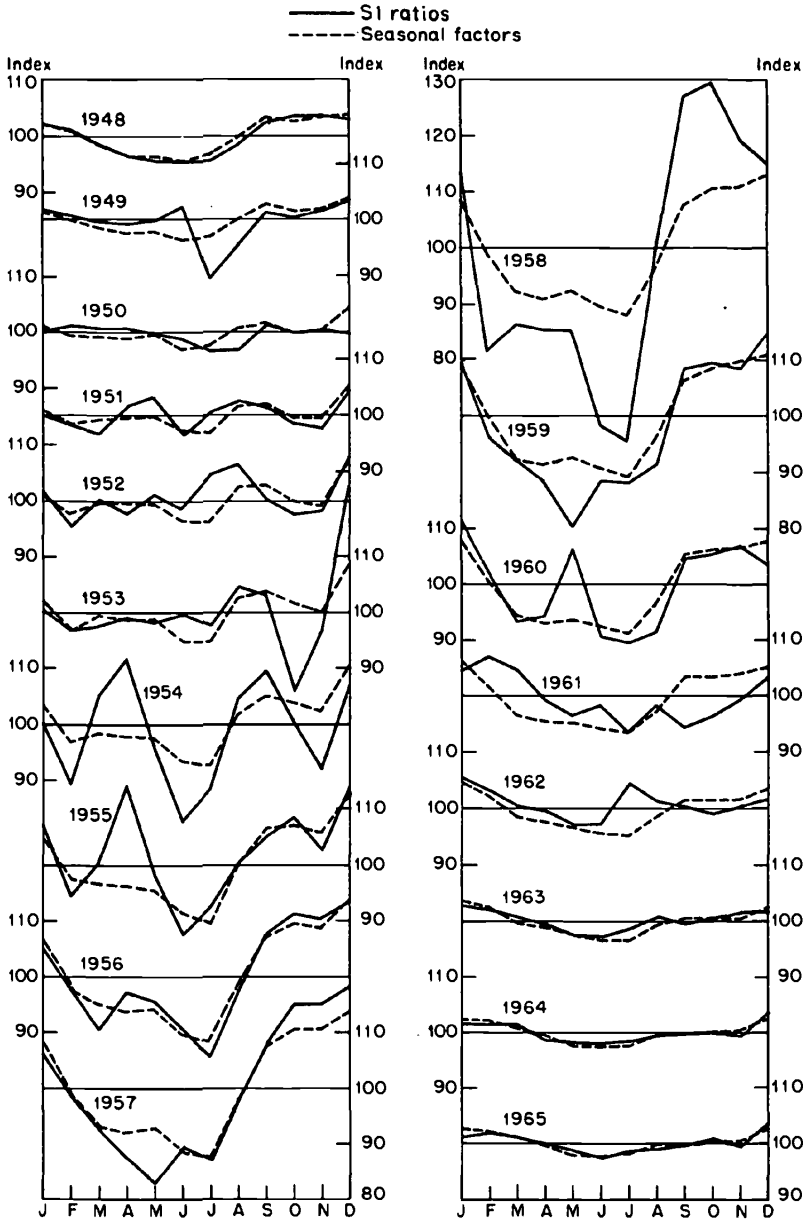
The factor curves, the broken lines of Chart 7, are for the most part dampened versions of the corresponding SI ratios, although at times the factor curve for one year betrays the influence of its predecessor more than that of its contemporary SI ratios. In this regard the factor curves ignore certain abrupt movements of the SI ratios, as in April 1955, the program being designed to sidestep points it regards as extreme.<sup>8</sup>

<sup>7</sup> Before 1957 the movement between September and December was not monotonic.

<sup>8</sup> Briefly, an extreme point is one that falls outside the range of 1.5 standard deviations, the latter computed for the entire set of data several times to

CHART 7

SI Ratios and Seasonal Factors for Treasury Bill Rates, 1948-65



The relative stability of the seasonal pattern is in part a measure of the adjustment's effectiveness since the program does not attempt to directly fit the solid lines in Chart 7. Instead it smooths the SI ratios month by month as shown in Chart 8. As a given month's factors evolve through the years, any persistent change in their relation to another month's factors, that is, any change in the ordering of the twelve factors, will change the seasonal patterns given in Chart 7. Barring the extremes, the factor curves in Chart 8 fit the SI ratios quite closely although severely dampening their movement. One may wish to quarrel with the fit at a few places, but it will soon be shown that any reasonable alterations would have little quantitative importance. The two or three years preceding 1954 and following 1960 appear overly dependent on the years in between; and correspondingly, the peak period is excessively dampened. Aside from the transitional months, February and August, the monthly factor curves trace out the bell-shaped curves noted earlier, inverted in the low months, of course, and most of them remain above 100 or below 100 throughout the period. While most of the curves as of 1965 roughly coincide with the 100.0 line, the curves for July and December are still about 2 per cent under and over the line, denoting the persistence of a small seasonal variation in Treasury bill rates. However, the pattern for the last four years differs somewhat from that in earlier years: the trough appears in June instead of July; the January factors become at least as prominent as the ones for December; and, in the last two years, the November factors dip below the 100.0 line. Similar changes will be shown to have occurred in yields on bankers' acceptances as well.<sup>9</sup>

#### OTHER SHORT-TERM RATES

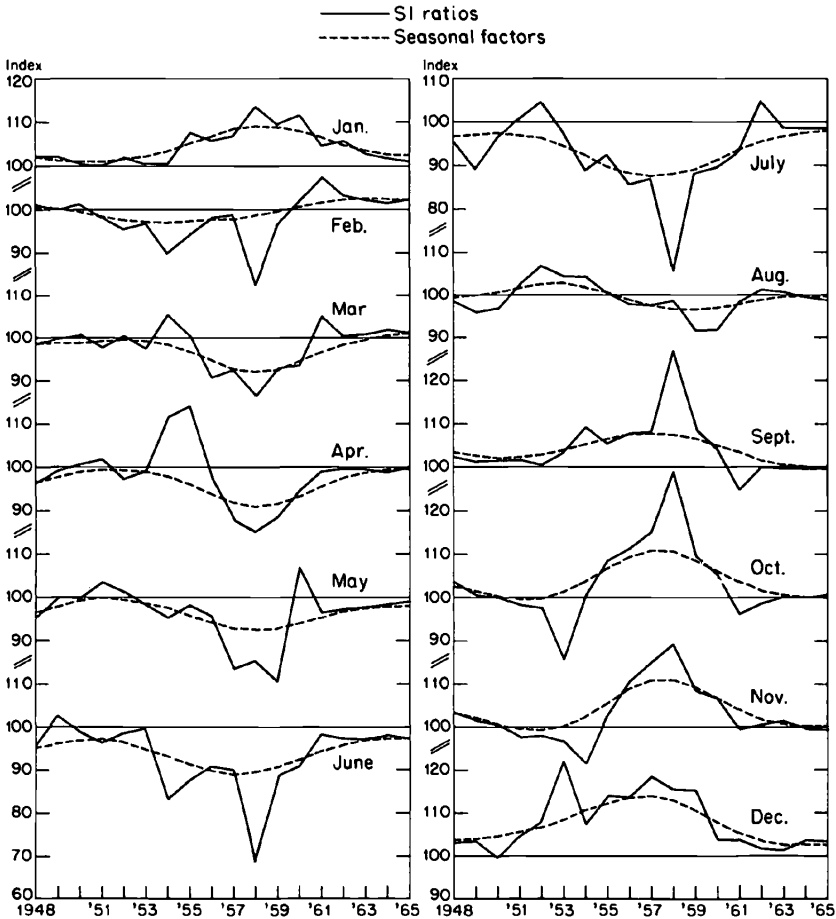
The seasonal patterns of the other short-term rates considered, except for commercial paper rates prior to 1956, are very similar to the one for Treasury bill rates. The similarity is greatest in the peak

eliminate the effect on it of the extreme points. The extremes are weighted linearly from 1 to 0 as they fall between 1.5 and 2.5 standard deviations. An extreme SI ratio different from 100 by two standard deviations is weighted 0.5. See X-11 Manual, *op. cit.*, pp. 4-5.

<sup>9</sup> The appendix to this chapter uses alternate adjustment procedures to evaluate the X-11 adjustment of Treasury bill rates.

CHART 8

Variation of Monthly Factor Curves and SI Ratios of Treasury Bill Rates Through Years, 1948-65



period, when the patterns for all the series have their greatest stability. Table 2 lists the simple correlation coefficients between the twelve seasonal factors for Treasury bill rates and those for the other series over selected years. In 1957 the correlations were all in excess of .95, indicating virtual identity among the four patterns; by 1965, however, the correlations were much less, and could be in part the spurious aftermath of the earlier similarity.

TABLE 2

*Sample Correlation Coefficients Between the Seasonal Factors  
for Treasury Bill Rates and Those for Other Short-Term  
Rates for Selected Years*

	1953 (1)	1957 (2)	1963 (3)	1965 (4)
Bankers' acceptances	.6430	.9742	.7926	.5283
Commercial paper	.1380	.9724	.8917	n.a.
9-12 month Treasury securities	.8447	.9510	.7700	.6492

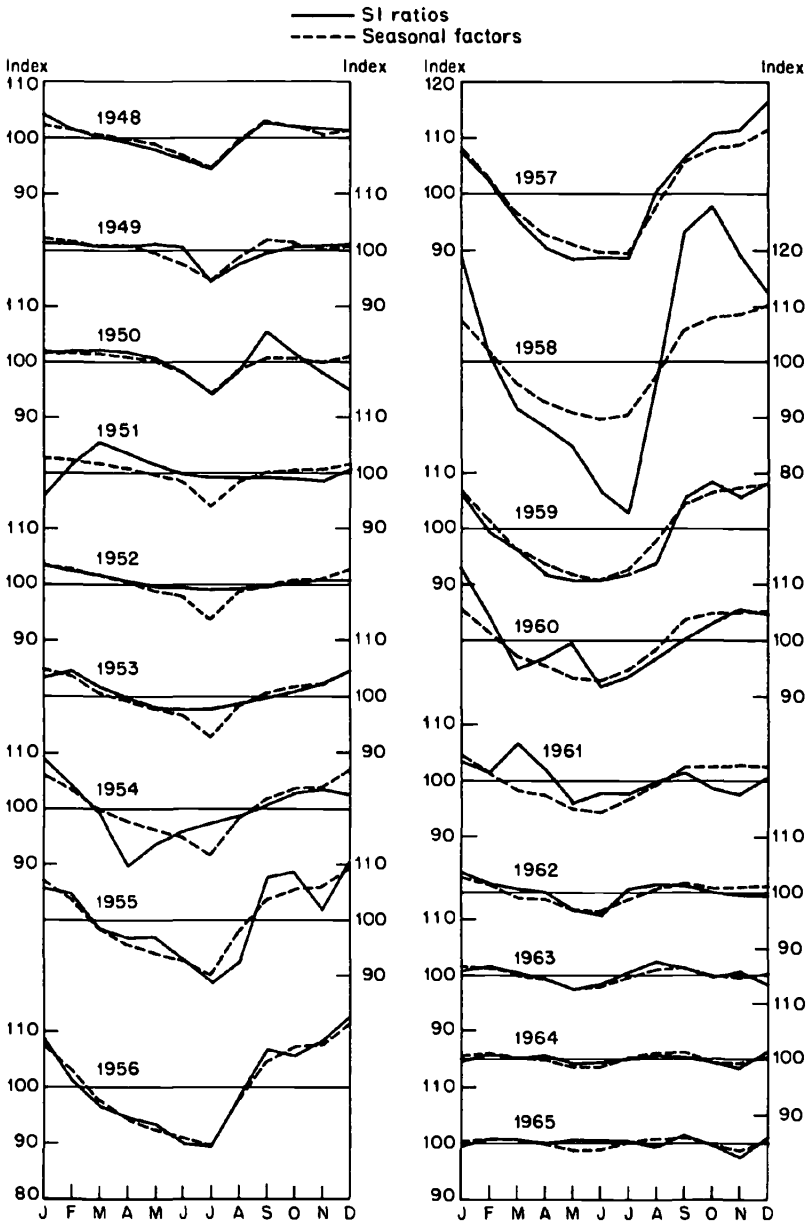
NOTE: The numbers are all simple correlation coefficients between the seasonal factors of Treasury bill rates and the factors in the corresponding years for the other three series.

n.a. = not available.

Closest in pattern and evolution to that of Treasury bill rates is the seasonal movement of yields on bankers' acceptances. As in the case of bills, the seasonal pattern for yields on bankers' acceptances is fairly stable from 1948 through 1950, but, again like bills, the fit is less adequate from 1951 through 1954. In Chart 9 the estimated factors are shown to grossly exaggerate the trough in July. Seasonality exists in the period (July, for example, is always below the 100.0 line and December above), but its pattern is less stable than and differs from the patterns of other years. During the peak period its pattern is the familiar high in January, declining past neutral February through the spring lows to a trough in July, then climbing steeply upward to September and, more gradually, to a December peak. Although its peak amplitude, at 12 per cent on either side of the corresponding trend-cycle values, is somewhat less than the peak in the bill rate seasonal pattern, the seasonal amplitude on yields on bankers' acceptances is more prominent in the total variation of the series (columns 3 and 5 of Table 1), and the estimated factors are more faithful to the SI ratios (Chart 9). Beginning in 1959 the pattern begins to change somewhat, the trough shifts from July to June and the peak from December to January. These changes, as well as the dip in November, are virtually identical to those that occurred somewhat later in Treasury bill rates. Chart 9 reveals the low and declining

CHART 9

SI Ratios and Seasonal Factors for Bankers' Acceptance Rates, 1948-65



seasonal amplitude in yields on bankers' acceptances in the sixties. Although the last year of the adjustment period is always tricky, the factors for 1965 appear to signal the end of the seasonal component.

Because its pattern is less stable than the patterns of the two short-term series described above, the seasonal variation of commercial paper rates is more difficult to isolate. Chart 10 plots the SI ratios and corresponding factors for commercial paper rates. From 1948 through 1950 the seasonal pattern, except for a high in October, is virtually the mirror image of the pattern in the late fifties—the first half year in the earlier period being above the 100.0 line, the second half below. The pattern evolves through a transition period in 1951 to a pattern that extends through 1955, and is quite similar to the one for long-term rates. Beginning with a low in January the factors drop to a trough in March, turn up through May or June to a peak in October and then sharply down. The pattern in 1955 already blends into the new pattern that is characteristic of the other short-term series: From a high in January the factors turn down through neutral February and the springtime lows to a trough in July then go up through neutral August to a peak in December. This pattern persists throughout the period of peak seasonality in the late fifties, during which, however, the seasonal amplitude never exceeds 8 per cent of the corresponding average values of the series. Beginning in 1962 the pattern appears to drift back towards the earlier one that resembled the pattern of long-term series. Nevertheless, the seasonal amplitude persists through the end of the period.

As in the case of the other short-term series, there is a repetitive ripple in the yields on nine- to twelve-month Treasury securities from 1948 through 1950, but, as Chart 11 shows, the factors do not fit the SI ratios as well as in the case of the other short-term series. Several years of erratic movement follow before the final pattern emerges in 1955. Choppy at first, it evolves rapidly into full shape in 1957 and slowly down again but persisting through the end of the period. Indeed, in 1965 the amplitude is greater and the pattern more discernible than in the case of bill rates.

The pattern during the late fifties is very similar to that of the other short-term series except that the trough comes a month earlier (in June) and a fall plateau replaces the December peak. This change in pattern is in the direction of the long-term series.

CHART 10

SI Ratios and Seasonal Factors for Commercial Paper Rates, 1948-65

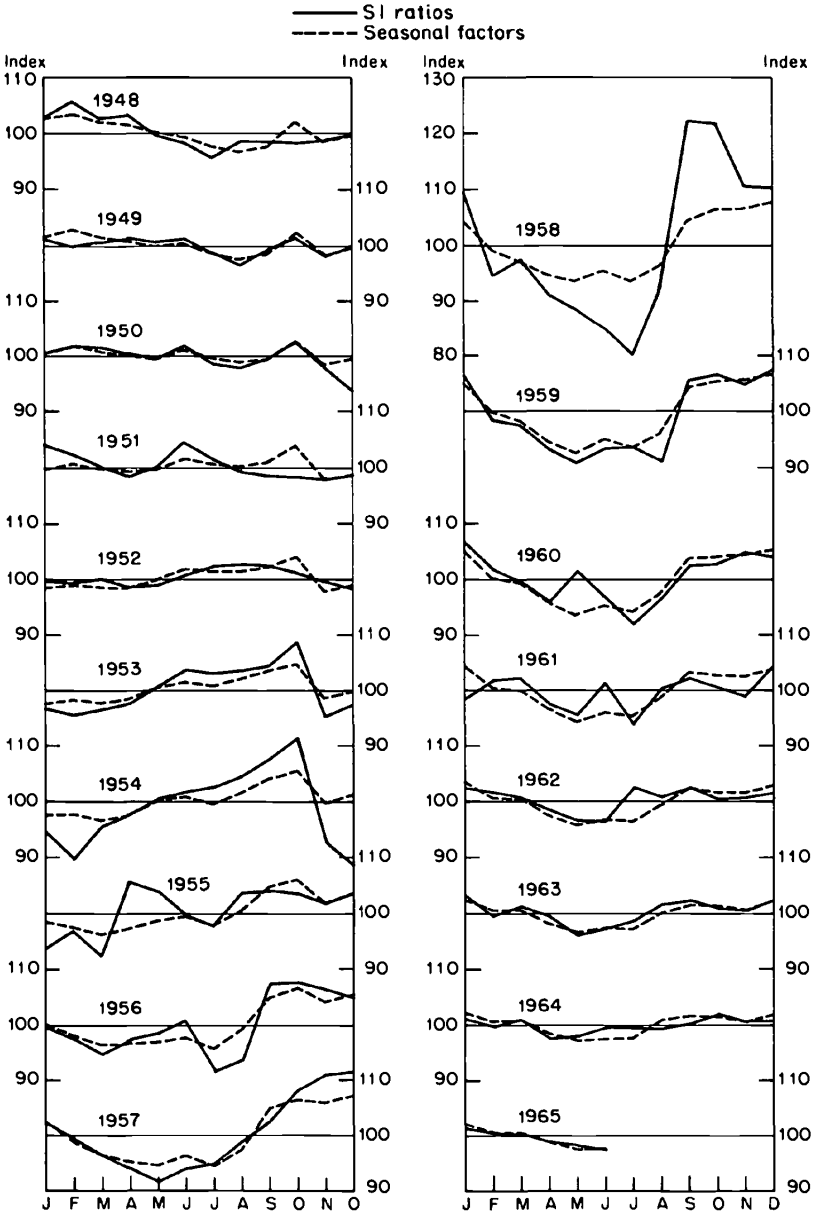
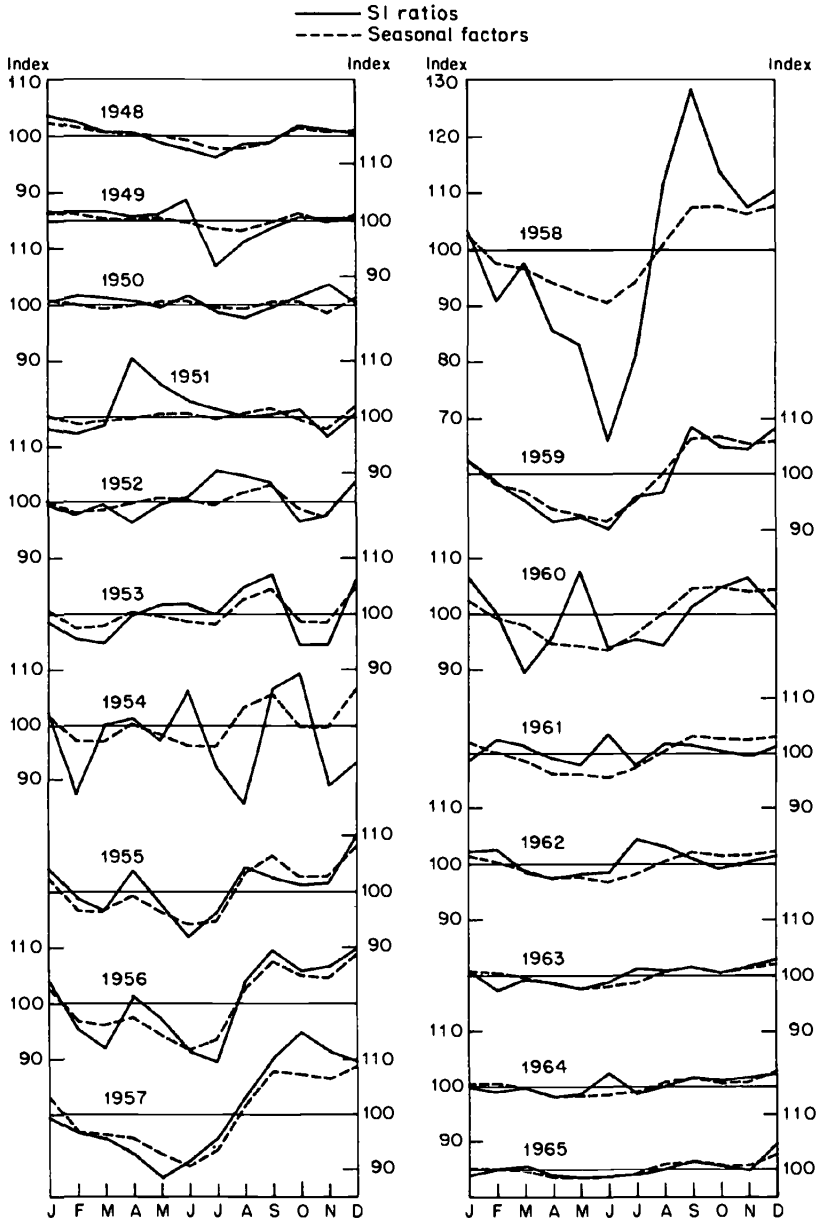


CHART 11

SI Ratios and Seasonal Factors for Yields on Nine- to Twelve-Month U.S. Government Securities, 1948-65



## SUMMARY OF SHORT-TERM SERIES

While the seasonal patterns of short-term rates are not entirely uniform throughout the postwar period, for the most part they have in common springtime lows and a midyear trough, as well as fall highs and a December peak continuing with some diminution through January. This pattern describes all four series in the period of greatest seasonality, 1956 through 1960. Treasury bill rates and yields on bankers' acceptances sustain this pattern throughout the 1948-65 period although not with as much stability and amplitude as during the 1956-60 period. The patterns for commercial paper rates prior to 1956 are quite different from those of the other short-term rates and, in fact, resemble those of the long-term rates that are described below. There are actually two distinct patterns for commercial paper rates during 1948-55, a fact making the adjustment for this period of questionable value. While the pattern for nine- to twelve-month Treasury securities prior to 1955 is quite similar to the one for Treasury bills (column 1 of Table 2), the factors are not sufficiently faithful to the SI ratios, in this study's view, to warrant an adjustment. In the later period, however, the seasonal influence is unambiguous. Chart 12 plots the time series of seasonal factors of the four short-term series considered in this study.

After 1960 the seasonal amplitudes of all the short-term series rapidly decline. While some seasonality persists after 1963, an adjustment will unavoidably introduce some additional error into the series. Whether the elimination of the true seasonal is worth the increased danger of introducing error as a result of adjusting for a spurious seasonal factor is an issue the user must decide. Table 3 lists the periods during which, in this study's view, the seasonal is worth adjusting for.

## LONG-TERM SECURITIES

## SUMMARY STATISTICS

The seasonal amplitudes of yields on long-term bonds are not high. In only two of the thirteen cases listed in Table 4 does the highest estimated factor for a given bond exceed 4 per cent and in only four

**CHART 12**  
**Seasonal Factors for Short-Term Interest Series, 1948-65**

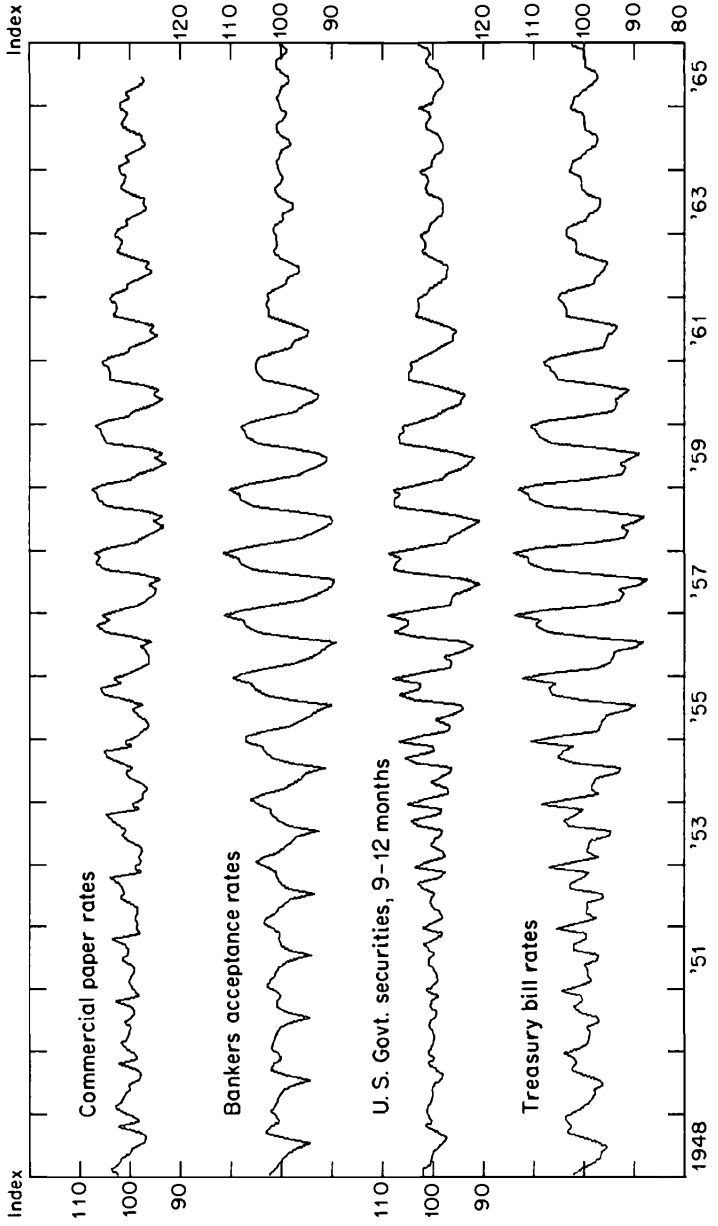


TABLE 3

*Suggested Periods for Adjusting Short-Term Series*

Security	Adjustment Period
Treasury bill rates	1948-65
Bankers' acceptance rates	1948-63
Commercial paper rates	1956-65
9-12 month Treasury security rates	1955-65

cases 3 per cent (column 8). Notwithstanding the low amplitudes, the computed *F*-statistics signify stable seasonality in all but two cases and in most cases by a wide margin (column 6). Finally, again notwithstanding the low peak amplitudes, as well as the seasonal components' relatively low average month-to-month percentage changes without regard to sign (column 4), the estimated percentages of total variation of the series due to the seasonal components (column 3) are roughly comparable and a little higher than the corresponding figure for Treasury bill rates.

These summary statistics are likely to be more reliable than the corresponding statistics for short-term securities because in the long-term bond rates, exclusive of the two Treasury series, the seasonality persisted throughout most of the study period, although with some changes in both pattern and amplitude. The summary statistics for three- to five-year and long-term Treasury securities, however, largely reflect the seasonal flourish in the late fifties. Both its small amplitude and its shifting pattern likely contribute to the persistence of seasonality in long-term bonds (as of 1965 the seasonal influence in most of the series was minute but discernible) since these characteristics in effect obscure the seasonal variation and thereby lessen the likelihood of investors trading them away.<sup>10</sup>

<sup>10</sup> A small amplitude does not by itself impugn the significance of the seasonal component. The seasonal amplitude for the series on the stock of money, for example, is never more than 3 per cent; although the value of the *F*-statistic in the postwar period is a robust 281. A comparison between column 5 in Tables 1 and 4 shows that relative to the variation of the cyclical component the seasonal amplitude of long-term bond rates is typically greater than that of short-term rates.

**TABLE 4**  
**Measures of the Relative Importance of the Seasonal Components in the**  
**Variation of Yields on Long-Term Bonds, 1948-65**

Series	Percentage of Total Variation of Series Due to Each Component		Average Month-to-Month Percentage Changes Without Regard to Sign of Seasonal Component $S$ (4)	Ratio of Column 4 to Corresponding Figures for the Cyclical Component $S/\bar{C}$ (5)	F-Test for Stable Seasonality (6)	Date and Factor of Seasonal High and Low for Whole Period <sup>a</sup>				
	I (1)	C (2)				S (3)	High		Low	
							Date (7)	Factor (percent-age) (8)	Date (9)	Factor (percent-age) (10)
Municipals <sup>b</sup>										
highest rating	44.23	34.78	21.00	0.78	2.855 <sup>c</sup>	Sept. 1958	104.7	Feb. 1958	96.6	
lowest rating	45.97	36.30	17.72	0.70	1.515	Sept. 1958	102.3	Apr. 1960	97.7	
high grade (Standard & Poor)	55.38	26.74	17.88	0.82	2.390 <sup>c</sup>	Sept. 1958	103.5	Feb. 1955	97.1	
Railroads										
highest rating	34.65	45.46	19.89	0.66	7.230 <sup>c</sup>	Sept. 1957	101.8	Mar. 1957	98.3	
lowest rating	36.92	38.36	24.71	0.81	7.111 <sup>c</sup>	Dec. 1956	101.6	Mar. 1954	97.9	

Series	Percentage of Total Variation of Series Due to Each Component			Average Month-to-Month Change Without Regard to Sign of Seasonal Component $\frac{S}{S}$ (4)	Ratio of Column 4 to Corresponding Figures for the Cyclical Component $\frac{S/C}{S/C}$ (5)	F-Test for Stable Seasonality (6)	Date and Factor of Seasonal High and Low for Whole Period <sup>a</sup>			
	I (1)	C (2)	S (3)				High		Low	
							Date (7)	Factor (percent-age) (8)	Date (9)	Factor (percent-age) (10)
Corporates										
highest rating	37.74	38.63	23.62	.54	.78	5.806 <sup>c</sup>	Sept. 1957	102.2	Mar. 1957	97.5
lowest rating	31.34	45.72	22.94	.43	.72	7.851 <sup>c</sup>	Oct. 1957	101.7	Mar. 1955	98.5
Public Utilities										
highest rating	34.60	38.92	26.48	0.61	0.82	7.290 <sup>c</sup>	Oct. 1958	103.2	Mar. 1958	97.2
lowest rating	38.26	41.22	20.52	0.44	0.70	7.078 <sup>c</sup>	Oct. 1957	101.7	Mar. 1955	98.5
Industrials										
highest rating	46.42	29.75	23.83	0.66	0.89	4.130 <sup>c</sup>	Sept. 1957	102.3	Mar. 1958	97.6
lowest rating	29.12	53.34	17.54	0.36	0.57	6.974 <sup>c</sup>	Oct. 1957	101.4	Mar. 1958	98.6
U.S. Treasury										
long-term 3-5 years	7.51	40.42	2.08	0.50	0.55	1.787	Sept. 1957 <sup>b</sup>	102.8	Apr. 1957	98.0
	3.73	37.76	8.51	1.45	0.71	3.642 <sup>c</sup>	Sept. 1957	105.8	Apr. 1959	95.2

Although there is a general bell-shaped pattern to the seasonal amplitudes of the long-term securities during the postwar period, the relative rise in the late fifties is not as pronounced as in the case of the short-term securities nor is it as pronounced in the private long-term rates as in the Treasury rates. Table 5 lists the variances of the seasonal factors of a given year for selected years for all the securities considered in this study. The variance of the twelve monthly factors is a convenient measure of the over-all seasonal amplitude for a given year. The bell-shaped pattern refers to the increase in the securities from 1953 to 1957 and their decrease to 1963. (The bell is actually quite symmetrical, a fact that is hidden by the different spans of the two periods.) Except for the railroad (lowest quality) securities, the table supports the generalization stated above.

There is a curious consistency in the relations among the seasonal factors of the several long-term series, a consistency for which there is no mechanical explanation. In Table 5, for example, the seasonal amplitude in 1954 for yields on lowest rated security groups is in every case less than the amplitude for the corresponding highest rated group; except for railroad bonds, this relationship holds in 1963 and 1965 as well.<sup>11</sup> Table 6 lists the correlation coefficients for the twelve monthly factors in 1957 of each of the long-term groups of securities with those of each of the other groups. In almost all cases the coefficients exceed .7; in most they exceed .8; and in many they exceed .9. The high correlations denote the substantial uniformity in the seasonal patterns of the long-term yields during the period of peak seasonality. Moreover, in virtually every case the correlation between

<sup>11</sup> Column 3 of Table 4 shows that the seasonal accounts for a smaller part of the total variation of low quality than of high quality bonds and the cyclical component accounts for a higher proportion (except for rails). This study is unable to explain the phenomenon.

#### NOTES TO TABLE 4

<sup>a</sup> A moving seasonal component was estimated for the four series. Columns 7 and 9 list the dates when the amplitudes of the seasonal component was greatest and columns 8 and 10 the values of the estimated seasonal factors for these dates.

<sup>b</sup> The estimated seasonal factor for September 1958 was also 102.8.

<sup>c</sup> Significant at 1 per cent level.

TABLE 5

*Variance of Seasonal Factors of a Given Year for Selected Years  
for All Securities Considered in this Study*

Security	1953	1957	1963	1965
<b>Long-Term bonds</b>				
<b>Municipals</b>				
highest rating	2.789	6.386	1.395	1.279
lowest rating	1.182	2.085	0.734	0.605
high-grade (Standard & Poor)	2.914	3.614	1.624	1.445
<b>Corporates</b>				
highest rating	0.516	2.208	0.224	0.145
lowest rating	0.664	1.132	0.071	0.053
<b>Industrials</b>				
highest rating	0.557	2.286	0.398	0.302
lowest rating	0.588	1.014	0.112	0.075
<b>Railroads</b>				
highest rating	0.386	1.350	0.115	0.062
lowest rating	1.300	0.846	0.211	0.223
<b>Public Utilities</b>				
highest rating	0.762	3.580	0.260	0.144
lowest rating	0.381	2.490	0.094	0.075
<b>U.S. Treasury</b>				
long-term	0.231	2.161	0.253	0.096
3-5 years	2.756	14.455	0.992	0.308
<b>Short-Term securities</b>				
U.S. Treasury bills	15.453	92.910	5.080	2.996
Commercial paper	5.684	25.341	4.252	n.a.
Bankers' acceptances	12.816	67.158	1.603	0.734
U.S. Treasury 9-12 months	6.843	42.367	2.425	1.977

NOTE: Each number signifies the variance of the twelve seasonal factors computed separately for each of the years and securities shown.

n.a. = not available.

any group of securities, say corporates and industrials, is greater for comparisons among security groups of the same quality rating. Along the last row, for example, the correlation between the factors for the

TABLE 6

*Coefficients of Correlation Among the Seasonal Factors in 1957 for All the Long-Term Securities Considered*

Series	Municipals			Corporates			Industrials			U.S. Treasury			Railroads			Public Utilities		
	highest rating	lowest rating	rating	highest rating	lowest rating	rating	highest rating	lowest rating	rating	3-5 years	long-term	rating	highest rating	lowest rating	rating	highest rating	lowest rating	rating
high-grade (Standard & Poor)																		
Municipals																		
highest rating	.9627																	
lowest rating	.9385	.9596																
Corporates																		
highest rating	.9644	.9349	.9134															
lowest rating	.7226	.7040	.7568	.8263														
Industrials																		
highest rating	.9749	.9307	.9067	.9660	.7223													
lowest rating	.7320	.7178	.7529	.8196	.9765	.7237												
U.S. Treasury																		
long-term	.9222	.9153	.8942	.8793	.7708	.8438	.8062											
3-5 years	.6553	.8514	.8163	.9036	.7914	.8440	.8251	.9400										
Railroads																		
highest rating	.9370	.8943	.9036	.9568	.8654	.8923	.8559	.9163	.9346									
lowest rating	.7920	.6986	.7568	.8752	.9126	.8084	.9168	.7567	.8334	.8993								
Public Utilities																		
highest rating	.9156	.9194	.8980	.9741	.8905	.9057	.8749	.8860	.9015	.9469	.8597							
lowest rating	.6860	.7252	.7452	.7967	.9650	.6962	.9512	.7435	.7369	.8015	.8152	.8774						

NOTE: Each coefficient, based on twelve observations, is computed by cross correlating the twelve factors of the vertical and horizontal series. Diagonal elements are omitted.

lowest rated version of the public utility group and those for, say, industrials-lowest rating, is greater than the correlation between the former and industrials-highest rating. In the row above, the correlations between the factors for the highest rated public utility group are greater in cases where the highest rated version of the paired group is considered in place of the lowest rated version of the same group. The correlations, of course, are not as high in years outside the period of peak seasonality; nor is the characteristic just described as conspicuous. These figures are shown in Tables 7 and 8.

By far the lowest correlations are with the long-term Treasury securities in 1965. These low coefficients help substantiate the conclusion that there is no seasonality in the series at the end of the study period. Evidence of uniformity is never more than suggestive. However, the more uniformity, the greater are the similarities among independently calculated results and the less likely are explanations of any given result that depend on alleged accidents or quirks in the computation. In this sense the similarity of the seasonal patterns and the curious relations among security groups of homogeneous rating constitute *prima facie* cases for the existence of a seasonal pattern, indictments, so to speak, on which it now behooves this study to obtain a conviction.

Before 1955 the typical seasonal pattern for yields on long-term securities describes low points in the first four months of the year and highs for the remainder, excepting a slight dip below 100.0 in November. The trough usually appeared in February or March, while the peak varied between July and December. The pattern changed in the 1954-56 period to one with a slight high in January, a rapid fall to a trough usually in March, continued lows through July, then a steep incline to a peak in September, and finally a gradual decline to December, still above the line. Table 9 lists the average seasonal factors for yields in long-term securities for selected years, computed by arithmetically averaging, the monthly factors for a given year, one at a time, of the thirteen long-term series. The recorded January factor, for example, is the average of all the January factors for the given year. In addition to the change in the seasonal pattern, the differences in the seasonal amplitude between the peak period in the late fifties and that before and after the period are revealed as well.

While Table 9 adequately describes the general seasonal pattern, its

TABLE 7

## Coefficients of Correlation Among the Seasonal Factors in 1953 for All the Long-Term Securities Considered

Series	Municipals		Corporates		Industrials		U.S. Treasury		Railroads		Public Utilities	
	highest rating	lowest rating	highest rating	lowest rating	highest rating	lowest rating	long-term	3-5 years	highest rating	lowest rating	highest rating	lowest rating
high-grade												
(Standard & Poor)												
Municipals	.9609											
highest rating	.7981	.8087										
lowest rating												
Corporates	.8679	.7868	.7454									
highest rating	.7224	.5943	.5041	.8114								
lowest rating												
Industrials	.8880	.8268	.8020	.9185	.5921							
highest rating	.6384	.5045	.4577	.7622	.9669	.5380						
lowest rating												
U.S. Treasury	.7881	.7092	.8303	.7772	.6281	.7944	.5338					
long-term	.3472	.1879	.3052	.6097	.5854	.4269	.5906	.5879				
3-5 years												
Railroads	.7056	.5705	.4289	.8722	.9021	.7061	.8373	.5518	.5505			
highest rating	.7316	.6001	.5127	.8660	.9774	.6681	.9407	.6040	.5792	.9619		
lowest rating												
Public Utilities	.8468	.7739	.7470	.9737	.8165	.8522	.7907	.7718	.6739	.8037	.8362	
highest rating	.5580	.4714	.4266	.6534	.9171	.3765	.9331	.4897	.6088	.7142	.8548	.7272
lowest rating												

NOTE: Same as Table 6.

TABLE 8

## Coefficients of Correlation Among the Seasonal Factors in 1965 for All the Long-Term Securities Considered

Series	Municipals		Corporates		Industrials		U.S. Treasury		Railroads		Public Utilities		
	high-grade (Standard & Poor)	highest rating	lowest rating	highest rating	lowest rating	highest rating	lowest rating	long-term	3-5 years	highest rating	lowest rating	highest rating	lowest rating
Municipals													
highest rating	.8655												
lowest rating	.8308	.7063											
Corporates													
highest rating	.6837	.7054	.8218										
lowest rating	.7762	.7062	.8174	.9384									
Industrials													
highest rating	.5769	.5751	.6732	.9535	.8958								
lowest rating	.5012	.5032	.3760	.6152	.6248	.6149							
U.S. Treasury													
long-term	-.0939	-.3798	-.0377	-.1206	.0191	.0169	-.4666						
3-5 years	.2306	.0658	.5929	.5911	.5510	.6210	.1167	.5138					
Railroads													
highest rating	.3308	.1636	.6548	.6029	.5654	.5585	.1542	.2523	.8091				
lowest rating	.7303	.6211	.8418	.8538	.9189	.7912	.3435	.1191	.5646	.6900			
Public Utilities													
highest rating	.7939	.7504	.8317	.9216	.8983	.8833	.6508	-.1121	.4939	.3914	.7607		
lowest rating	.5433	.3077	.4219	.6064	.7205	.7434	.4972	.4636	.5530	.4100	.5857	.6382	

NOTE: Same as Table 6.

TABLE 9

*Average Seasonal Factors for Yields on Long-Term  
Securities for Selected Years*

	1953	1957	1963	1965
January	99.3	100.4	100.1	99.8
February	98.7	98.4	99.6	99.5
March	98.8	97.9	99.3	99.6
April	99.3	98.3	99.3	99.5
May	100.4	98.6	99.3	99.4
June	101.1	99.2	100.0	100.0
July	100.7	99.4	100.4	100.4
August	100.5	103.1	100.3	100.2
September	101.1	102.5	100.7	100.6
October	100.5	101.9	100.5	100.4
November	99.8	101.2	100.3	100.1
December	100.4	101.0	100.4	100.3

NOTE: The figures for each month are computed by arithmetically averaging the seasonal factors for that month of the thirteen long-term securities considered in this study.

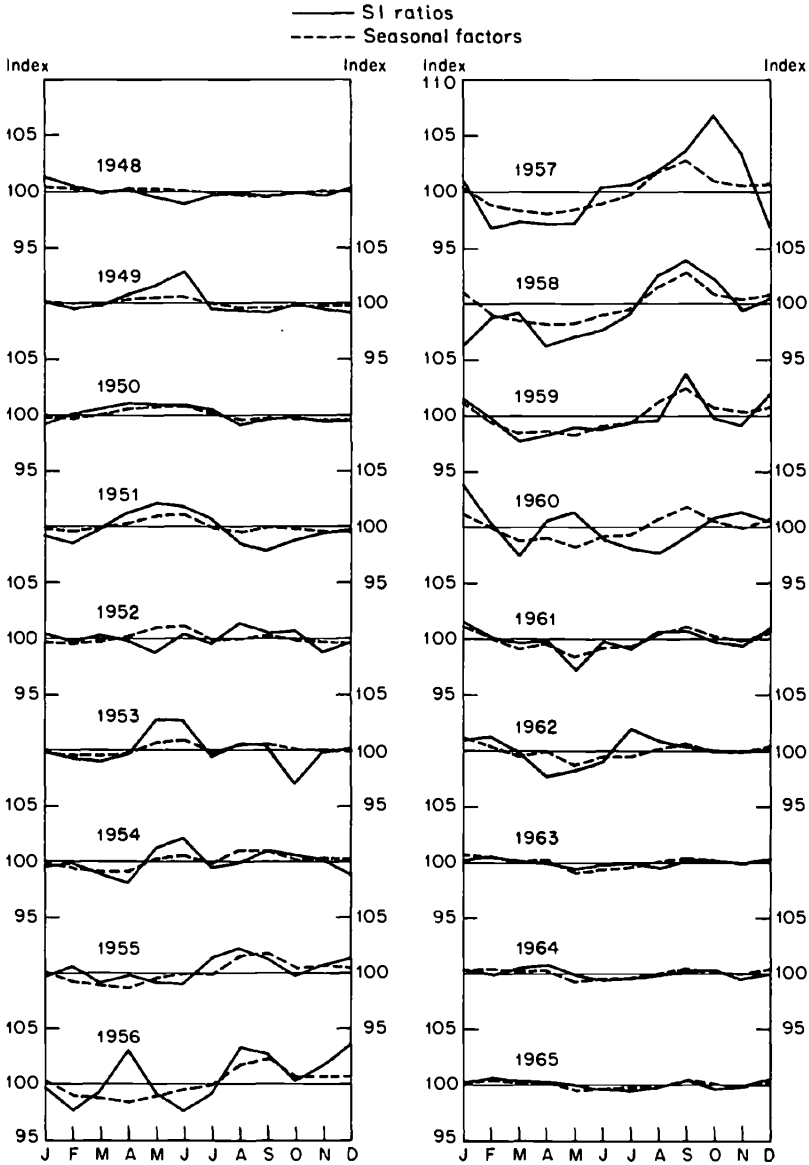
evolution, and the order of magnitude involved, it necessarily obscures important differences in the several series. The high correlations in Table 6 reveal a similarity in the patterns of most series during the late fifties; although the considerable differences in amplitude revealed in Table 5 are not, of course, accounted for. The somewhat lower correlations in 1953 and the considerably lower ones in 1965 lessen the usefulness of the computed average pattern outside the 1955-60 period. In addition to the variety of patterns and amplitudes in the long-term series, there are also differences in the quality of the estimates of the seasonal factors. To decide on the extent of appropriate adjustment for each series it is therefore necessary to examine the familiar charts of factors and SI ratios, which we do now.

#### GRAPHIC ANALYSIS OF SEASONAL VARIATION

*U.S. Treasury securities.* Chart 13 plots the seasonal factors and SI ratios for long-term U.S. Treasury securities (the scale is twice

CHART 13

SI Ratios and Seasonal Factors for Yields on Long-Term U.S. Bonds, 1948-65

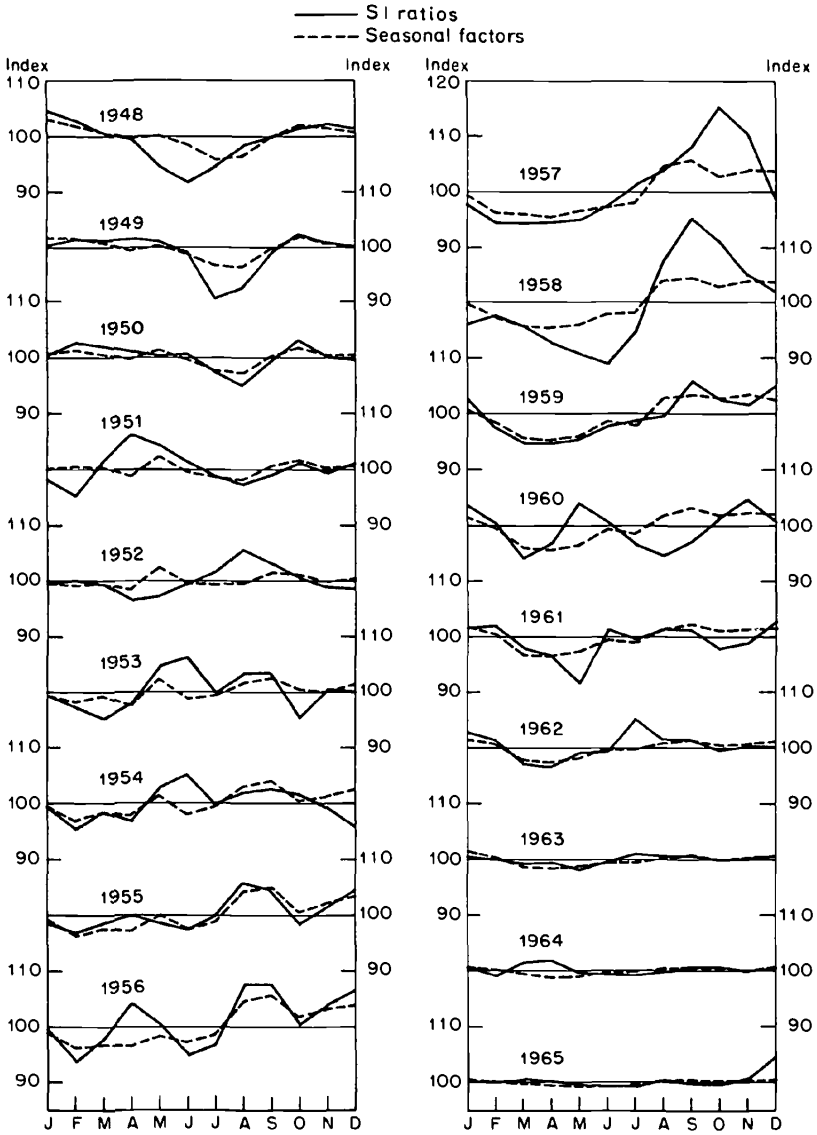


that used for short-term securities). The pattern prior to 1953 is very similar to the early pattern of commercial paper rates; the peak coming in midyear and the trough in the fall. Although the amplitude is small, the pattern is quite real. The reason for the asserted reality resides not only in the similarity between the curves connecting the SI ratios and the ones connecting the factors but also in the position of the factors, falling as they do between the SI ratios and the 100.0 level. The latter result is most conspicuous for 1951 and has the effect of both dampening the adjusted series and minimizing the possibility of the adjustment contributing to the random fluctuation of the series. As in the case of the Treasury bill rates during the late fifties, the assurance that all that is removed is seasonal comes at the expense of understating what seasonal influence there is. The data for 1952 provide a good example of the dilemma involved in adjusting a series with a rapidly shifting pattern: the two curves are virtually unrelated. Use of the estimated factors may then introduce random errors into the series. In 1953 the pattern assumes the shape of the average pattern of Table 9 and, hence, the high correlations in Table 7 between the long-term Treasuries and the other securities. In the following years the pattern rapidly bends into the one typical of all the rates in the late fifties. Even in this period the fit is not very good, although the pattern is clearly there; and the amplitude is among the highest of the long-term security groups. One notices, even in the late fifties, how the low part of the pattern is gradually extended through the summer, as with the short-term securities. By 1965 the pattern is very different from the average pattern and, in fact, inversely correlated with those of most of the long-term series (Table 8). It is clear, then, that the low  $F$ -statistic noted earlier for this series is due both to a constantly shifting pattern resulting in a mediocre fit and an unstable pattern even within the periods it describes. That a seasonal pattern existed in the late fifties and even that a systematic ripple persisted to the end of the study period is, in this study's view, established. Whether there is reasonable cause to adjust the series outside the 1955–59 period is somewhat dubious; and whether it would be meaningful to adjust a series for such a short period is equally dubious. This series is perhaps one that may profitably be left alone.

Chart 14 plots the seasonal factors and SI ratios for U.S. Treasury

CHART 14

SI Ratios and Seasonal Factors for Yields on U.S. Government Three- to Five-Year Securities, 1948-65



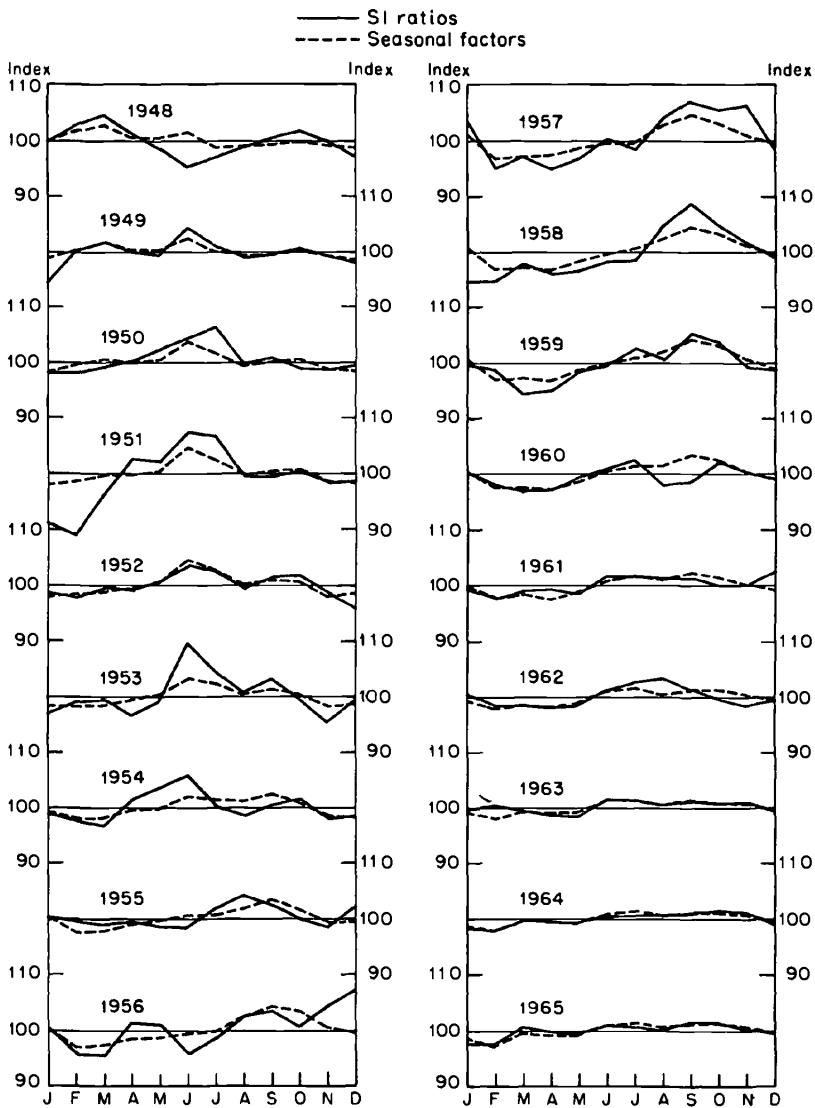
securities with three- to five-year maturities. There is a fairly clear pattern for the years 1948–60 followed by several years of erratic movement—the pattern for 1953 is typical. In 1955 the pattern assumes the shape it maintains for the remainder of the decade. In this period the seasonal amplitude of this series is somewhere between the typical amplitude of short- and long-term securities. From 1960 on, however, the seasonal pattern is too unstable to justify an adjustment.

*Municipal securities.* Of all the long-term securities the  $F$ -statistics are among the lowest and the amplitudes among the highest for the yields on the three municipal bond series considered. The  $F$ -statistics, however, are not brought down, as in the case of long-term Treasuries, by constantly shifting patterns of small amplitude and erratic transition periods; nor is the  $F$ -statistic spuriously high as a result of the attenuation of the peak seasonal in the late fifties. Chart 15 plots the seasonal factors and SI ratios for yields on municipal bonds of highest rating. Excluding the first two years, the pattern in the early years is quite similar to the general pattern of this period described in Table 9: lows during the first four months, a June peak, but then, unlike the general pattern, a rapid decline to lows in the last two months. The large gaps between the curves, implied in the unusually large irregular component recorded in column 1 of Table 4, reflect changes in the seasonal amplitude of the series rather than an unstable pattern. Although the greater dispersion of a given month's SI ratios results in less reliable estimates of the seasonal factors, the diagrams reveal the SI ratios for given months to fluctuate about means that clearly remain above or remain below the 100.0 line. From 1949 through 1954, every June is a high, and every January and December are lows. In 1955 the new pattern emerges, again similar to the general pattern of the late fifties, with a trough in February or March and a peak in September. The pattern changes somewhat after 1959, but persists in similar form through the end of the study period.

In this case, therefore, and in the case of the other municipal groups not shown here, a basis for seasonal adjustment exists in spite of the feeble  $F$ -statistics. There is an important difference between the effects of random variations in seasonal amplitude and those in the seasonal pattern on identifying the presence of seasonality. In the

CHART 15

SI Ratios and Seasonal Factors for Yields on Municipal Bonds, Highest Rating, 1948-65



former case, by narrowly defining extreme points, the program can dampen the estimated seasonal amplitudes, imposing a downward bias on the estimated means, and thereby reducing the risk of wrongly affirming the presence of seasonality. In the latter case the danger is greater that the program will impose a seasonal pattern on the series.

*Private long-term securities.* Although there are differences in detail it is convenient to describe the seasonality of yields on private long-term securities for the group as a whole. This group's seasonal pattern differs from that of the municipals by its somewhat smaller amplitude and its greater stability. Table 4 records higher *F*-statistics, lower residual variation, and smaller peak amplitudes (columns 6, 1, 8, and 9 respectively), and Table 5 shows the seasonal amplitudes of the private groups to be less than those of the municipals with very few exceptions throughout the study period.<sup>12</sup> The curves connecting the SI ratios in Chart 16, drawn for yields on corporate securities of lowest rating, are clearly less choppy than the corresponding curves for municipal securities, and the amplitude of the corporates is smaller. The patterns are somewhat erratic through 1951, but thereafter they are quite similar to the first pattern listed in Table 9. In 1955 the January factor starts to rise and the low period extends into the summer, typical of the general pattern in this period. This pattern persists to the end of the study period although its amplitude at the end is barely perceptible.

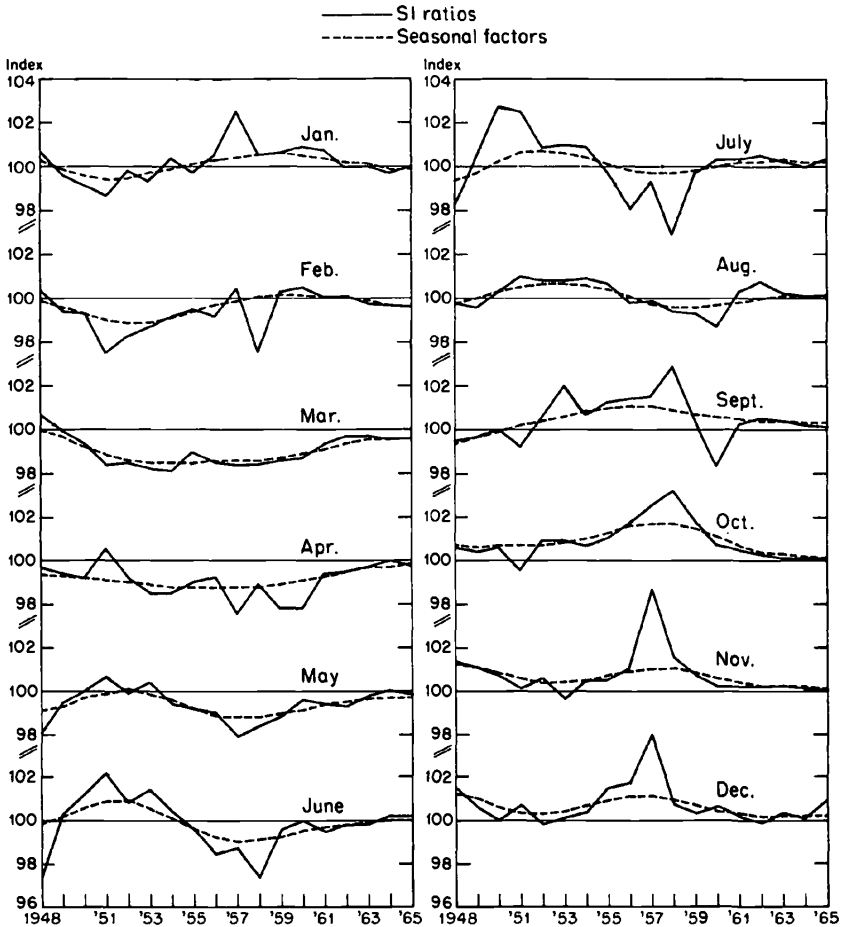
The stability of the pattern and therefore the evidence of seasonality is further illustrated in Chart 17, where the SI ratios and the factors are plotted one month at a time across the years. There are five months for which the direction of seasonal change is consistent throughout the study period: March, April, September, October, and November. Beginning in 1950 the SI ratios for March are consistently below the 100.0 line; their amplitudes in the early fifties are as great as in the late fifties, tapering off in 1960 but persisting to the end of the study period. The factor curve is virtually identical with the curve connecting the SI ratios. Again for April, the SI ratios remain consistently below the 100.0 level, crossing it briefly in 1951 and

<sup>12</sup> It may be that an explanation of the difference in amplitude between the two groups of securities lies in the greater ease with which private corporations can time their borrowing to correspond with periods of seasonally low yields.



CHART 17

SI Ratios and Seasonal Factors for Yields on Corporate Bonds, Lowest Rating, One Month at a Time, 1948-65



again in 1961. Except for these two years the factor curve either coincides with the SI curve or rises above it, in any case it does not exaggerate the seasonal variation. In September, barring 1951 and 1960, and again in October the SI ratios are consistently above 100.0. Of the four months only October evinces a greater amplitude in the late fifties; and the factor curve virtually nullifies the increase. The curves for the middle months, May through August, and the one for

January all cross the 100.0 line at about 1954, signifying the changed pattern.

While there are differences in the patterns of the various groups of long-term securities a detailed account of each of the series would be almost as tedious to write as it would be to read. In lieu of that Chart 18 plots time series of the seasonal factors for all the long-term series considered in this study.

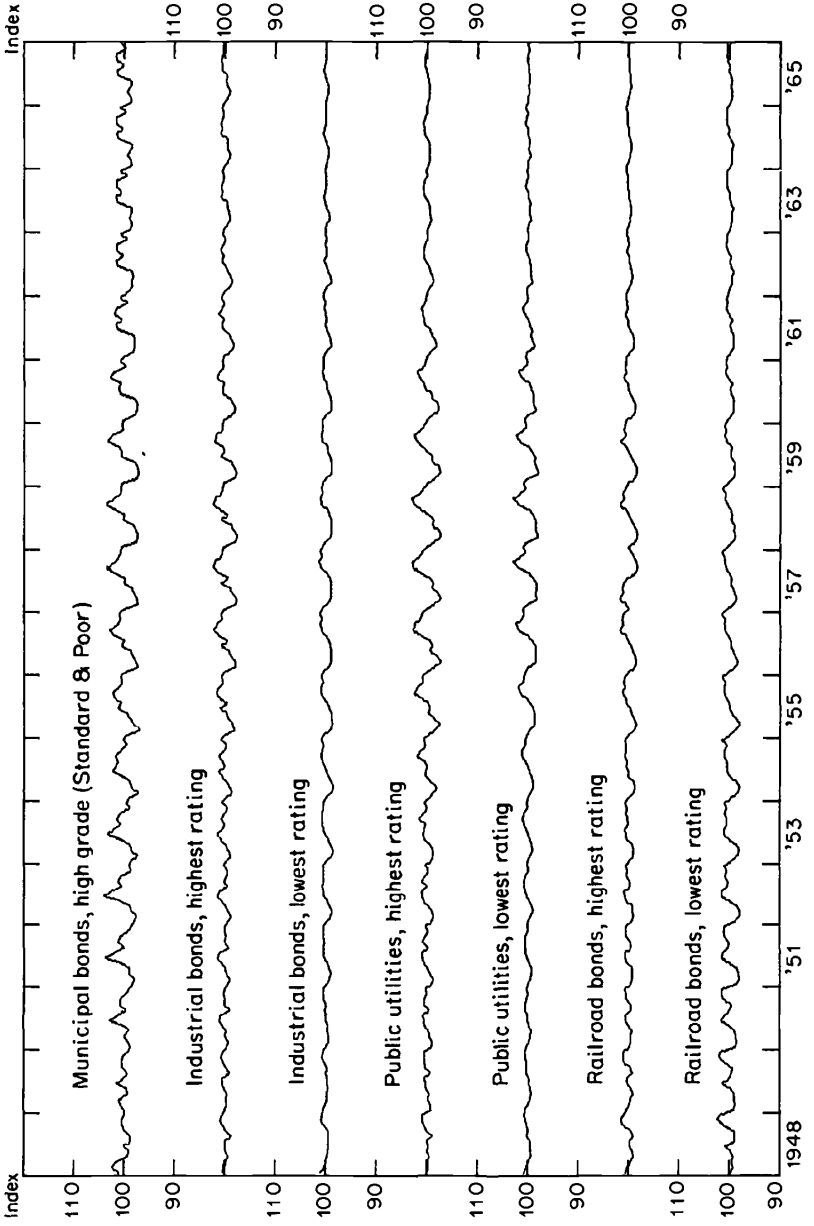
#### SUMMARY OF LONG-TERM BONDS

In spite of the low seasonal amplitudes in long-term bonds, both the summary statistics and the diagrams confirm the presence of seasonality in these series throughout most of the sample period. Starting about 1950, the seasonal factors are typically below the 100.0 line in the first four months of the year, the trough usually occurring in March, rise above the line in midyear and beyond it to a peak in September or October then fall back to the 100.0 line in November and December. Starting in 1955 the midyear months stay below the 100.0 line, and the months at either end rise a little. The key seasonal months—March, April, September, and October—are largely unaffected by this change. The prominent bell-shaped pattern in the seasonal factors described for the short-term rates is less prominent in the case of long-term bonds, although the seasonal factors for some of the series clearly evolve in this manner.

As in the other series considered in this study there are some years for which there appear to be no seasonal movement at all or for which, whether present or not, the seasonal is too small and uncertain to be measured. There is no accepted method for reliably choosing the years for which an adjustment is appropriate. Nor do the necessary conditions prevail for reliable inference from the summary statistics. Although there is a large subjective element involved in the method used in this study, the primary criterion has been the apparent stability in the seasonal patterns regardless of their amplitudes. But the judgment involved in this study is restricted to the choice of accepting the results of the X-11 program. Additional judgment is required to improve this adjustment. Table 10 presents the suggested dates for accepting the machine adjustment. For convenience the table includes the findings for all seventeen series considered in this study.

CHART 18

Seasonal Factors for Long-Term Interest Rates, 1948-65



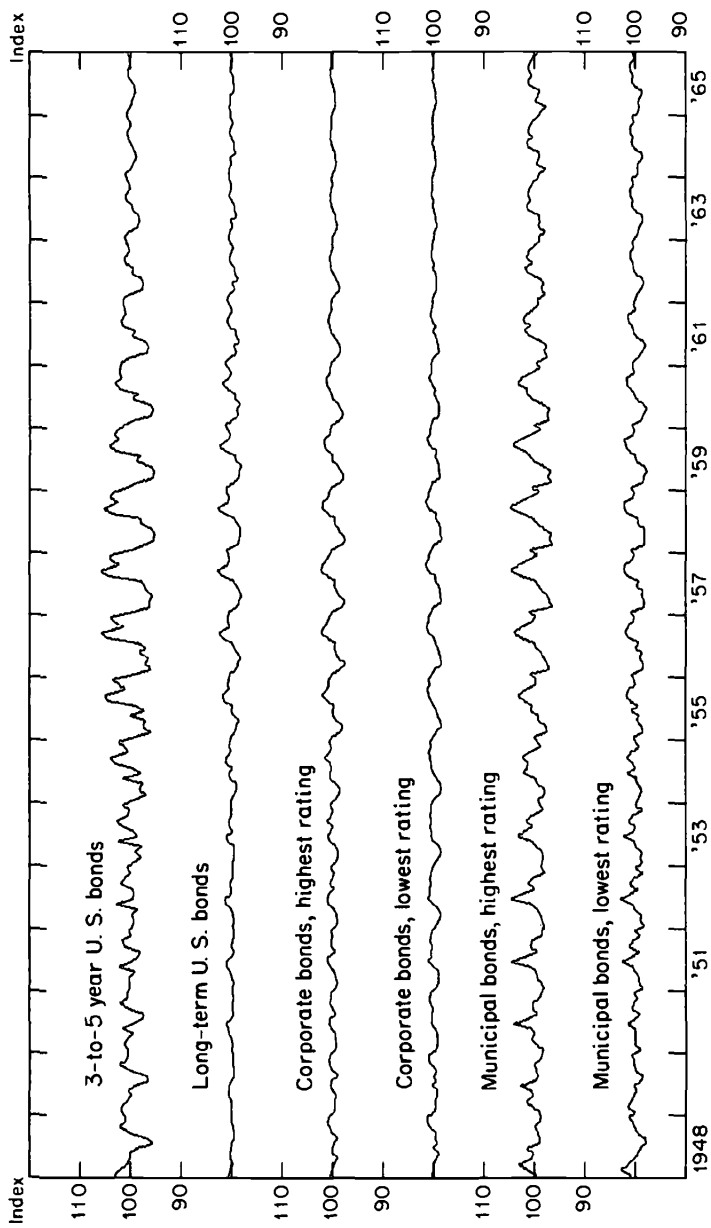


TABLE 10

*Suggested Periods for Accepting Seasonally  
Adjusted Interest Rate Series*

Series	Period
Treasury bills	1948-65
9-12 month Treasury securities	1955-65
3-5 year Treasury securities	1955-59
Long-term Treasury securities	1955-59
Commercial paper rates	1956-63
Bankers' acceptances	1948-63
Industrial bonds <sup>a</sup>	1956-65
Industrial bonds <sup>b</sup>	1950-65
Public utility bonds <sup>a</sup>	1950-65
Public utility bonds <sup>b</sup>	1951-61
Corporate bonds <sup>a</sup>	1952-65
Corporate bonds <sup>b</sup>	1952-65
Railroad bonds <sup>a</sup>	1953-63
Railroad bonds <sup>b</sup>	1950-65
Municipal bonds <sup>a</sup>	1950-65
Municipal bonds <sup>b</sup>	1950-65
Municipal bonds <sup>c</sup>	1950-65

<sup>a</sup>Highest rating.

<sup>b</sup>Lowest rating.

<sup>c</sup>High-grade Standard and Poor.

#### APPENDIX: ALTERNATE ADJUSTMENTS OF TREASURY BILL RATES

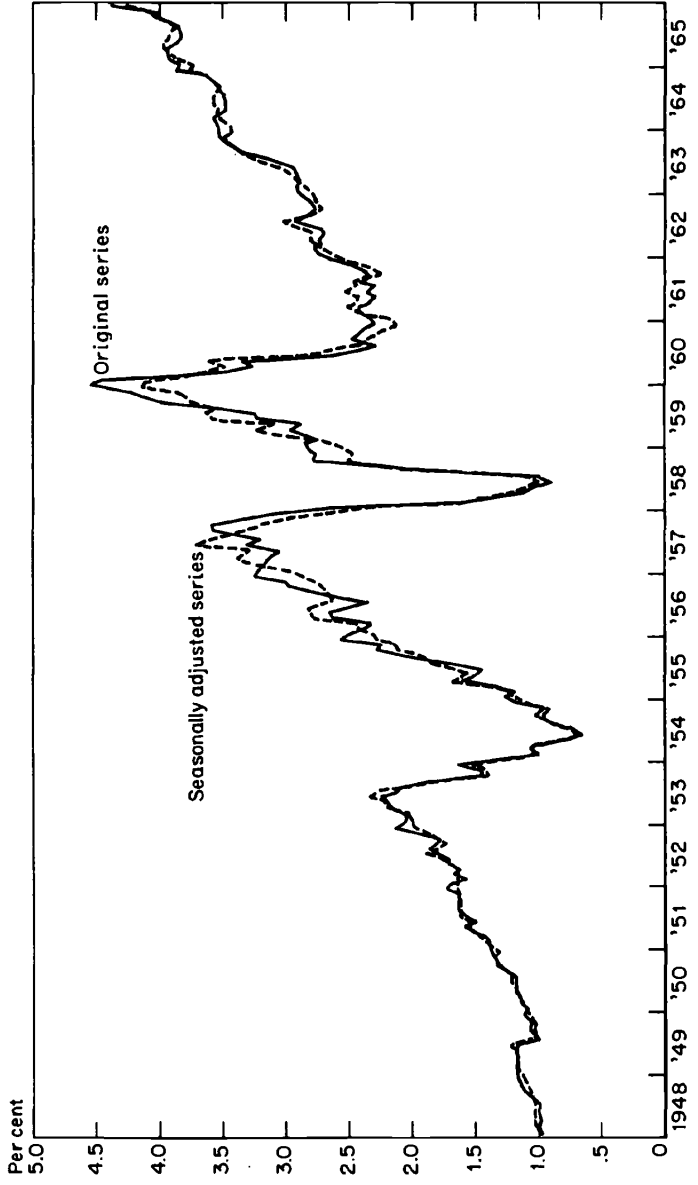
There are several reasons for the differences between the curves of SI ratios and seasonal factors in Charts 7 and 8. Most important among them is, of course, the seasonal factors' intended elimination of the irregular component through its smoothing out the nonrecurring variation. One-time jumps, therefore, as in May 1960, appropriately result in gaps between the two curves. It is easy to identify these atypical (or irregular) jumps when they produce inversions (in the

case of May 1960 from a low to a high), but the identification is more difficult when the atypical events exaggerate the usual fluctuation without seriously distorting the pattern. In 1958, for example, the two curves are far apart. There are no inversions, but the July trough is well below its value in adjacent years, and the October peak is high—both absolutely and relative to adjacent months. Short-term interest rates were dropping sharply in the spring of 1958 (see Chart 19), and investors, anticipating a continuing decline, borrowed heavily to purchase maturing Treasury bonds that earned rights to a new issue in June. Short-term capital gains on the new issue appeared certain as all the indicators agreed the decline in rates would continue. When short-term rates suddenly turned upward in June, they left many highly leveraged investors on the wrong end of the fulcrum. In addition to distorting the SI ratios, the resulting chaos invited a congressional investigation.<sup>18</sup>

While it is easy and proper to shield the seasonal adjustment from one-time changes in the pattern of SI ratios, it is much harder to apportion brief changes (whether lasting one or a few years) in the amplitude of a fixed pattern of SI ratios. There is no requirement that the seasonal amplitude be stable from year to year or even that it change only gradually. For example, the seasonal in the consumption of soft drinks is related to the weather; a particularly warm summer will result in an atypically large seasonal amplitude. An adjustment that is constrained to gradual changes in amplitude will arbitrarily truncate the seasonal amplitude in the above example. Alternatively, one may argue that seasonal analysis is concerned not with the relation between climate and soft drink consumption but only with systematic intrayear changes in consumption regardless of cause. This point of view is predicated on the idea that seasonals are important because they are predictable by mechanical projections of past behavior; any variation that is not predictable in this way, even when it is caused by the same variables that cause the predictable variation, is not seasonal. In cases, such as interest rates, where there are abrupt changes in the amplitude of seasonal, or, if you will, intrayear,

<sup>18</sup> Part of the results of this investigation are recorded in a fascinating study of the crisis. See *The Treasury-Federal Reserve Study of the Government Securities Market*, Part II, Washington, D.C., February 1960.

CHART 19  
Original and Seasonally Adjusted Series of Treasury Bill Rates, 1948-65



variation, the choice of the appropriate adjustment depends on which of the two concepts of a seasonal movement is intended.

To illustrate the practical significance of this issue the study experimented with alternate adjustments of Treasury bill rates. The Kuznets amplitude ratio method of adjustment is convenient for dealing with an abruptly changing seasonal amplitude provided the month-to-month pattern of factors is fixed, and only the amplitude changes from year to year. The factors are computed by regressing, one year at a time, the twelve modified SI ratios<sup>14</sup> on the set of constant factors obtained by averaging, one month at a time, the modified SI ratios over the whole period. The more similar the particular year's pattern is to the average pattern the higher will be the correlation coefficient; when it is equal to one the regression coefficient will exactly measure the proportionate difference in amplitude between them.<sup>15</sup> Table 11 lists the coefficients of correlation and regression, as well as the latter's *t*-value for the eighteen regressions from 1948 through 1965. All three statistics reveal the bell-shaped pattern in the seasonal amplitude that was noted earlier. Again, the seasonal in 1948 is unusually clear, its pattern being highly correlated with the average pattern. In the following two years the seasonal is still inexplicably strong, although less than in 1948, the pegged prices notwithstanding. After two dormant years in 1951 and 1952 the seasonal emerges again in 1953 and more strongly in 1954, although the erratic pattern noted earlier is reflected in a correlation coefficient that is lower than in the following years. The peak seasonal continues through 1960, after which it drops off sharply; and after 1961 the seasonal is very small.

The regression coefficients listed in Table 11 are convenient for evaluating the hypothesis that there is a relation between the sea-

<sup>14</sup> That is, modified to eliminate extreme values. It is clearly important to eliminate the effect of extreme values on a regression computed with twelve observations, although the X-11 modifications, used here, will in some cases dampen the very changes in amplitude the Kuznets method is designed to reveal.

<sup>15</sup> In commenting on this method Burns and Mitchell (*op. cit.*, pp. 48-49), recommend not accepting the estimate when the correlation coefficient falls below .7. It is clear that when the pattern changes, as in the case of commercial paper rates, the method has no value. The pattern for bill rates, however, is fairly stable; therefore, a low correlation coefficient signifies doubtful seasonality.

sonal and cyclical components. Chapter 2 noted that the X-11 method of smoothing the SI ratios over adjacent years to compute the factors necessarily obscures any relation that may exist between its seasonal and cyclical components. There is nothing, however, to prevent the Kuznets amplitude-ratios (the regression coefficients in Table 11) from varying with the cyclical component of the series. There is clearly nothing in columns 3 and 7 to reveal any relation between the prominence of the seasonal component and the level of the series as shown in Chart 19. The erratic movements in the SI ratios in 1954 and 1958 suggest that the irregular component may be more prominent during cyclical troughs, a proposition that may or may not reflect on the capacity of the Henderson curve to capture sharp turning points.

Columns 4 and 8 of Table 12 list the seasonal factors computed with the Kuznets method.<sup>16</sup> These factors are simply the computed values of the regressions described earlier. The X-11 factors (columns 1 and 5) clearly dampen the changes in seasonal amplitudes. Excluding 1948 and 1949, the Kuznets factors show larger seasonal amplitudes from 1956 through 1960 than the X-11 factors and smaller amplitudes elsewhere. The differences are greatest in 1957 and, expectedly, in 1958. Whereas the X-11's July factors for these years are, respectively, 87.4 and 87.9, the corresponding Kuznets factors are 85.0 and 80.4; for December the X-11 factors are 113.9 and 113.0, respectively, for 1957 and 1958, and the corresponding Kuznets factors, 118.7 and 121.6.

The adjustment's sensitivity to abrupt changes in amplitude is not solely a question of preference. One condition for a perfect adjustment is the absence of seasonality in the adjusted data.<sup>17</sup> Perfection aside, the seasonality that remains in the adjusted data betrays the quality of the original adjustment. In other words, even allowing a gradually

<sup>16</sup> The other figures in Table 12 and curves in Chart 10 are explained below.

<sup>17</sup> While necessary, this condition is not sufficient. The adjusted data must also remain faithful to the original in all respects other than seasonality. The trend-cycle values, for example, are free of seasonality but do not otherwise qualify as properly adjusted values of the original series. Lacking perfection, an adjustment method should evince a convergence toward no-seasonality upon successive adjustments of the data, that is, adjustments of the adjusted data. The X-11 appears to satisfy this requirement, although the present study has not considered this issue in any detail.

TABLE 11

Summary Statistics for Kuznets' Amplitude-Ratio Method Regressions,<sup>a</sup> 1948-65

Year (1)	Correlation Coefficient (2)	Regression Coefficient (3)	<i>t</i> = value (4)	Year (5)	Correlation Coefficient (6)	Regression Coefficient (7)	<i>t</i> = value (8)
1948	.9346	.7106	8.3102	1957	.9495	2.4562	9.5708
1949	.8826	.4643	5.9367	1958	.9434	3.0079	8.9945
1950	.7073	.2759	3.1642	1959	.9619	2.2094	11.1281
1951	.3046	.1785	1.0114	1960	.8772	1.4544	5.7780
1952	.3935	.3350	1.3534	1961	.7420	.6177	3.5004
1953	.5977	.4930	2.3575	1962	.6668	.4048	2.8294
1954	.7859	1.0771	4.0198	1963	.7134	.2911	3.2193
1955	.9130	1.5308	7.0785	1964	.6991	.2750	3.0922
1956	.9529	1.9761	9.9303	1965	.6745	.2589	2.8890

<sup>a</sup>The regression form for *i*th year is as follows: Modified SI ratios for year *i* = *a* + *b* (constant seasonal factors) + U.

Since the mean value for both variables is 100, the constant term, *a*, is equal to 100 (1-*b*). There are twelve observations for each regression.

TABLE 12  
*Seasonal Factors in Treasury Bill Rates for July and December Computed with the X-11 on the Original and  
 Seasonally Adjusted Series, and with the Kuznets Method*  
 (per cent)

	July				December			
	Original Seasonal Factors (1)	Adjustments of Seasonal Factors (2)	Implicit Factors (3)	Kuznets Factor (4)	Original Seasonal Factors (5)	Adjustments of Seasonal Factors (6)	Implicit Factor (7)	Kuznets Factor (8)
1948	96.8	100.0	96.9	95.1	103.7	99.8	103.5	104.9
1949	97.0	970.9	97.8	96.9	103.9	99.4	103.3	103.3
1950	97.3	102.1	99.3	98.3	104.6	99.3	103.8	102.1
1951	96.9	102.9	99.8	99.0	105.5	99.1	104.5	101.4
1952	96.2	103.5	99.6	98.1	106.9	99.2	106.0	102.7
1953	94.4	103.1	97.3	97.1	108.4	99.4	107.8	103.8
1954	92.4	102.3	94.5	92.9	110.5	100.1	110.6	107.7
1955	89.8	100.7	90.5	91.0	112.2	100.9	113.2	112.0
1956	88.3	99.8	88.1	87.8	113.8	102.0	116.1	114.8
1957	87.4	99.1	86.6	85.0	113.9	102.4	116.7	118.7
1958	87.9	99.1	87.1	80.4	113.0	102.4	115.8	121.6
1959	89.0	99.0	88.2	85.0	110.5	101.4	112.1	115.2
1960	91.1	99.6	90.8	89.9	107.9	100.3	108.2	109.8
1961	93.3	100.2	93.5	95.9	105.1	99.1	104.2	104.3
1962	95.3	100.8	96.1	97.5	103.5	98.5	101.9	103.1
1963	96.7	101.3	98.0	98.2	102.7	98.3	101.0	102.2
1964	97.6	101.7	99.3	98.3	102.7	98.4	101.1	102.0
1965	98.0	101.9	99.9	98.3	102.6	98.3	100.9	101.9

NOTE: Columns 1 and 5 are the original factors computed with the X-11. Columns 2 and 6 are the factors computed for adjusting the seasonally adjusted series. Columns 3 and 7 are the implicit factors obtained with the double adjustment.

changing seasonal, the moving average of the X-11 may not be sufficiently elastic to expunge all the variation that by its own criteria (i.e., its response to a second round) are seasonal. The seasonally adjusted data will in this case retain some remnants of the seasonal pattern. The experiment with the Kuznets method suggests what these remnants will look like.

To illustrate the point this study ran the seasonally adjusted data once again through the X-11. Columns 1 and 5 of Table 12 list the factors for July and December, respectively, obtained with the original adjustment; columns 2 and 6 list the factors with the second adjustment; and columns 3 and 6 the implicit factors computed by dividing the twice-adjusted series (i.e., the series obtained by adjusting the seasonally adjusted series) into the original series. When the factor in column 2 is above 100.0, the original estimate of the July factor is too low, that is, the seasonal decline is exaggerated; the implicit factor in column 3 is in this case greater (i.e., closer to 100.0) than the original factor in column 1. Conversely, when the factor in 2 is below 100.0 the original estimate is too high (i.e., the seasonal trough underestimated) and the factor in 3 is below the factor in 1. When the factor in 6 is below 100.0, the original estimate of the seasonal peak is exaggerated and correspondingly reduced in 7; when above the original estimate it is too low and therefore increased in 7. From 1956 to 1960 the machine understated the seasonal low in July. Some, but very little, seasonal low remains in the adjusted data. In the other years, however, the machine converted the original seasonal lows in the July data to seasonal highs<sup>18</sup> in the adjusted data. In December the seasonal from 1954 to 1960 is understated in the original adjustment and in the remaining years overstated.

Table 13 puts the same story a little differently. In 1953 the factors computed in the second adjustment (column 2) denote, with some exceptions and with a smaller amplitude, seasonal movements in the opposite direction from those implied in the original adjustment. In 1958 the seasonal timing implied in both columns 5 and 6 is the same, reflecting the original adjustment's failure to fully re-

<sup>18</sup> "High" and "low" denotes positions above and below 100.0, respectively; "peak" and "trough" denotes highest and lowest, respectively. The Kuznets experiment implies the same result for July but dates the period of understatement in 1956 instead of 1954 for December.

TABLE 13

*Seasonal Factors for Selected Years Computed for Both the Original and the  
Seasonally Adjusted Treasury Bill Rate Series*

	1953				1958				1965			
	Original (1)	Ad- justed (2)	Implicit (3)	Kuznets (4)	Original (5)	Ad- justed (6)	Implicit (7)	Kuznets (8)	Original (9)	Ad- justed (10)	Implicit (11)	Kuznets (12)
Jan.	102.1	98.9	101.0	102.3	108.9	101.6	110.6	112.3	102.3	99.7	102.0	101.1
Feb.	97.0	98.5	95.6	100.0	98.4	99.9	98.3	98.1	102.0	100.5	102.5	99.9
March	99.2	100.8	100.0	98.9	92.1	98.4	90.6	91.8	100.8	101.7	102.5	99.3
April	98.9	101.1	100.0	98.6	91.0	98.1	89.3	89.7	99.3	101.4	100.6	99.2
May	98.7	100.7	99.4	98.5	92.6	95.0	87.9	89.4	97.9	101.4	99.3	99.1
June	94.9	100.1	95.0	97.3	89.4	99.7	89.1	81.9	97.3	101.4	98.7	98.5
July	94.4	103.1	97.3	97.1	87.9	99.1	87.1	80.4	98.0	101.9	99.9	98.3
Aug.	102.9	102.2	105.2	99.9	96.2	99.7	96.0	97.8	99.8	100.7	100.1	99.9
Sept.	108.9	98.9	102.7	102.0	107.4	100.4	107.8	110.5	99.8	97.7	97.4	100.9
Oct.	101.3	99.2	100.6	102.0	110.6	101.7	112.5	110.5	100.0	97.3	97.3	100.9
Nov.	100.0	97.7	97.7	101.9	110.9	102.8	114.0	110.1	100.1	97.9	98.0	100.9
Dec.	108.4	99.4	107.8	103.8	113.0	102.4	115.8	121.6	102.6	98.3	100.9	101.9

CHART 20

Unmodified SI Ratios, Seasonal Factors, Implicit Seasonal Factors, and Kuznets' Factors for Treasury Bill Rates

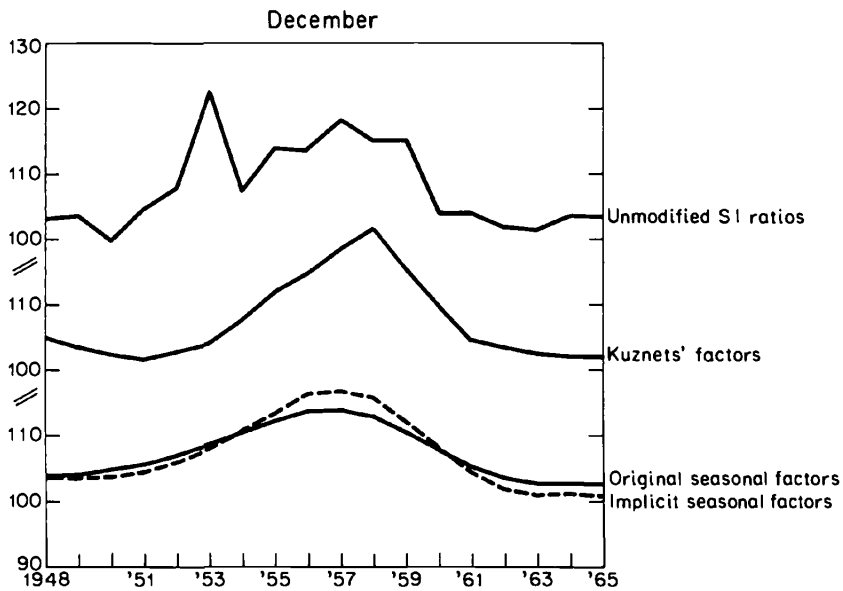
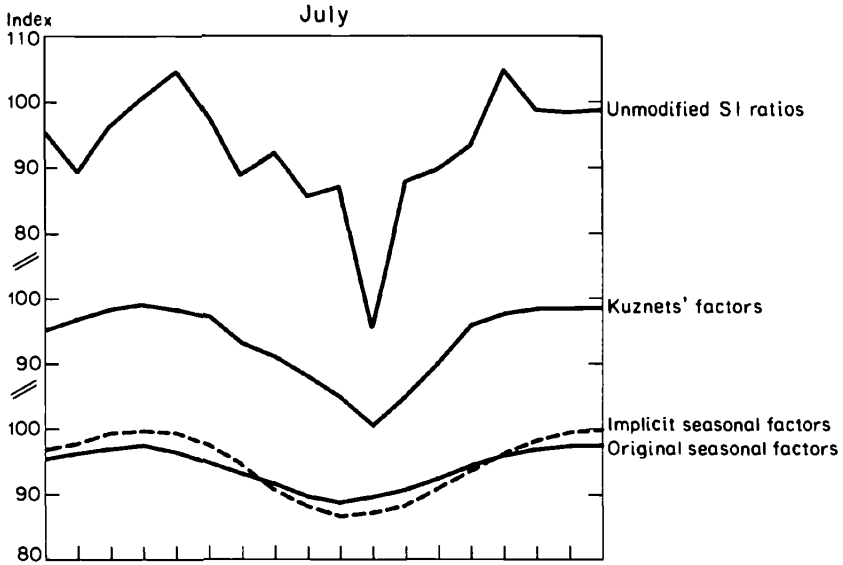
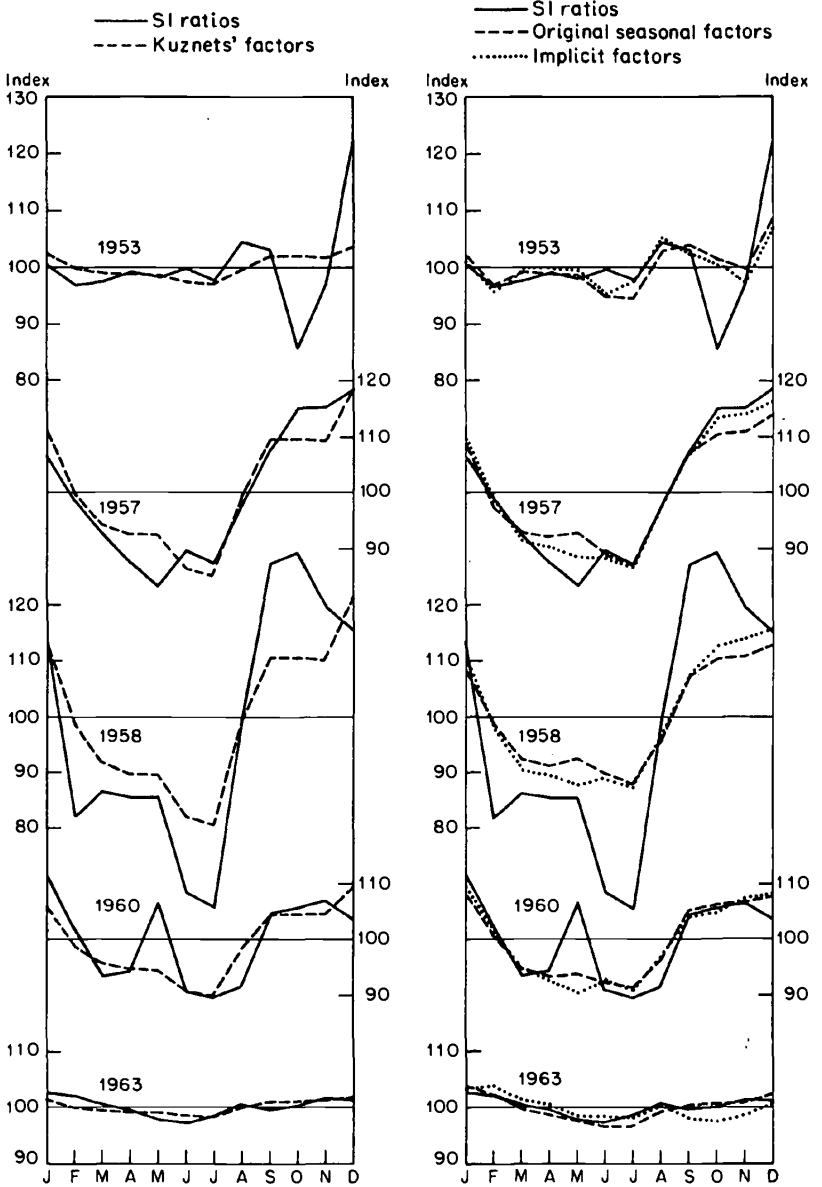


CHART 21

SI Ratios, Seasonal Factors, Kuznets' Factors, and Implicit Factors for Treasury Bill Rates; Selected Years



move the seasonal variance in the original series. In 1965, again the directions are reversed.

While these results confirm the expected faultiness in the machine's handling of a moving seasonal component, they emphasize even more that the magnitude of error is usually quite small; for the bill rate, with one exception, it is never more than 3 per cent of the original series and usually much less.<sup>19</sup> However, during the peak period the Kuznets factors differ from the original X-11 factors by up to 7 percentage points. The order of magnitude involved is shown in Chart 20 and again in Chart 21, where both the original and the implicit factors, as well as the Kuznets factors, are plotted against the SI ratios. The implicit factors appear to follow the SI ratios slightly better than do the original factors; whether they are therefore preferable is a question on which one can argue both sides. Although the double adjustment method is used in this study for illustrative purposes, further experimentation may demonstrate its usefulness as a method of adjustment when there are abrupt changes in the seasonal amplitude. Whether the single or the double adjustment is preferred there is no compelling reason to reject the machine adjustment for any part of the sample period, although some users may prefer to take the adjusted data back only to about 1953.

<sup>19</sup>The *F*-test for stable seasonality in the second adjustment indicates insignificant seasonality at any level. In adjusting the adjusted data the machine confronts the same problem of a moving seasonal and therefore likely underestimates the extent of the original maladjustment. The magnitudes involved there, however, are small, and one may safely ignore this point. In fact the opposite danger exists that the machine may confuse the relatively much greater irregular component with the seasonal one and exaggerate the extent of the original maladjustment. There is no justification, for example, for the implicit lows in September, October, and November of 1965 (column 11, Table 13).