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CHAPTER 8

Cycle-by-Cycle Variability in Cyclical Behavior

I Problems Raised by Cycle-by-Cycle Variability

So far we have dealt almost exclusively with the average cyclical behavior of individual series, or groups of series. Chapter 4 presents average reference-cycle patterns; Chapter 5 treats 'characteristic' cyclical timing; Chapter 6 considers indexes of conformity to all the business cycles covered by a series; Chapter 7 takes up average reference-cycle amplitudes. The variability in cyclical behavior on which stress has been laid is variability *among* series in average behavior. Little has been said about a more fundamental type of variability—that found from cycle to cycle *within* series.

An ultrasimple measure of this behavior trait is presented by the vertical lines accompanying each reference-cycle pattern of Chart 1 that covers more than a single cycle. We shall now inquire into the economic meaning of these bleak symbols. They are briefly referred to in Chapter 4 as representing the average deviations of the standings of a series in individual cycles from the mean standings that constitute the average reference-cycle pattern. That is an accurate but not an illuminating statement. Before we begin using average patterns as prefabricated materials for constructing a model business cycle, we should learn what we can about the relations between average cyclical behavior and cycle-by-cycle behavior.

What types of movement are responsible for the average deviations from our reference-cycle patterns? Why do the average deviations differ so widely from one series to another? What bearing have they upon the use of our cyclical patterns and of the measures derived from them—the reference-cycle amplitudes and the average rates of change per month from stage to stage of business cycles?

II Cycle-by-Cycle Variability within Series

A Components of Average Deviations

The single-cycle patterns of a series, therefore the average reference-cycle pattern, and the average deviations from the latter may include contributions from all types of movement recognized by time-series analysis.

1) Our efforts to remove seasonal variations from our series before converting the original data into reference-cycle relatives are fallible. Doubtless the adjustments are here too large, there too small. Probably we have failed to recognize some genuine cases of seasonality, and perhaps we have introduced spurious seasonal movements into some series by making adjustments when none are needed. Once committed, errors of all these kinds are carried into the single-cycle patterns, are reflected at least faintly in the average pattern, and reappear in the average deviations.

2) Errors in the reference dates have similar effects. They do most damage to reference-cycle standings at stages I, V, and IX—also, of course, to leads, lags, and decisions about the variety of cyclical timing. A wrong peak date will warp the reference-cycle pattern of virtually every series in our whole collection that includes the date in question, while a wrong trough date will warp the patterns of two cycles.[1]

3) Grouping months into reference-cycle stages is a 'smoothing' operation with a variable span—seldom less than 3 months or more than 15.[2] Smoothing is the standard device for 'eliminating' irregular movements from time series; more accurately, for redistributing irregularities among the entries according to some scheme set by the smoothing formula employed. Of course, we cannot assume that the variable-span smoothing incident to the preparation of our single-cycle patterns disposes of the most formidable difficulty in time-series

[1] The text refers to cycles taken as units running from one trough to the next. If a peak-to-peak analysis is used, errors in trough dates disturb the pattern of one cycle and errors in peak dates of two cycles.

[2] For full details, see *Measuring Business Cycles*, Appendix A.

analysis. Irregular movements must be prominent components of the average deviations in many series, if not also of many average patterns.

4) Reasons will be shown presently for thinking that virtually all of the intracycle trends retained in our reference-cycle patterns change over time. If so, they too give rise to differences among single-cycle patterns. When these secular shifts become prominent in comparison with the cyclical movements, we break a series into segments, and strike two or more sets of averages. But within each segment there remain cycle-by-cycle differences in the trend component.

5) Along with all these other elements, wanted and unwanted, the single-cycle patterns include what we call 'cyclical movements'—the changes in individual series that correspond to (and, when taken all together, constitute) business cycles. We have no assurance that these cyclical movements tend to be uniform. Possibly they tend to grow progressively more violent, as Karl Marx predicted; possibly they tend to subside into brief and mild expansions followed by long and moderate contractions, as Thorstein Veblen surmised; perhaps they tend to alternate in character according to a rhythm of their own, as suggested by some long-cycle theorists; perhaps they tend to vary in ways and for reasons yet unguessed. Whatever the future may teach us, for the present we should not exclude the hypothesis that our average deviations are traceable in part to cycle-by-cycle variations in the movements the average reference-cycle patterns are meant to represent.

6) We know that sudden shifts occasionally occur from one well established cyclical pattern to another, which is then followed consistently for a while. Major discontinuous shifts of this sort lead us to subdivide series; for minor discontinuities we make no adjustment. Presumably the minor discontinuities are numerous, and leave their imprint on the cyclical patterns.

7) While the inquiry into "Cyclical Changes in Cyclical Behavior" in Chapter 11 of *Measuring Business Cycles* did not yield any satisfactory evidence that the fluctuations marked off by our reference dates are integral parts of long cycles, we

are satisfied that long waves occur in the building industry and certain other processes. Further, we suppose that diligent inquiry and the use of special tools would lengthen the list of activities in which long waves are found. Presumably our patterns of single cycles in many series tend to differ according as the 'long range' conditions that affect building construction favor a rise or a fall. We shall not know how common or how important such influences are until a thorough search for long cycles has been made, and the timing relations among the waves in different sectors of the economy established.

Thus, any single-cycle pattern in any series may be the net resultant of at least six or seven factors. As long as we confine attention to a single cycle, we cannot do much more toward segregating these components than has already been accomplished by adjusting for seasonals, eliminating the intercycle component of trends, and smoothing out some irregularities. For example, who could say what cyclical behavior is characteristic of imports and exports, of steel production and sugar refining, of stock sales and call loan rates, if he had only the reference-cycle patterns of 1927–33 or only those of 1933–38 at his disposal? And how would hypotheses based exclusively upon single-cycle studies of the first of these cycles compare with hypotheses based solely upon the second?

It was this inscrutability of single cycles that forced upon us the arduous task of collecting data covering as many cycles as feasible, and of devising ways of finding what happens on the average. We argued in the final chapter of *Measuring Business Cycles* that movements peculiar to single cycles, from whatever source they arise, tend to fade out of cyclical averages, while movements common to the species become more prominent the more cycles we cover. So far as averaging achieves this end of clarifying the combined cyclical and intracycle trend movements, it may be made to clarify also our views about the other factors that make single-cycle patterns what they are.

Of course the deviations of a single-cycle pattern from an average pattern are net resultants of numerous movements, and hardly less inscrutable than the single-cycle reference patterns. But, once again, averaging what happens in successive cycles enables us to take a long stride toward segregating elements that we might never pry apart so long as we dealt with single cases. Just as average cyclical patterns have much higher value for economic analysis than single-cycle patterns, so average deviations have much higher value than single-cycle deviations. Indeed, certain mathematical implications of our technique, plus the relative richness of our sample, enable us to learn more than we foresaw about the noncyclical features of economic changes, and in the process to learn more also about the cyclical features.

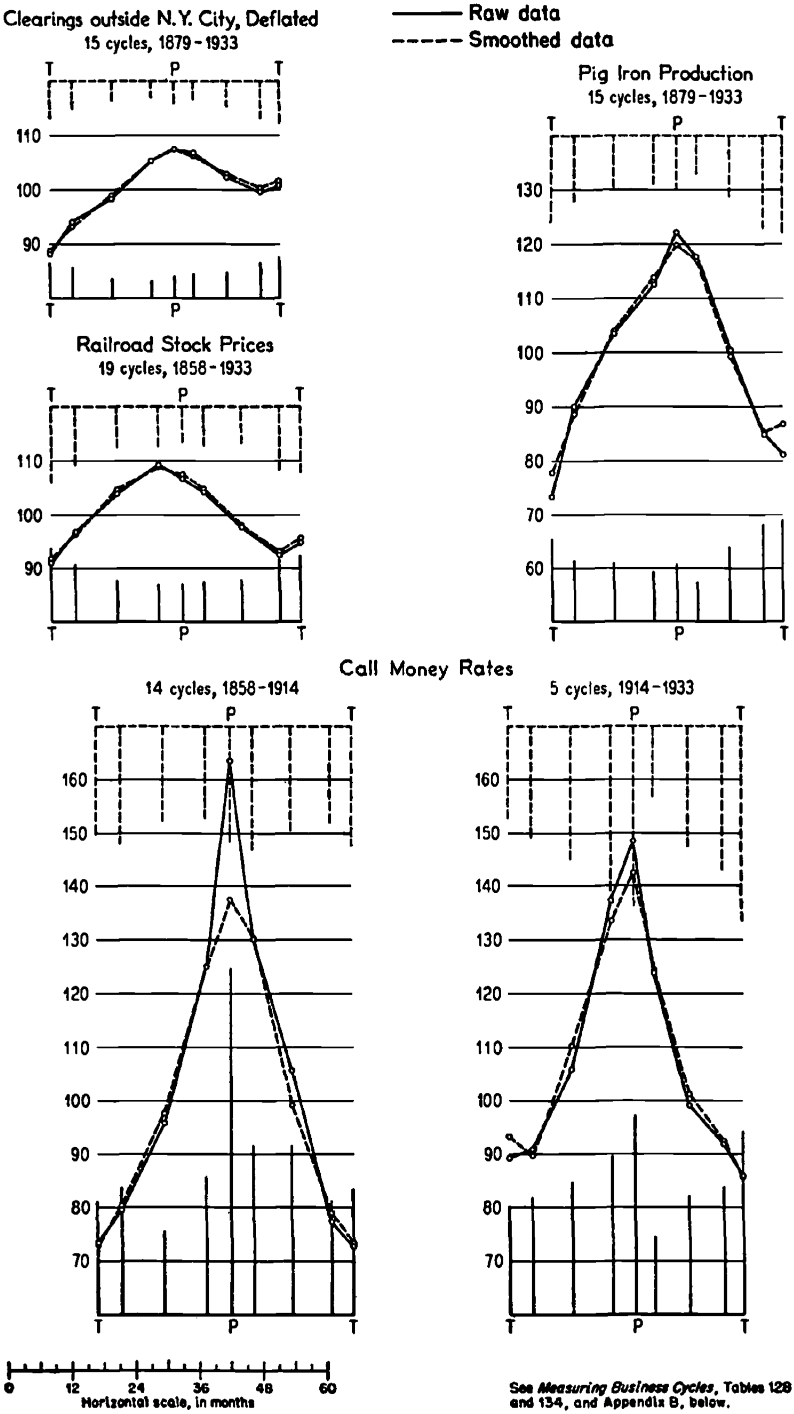
B CONTRIBUTION OF IRREGULAR MOVEMENTS

What we have learned about the residue of irregular movements in our measures stems mainly from the experiments reported in Chapter 8 of *Measuring Business Cycles*. Realizing that the variable-span averages of our nine-point cyclical patterns merely moderate the irregularities of monthly series, we probed the effect of smoothing the original data before beginning our analysis. An earlier National Bureau investigation supplied excellent materials. In his studies of interest rates, bond yields, and stock prices, Frederick R. Macaulay developed a method of "graduating monthly data in such a manner as to eliminate seasonal and erratic fluctuations and at the same time save all trend and the nonseasonal cyclical swings".[3] We chose four long series that Macaulay had smoothed—series characterized by wide differences in cyclical behavior—analyzed his form of the figures in our usual fashion, and compared the results with the measures we had obtained from what we called by way of contrast the 'raw' data. Chart 6 and Table 19 present the results of chief immediate interest.

The chart shows that the effects of systematic smoothing on

[3] Macaulay's formula is a "43-term summation, approximately fifth-degree parabolic graduation". For methods of computing and weight diagram, see his *Smoothing of Time Series* (National Bureau, 1931), especially pp. 24–6, 73–5, and the references there given.

Chart 6
Average Reference-Cycle Patterns of Raw and Smoothed Data of Four Series



See *Measuring Business Cycles*, Tables 128 and 134, and Appendix B, below.

Table 19

Effects of Smoothing Four Monthly Series upon Their Average Reference-Cycle Patterns and the Average Deviations from Them

(Effects measured in percentages of reference-cycle bases)

Series[a]	Reference-cycle stages I	II	III	IV	V	VI	VII	VIII	IX
Clearings outside N.Y.C., deflated, 15 cycles, 1879–1933									
Mean duration of stage, mos.	3.0	7.6	7.1	7.6	3.0	6.2	6.5	6.2	3.0
Change made by smoothing in									
Av. standing	+0.5	-0.9	+0.5	0.0	0.0	-0.5	+0.5	+0.6	+0.9
Av. deviation	+0.2	-0.5	+0.1	0.0	+0.2	-1.0	0.0	+0.3	+0.2
R.R. stock prices, 19 cycles, 1858–1933									
Mean duration of stage, mos.	3.0	8.1	7.6	8.1	3.0	6.9	7.3	6.9	3.0
Change made by smoothing in									
Av. standing	+0.7	-0.5	+0.7	-0.1	+0.6	+0.3	+0.1	+0.5	+1.0
Av. deviation	+0.3	+0.1	-0.2	+0.4	-0.3	-0.2	-1.0	+0.3	+0.1
Pig iron production, 15 cycles, 1879–1933									
Mean duration of stage, mos.	3.0	7.6	7.1	7.6	3.0	6.2	6.5	6.2	3.0
Change made by smoothing in									
Av. standing	+4.5	-1.4	+0.3	+1.3	-2.4	-0.5	-1.1	+0.2	+5.6
Av. deviation	+0.6	+0.8	-1.2	-0.4	-0.6	-0.1	-2.6	-1.0	-1.1
Call money rates, 14 cycles, 1858–1914									
Mean duration of stage, mos.	3.0	8.2	7.7	8.2	3.0	7.2	7.4	7.2	3.0
Change made by smoothing in									
Av. standing	-0.2	+1.0	+1.9	0.0	-26.2	+0.1	-6.4	+1.7	+0.5
Av. deviation	-1.1	-1.5	+2.3	-8.6	-43.0	-8.3	-12.0	-3.1	-1.0
Call money rates, 5 cycles, 1914–1933									
Mean duration of stage, mos.	3.0	7.6	7.4	7.6	3.0	6.2	6.8	6.2	3.0
Change made by smoothing in									
Av. standing	+4.0	-1.2	+4.5	-3.7	-6.0	+0.3	+2.0	+0.4	-0.1
Av. deviation	-2.9	-0.9	-0.4	+1.4	-3.7	-1.1	+0.8	+3.4	+2.3

[a]See Appendix B for sources of data.

average reference-cycle patterns are negligible in deflated clearings and railroad stock prices. In iron production, smoothing raises the troughs about 5 points and reduces the peak half as much. (A 'point' here is 1 percent of the cycle bases.) At other stages the maximum effect is 1.4 points. But in call loan rates before 1914 smoothing reduces the peak of the average pattern by 26 points. Under the Federal Reserve System this reduction is still 6 points. In other stages, smoothing shoves the call loan pattern up or down by an average of 1.5 points in 1858–1914, and of 2 points in 1914–33. To what are these wide differences in the effects of smoothing due?

Any moving average with a considerable span will change raw data patterns much or little according as the latter present brief and violent or long and gentle movements. Macaulay's formula raises the troughs of the pig iron pattern appreciably because these troughs are frequently V-shaped. It affects the troughs of call loan rates relatively less because these troughs are typically U-shaped. It affects the pig iron peak less than pig iron troughs because the slopes on the two sides of the peak are somewhat less precipitous than those on the two sides of the troughs. It reduces the peak of call loan rates before 1914 drastically because this peak is a veritable spike in the raw data, shooting up 38 points in the 4.6 months of stage IV–V and falling 33 points in the 4.1 months of stage V–VI. It affects the call loan peak after 1914 much less than before, but still considerably, because the movements of the raw data pattern in stages IV–V and V–VI, though moderated, remain large (+11, −25). It makes little change in the reference-cycle patterns of deflated clearings and railroad stock prices at any stage because their raw data patterns have few violent movements to be rounded off.[4]

Now brief and violent movements in an *average* reference-cycle pattern are likely to be cyclical phenomena, especially when the series in question covers a dozen or more cycles and a considerable volume of transactions. It would be strange indeed if a violent random perturbation recurred in the same form in a long series of broad coverage within the same stage of enough reference cycles to produce such a spike in the average pattern as we find in call loans, or even such troughs as we find in pig iron. And when, as in these cases, cyclical explanations of the brief but violent movements are at hand, we may be confident that the smoothing is not to be credited with mitigating an erratic movement, but debited with blunting cyclical turns.[5]

[4] For a fuller analysis, see *Measuring Business Cycles,* Ch. 8, especially Sec. IV.

[5] The explanation of the call loan spike before 1914, for example, runs as follows: Whenever the country's banks needed additional currency in large amounts, their demands centered upon a small number of banks in New York City. The first recourse of these banks when demands for currency were large was to call in their loans to stockbrokers. The brokers, threatened with ruinous losses if many of their customers had to sell securities all at once, bid desperately for funds, and the rates they are reported to have paid sometimes shot up above 100 percent for a few hours or days.

Let us examine next the effects of smoothing on average deviations. When they are averaged over all of the 9 stages of reference cycles, we find that smoothing has effected a net reduction equal to 0.1 percent of the reference-cycle bases in clearings, railroad stock prices, and call loans after 1914, 0.6 percent in iron production, and 8.5 percent in call loans before 1914. What makes all except the last of these changes slight is that at some stages smoothing increases the average deviations instead of reducing them. Increases occur at five stages of the patterns for clearings and railroad stock prices, at four stages of the call money pattern after 1914, at two stages of the iron pattern, and even at one stage of the earlier pattern for call money. Altogether, Table 19 shows 26 decreases in average deviations, 2 cases of no change, and 17 increases.

We are prone to think of smoothing as operating chiefly upon random movements, and of random movements as brief

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Such rates did not bankrupt the borrowers, because call loans could be refunded at lower rates as soon as the stringency relaxed (100 percent interest for 3.65 days is only 1 percent of a loan).

Heavy demands for money to move the crops were made on New York every autumn; but they could be foreseen and prepared for. Such effects as these seasonal requirements had upon call rates are virtually eliminated before our raw data patterns are computed. Not so the effects of the panicky demands for currency that frequently accompanied business-cycle recessions in those days. It is these movements that produce the call money spike at stage V. Even our customary 3-month averages at the peaks understate the average cyclical rise of call loan rates.

I may add that the average pattern before World War I is much influenced by the extraordinary peak of October 1873 when the autumnal demand for crop moving funds coincided with a severe financial panic. But the omission of that cycle from the average would leave a 22 point rise between stages IV and V (instead of 38) and a 26 point fall between stages V and VII (instead of 58).

The less lofty but still considerable raw data peak of the pattern for 1914–33 also has a simple economic explanation. The structural change in banking effected by the Federal Reserve System has greatly reduced, but not wholly removed, money market stringencies in New York at times of recession. The peak reference-cycle standing of call loans in 1929 was 215.

Finally, our series on call money rates is compiled from monthly averages, and does not reveal the extreme fluctuations of the daily quotations. Our highest entry is 61.23 percent in September 1873; entries above 10 percent occur in 36 months scattered over 22 years from 1857 to 1920. The highest entry of later date is 9.23 percent in July 1929. For our data up to January 1937, see F. R. Macaulay, *Interest Rates, Bond Yields and Stock Prices in the United States since 1856* (National Bureau, 1938), pp. A142–61.

perturbations not closely correlated with one another. On this basis, probability theory suggests that smoothing will reduce average deviations most at stages I, V, and IX of our reference-cycle patterns. At these turning stages our standard measures span only 3 months, while at the other six stages they span on the average more than twice as many months, and so give much better chances for the mutual offsetting of brief random movements. In the data that have been smoothed in advance by Macaulay's 43-term formula, this disparity in the duration of stages counts for little; certainly for much less than in our standard measures. Hence one might expect that the reductions in the average deviations effected by smoothing would be appreciably greater on the average at stages I, V, and IX than at stages II–IV or VI–VIII. But that is not what Table 19 tells us. In clearings, railroad stock prices, and iron production, smoothing reduces the average deviations rather less on the whole at the turning stages than at the stages dominated by expansion and by contraction.[6] Only in call

[6] The following table tells its own story.

EFFECTS OF SMOOTHING AT THREE GROUPS OF STAGES UPON AVERAGE DEVIATIONS FROM AVERAGE REFERENCE-CYCLE PATTERNS

	REFERENCE-CYCLE STAGES		
	I, V, IX	II, III, IV	VI, VII, VIII
Deflated clearings			
Signs regarded	+0.20	−0.13	−0.23
Signs disregarded	0.20	0.20	0.43
R.R. stock prices			
Signs regarded	−0.03	+0.10	−0.30
Signs disregarded	0.23	0.23	0.50
Iron production			
Signs regarded	−0.37	−0.27	−1.23
Signs disregarded	0.77	0.80	1.23
Call money before 1914			
Signs regarded	−15.03	−2.60	−7.80
Signs disregarded	15.03	4.13	7.80
Call money after 1914			
Signs regarded	−1.43	+0.03	+1.03
Signs disregarded	2.97	0.90	1.77

As in Table 19, from which the summary is compiled, the measures are expressed in percentages of reference-cycle bases.

Striking separate averages for the trough and peak stages would bring out again what has already been said about the unlike effects of smoothing upon call money peaks and troughs. In the other 3 series this elaboration would show inconsequential differences. The mathematically instructed will find the results of our smoothing tests easier to grasp if they read Millard Hastay's algebraic analysis in the first Technical Note appended to this chapter.

money is the expectation borne out by the figures, and there we have shown that the movements chiefly affected by smoothing are cyclical in character.

We do not conclude from these experiments that irregular movements are a minor factor in economic fluctuations, or that our standard analysis eliminates all but a small percentage of irregular movements from reference-cycle patterns and average deviations. The lesson is rather that we should discriminate between two types of irregular movements—those which last only a month or two, and those which persist for several quarters or even years.

Brief movements are virtually erased when they are mild, and much modified when they have considerable amplitudes, by the simple smoothing involved in our technique, so that only slight residues are left for more systematic smoothing to iron out. If a brief random movement during a cycle is so violent that it would distort the average reference-cycle pattern of a series, we exclude that cycle.

Long-continued irregular movements, on the other hand, survive not only the variable-span smoothing of our standard analysis, but also more systematic smoothing, provided the latter is not so drastic as to erase almost all cyclical movements. For example:

> Smoothing by Macaulay's method will not remove the great bulges in American price and value series in 1862–67 and 1915–21. It will moderate the effects of two bad harvests upon agricultural prices less than it will moderate the effects of speculative maneuvers associated with monthly crop reports, or even the effects of two bad seasons separated by a good season. Random effects of considerable size thus remain in the smoothed forms of the series we have used in our tests, and contribute toward making the average deviations nearly as large in the results obtained from the smoothed as in those obtained from the raw data.[7]

A methodological conclusion follows: If major wars and two bad harvests in succession typify the irregular movements that cut a considerable figure in the average deviations from

[7] *Measuring Business Cycles*, p. 362.

mean reference-cycle patterns made from smoothed data, and *a fortiori* in the deviations from our standard patterns, we should be able to formulate specific hypotheses concerning the irregularities of most moment to us. We need not give up the concept that every economic activity is influenced at all times by a host of factors that cannot be classified as seasonal, cyclical, or secular; but we can suppose that the net effects of this host upon our standard measures of time series as comprehensive as the four used in our test are relatively slight in comparison with the effects produced by identifiable events. Then we can concentrate upon the latter.[8]

It should be observed that the conclusions of this section rest upon a narrow statistical base. Only 4 series were used in our smoothing tests. However, the average deviations from their reference-cycle patterns run the gamut from 3 percent of the reference-cycle bases at stage IV of the clearings pattern to 65 percent at stage V of the pattern for call money before 1914. All 4 series are exceptionally long—a circumstance that favors regularity in reference-cycle patterns. But the call money pattern after 1914 covers only 5 cycles, and when we broke each of the long series into three segments we found no reason to change our conclusions materially.[9] Yet it is fortunate that the next two sections, which will lead by a roundabout path back to the irregular component in average deviations, have a much broader foundation.

C CONTRIBUTION OF SECULAR MOVEMENTS

Our method of eliminating the portion of a secular trend that represents shifts in the level about which a series fluctuates from one cycle to the next "implies that if the secular trend were represented . . . by a continuous line, that line would be

[8] Certainly our smoothing tests lend little support to the assumption that the irregular component of our average deviations varies inversely as the duration of reference-cycle stages. Presumably there is some such effect; but our efforts to demonstrate it have so far been unavailing.

[9] See Chart 51 in *Measuring Business Cycles* and the textual comments upon it.

a flexible curve cutting through successive specific cycles".[10] We do not draw such lines, but we do measure the intercycle component of trend by computing percentages of change per month from one specific-cycle base to the next. These measures vary from cycle to cycle within series, and also among series, in much the same fashion as do our measures of cycle durations and amplitudes. The transfer from specific-cycle to reference-cycle analysis changes the individual cycle bases little or much according as a series conforms closely or loosely to business cycles and I–V timing; but the transfer does not diminish the differences among shifts from one cycle base to the next. Finally, the readily demonstrable variability in the intercycle component of trend implies corresponding variability in the intracycle component; for the intracycle component is the full trend of a series between stages I and IX of a cycle, just as the intercycle component is the full trend from one cycle to the next. Thus our reference-cycle standings include a trend component that is far more variable from cycle to cycle than the term 'secular trend' suggests to many minds. What are for our purposes secular movements must cut a considerable figure in the average deviations from our average measures of cyclical behavior.[11]

Geoffrey H. Moore has found a way to determine approximately how much the intracycle trend component contributes to the average deviations from average specific-cycle or average reference-cycle standings.[12] His method rests upon the

[10] *Measuring Business Cycles*, p. 39, note 5.

[11] How we come to use what are in effect highly flexible trends is set forth in Technical Note II appended to this chapter. The gist of the matter is that shifts in cycle bases are to us secular movements. Presumably these highly variable shifts are influenced by all the 'forces', or 'factors', or 'movements' that affect the monthly, quarterly, or annual data used in computing bases. To account for the alterations, large or small, in the levels on which successive specific and successive business cycles run is a problem of the first magnitude. When economists get around to treating this problem, or group of problems as it will probably turn out to be, they may find the National Bureau's specific-cycle bases a highly valuable collection of observations. Perhaps even more useful will be the corresponding collection of reference-cycle bases.

[12] A similar method was suggested independently by Simon Kuznets.

fact that our standard analysis tends to make the secular contribution to the average deviations largest at the 'ends' of the cycles and smallest in their middles.

The reason is simple. In effect, we represent the trend of a series, not by a continuous line, but by a succession of horizontal base lines, each one cycle long. When the trend rises, each base line is higher than its predecessor; when the trend falls the lines are progressively lower.[13] In series with rising trends, each base line tends to be farthest above the corresponding continuous trend line at the beginning of a cycle, to coincide with the continuous trend at the cycle's midpoint, and to be farthest below the continuous trend at its end. With appropriate reversals, this rule holds also for declining trends. The rule applies most clearly and strictly to linear trends; virtually it applies also to most curvilinear trends met in practice, for the intracycle segments of such trends seldom differ so much from straight lines as to alter the stages of maximum and minimum divergence from our horizontal bases. What is more to the point, the rule applies roughly even to the highly flexible trends that correspond to the ever varying shifts in our cycle bases.

Whatever cycle-by-cycle changes occur in the intracycle trend component influence our measures most at the stages where our horizontal base lines diverge most from flexible but continuous trend lines, and influence our measures least where the divergence is least. Theoretically, indeed, the trend component should be wholly eliminated from our reference-cycle measures at the points where the 'true' trend coincides with our horizontal cycle bases; that is, the secular component in the average deviations should be zero at these points of intersection. The longer a cycle is, the greater tends to be the difference between the value of the trend component at the midpoint of the cycle and at its 'ends'.

Our practice of marking off business cycles by their troughs makes troughs the 'ends' of a cycle, while the peak stage more

[13] For illustrations see Charts 14 and 18 of *Measuring Business Cycles*, which represent the derivation of specific- and reference-cycle patterns.

often than any other approximates the midpoint. If we mark off cycles by successive peaks, these peaks become the 'ends', and the midpoints are more likely to occur in troughs than in any other stage. Appendix Tables B3 and B4 in *Measuring Business Cycles* give the cycle-by-cycle patterns of 7 series covering 14 or more reference cycles both on a trough-to-trough (T–T) and on a peak-to-peak (P–P) basis. As Table 29 (see Technical Note II at the end of the chapter) shows, the shift from a T–T to a P–P basis has one of the expected effects: without exception, P–P analysis reduces the average deviations at troughs and raises them at peaks. But the test does not show a second expected effect: average deviations are not smallest at cyclical peaks on the T–T basis, or smallest at the troughs on the P–P basis, as they would be if trends were the sole or the controlling factor at work. Only in railroad stock prices do the minimum deviations occur at the T–T peak; in no series does the minimum occur at the trough in either form of analysis. Thirteen out of 14 times the influence of trends upon the location of *minimum* deviations is blocked by some other factor; but 12 times out of 14 the *maximum* deviations occur in the stage where trends tend to place them—at the trough in T–T, at the peak in P–P analysis.[14]

D CONTRIBUTION OF CYCLICAL MOVEMENTS

This striking difference is due to another feature of our technique. Whether the analysis is T–T or P–P, our reference-cycle relatives tend to minimize the deviations arising from conforming cyclical movements at the midpoints of expansions and contractions, and to maximize them at troughs and peaks impartially.

Suppose (1) that the T–T pattern of every series in every cycle consists of a straight line sloping upward from trough to peak and a straight line sloping downward from peak to

[14] One of the exceptions to the latter rule is that call money rates have their maximum average deviation in stage V, even in the T–T analysis; another is that pig iron production has its maximum deviation in stage VI in the P-P analysis. In the T–T analysis, the deviation of outside clearings in stage II equals that at the trough.

trough; (2) that the standings at the initial and terminal troughs of a cycle are identical; (3) that the slopes of these lines, and therefore the amplitude of rise and fall, differ from cycle to cycle; and (4) that, while the durations of successive cycles differ, expansion and contraction are of equal length in each cycle.

It follows from our assumptions that in every cycle covered by a series, the stage standing is farthest below the cycle base at the troughs and farthest above it at the peak. The maximum differences among the standings occur where the standings themselves diverge most from the horizontal base. But in rising from their low values at stage I to their high values at V, all the straight lines intersect the base in the middle of expansion, and the down-sloping lines from V to IX intersect the base in the middle of contraction. That is, in stages III and VII the standings in every cycle are 100, and the average deviations zero. This argument applies also to straight-line patterns that on the T–T basis fall from stage I to V and rise from V to IX, that is, to inverted series.

Of the assumptions used here, the third is true to fact: the rise and the fall of reference-cycle relatives differ from one cycle to another. The other assumptions distort the observed facts. The movements from I to V and from V to IX seldom follow straight lines; often stages I and IX are not troughs and stage V is not the peak of a pattern; some series have reference-cycle patterns so irregular that one cannot identify either peaks or troughs; expansions and contractions differ in duration; and the standings at stages I and IX commonly diverge. Yet the movements we observe during successive reference cycles in the many series of our sample seem to be distributed not very unevenly around the ultrasimple model we have assumed. If the cyclical component in cycle-by-cycle variability stood in splendid isolation, the average deviations would be least in stages III and VII, greatest in stages I, V, and IX.

E STATISTICAL TESTS

As the reader may have remarked, the effects of cycle-by-cycle changes in intracycle trends and in cyclical amplitude upon the average deviations agree at the troughs and oppose each other at the peaks of business cycles, when we analyze series on our standard T–T plan. At the troughs our methods tend to maximize the deviations of both components; at the peaks they tend to maximize cyclical and minimize secular deviations. These tendencies are frequently overborne by irregular movements; but such 'disturbing circumstances' are as likely to reenforce as to counteract the systematic effects of our technique. Hence in fairly long series, such as were used in our tests, or in a considerable collection of short series, we may expect to find the maximum deviations most frequently in stages I and IX, but we do not know whether to expect the minimum deviations in stages III and VII, where the cyclical component tends to put them, or in stage V, which is favored by the secular component, or perhaps in one or another of the intermediate stages.

Our 7 test series in Table 29 are too small a sample to settle the open issue. Their T–T analyses scatter the minimum deviations over all the stages from III to VII with fine impartiality.[15] So we turn to the full sample of 794 series presented in Table 3, or rather, this sample minus the 6 series that cover only one cycle and so have no cycle-by-cycle differences to measure. These numerous T–T analyses give a definite answer. Table 20 shows that averages of the average deviations are smallest in stages IV and VI. They decline regularly from one maximum in stage I to a minimum at IV, rise rather sharply in V, fall as sharply to a second minimum in VI, then rise regularly to a second maximum in IX. Of course, departures from this average scheme appear among the 29 groups for which we have computed separate averages. For example, maximum devia-

[15] The minimum deviations appear at the following stages in the several series:

III	Call money rates
IV	Clearings outside N.Y.C., deflated
V	Railroad stock prices
VI	Pig iron production and shares traded
VII	Freight car orders
IV, VI	Railroad bond yields

For full details, see Technical Note II, Table 29.

Table 20

Mean Average Deviations from Average Reference-Cycle Patterns, by Groups of Series

Group	No. of Series[a]	Mean Average Deviation at Reference-Cycle Stages								
		I	II	III	IV	V	VI	VII	VIII	IX
Retail sales	10	6.2	5.4	4.4	4.9	4.9	3.6	3.8	7.6	9.6
Wholesale sales	14	11.2	9.7	9.1	7.5	10.0	8.5	8.2	11.2	12.6
New orders from manufacturers	17	28.8	32.1	25.5	31.9	40.9	22.1	27.6	22.4	29.5
Construction contracts										
Private	26	36.3	38.1	36.4	29.6	47.3	32.1	26.4	30.9	32.4
Public	16	30.7	18.9	17.9	12.4	24.7	20.2	15.4	27.2	37.3
Inventories										
Positive timing	18	13.3	11.5	10.0	12.2	10.9	7.6	11.9	14.5	16.4
Irregular timing	18	21.7	16.6	11.0	10.0	13.5	13.1	12.6	16.7	20.6
Inverted timing	24	22.6	18.1	14.6	12.0	11.0	13.3	16.8	20.1	24.9
Production										
Foodstuffs	47	10.8	8.4	6.1	5.6	7.0	6.7	7.7	8.4	9.5
Other perishables	29	10.9	8.0	8.0	6.3	9.6	8.4	7.3	11.6	11.4
Semidurables	28	14.1	9.5	8.9	8.9	9.3	5.0	10.9	11.2	11.7
Durables	53	25.5	20.4	18.2	14.3	18.1	17.0	16.5	22.6	22.6
Employment										
Perishable goods industries	8	8.5	5.5	4.6	4.2	3.9	3.6	3.4	6.2	6.4
Semidurable goods industries	13	9.0	5.9	4.9	4.4	4.5	2.9	5.0	5.9	6.7
Durable goods industries	9	18.1	12.8	10.4	7.9	10.6	8.5	10.3	14.8	14.4
Hours of work per week	9	5.5	4.3	4.1	3.4	4.0	2.3	2.2	5.2	5.8
Earnings per week, month, or year	10	5.8	4.8	3.1	2.9	4.9	5.2	5.9	7.9	9.1
Payrolls										
Perishable goods industries	8	12.0	8.5	5.9	5.4	6.5	6.2	5.8	11.3	13.5
Semidurable goods industries	13	17.4	10.4	8.1	7.5	8.5	5.7	6.0	10.8	13.3
Durable goods industries	6	23.8	19.3	15.4	14.2	19.1	13.5	14.7	21.6	22.8

Table 20 (concl.)

Prices of commodities										
Farm products & foods	51	16.0	12.4	9.1	8.4	10.1	8.9	7.8	11.3	13.6
Other perishables	22	10.6	8.7	8.0	6.8	8.1	7.5	7.6	8.9	9.9
Semidurables	18	14.4	10.5	8.7	7.3	8.2	7.0	8.3	9.8	10.9
Durables	45	11.5	9.0	6.4	5.3	6.4	6.3	7.6	9.2	9.4
Interest rates										
Short-term	11	30.7[b]	13.0	13.2	15.4	18.7	8.2	15.6	12.9	13.9
Long-term & bond yields	12	7.9	5.6	3.0	3.0	3.2	3.3	3.6	4.8	4.6
Security issues, corporate	14	41.6	32.4	35.5	45.2	56.4	41.3	26.5	43.0	53.1
Bank clearings or debits	8	10.4	10.0	7.6	9.1	9.3	7.3	7.6	13.1	16.2
Indexes of business activity	11	8.2	6.6	5.8	5.1	6.1	4.7	5.4	8.3	8.8
Summaries										
All series on										
Construction contracts or permits	58	31.0	28.2	26.0	21.2	33.1	23.8	20.8	27.8	31.4
Production	183	15.8	12.2	10.9	9.1	11.2	9.6	10.5	13.7	13.9
Employment	37	11.3	7.8	6.2	5.1	6.0	4.6	6.1	8.5	8.7
Payrolls & other income payments	30	17.0	11.8	9.0	8.4	10.4	7.7	8.0	13.6	15.7
Prices of commodities	147	13.1	10.1	7.8	6.8	8.0	7.3	7.5	9.8	11.0
All series on										
Flow of commodities, services, or income	472	17.7	14.6	12.6	11.2	14.3	11.3	11.9	15.6	17.6
Prices of commodities or services	168	12.1	9.4	7.1	6.2	7.5	6.9	7.1	9.3	10.5
Financial activities	135	17.6	12.3	10.3	11.5	13.9	11.3	11.5	14.7	17.4
General business activity	13	8.7	7.3	6.2	5.9	6.8	5.2	5.8	9.5	10.5
All series in sample	788	16.4	13.0	10.9	10.1	12.7	10.2	10.7	14.0	15.9

[a] See Table 8, note a. Six series that cover only one cycle are omitted here.

[b] The high figure at this stage is largely due to the sharp decline of short-term interest rates after 1933. Their level at the 1933 trough, when expressed as a percentage of the 1933–38 cycle, is far above the standing at stage 1 in previous cycles.

tions appear at stage V in new orders from manufacturers, contracts for construction (all series and the private subgroup), and issues of corporate securities—three groups that relate to investments and in which cyclical are much larger than secular changes. In one group, production of perishable goods other than foods, the maximum occurs in stage VIII. In all the other groups we have averaged, the maximum occurs in one of the troughs. A similar concentration appears in the location of the minimum deviations: in the 29 groups they occur 22 times in stages IV or VI, 5 times in stage VII, once in stages III and IV, and once in stage V. Thus the evidence of the groups amply supports that of the grand average of all series, and so also does that of the groups of groups near the end of the table.

F DECOMPOSITION OF AVERAGE DEVIATIONS

When pushed to its logical limits, the preceding analysis enables us to decompose the average deviations from the average reference-cycle patterns of our full sample into their irregular, secular, and cyclical components. This operation strains one's credulity; but let us perform it hardily and assess the results later.

By way of preparation we enter under each reference-cycle stage the full average deviation we shall presently break into parts. Below these figures we put down the way in which each component is supposed to behave 'under the bludgeonings of chance' and the pressures of our analytic methods.

Table 21

MEAN AVERAGE DEVIATIONS FROM AVERAGE REFERENCE-CYCLE PATTERNS OF FULL SAMPLE, AND THEORETICAL EXPECTATIONS CONCERNING STAGE-BY-STAGE CHANGES IN THEIR COMPONENTS [a]

	REFERENCE-CYCLE STAGES								
	I	II	III	IV	V	VI	VII	VIII	IX
Av. deviation	16.4	13.0	10.9	10.1	12.7	10.2	10.7	14.0	15.9
Component									
Irregular	C	C	C	C	C	C	C	C	C
Secular	M	F	F	F	O	R	R	R	M
Cyclical	M	F	O	R	M	F	O	R	M

[a] C stands for constant, M for maximum, F for falling, R for rising, and O for zero.

Note first how symmetrically the average deviations are arrayed on either side of the reference-cycle peak. The following rearrangement of the figures makes this symmetry clearer.

REFERENCE-CYCLE STAGES	AVERAGE DEVIATIONS	DIFFERENCE BETWEEN AV. DEVIATIONS AS % OF Reference-Cycle Bases	Mean of Deviations
I and IX	16.4 and 15.9	0.5	3.1
II and VIII	13.0 and 14.0	1.0	7.4
III and VII	10.9 and 10.7	0.2	1.9
IV and VI	10.1 and 10.2	0.1	1.0
Average	12.6 12.7	0.4	3.4

This close approach to perfect balance fits neatly the theoretical expectations set forth in preceding sections and summarized in Table 21. If the implications of our theoretical model were fully realized, and if the stage-by-stage changes in the average deviations proceeded at uniform rates per month, we could start with stage V and argue as follows: Since the rise in the secular component of the average deviations from zero at the cyclical peak to a maximum value at stage IX matches in rate and duration the preceding fall from a maximum at stage I to zero at the peak, the maximum secular values at stages I and IX are equal, and so also are all intermediate secular values equidistant from stage V. On the same assumptions, a similar argument applies to the cyclical component. Since the fall in this component from a maximum value at the peak to zero at stage VII is supposed to equal in rate and duration the preceding rise from zero at stage III to the maximum at V, the intervening cyclical values at VI and IV are equal. So also are the cyclical values at stages VIII and II, and at IX and I; for the rise from zero at VII to a maximum at IX is supposed to equal in rate and duration the fall from a maximum at I to zero at III.

The perfect balance thus theoretically maintained on the two sides of the cyclical peak by each component insures that the two components combined maintain such a balance. But the preceding argument does not explain why the *full* average deviations should be so nearly equal at the stages paired as the above figures show them to be. For this argument tells us nothing about the average deviations arising from irregular movements, which may well exceed the deviations arising from both of the other components combined. Our assumption that the irregular component remains constant from stage to stage of reference cycles rests upon an independent foundation. This component includes all changes in economic activities that are not correlated with the secular-cyclical fluctuations in which our interest centers.[16] To say that these movements are not correlated in time with the movements we are trying to measure is to say that we have no reason to expect that the irregular component in the average deviations will be larger at some stage of business cycles than at other stages; which is equivalent to the expectation that this component will have the same value at all stages.[17]

While this expectation is independent of our expectations concerning the other components, the evidence at hand seems to confirm it. A restatement of the preceding findings makes this clear. The stage-by-stage changes in the full average deviations comply so closely with theoretical expectations concerning the secular and cyclical components, taken by themselves, that only small changes are left to be explained by other factors. Apparently, all nonsecular-cyclical movements have a small part in producing stage-by-stage differences in average

[16] In terms of Section A above, we classify with irregular movements in the usual sense (if there be one) seasonal residues and overadjustments, the effects of reference-date errors, elements of long cycles and discontinuous changes in cyclical behavior, and perhaps other kinds of nonsecular-cyclical movements that may some day win names and recognition.

[17] To assume that the irregular component is unevenly distributed over reference-cycle stages would not be inconsistent with our assumption that the irregular component tends to be distributed evenly over time. For these stages are unequal in duration; and, when we are dealing with samples that are limited in both economic coverage and time, departures from uniform distribution will occur, and are most likely when irregular movements have least chance of canceling one another—that is, in the short stages I, V, and IX. But in Section B above we found evidence that, in practice, the average deviations are not appreciably influenced by the duration of stages, apart from the deviations due to certain violent cyclical movements. And this observation is made less surprising by Hastay's theoretical analysis in Technical Note I, at the end of the chapter.

deviations, however large a part they may have in producing these deviations at every stage.

One other feature of our technique should be noted before we try to determine the actual magnitudes of the three components. The difference between the average deviations at the stages I have paired increases from 0.1 percent of the reference-cycle bases at stages IV and VI to 0.2 percent at III and VII, and to 1.0 percent at II and VIII; then the difference drops to 0.5 percent at stages I and IX. This drop is due to our practice of taking the terminal trough of one cycle as the initial trough of its successor; that is, the same seasonally-adjusted data are used in computing the reference-cycle standing of a series at stage IX of a cycle and stage I of the following cycle. It may seem, in view of this technical fact, that the average deviations at stages I and IX should be even closer than we find them. However, two factors oppose equality. (1) The standing at stage I of the first cycle in a series has no mate at a preceding stage IX, and the standing at stage IX of the last cycle has no mate at stage I of a subsequent cycle. In short series these non-overlapping entries have a considerable influence, and Table 3 shows that over half the series in our sample cover less than six cycles. When in longer series we have to drop a cycle or two between the first and last one covered, at least two standings in stage I have no mates in IX, and at least two in IX are without mates in I. (2) Every change in cycle bases produces a difference between the standings at stage IX of one cycle and at stage I of the next cycle; for these two standings are ratios of the same magnitude to unequal magnitudes. The last section of Technical Note II, appended to this chapter, shows how large these effects can be and frequently are.[18]

G ESTIMATES OF COMPONENTS

We can get a basal estimate of the secular component of the average deviations by subtracting the average deviation at stage V from the average deviations at stages I and IX. The

18 The average deviations from average reference-cycle standings are lower at stage IX than at stage I in 16 of the 29 minor groups in Table 20, and higher in 13 of these groups.

cyclical component is supposed to be at its maximum (that is, at the same value) in all three stages and the irregular component is supposed to remain constant from stage to stage; the only difference is that the secular component is zero in V and at its maximum values in I and IX. Thus the differences between the full average deviations at the peak and at the two troughs, 3.7 and 3.2 percent respectively of the reference-cycle bases, express the maximum deviations due to changes in intracycle trends (Table 21).

By accepting these figures and the corresponding value of zero at the peak, assuming that the stage-to-stage changes proceed at a uniform rate per month, and choosing an appropriate measure of business-cycle durations, we can estimate the numerical value of the secular components in the mean average deviations from the average reference-cycle patterns of our full sample at each of the nine stages. The most inclusive of our monthly duration measures, based upon the American reference dates from 1854 to 1938, gives an average of 47.7 months. Within that span, let us say, the secular component falls from 3.7 percent of the reference-cycle bases to zero and then rises to 3.2 percent, a total movement of 6.9 percent, which means a shift of 0.145 percent each month. The monthly intervals between stages are:

I–II	II–III	III–IV	IV–V	V–VI	VI–VII	VII–VIII	VIII–IX
4.8	8.3	8.3	4.8	3.9	6.9	6.9	3.9

Thus estimated, the secular component of the mean average deviations of our full sample, expressed in percentages of reference-cycle bases, comes out as follows:

I	II	III	IV	V	VI	VII	VIII	IX
3.7	3.0	1.8	0.6	0.0	0.7	1.6	2.6	3.2

Next we estimate the irregular component. That is easily done now that we have the secular estimates. Since the cyclical component is supposed to be zero at stages III and VII, the irregular component at these stages will be the full average deviation minus the secular component:

STAGE	FULL AVERAGE DEVIATION	SECULAR COMPONENT	IRREGULAR COMPONENT
III	10.9	1.8	9.1
VII	10.7	1.6	9.1

Of course, the results accord well with our assumption that the irregular component is uniform from stage to stage.

To complete the operation, we obtain the cyclical component by subtracting the sum of the above estimates of the secular and irregular components from the full average deviations at each stage.

Table 22

ESTIMATES OF THE SECULAR, CYCLICAL, AND IRREGULAR COMPONENTS IN MEAN AVERAGE DEVIATIONS FROM AVERAGE REFERENCE-CYCLE PATTERNS OF FULL SAMPLE

	REFERENCE-CYCLE STAGES [a]								
	I	II	III	IV	V	VI	VII	VIII	IX
Av. deviation	16.4	13.0	10.9	10.1	12.7	10.2	10.7	14.0	15.9
Component									
Irregular	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1
Secular	3.7	3.0	1.8	0.6	0.0	0.7	1.6	2.6	3.2
Cyclical	3.6	0.9	0.0	0.4	3.6	0.4	0.0	2.3	3.6

[a] All entries are expressed in percentages of reference-cycle bases.

H SOME CONCLUSIONS

It would be easy to refine and enlarge upon Table 22. For example, the procedure followed in making it throws what stage-by-stage departures from regularity there are in the average deviations of our full sample into the cyclical component. They might more plausibly be assigned to the irregular component. Also, the differences between the deviations at troughs and peaks are ascribed wholly to the trend component—a position that taxes credulity. We might try several duration scales, derivable from our reference dates or by averaging the time span of the series in our samples. We might make calculations like those in Table 22 for various parts of our sample—say for all series on physical production, all on commodity prices, etc. But such betterments and additions would carry us into the realm of rapidly diminishing returns and, what is worse, suggest an unwarranted confidence in the representative value and precision of our results.

Table 22 rests upon a set of bold assumptions and an imperfect set of data. At best, the assumptions represent central tendencies that should appear in a large and properly drawn sample of measures made by our methods from time series subject to cyclical fluctuations, to shifts in the levels about which successive cycles occur, and also to ever changing complexes of other movements that bear irregular relations in time to the cyclical-secular complex. The statistical observations show clearly certain effects logically to be expected from the secular and cyclical components. They do not demonstrate the zero influence of secular changes at stage V, or the uniform rate of change in the secular factor from I to V and from V to IX, or the zero influence of cyclical changes at stages III and VII, or their maximum influence at troughs and peaks. These features of the scheme are deduced from certain properties of secular and cyclical movements, from the mathematical implications of our technique, and the observation that the modal type of timing among our series is that which fits the reference dates. From the statistical side, all we can say is that the evidence is compatible with broad expectations. If so much is granted, we can add that observation gives some warrant for the uniform distribution of noncyclical-secular effects among the nine stages. Thus our neat scheme, while not a set of actual measures, is more than a flight of fancy.

We may sum up the findings in the following propositions: The cyclical component varies from cycle to cycle; its variability declines sharply from stage I to stage III, rises sharply in III–V, then falls again in V–VII and rises in VII–IX. Intracycle trends also vary from cycle to cycle. Their contribution to the average deviations from reference-cycle patterns declines from stage I to stage V and rises from V to IX. Other types of cycle-by-cycle differences in our measures are not closely correlated with the changes in cyclical-secular movements, and tend to be evenly distributed among the nine stages. In magnitude they seem to exceed the differences attributable to the cyclical and secular components taken together, but the excess shown by

Table 22 cannot be taken at face value.[19] In any case it is not large at the troughs of the cycle where our methods maximize the variability of both the cyclical and secular components.

Since the stage-by-stage differences in average deviations arise in large measure from technical features of our analytic methods, we must be cautious about reading economic meanings into them. For example, the fact that these deviations reach major maxima at stages I and IX, and a minor maximum at stage V, does not prove that cyclical behavior is more variable from cycle to cycle at troughs than at peaks, and more variable at peaks than during expansions and contractions. Yet, by taking due precautions, we can make the stage-by-stage differences in average deviations yield economically significant information. An example of how that may be done is afforded by Table 23, which was drawn up by Geoffrey H. Moore. Since our methods are applied in virtually uniform fashion to all monthly and quarterly series, differences from series to series in the results must be due to the unlike ways in which different sets of data respond to the same treatment. To be

[19] Our calculations assume that the several components of variation can be treated as additive, whereas in fact such a relation does not hold in general among average deviations. However, an additive relation does hold among variances, provided the several components of variation can be treated as uncorrelated; and this suggests that a better approach would be to square the mean average deviation for each of the nine stages, then perform the decomposition described in the text, finally, extract square roots to put the results on an average deviation basis. Using this approach, Millard Hastay obtained the following interesting results:

TYPE OF COMPONENT	REFERENCE-CYCLE STAGES I	II	III	IV	V	VI	VII	VIII	IX
Irregular	8.2	8.2	8.2	8.2	8.2	8.2	8.2	8.2	8.2
Secular	10.4	9.4	7.3	4.3	0.0	4.2	6.8	8.7	9.6
Cyclical	9.7	3.8	0.0	4.1	9.7	4.4	0.0	7.3	9.7

Of course, it is not strictly permissible to pass directly from average deviations to variances. Further, the new results may differ markedly from those we would get if we worked throughout with standard deviations for individual series.

Table 23

Ratios of Mean Average Deviations at Reference Peaks to Mean Average Deviations at Reference Troughs, by Groups of Series[a]

GROUP	NO. OF SERIES	MEAN AV. DEVIATION AT STAGE V AS RATIO TO THAT AT Stage I	Stage IX	MEAN RATIO	RANK OF MEAN RATIO
Retail sales	10	.79	.51	.65	9
Wholesale sales	14	.89	.79	.84	24
New orders from manufacturers	17	1.42	1.39	1.40	29
Construction contracts					
Private	26	1.30	1.46	1.38	28
Public	16	.80	.66	.73	18.5
Inventories					
Positive timing	18	.82	.66	.74	20
Irregular timing	18	.62	.66	.64	8
Inverted timing	24	.49	.44	.46	1
Production					
Foodstuffs	47	.65	.74	.70	14
Other perishables	29	.88	.84	.86	25
Semidurables	28	.66	.79	.72	16.5
Durables	53	.71	.80	.76	21
Employment					
Perishable goods industries	8	.46	.61	.54	3
Semidurable goods industries	13	.50	.67	.58	6
Durable goods industries	9	.59	.74	.66	10.5
Hours of work per week	9	.73	.69	.71	15
Earnings per week, month, or year	10	.84	.54	.69	13
Payrolls					
Perishable goods industries	8	.54	.48	.51	2
Semidurable goods industries	13	.49	.64	.56	4.5
Durable goods industries	6	.80	.84	.82	23
Prices of commodities					
Farm products & foods	51	.63	.74	.68	12
Other perishables	22	.76	.82	.79	22
Semidurables	18	.57	.75	.66	10.5
Durables	45	.56	.68	.62	7
Interest rates					
Short-term	11	.61	1.35	.98	26
Long-term & bond yields	12	.41	.70	.56	4.5
Security issues, corporate	14	1.36	1.06	1.21	27
Bank clearings or debits	8	.89	.57	.73	18.5
Indexes of business activity	11	.74	.69	.72	16.5

[a] Based upon Table 20; see notes attached to that table.

Table 23 (concl.)

Summaries	NO. OF SERIES	MEAN AV. DEVIATION AT STAGE V AS RATIO TO THAT AT Stage I	Stage IX	MEAN RATIO
All series on				
Construction contracts or permits	58	1.07	1.05	1.06
Production	183	.71	.81	.76
Employment	37	.53	.69	.61
Payrolls & other income payments	30	.61	.66	.64
Prices of commodities	147	.61	.73	.67
All series on				
Flow of commodities, services, or income	472	.81	.81	.81
Prices of commodities or services	168	.62	.71	.66
Financial activities	135	.79	.80	.80
General business activity	13	.78	.65	.72
All series in sample	788	.77	.80	.78

RANKING OF GROUPS OF SERIES ACCORDING TO MEAN RATIO JUST GIVEN

Minor group	*Mean ratio*	*Major group*	*Mean ratio*
Inventories, inverted timing	.46		
Payrolls, perishables	.51		
Employment, perishables	.54		
Interest rates, long-term	.56		
Payrolls, semidurables	.56		
Employment, semidurables	.58		
		Employment	.61
Prices, durables	.62		
Inventories, irregular timing	.64	Payrolls & other income payments	.64
Retail sales	.65		
Employment, durables	.66	Prices of commodities or services	.66
Prices, semidurables	.66		
		Prices of commodities	.67
Prices, farm & food	.68		
Earnings per week, etc.	.69		
Production, foodstuffs	.70		
Hours of work per week	.71		
Production, semidurables	.72		
Indexes of business activity	.72	General business activity	.72
Construction contracts, public	.73		
Bank clearings or debits	.73		
Inventories, positive timing	.74		
Production, durables	.76	Production	.76
		ALL SERIES IN SAMPLE	.78
Prices, other perishables	.79		
		Financial activities	.80
		Flow of commod., serv., or incomes	.81
Payrolls, durables	.82		
Wholesale sales	.84		
Production, other perishables	.86		
Interest rates, short-term	.98		
		Construction contracts or permits	1.06
Security issues, corporate	1.21		
Construction contracts, private	1.38		
New orders from manufacturers	1.40		

more specific: our statistical operations tend to make the average deviations larger at the troughs than at the peaks of business cycles. But the degree in which this tendency manifests itself in the results varies from series to series according to individual trends, individual cyclical movements at peaks and at troughs, and according to the irregular factors that impinge upon individual series at these stages. Hence, when we compute the *ratios* of average deviations at peaks to average deviations at troughs, we find differences from series to series that are not mathematical consequences of our method. They must be due to dissimilarities of secular, cyclical, or irregular movements, taken in the senses assigned to these categories by our analytic technique. And when we compute these ratios, not for single series but for groups, we can go further and claim that the differences among groups are less the result of chance irregularities than of differences in the combined cyclical-trend components in which our interest centers.

Thus interpreted, Table 23 becomes highly interesting. For example, the last column shows that the groups having the highest variability at business-cycle peaks in relation to their variability at troughs are three representing the volume of investments to which private parties are committing themselves in the near future. How significant that finding may be for the theory of cyclical recessions and for the theory of revivals must be left for the present to the reader's imagination. Hardly less interesting is it to find the maximum variability at troughs in relation to peaks in inverted inventories—a business factor that makes the most trouble at the end of contractions. The difference between the role played by the durability of commodities in determining the cyclical behavior of series on prices and production, to which attention was directed in the chapter on reference-cycle amplitudes (Sec. IVF), reappears in this table on deviations. And this is only a beginning of the seemingly significant results that the reader can find in Table 23 if he observes with care. Our own efforts to exploit these materials, explaining what the table shows and making it explain other features of business cycles, will come later—when we shall be journeying round the cycle stage by stage. Still more useful will such studies of the average deviations prove when someone undertakes a systematic investigation of differences among business cycles. What is pertinent—and important—here is merely the demonstration that the technical effects of our analytic procedure upon the average deviations from mean reference-cycle standings do not render these measures useless. On the contrary, we can observe in them features of cyclical behavior that might otherwise escape attention—features that will enrich and enlarge our knowledge of what happens during business cycles.

What we have learned about the average deviations should reduce our troublesome misgivings concerning the representative value of reference-cycle patterns. Both the cyclical and the secular components in average deviations appear clearly, not only in the full sample, but also in most of the 29 groups. This finding means that, even in fairly small groups, the noncyclical-secular components are usually distributed evenly enough to let the systematic effects shine through. If this 'evening out' of irregular movements occurs among the nine stages of reference cycles, a 'canceling out' will probably occur when standings at the same stage of different cycles are combined to get one point of an average reference-cycle pattern. Indeed, the noncyclical-secular components of standings at the same stage of successive cycles are less likely to be intercorrelated than are these components of successive stages in the same cycle; that is, 'canceling out' is likely to be fuller than 'evening out'. And when we rise above the 29 groups to the full sample, we find some evidence of virtual stage-by-stage equality in the obscuring factors—evidence that is more impressive when coupled with 'evening out' in the group averages. But, once again, these reassuring conclusions apply in full force only to considerable samples of reference-cycle patterns. The representative value of the patterns of individual series we have still to consider.

I AVERAGE DEVIATIONS OF REFERENCE-CYCLE AMPLITUDES

So far the discussion has been confined to deviations from average standings at successive reference-cycle stages. A word should be added concerning the average deviations of the two sets of measures we derive from these standings.

Reference-cycle amplitudes are computed by taking the differences between the average standing of a series at whatever stage its timing variety indicates as the characteristic location of its peak and its standings in whatever preceding and following stages are indicated as its troughs. Obviously, the cycle-by-cycle variability of these differences depends upon the cycle-by-cycle variability of the standings at whatever may be the peak and trough stages of a series. Mathematical analysis suggests that the average deviation from the average amplitude of rise or fall may be approximately equal to the square root of the sum of the squares of the average deviations at the trough and peak standings, and hence larger than either the trough or the peak average deviation.

The mathematical grounds for this expectation have been stated at my request by Millard Hastay in Technical Note III, appended to this chapter. By way of seeing how our results fit the mathematical argument, we computed the average deviations from the average reference-cycle amplitudes of seven series in two ways: first, by direct computation, that is, measuring the amplitude of each cycle, casting up the sum, striking an average, and comparing with it the amplitudes of the individual cycles so as to get their average deviation; second, by squaring the average deviations at each of the two stages involved and taking the square root of the sum.[20]

Table 24 shows rather close agreement between expectations and observations. In most instances the average deviation from the mean amplitude is larger than the average deviation from

[20] For a more precise statement of this indirect method, see Technical Note III.

Table 24

Average Deviations from Average Reference-Cycle Amplitudes of Seven Series Computed Directly and Indirectly

			Average deviation from av. standing at stage of			Av. deviation from av. reference-cycle amplitude computed by					
						Direct Method			Indirect Method[b]		
Series[a]	No. of cycles	Expansion stages	Initial Trough	Peak	Terminal Trough	*Expansion*	*Contraction*	*Full cycle*	*Expansion*	*Contraction*	*Full cycle*
Clearings outside N.Y.C., deflated, 1879–1938	16	VIII–V	6.7	4.0	6.2	7.6	7.1	9.8	7.8	7.4	12.1
Pig iron production, 1879–1938	16	I–V	17.1	13.3	19.5	21.3	28.3	37.1	21.7	23.6	37.2
Freight car orders, 1870–1938	17	VIII–IV	34.8	38.2	38.2	52.1	58.7	85.2	51.7	54.0	92.2
Shares traded, N.Y. Stock Exchange, 1879–1938	16	VIII–IV	22.3	12.4	15.1	18.7	21.7	25.7	25.5	19.5	36.6
R.R. stock prices, 1858–1933	19	VIII–IV	16.5	7.0	11.5	15.8	16.7	26.8	17.9	13.5	24.5
Call money rates, 1858–1938	20	I–V	29.4	55.5	26.3	59.9	59.8	115.0	62.8	61.4	117.8
R.R. bond yields, 1858–1933	19	III–VII	3.3	3.2	8.0	6.2	6.3	6.7	4.6	8.6	10.8

[a] See Appendix B for sources of data.

[b] This involves squaring of average deviations of the standings at peaks and troughs, as explained fully in Technical Note III.

either of the average standings at the corresponding trough or peak. The two methods of computing the average deviation from the average amplitude yield results that, as a rule, are not far apart. The largest absolute discrepancies occur in the series on freight car orders and shares traded. Five of the 21 discrepancies are less than 1.0 percent, and 18 are less than 5 percent.

What has been said about the average deviations from reference-cycle amplitudes applies also, though with an important exception, to the average deviations from average rates of change per month from one reference-cycle stage to the next. One of the assumptions underlying the expectation that the average deviation of the reference-cycle amplitudes will approximate the square root of the sum of the squares of the average deviations at trough and peak stages is that variations in peak and trough standings are uncorrelated in combinations of successive peaks and troughs. (See Technical Note III.) Such an assumption is less likely to be approximated when the arrays of standings involved in the computation are at adjacent stages than when the arrays are separated by the full or approximate duration of a reference expansion or contraction. Hence we cannot expect that the average deviations from the average rates of change per month will bear as close a relation to the average deviations from the standings at the stages compared as Table 24 suggests.

III Cycle-by-Cycle Variability among Series

So far our interest has focused on average deviations of single series or groups of series. We have inquired what types of time-series movements contribute to the average deviations of reference-cycle patterns, what is the relative importance of three types of movements, and how the deviations differ from stage to stage. On turning from differences within series to differences among series in cycle-by-cycle behavior, we must ask two more questions about average deviations. Instead of dissecting deviations, we now accept them as wholes in an effort to learn how and why they differ from series to series, and what bearing their varying magnitudes have upon the representative value of average reference-cycle patterns and their derivatives.

A INFLUENCE OF AVERAGE AMPLITUDES

Among the factors that influence the size of full average deviations from average reference-cycle standings, first place belongs to specific-cycle amplitudes. Obviously, a series that rises and falls by only a few percent of its average value in successive cycles can show only small differences among its cycle-by-cycle standings at its initial troughs, its peaks, and its terminal troughs. An erratic movement at some other stage may produce a greater deviation; but large departures from their characteristic behavior are rare among series with low amplitudes, aside from war-begotten price gyrations, and these we exclude from the averages. On the other hand, series that typically rise and fall by 50 or 100 percent of their average value during a cycle are likely to show correspondingly large differences in cycle-by-cycle standings as we measure them, both at the turning and the intermediate stages.

This relation between average deviations and average amplitudes in the specific-cycle analysis persists with certain modifications when series are analyzed on a reference-cycle basis. That reference-cycle amplitudes and average deviations are correlated can be seen by glancing over the reference-cycle patterns of Chart 1. So close is the connection between the two variables that what was done in Chapter 7 toward explaining differences among series in amplitude applies in large part to the deviations also.

So, too, does what was said in the second section of that chapter concerning the problem of bias in our sample. The fact that about half of the series we have analyzed cover only the period between the two world wars, or parts of it, means not only that our results give an exaggerated impression of the violence of cyclical fluctuations in the American economy since the 1850's, but also that the results exaggerate long-run differences in cycle-by-cycle behavior.[21]

B INFLUENCE OF REGULARITY IN CYCLICAL TIMING

While amplitudes dominate deviations from average reference-cycle patterns, their sway is modified by other factors, of which degree of regularity in cyclical timing seems to be the most pervasive. Regular cyclical timing cannot confer large specific-cycle amplitudes upon a series, but it tends to preserve most of these amplitudes, whether large or small, when the series is analyzed on a reference-cycle basis. At the same time, the regularity that tends to keep reference-cycle amplitudes up to the specific-cycle level tends to keep average deviations from reference-cycle patterns down to the level of average deviations from specific-cycle patterns. Irregular timing, on the contrary, tends to make reference-cycle amplitudes decidedly lower and reference-cycle deviations higher than their specific-cycle counterparts.

The reasons why reference-cycle analysis changes specific-cycle amplitudes and deviations in opposite ways are simple. We have previously shown that irregular timing with respect to business cycles means that the specific-cycle peaks of a series are scattered among the nine reference-cycle stages in haphazard fashion.[22] Hence, no reference stage has as high an

[21] This effect is illustrated by the series used in our smoothing tests (see above, Sec. IIB). In call money rates the variability is smaller after than before 1914 because the passage of the Federal Reserve Act led to a 'structural change' in the New York money market. See above, note 5.

MEAN AV. DEV. FROM AV. REFERENCE-CYCLE STANDINGS AT ALL NINE STAGES

	Before World War I			After World War I		
	Period	No. of cycles	Mean av. dev.	Period	No. of cycles	Mean av. dev.
Deflated clearings	1879–1914	10	4.1	1919–1938	5	6.2
R.R. stock prices	1858–1914	14	6.6	1919–1933	4	18.1
Pig iron production	1879–1914	10	9.0	1919–1938	5	19.7
Call money rates	1858–1914	14	29.6	1919–1938	5	22.2

Here, as elsewhere, stages I and IX are given a weight of one-half each in striking nine-stage averages of the average deviations.

[22] See above, Ch. 7, Sec. IVA.

average standing as stage V of the specific cycles. Similarly, the specific-cycle troughs are scattered among several or all the reference-cycle stages, and none of the latter has an average standing so low as stages I and IX of the specific cycles. Lower peaks and higher troughs in the reference-cycle pattern yield lower amplitudes. But the scattering of the specific-cycle peaks and troughs among the nine reference-cycle stages means that each stage is likely to include some very low and some very high standings along with a larger number of intermediate size. Then the average deviation from the average standing at each stage tends to be larger than when all the initial troughs are assembled in a single stage, all the peaks in a second stage, all the terminal troughs in a third; and when each of the three steps between the initial trough and the peak, also each of the three steps between the peak and the terminal trough, is put with its fellows from other cycles.

How all this works out in practice may be illustrated by comparing a series with highly regular and one with decidedly irregular timing. Pig iron production has conformity indexes of +100, +100, +100, and its specific-cycle turns correspond invariably to our reference dates. Despite an average lead of three months at revivals and an average lag of two months at recessions, we treat the series as having I–V timing. Regular as is the behavior of iron production on this timing basis in cycle after cycle, only 8 of the 17 specific-cycle troughs occur within the three months centered on a reference-cycle trough date, and only 6 of the 16 specific-cycle peaks occur in stage V of a reference cycle. The value of total exports from the United States has much less regular timing; its indexes of conformity are +62, 0, +27. It undergoes 21 specific cycles in approximately the time occupied by 16 business cycles. Of its specific-cycle troughs only 11 correspond to reference troughs, and of these only 3 occur in stage I (or IX) of a reference cycle. Of its 21 specific-cycle peaks, 6 correspond to reference peaks and only 1 of them comes in reference-stage V.[23]

[23] For our technical rules concerning correspondence between specific-cycle and reference turns, see *Measuring Business Cycles*, pp. 118–26.

When these series are analyzed on a reference-cycle basis, both have their amplitude reduced and the average deviations from their cyclical patterns increased. In the series with regular timing, the reduction in amplitudes is from a rise and fall of 128 percent of the specific-cycle bases to a rise and fall of 99 percent of the reference-cycle bases—a decline of 23 percent of the larger amplitude. In the series with irregular timing, the reduction is from a rise and fall of 62 to one of 16—a decline of 74 percent. The average deviations from the average standings in the nine stages of the specific cycles of iron production are 12.8 percent of the cycle bases; the corresponding figure in the reference-cycle analysis is 13.7—an increase of 7.0 percent. In total exports the increase is from average specific-cycle deviations of 7.9 to average reference-cycle deviations of 10.6 —an increase of 34.2 percent. It should be noted that the increase in average deviations produced by the shift from specific to reference cycles is much smaller than the concomitant decrease in amplitudes, not only when measured in percentages of cycle bases, but also when measured in percentages of the specific-cycle results. A casual survey—no close examination has yet been made—suggests that in this respect iron production and exports are typical.

Identity of average amplitude and conformity does not guarantee identity of average deviations. Table 25 shows the average deviations from average reference-cycle standings at all nine stages for series with virtually perfect conformity and with amplitudes that round off at 25, at 50, and at about 100 percent of the reference-cycle bases. Within each of the three identical amplitude-conformity groups there are appreciable differences in the column for average deviations. The main reason for the differences is obvious. When the average amplitude is small and the cyclical timing regular, large average deviations cannot occur in the peak and trough stages of a series. But the opposite is not true: small average deviations can occur in the peak and trough stages when the average amplitude is large. The relation between average amplitudes and average deviations is likely to be especially loose in short

Table 25

MEAN AVERAGE DEVIATIONS FROM AVERAGE REFERENCE-CYCLE PATTERNS OF THREE GROUPS OF SERIES HAVING VIRTUALLY IDENTICAL REFERENCE-CYCLE AMPLITUDES AND CONFORMITY

	SERIES[a]	PERIOD COVERED	NO. OF CYCLES	AV. AMPLITUDE	MEAN CONFORMITY[c]	MEAN AV. DEVIATION AT REF.-CYCLE STAGES[d]
1	Cotton spindles active, noncotton-growing states	1914–38	6	25.4	84	7.7
2	Av. hours worked per week, males, skilled & semiskilled, mfg.	1921–38	4	25.3	100	4.9
3	Av. hours worked per week, mfg., total	1921–38	4	24.8	100	4.6
4	Factory payrolls, boots & shoes	1919–38	5	50.4	100	6.7
5	Mining production index, total	1914–27	4	50.2	100	7.0
6	Wholesale prices, lumber	1921–38	4	50.1	100	7.5
7	Wholesale sales, hardware	1919–27	3	49.7	100	6.5
8	Mining production index, metals	1914–38	6	102.8	100	18.8
9	Suspended commercial banks, no.	1921–33	3	101.8[b]	100	28.2
10	Production index, dur. producer goods	1919–38	5	101.3	100	15.2
11	Production, tin & terne plate	1924–38	3	100.9	100	10.1
12	Production index, durable goods, total	1919–38	5	100.3	100	15.9
13	Shipments, steel sheets	1919–33	4	100.3	100	18.0
14	Constr. contracts, total bldg., floor space	1919–38	5	100.2	100	27.5
15	Production, pig iron	1879–38	16	99.1	100	13.6

[a] See Appendix B for sources of data.
[b] Average of 2 reference cycles, with III–VII as the expansion stages.
[c] Average of indexes of conformity to reference expansions and reference contractions.
[d] Average deviations of standings at stages I and IX receive a weight of one-half each.

series covering periods affected by powerful irregular factors —such as are heavily represented in Table 25.[24] Yet it may be noted that Table 25, devised to show differences in average deviations among series having virtually identical average amplitudes and conformity, itself bears witness to the dominant influence of the factors whose omnipotence it denies. The mean average deviation of the 25 percent amplitude group is 5.7, that of the 50 percent group 6.9, and that of the 100 percent group 18.4. Only one series in the table has deviations that belong in the next higher group, and that is the only series with less than perfect conformity.

[24] To illustrate: Tin and terne plate production gets its average of 100.9 from three reference cycles with amplitudes of 72.5, 73.9, and 156.3. The number of suspended banks, with the unusual timing scheme III–VII, allows us to measure the patterns of three reference cycles but the amplitudes of

C STATISTICAL TESTS

More systematic evidence concerning the relations among deviations, amplitudes, and conformities is offered by Table 26. In making this table the 29 groups of series used in Table 20 were ranked, first according to their average deviations, second according to their average amplitudes, third according to their ratios of average deviations to amplitudes, and fourth according to their mean conformity to reference expansions and contractions. In each ranking all four measures of the groups were recorded. Then the 29 groups were divided into 3 nearly equal sets. The table presents averages of these sets, and the ranges of the measures from which the averages are

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only two. The first (1921–24) was accompanied by a severe epidemic of bank failures; the second cycle (1924–27) was marked by relative banking tranquility. The amplitudes of bank suspensions in these two cycles (168.7 and 34.9) yield an average (101.8) that is 0.9 points higher than for tin and terne plate production; but the average deviations are more than twice as large (28.2 compared with 10.1). Needless to say, the averages of these very short series, though better than no information (especially when supplemented by other series of similar character), have slight claim to representative value.

Another illustration that may be helpful concerns the effects of irregular timing upon the relation between average deviations and average amplitudes. The following figures are nearly self-explanatory.

	IRREGULAR INVENTORIES	POSITIVE INVENTORIES	INVERTED INVENTORIES
Number of series	18	18	24
Av. reference-cycle amplitude [a]	33.6	44.3	68.2
Av. index of conformity to business cycles [a]	24	69	64
Mean av. deviation at reference-cycle stages [b]	14.3	11.7	16.2
Mean av. deviation as % of av. amplitude	42.6	26.4	23.8

[a] Taken without regard to sign.
[b] Average deviation at stages I and IX receive a weight of one-half each

The irregular inventories have larger deviations than the positive, despite the fact that the latter have substantially higher reference-cycle amplitudes. But the effect of irregular timing in raising deviations cannot equal the effect of a doubling of reference-cycle amplitudes, which appears on comparing the irregular with the inverted inventories. In the last line of the table, however, the *ratio* of deviations to amplitudes is lowest in the inverted and much the highest in the irregular group.

made, so that one can judge how much or how little the sets overlap one another. Correlation coefficients at the end of the table summarize the relations among the rankings in a more compact way.

The first ranking indicates a close association between average deviations and average amplitudes, but not between average deviations and the other two behavior traits taken separately.

The ranking by average amplitudes shows again, from the opposite viewpoint, the close association between amplitudes and deviations. But amplitudes are somewhat more closely related than deviations to the other two measures. The larger the amplitudes the higher tend to be the mean conformity indexes and the lower tend to be the ratios of average deviations to amplitudes. However, these tendencies are often balked by other factors.

The ranking by ratios of average deviations to average amplitudes confirms the moderately inverse relation to amplitudes. The new feature is the close inverse relation to conformity indexes. This finding confirms the foregoing observation that close conformity to business cycles has opposite effects upon reference-cycle amplitudes and deviations from reference-cycle patterns, for close conformity tends to preserve full specific-cycle amplitudes in the reference-cycle analysis and to minimize cycle-by-cycle deviations from average standings. Of course, holding amplitudes up and deviations down is equivalent to decreasing the ratio of deviations to amplitudes.

The fourth ranking—that by conformity indexes—merely repeats in turn what the previous rankings told us. Conformity indexes, taken by themselves, have no regular relation with average deviations, a positive relation with average amplitudes, but a close inverse relation with ratios of the first to the second.

The coefficients of rank correlation at the end of the table put the preceding conclusions in so precise a form that they tempt one to overrate the representative value of the findings. But they add one interesting item: the relations among the traits we have been studying appear to be substantially closer when one correlates deviations with amplitudes and conform-

Table 26

Summary of Relations among Average Deviations of Reference-Cycle Patterns, Average Amplitudes, and Regularity of Cyclical Timing in 29 Groups of Series[a]

	Av. deviation of reference-cycle pattern[b]	*Av. reference-cycle amplitude*	*Percentage ratio of av. deviation to av. amplitude*	*Mean conformity*[c]
1) Groups ranked by av. deviations				
Averages				
Lowest 10 groups	5.8	24.8	25.6	69.9
Middle 10 groups	9.7	43.4	24.7	67.5
Highest 9 groups	23.1	91.0	28.8	63.4
Ranges				
Lowest 10 groups	3.9– 7.8	12.8– 46.3	13.6–49.0	40–100
Middle 10 groups	8.2–11.7	24.1– 62.8	16.2–41.8	40– 92
Highest 9 groups	14.3–41.0	33.6–166.0	18.5–50.1	33– 96
2) Groups ranked by av. amplitudes				
Averages				
Lowest 10 groups	6.3	22.0	29.7	61.3
Middle 10 groups	11.3	41.7	27.7	64.3
Highest 9 groups	20.7	96.0	20.8	76.6
Ranges				
Lowest 10 groups	3.9–10.4	12.8– 27.8	19.1–49.0	40–100
Middle 10 groups	6.3–21.3	29.5– 49.5	13.6–50.1	33– 98
Highest 9 groups	8.8–41.0	51.4–166.0	16.2–32.8	55– 96
3) Groups ranked by ratios of av. deviation to av. amplitude				
Averages				
Lowest 10 groups	10.6	58.4	18.0	84.1
Middle 10 groups	12.7	56.4	23.6	69.0
Highest 9 groups	14.5	39.3	38.4	46.0
Ranges				
Lowest 10 groups	3.9–29.0	20.4–147.6	13.6–19.6	68–100
Middle 10 groups	4.9–34.4	18.2–166.0	20.5–27.8	55– 80
Highest 9 groups	4.1–41.0	12.8–124.9	28.9–50.1	33– 62
4) Groups ranked by mean conformity				
Averages				
Lowest 10 groups	11.4	32.6	36.4	46.3
Middle 10 groups	12.3	50.4	23.3	70.3
Highest 9 groups	14.0	74.5	18.4	86.6
Ranges				
Lowest 10 groups	4.1–21.3	12.8– 68.2	23.8–50.1	33– 58
Middle 10 groups	4.9–41.0	18.2–124.9	17.1–32.8	62– 74
Highest 9 groups	3.9–34.4	20.4–166.0	13.6–20.7	76–100

Table 26 (concl.)

Coefficients of rank correlation for the 29 groups of series, when the factors related are:

Average deviations and	
Average amplitudes	.79
Ratios of average deviations to average amplitudes	.15
Mean indexes of conformity	−.08
Average amplitudes and	
Ratios of average deviations to average amplitudes	−.43
Mean indexes of conformity	.42
Ratios of average deviations to average amplitudes and Mean indexes of conformity	−.88
Average deviations and average amplitudes, with effect of mean indexes of conformity eliminated	.91
Average deviations and mean indexes of conformity, with effect of average amplitudes eliminated	−.74
Average deviations and both average amplitudes and mean indexes of conformity	.91

[a] For a list of the groups, see Table 20.

[b] Average deviations at stages I and IX receive a weight of one-half each.

[c] Average of indexes of conformity to reference expansions and reference contractions.

[d] This coefficient is larger by less than a half unit in the second decimal place than the coefficient which eliminates the effect of mean conformity.

ity together. It was with this last combination that the preceding analysis began.

IV The Representative Value of Cyclical Patterns

Though the preceding analyses offer merely a rough sketch of a field that should be surveyed thoroughly, they demonstrate that the deviations from reference-cycle standings have a rationale of their own, and can be made to contribute toward the understanding of business cycles. These deviations correspond to the 'disturbing circumstances' economic theorists impound in *ceteris paribus* clauses, or exclude by discussing what happens 'in the long run'. For such logical devices a statistical inquirer substitutes averages that he hopes bring out the central tendencies of his arrays. There is, however, an important psychological difference between the two procedures. 'Disturbing circumstances' is too vague a concept to excite much interest. Seldom does a theorist feel impelled to hunt for the exceptions to his rules and inquire how they come about. Of course a statistician may rest content with what his averages tell. But in the statistical approach deviations from averages constitute a standing challenge to scientific curiosity, inciting further explorations, which may lead to the discovery of unsuspected regularities among what had seemed annoying exceptions to orderly relations. Presently the newly discovered regularities may be incorporated into the older generalizations; this more adequate formulation stimulates fresh inquiries into the discrepancies that still appear between 'theory' and 'fact', and the spiral of research mounts to a higher level.

At present I can follow only one round of this spiral. Having shown that the average deviations are complexes made up of cyclical and secular as well as irregular elements, and pointed out the chief factors that influence the magnitude of the deviations, I pass on to the study of averages. Later I should return to cycle-by-cycle differences, and use whatever the averages have taught in an effort to find out how business cycles come to differ from one another so much as they patently do. But that task remains for later times and other hands.

Meanwhile what we can discover here and now about average behavior should be the richer and the truer for our brief study of deviations from it. First and foremost we should keep in mind that cycle-by-cycle variability is the one trait most typical of cyclical behavior. In the coming comparisons of reference-cycle patterns and summaries of reference-cycle standings or amplitudes, I shall seldom include the average deviations, for a reconnaisance survey of a complicated province must be confined to salient features. But the reader should remember that the real terrain is not so simple as my sketch maps make it look. Second, this study of average deviations has provided a practical demonstration of the tests of consilience on which such heavy stress was laid in the last chapter of *Measuring Business Cycles* and in Chapter 3 of this book. Rather subtle implications of our technique that can seldom be traced in individual series began to appear when we assembled series in relatively homogeneous groups, and became clearer still when we combined all the heterogeneous groups that constitute our sample. That experience should warn us against relying exclusively upon what we might infer from a few series, however comprehensive they may be. Rather must we put our trust in findings borne out by groups of series, the broader and more varied the better. Third, close agreement between the average deviations in stages at which the cyclical-secular components are similar (I and IX, II and VIII, III and VII, IV and VI) indicates a remarkable degree of 'evening out' of noncyclical-secular components at different stages in our whole sample, and a similar effect can be traced, though with less confidence, in numerous groups. We drew, and now have use for, the conclusion that the chances are better still of a 'canceling out' of noncyclical-secular components when standings during the same stage of successive cycles are averaged. Fourth, on turning to averages of average deviations at all stages, we noted that their magnitude rises with, but on the whole more slowly than, the average amplitude. That is, deviations so large as to arouse grave misgivings about the representative value of a reference-cycle pattern are often (by no means always) moderate in comparison with the average rise and fall. On the other hand, small deviations are sometimes large percentages of modest amplitudes. But on this crucial topic I should be more specific.

The reference-cycle patterns are meant to approximate the combined cyclical and intracycle trend components in economic fluctuations; that is, to extricate from their matrices the movements attributable to the alternating tides of expansion and contraction acting upon elements of the economy that keep changing the levels about which they fluctuate. Always the cyclical tides flow and ebb in a complex of conditions peculiar to the national and international scene during some segment of history—conditions that affect different factors in the economy in unlike ways and degrees. It is precisely because the movements of a series during successive business cycles are not alike that, to approximate cyclical behavior, we adopt the laborious method of averaging movements in as many cycles as feasible. The greater the cycle-by-cycle diversity of movements, the more do we need an average to help us judge whether a 'central tendency' toward cyclical regularity can be descried in the jumble. When such a tendency does appear, the larger the average deviations from the average pattern, the more trustworthy is this pattern as a measure of cyclical behavior in comparison with the movements during any single cycle.

Yet it is also true that the larger the average deviations, the harder it is to derive a trustworthy measure of the cyclical-secular component of economic movements. Distrust of a reference-cycle pattern subject to large average deviations is justified, not because the average pattern differs widely from the movements in individual cycles, but because the average may be too much like the pattern of some one or two especially violent cycles. Under these circumstances, we become more eager than ever to cover more cycles and to find closely related series for comparison. Even when the quest for further evidence succeeds, we sometimes end with the conviction that the best measures we can make are exceedingly rough approximations. Yet, at worst, it is no small advantage to have measures that tell what economic activities are unpredictable in cyclical behavior and what have responded to successive cycles in a relatively uniform fashion.

Technical Note I

EFFECT OF MACAULAY'S SMOOTHING FORMULA ON THE VARIATION OF A RANDOM SERIES

In discussing my observations upon the way in which smoothing alters the average deviations from mean reference-cycle standings, Millard Hastay suggested that the effect of Macaulay's formula on the variation of a random series might be investigated algebraically. At my request, he worked out this suggestion, and summarizes his findings below.

Memorandum by Millard Hastay

Consider first a purely random time series without trend or cycles or other systematic component. The argument is simplified without affecting the conclusions by assuming that the average level of this series is zero. Assume further that the variability of the observations tends to remain constant through time. We may then represent the series as follows:

$$\ldots, x_{-1}, x_0, x_1, x_2, x_3, \ldots, x_i, \ldots, x_n, \ldots$$

where the time origin is taken at any convenient point and our assumptions imply the following expected values (E):

(1) $E(x_i) = 0$ for all i	(zero trend level)
(2) $E(x_i^2) = \sigma^2(x_i) = \sigma^2$ for all i	(constant variation)
(3) $E(x_i x_j) = 0$ if $i \neq j$	(simple randomness)

Suppose such a series to be smoothed by a weighted moving average with weights $w_{-n}, \ldots, w_0, \ldots, w_n$ and period $2n + 1$. (The assumption of an odd period is not essential, and is made simply in view of the application to Macaulay's formula below.) In this fashion a new series $X_0, X_1, X_2, \ldots$ is generated, where

$$X_0 = w_{-n}x_{-n} + w_{-n+1}x_{-n+1} + \cdots + w_{-1}x_{-1} + w_0x_0 + w_1x_1 + \cdots + w_ix_i + \cdots + w_nx_n$$

$$X_1 = w_{-n}x_{-n+1} + w_{-n+1}x_{-n+2} + \cdots + w_{-1}x_0 + w_0x_1 + w_1x_2 + \cdots + w_ix_{i+1} + \cdots + w_nx_{n+1}$$

$$\vdots$$

$$X_i = w_{-n}x_{-n+i} + w_{-n+1}x_{-n+1+i} + \cdots + w_0x_i + \cdots + w_{n-i}x_n + \cdots + w_nx_{n+i}$$

etc.

Like the original series, this series will have average level zero, for

$$E(X_i) = E\left(\sum_{j=-n}^{n} w_j x_{j+i}\right) = \sum_{j=-n}^{n} w_j E(x_{j+i}) = 0$$

The variance is therefore easily computed

$$\begin{aligned}\sigma^2(X_0) &= E(X_0^2) = E(\sum w_j x_j)^2 \\ &= E\left[\sum_{j=-n}^{n} w_j^2 x_j^2 + 2\sum_{j=i+1}^{n}\sum_{i=-n}^{n-1} w_i w_j x_i x_j\right] \\ &= \sum w_j^2 E(x_j^2) + 2\sum_{i<j} w_i w_j E(x_i x_j)\end{aligned}$$

But by assumption $\quad E(x_j^2) = \sigma^2$

$$E(x_i x_j) = 0, \quad i \neq j$$

Thus

$$\sigma^2(X_0) = \sigma^2 \sum_{j=-n}^{n} w_j^2 \tag{4}$$

Since X_0 might be any term of the series, this result is plainly general, whence the smoothed series too exhibits homogeneous variation over time.

However, the third feature of the original series is not reproduced in the smoothed one, for successive items are serially correlated. Since the smoothed series varies homogeneously with time, the serial correlations will be a constant multiple [namely, $1/\sigma^2(X)$] of the corresponding covariances and it therefore suffices to determine the latter. By the definition of covariance with lag i months

$$\begin{aligned}(5)\quad \sigma(X_0X_i) = E(X_0X_i) &= E\left[\left(\sum_{j=-n}^{n} w_j x_j\right)\left(\sum_{j=-n}^{n} w_j x_{j+i}\right)\right] \\ &= E\left[\sum_{j=-n}^{n-i} w_j w_{j+i} x_{j+i}^2 + \sum_{\substack{j=-n \\ j\neq k+i}}^{n}\sum_{k=-n}^{n} w_j w_k x_j x_{k+i}\right] \\ &= \sum_{j=-n}^{n-i} w_j w_{j+i} E(x_{j+i}^2) + \sum_{j}\sum_{\substack{k \\ j\neq k+i}} w_j w_k E(x_j x_{k+i}) \\ &= \sigma^2 \sum_{j=-n}^{n-i} w_j w_{j+i}\,, \text{ since } \underset{j\neq k+i}{E}(x_j x_{k+i}) = 0\end{aligned}$$

This argument is plainly independent of the term chosen as X_0, while the lag i might be anything from one to $2n$ months. Thus the serial correlations of the smoothed series are independent of the time origin; i.e., for a fixed lag they tend to be the same anywhere in the series.

Formulas (4) and (5) have been used to compute the variance and lagged covariances of a series derived by smoothing a purely random series with Macaulay's "43-term approximately 5th-degree parabolic graduation", utilizing the implicit weights given in Table A.

Table A

WEIGHTS IMPLIED BY MACAULAY'S 43-TERM APPROXIMATELY 5TH-DEGREE PARABOLIC GRADUATION FORMULA

Month	*Weight*	*Month*	*Weight*
0	+.12042	−11, +11	−.02135
−1, +1	+.11739	−12, +12	−.01854
−2, +2	+.10937	−13, +13	−.01271
−3, +3	+.09667	−14, +14	−.00625
−4, +4	+.07917	−15, +15	−.00083
−5, +5	+.05854	−16, +16	+.00292
−6, +6	+.03750	−17, +17	+.00469
−7, +7	+.01698	−18, +18	+.00417
−8, +8	−.00063	−19, +19	+.00312
−9, +9	−.01323	−20, +20	+.00187
−10, +10	−.01979	−21, +21	+.00073

No essential generality is lost by assuming that the random series has variance $\sigma^2 = 1$; in fact, the results for this case, summarized in Table B, constitute a kind of canonical form for all random series such that

$$\begin{aligned} \sigma(x_i x_j) &= 0, \qquad i \neq j \\ \sigma^2(x_i) &= \sigma^2 \qquad \text{for all } i \end{aligned}$$

when Macaulay's formula is used in the graduation.

Table B

VARIANCE AND LAGGED COVARIANCES OF A SMOOTHED SERIES DERIVED BY GRADUATING A PURELY RANDOM SERIES OF UNIT VARIANCE WITH MACAULAY'S 43-TERM FORMULA

$\sigma^2(X_i) = 0.1107$	$\sigma(X_i X_{i+4}) = 0.0647$
$\sigma(X_i X_{i+1}) = 0.1082$	$\sigma(X_i X_{i+5}) = 0.0594$
$\sigma(X_i X_{i+2}) = 0.1010$	$\sigma(X_i X_{i+6}) = 0.0452$
$\sigma(X_i X_{i+3}) = 0.0898$	$\sigma(X_i X_{i+7}) = 0.0268$

Consider next the effect of averaging successive items in a smoothed series of the above type. Let the average be denoted

$$\xi = \frac{X_1 + \cdots + X_N}{N}$$

Then by definition of the variance of a simple average

$$
\begin{aligned}
(6) \qquad \sigma^2(\xi) &= \frac{1}{N^2} E(X_1 + \cdots + X_N)^2 \\
&= \frac{1}{N^2} E\left[\sum_{i=1}^{N} X_i^2 + 2 \sum_{j=i+1}^{N} \sum_{i=1}^{N-1} X_i X_j \right] \\
&= \frac{1}{N^2} \left[N\sigma^2(X) + 2 \sum_{j=i+1}^{N} \sum_{i=1}^{N-1} \sigma(X_i X_j) \right] \\
&= \frac{1}{N^2} \left[N\sigma^2(X) + 2 \sum_{j=1}^{N-1} \sum_{i=1}^{N-j} \sigma(X_i X_{i+j}) \right] \\
&= \frac{\sigma^2(X)}{N} + \frac{2}{N^2} \sum_{j=1}^{N-1} (N-j)\sigma(X_0 X_j)
\end{aligned}
$$

This expression depends upon the N^2 possible variances and covariances between N successive items of the smoothed series; N of these equal the variance of a single smoothed item, $2(N-1)$ equal the covariance with lag 1, . . ., $2(N-j)$ equal the covariance with lag j, etc.

As a sample calculation, suppose $N = 3$ and the variance and covariances cf the smoothed series are as in Table B. Then $\xi = (X_1 + X_2 + X_3)/3$ and

$$
\begin{aligned}
\sigma^2(\xi) &= \tfrac{1}{9} [3\sigma^2(X) + 4\sigma(X_i X_{i+1}) + 2\sigma(X_i X_{i+2})] \\
&= \tfrac{1}{9} [3(0.111) + 4(0.108) + 2(0.101)] \\
&= 0.107
\end{aligned}
$$

Formulas (4), (5), and (6) permit us to study the relative effects of smoothing, of averaging, and of smoothing with subsequent averaging, on a purely random series. As before, it is convenient to assume that the random series has variance $\sigma^2 = 1$; also, in order that the results may be directly applicable to the effect of averaging over reference-cycle stages of different duration, we consider averages of 3, 6, and 7 successive items. The relevant findings are summarized in Table C.

Table C

VARIANCES OF ORIGINAL AND SMOOTHED TIME SERIES AND OF AVERAGES OF SUCCESSIVE ITEMS FROM THEM

Variance of	*Original series*	*Smoothed series*
Single item	1.00	0.111
Average of 3 successive items	0.33	0.107
Average of 6 successive items	0.17	0.096
Average of 7 successive items	0.14	0.092

As a measure of the effect of smoothing alone, we find that the variance of a single smoothed item is only 11.1 percent of the variance of an item in the unsmoothed series. The gain from subsequent averaging is relatively small. Averaging 3 successive items of the smoothed series achieves a further reduction of variance of only 4 percent; averaging 6 items, only 14 percent; averaging 7 items, 17 percent. In the original series the corresponding reductions, which measure the influence of averaging alone, reflect the rule that the variance of the average of successive items in a random series varies inversely as the period spanned. These reductions are:

3 items, 67 percent
6 items, 83 percent
7 items, 86 percent

The largest of these reductions is only slightly less than that achieved by the 43-item weighted average, namely, 89 percent.

These results are somewhat more intelligible when translated into terms of standard deviations, as is readily accomplished by taking square roots of the items in Table C. We find that smoothing alone reduces the standard deviation of the original series by 67 percent, while the effects of averaging on the standard deviations of the original and smoothed series are as shown in Table D. Application

Table D

Percentage Reduction of Standard Deviation Due to Averaging

Average of	*Original series*	*Smoothed series*
3 successive items	43	2
6 successive items	59	7
7 successive items	63	9

of these results to the average deviation is immediate if we may assume that the average deviation tends to be a fixed multiple of the standard deviation. Such a relation is readily demonstrated for the normal distribution and seems to hold with good approximation for symmetrical distributions that do not depart too widely from the normal type.

Now suppose that the original series had involved serial correlations, at least for lags of a few months. This would mean that the assumption $E(x_i x_j) = 0$ for $i \neq j$, on which formulas (4) and (5) are based, no longer holds for $(j-i)$ less than some small integer d, say 3 or 4. The usual conception of these correlations suggests that they be considered positive. If, then, we assume

$$Ex_i x_j > 0, \quad (j-i) < d$$

it follows that

$$\sigma^2(X) > 0.111$$

and that $\sigma(X_i X_j)$ will exceed the corresponding covariances in Table B. In other words, smoothing the original series would reduce its variance by less than 89 percent—by substantially less if the serial correlations of low order were fairly strong. The effectiveness of averaging the smoothed series would likewise be reduced, so that the variance of the average of 7 items might be little different from the variance of a single smoothed item.

To a lesser degree, a similar influence on averaging would appear in the original series; averages of short duration might be little less variable than a single item; those of longer duration would be affected to lesser extent by serial correlations but their variability would still be reduced by less than the rule of inverse proportion to duration suggests. For example, if the original series were constituted with serial correlations as high as those of a random series smoothed by Macaulay's formula, the standard deviation of a seven-item average would be only 7 percent less than that of a three-item average.

Technical Note II

Cycle-by-Cycle Variability in the Trend Components of Average Reference-Cycle Standings

1 *Type of Secular Trend Implied by National Bureau Measures*

In 1934 Edwin Frickey illustrated the "basic logical difficulty connected with the separation of secular and cyclical variations" by presenting "a list of the various mathematically-fitted secular trends which have . . . been calculated" for pig iron production in the United States. His list includes 23 such efforts, ranging from straight lines fitted to the data for different periods to such constructions as a third degree parabola fitted to logarithms of the data. To these mathematically fitted lines he added 6 moving-average trends with periods running down from 20 to 3 years.

With these materials in hand, Frickey determined the number and duration of the 'cycles' in pig iron production by "observing the number of complete swings" of the data about each trend line. The average duration of these 'cycles' varied between 3.3 and 40–45 years. The conclusion he drew was not that the 29 trends "are lacking in statistical or economic significance". On the contrary, Frickey believed that "many of them unquestionably possess such significance", though he did not try to determine which trends are useful for what purposes, or to discuss criteria for choosing among competitors for a given use. His positive conclusions were:

first, that the average length of 'cycle' for a series—and for that matter, the whole form of the supposed cyclical picture—may exhibit great variation depending upon the kind of secular trend which has *previously* been fitted; second, that the discovery, about a particular trend representation which has been set up for a given economic series, of oscillations which may conform more or less closely to a certain average length cannot in itself be taken as establishing the statistical or economic validity of such movements as cycles.[1]

By way of contrast, this notable paper facilitates understanding of the National Bureau's treatment of trends. Instead of first fitting lines of secular trend to time series, and afterwards observing the number and duration of specific cycles, we first identify the specific cycles in a series, and afterwards measure the secular movements indicated by the cycles. We can follow this order because our working definition of business cycles tells us the basic characteristics of the specific cycles we wish to observe in many series —"and for that matter, the whole form of the supposed cyclical picture". These specific cycles are "recurrent sequences of expansion, recession, contraction, and revival, lasting more than one year but not more than ten or twelve years" (*Measuring Business Cycles*, p. 11). Their statistical and economic 'validity' is involved in that of the basic concept we are testing.

Having found such cycles in a series, we wish to distinguish them from the seasonal, irregular, and secular movements with which they are intertwined. As part of this effort, we convert the original data of a series during each of its specific cycles into percentages of their average value during that cycle. It is from these averages—cycle bases in our jargon—that we make 'measures of secular movements'. We usually find rather marked variability in the percentage change per month from one cycle base to the next. Passing from specific to reference cycles does not make these secular changes any more uniform. In short, our 'secular' measures give firm statistical support to the statement in the text that, if the secular trend implied by our procedure were represented by a continuous line, that line would be a flexible curve. Yet our implicit

[1] My italics. See Edwin Frickey, "The Problem of Secular Trend", *Review of Economic Statistics*, October 15, 1934, pp. 199–206.

trend of pig iron production is no more flexible than some half dozen of the trends listed by Frickey, if we may judge flexibility by the average duration of the 'cycles' found by Frickey and by us.

2 *Effects of Preliminary Trend Adjustments upon Cycle Bases*

As the preceding section suggests, the usual methods of 'eliminating' secular trends from time series accomplish only a partial separation of cyclical fluctuations from shifts in the levels upon which these fluctuations occur. Experience in analyzing data that others have adjusted for trend has taught us to expect cycle-by-cycle differences in both their specific-cycle and reference-cycle bases. For our purposes, such differences are remnants of trends, and to dispose of them we must go through all the operations we perform upon unadjusted data. Meanwhile, the preliminary trend adjustment wipes out part of the intracycle components that we wish to retain in our measures.

By way of illustration, consider Table 27, which presents the reference-cycle bases of pig iron production before and after adjustment for trend by Frederick R. Macaulay. The chart of Macaulay's trend on page 272 of *Measuring Business Cycles* shows that it fits the data well, at least up to the 1930's, and to give the trend every advantage I omit the last cycle from the averages.[2] The manner in which the adjusted bases run now somewhat above, then somewhat below 100 percent of the mathematical trend is further testimony to the goodness of Macaulay's fit for some 50 years; the average of all the bases up to 1927 is 99.5 percent. Nevertheless, not much less than half of the cycle-by-cycle shifts in reference-cycle bases remain in the trend-adjusted figures, and have to be taken out for our purposes by a second operation.

Removing secular trends in two stages instead of one would have substantial advantages if we had clearer ideas about what fitted

[2] In *Interest Rates, Bond Yields and Stock Prices in the United States since 1856*, Macaulay wrote: "The mathematical equation used to describe the trend of pig iron production was fitted to the data fifteen or sixteen years ago. Upon taking up the series for the purposes of this book, we decided to use the curve already fitted, not only because it had remained so astonishingly good but also because of the interest attaching to it as an illustration of how growth curves seem sometimes to be more than mere fits to existing data" (p. 209, note).

Whether an extrapolation of the trend would fit the data for the 1940's better than the data for the early 1930's is doubtful. One convenience of the National Bureau's method of treating trends is that its cycle bases are less likely to be altered radically with the passage of time than the equations of trend lines.

Table 27

CHANGES IN THE REFERENCE-CYCLE BASES OF PIG IRON PRODUCTION BEFORE AND AFTER ADJUSTMENT FOR SECULAR TREND

REFERENCE CYCLE (trough-to-trough)	REFERENCE-CYCLE BASE: Unadj. data (*thous. gross tons per day*)	REFERENCE-CYCLE BASE: Adj. data (*% of trend*)	% CHANGE FROM PRECEDING BASE: Unadj. data	% CHANGE FROM PRECEDING BASE: Adj. data
March 1879 – May 1885	9.77	106.0	...	...
May 1885 – April 1888	14.20	104.3	+45.3	–1.6
April 1888 – May 1891	19.71	113.7	+38.8	+9.0
May 1891 – June 1894	20.76	95.1	+5.3	–16.4
June 1894 – June 1897	23.16	83.7	+11.6	–12.0
June 1897 – Dec. 1900	33.84	96.8	+46.1	+15.7
Dec. 1900 – Aug. 1904	45.11	102.7	+33.3	+6.1
Aug. 1904 – June 1908	61.02	111.7	+35.2	+8.8
June 1908 – Jan. 1912	65.79	99.2	+7.8	–11.2
Jan. 1912 – Dec. 1914	76.33	100.2	+16.0	+1.0
Dec. 1914 – April 1919	98.99	114.0	+29.7	+13.8
April 1919 – Sept. 1921	77.24	82.4	–22.0	–27.7
Sept. 1921 – July 1924	85.81	86.8	+11.1	+5.3
July 1924 – Dec. 1927	99.58	96.5	+16.0	+11.2
Dec. 1927 – March 1933	73.60	69.1	–26.1	–28.4
Average, excluding last cycle				
Regarding signs			+21.1	+0.15
Disregarding signs			24.5	10.8

trend lines represent. For example, if we could believe that Macaulay's trend (or any of its numerous rivals) represents the net effects of the secular 'forces' impinging upon the iron-steel industry, or its 'secular growth', we might ascribe differences between the shifts in cycle bases computed from trend-adjusted data and the shifts in cycle bases computed from unadjusted data to nonsecular factors and start hunting for them. Perhaps an investigation along these lines would yield valuable results even now despite the vagueness of the secular concept, especially outside the realm where the biological notion of growth is appropriate. But we cannot take on this adventure as a side issue of cyclical studies. What we do in effect is to throw into the box labeled 'secular movements' all changes in cycle bases from one specific or reference cycle to the next. In a statistical sense, this practice gives an unwonted definiteness to the secular concept—a definiteness limited only by the fuzzy edges surrounding our concepts of seasonal variations, specific cycles, and business cycles. If it does nothing else, this practice at least discloses the shifting levels on which business cycles and their component specific cycles run their rounds.

3 *Effects of Preliminary Trend Adjustments upon Average Deviations from Reference-Cycle Standings*

As remarked above, converting the data of a time series into percentages of a secular trend tends to reduce the intracycle trend component that we wish to retain in our measures. The plainest sign of this change is usually that cyclical patterns are made more nearly horizontal. We have analyzed 9 series before and after the data have been adjusted for trend by competent statisticians, and this effect appears in 7 of them.[3]

An operation that reduces cycle-by-cycle differences in any type of movement covered by our measures should reduce the average deviations of reference-cycle standings, unless it has an offsetting effect upon the variability of other components. Seeing no reason to suppose that converting data into percentages of the ordinates of a trend line will systematically increase the variations we classify as cyclical or irregular, we may expect that a series will usually have smaller average deviations after than before it is

[3] The change in the tilt of the patterns made by adjusting for trend should appear numerically as an alteration in the difference between the reference-cycle standings at stages I and IX. In practice this difference may be seriously influenced by the relative depths of the initial and terminal troughs covered by a series. The differences before and after adjusting for trend are as follows in the 9 series:

	AV. REFERENCE-CYCLE STANDING				DIFFERENCE BETWEEN STANDINGS AT I AND IX	
	Stage I		*Stage IX*			
SERIES	U	A	U	A	U	A
Pig iron production	73.3	80.0	81.1	73.7	+7.8	−6.3
Electric power production	85.6	98.2	103.2	91.5	+17.6	−6.7
Department store sales, deflated	93.0	96.3	93.2	92.8	+0.2	−3.5
Clearings outside N.Y.C., deflated	88.1	95.6	100.6	92.9	+12.5	−2.7
Clearings, 7 cities, Frickey	83.5	91.6	101.1	92.2	+17.6	+0.6
AT&T index, 1932 revision	86.8	92.1	90.8	85.7	+4.0	−6.4
AT&T index, 1944 revision	82.8	88.3	90.8	85.2	+8.0	−3.1
Axe-Houghton index	83.2	87.7	95.7	89.1	+12.5	+1.4
R.R. bond yields	102.0	100.3	100.2	100.8	−1.8	+0.5

U = unadjusted data; A = trend-adjusted data.

For fuller titles of several series and the time coverage of all, see Table 28.

adjusted for trend. But this effect is not likely to be very pronounced. For, as the text shows, the trend component in average deviations is supposed to approximate zero at stage V, and to be not far removed from zero at stages IV and VI. If the estimates of Table 22 are not grievously wrong, the trend component is a minor part of the whole even in stages I and IX—when it is at a maximum. And if a trend adjustment merely reduces the intracycle trend component by a fraction, as our pig iron illustration suggests, this alteration in a minor factor may not stand out clearly in a small sample. To repeat, our sample includes only 9 series in both trend-adjusted and unadjusted form.

The results presented in Table 28 answer expectations tolerably well. If we include all 9 stages in all 9 series, eliminating the trend reduces the average deviations in 54 percent of the stages and increases them in 40 percent (in 6 percent there is no change). But if we take only the stages in which the trend component has most effect (I and II, VIII and IX) the reductions rise to 72 percent; and if we take only the stages where the trend component has least effect (III, IV, V, VI, and VII) the reductions fall to 40 percent.

How the effects of removing the trend differ from series to series can be seen most readily in the following ranking.

SERIES	CHANGE IN AVERAGE DEVIATIONS, ALL STAGES	NO. OF STAGES IN WHICH AVERAGE DEVIATIONS ARE Reduced	Unchanged	Raised
R.R. bond yields	−1.4	9	..	..
Clearings, 7 cities, Frickey	−0.6	8	1	..
Pig iron production	−0.4	6	..	3
Clearings outside N.Y.C., deflated	−0.3	6	1	2
Department store sales, deflated	+0.1	4	..	5
AT&T index, 1932 revision	+0.1	4	..	5
AT&T index, 1944 revision	+0.1	2	2	5
Axe-Houghton index	+0.2	3	1	5
Electric power production	+0.9	2	..	7
Total		44	5	32

Perhaps these differences have an economic meaning, but I think it more probable that they arise from differences in statistical procedure by the trend fitters and perhaps in the period covered.

4 *Intracycle Trend Component in Average Deviations from Average Reference-Cycle Standings*

Moore's method of approximating the intracycle trend component in average deviations from average reference-cycle patterns is ex-

Table 28

AVERAGE DEVIATIONS FROM AVERAGE REFERENCE-CYCLE PATTERNS OF NINE SERIES
BEFORE AND AFTER ADJUSTMENT FOR SECULAR TREND

SERIES[a]	PERIOD COVERED	FORM OF DATA[b]	AVERAGE DEVIATION[c] AT REFERENCE-CYCLE STAGES: I	II	III	IV	V	VI	VII	VIII	IX	*All Stages*[d]
1 Pig iron production	1879-1933	U	15.5	11.4	10.9	9.4	10.7	7.3	13.9	18.2	19.0	12.4
		A	15.4	11.0	9.9	9.6	12.1	7.5	13.1	16.6	16.7	12.0
2 Electric power production	1919-1933	U	5.6	4.4	2.4	2.7	2.3	0.9	4.7	9.0	8.7	4.2
		A	7.2	6.5	4.8	3.4	3.2	2.4	3.4	8.4	9.6	5.1
3 Department store sales, deflated	1921-1938	U	5.9	4.2	3.1	2.9	1.5	1.0	1.2	6.4	8.9	3.5
		A	6.2	3.8	3.8	3.2	2.6	1.8	0.6	5.6	7.9	3.6
4 Clearings outside N.Y.C., deflated	1879-1933	U	6.6	5.7	3.7	3.2	4.0	4.5	4.8	6.6	7.6	5.0
		A	6.2	5.7	3.8	3.8	3.1	3.5	4.3	6.3	7.5	4.7
5 Clearings in 7 cities outside N.Y.	1879-1914	U	6.9	5.4	4.2	3.2	2.8	4.1	6.5	6.0	7.6	4.9
		A	6.1	4.8	4.0	3.2	1.6	2.7	6.0	5.7	7.3	4.3
6 A T & T index of industrial activity, 1932 revision	1900-1933	U	9.1	7.0	6.0	5.3	7.0	5.4	4.8	9.6	9.5	6.8
		A	9.0	7.4	5.8	5.8	7.6	5.5	5.1	9.3	8.9	6.9
7 A T & T index of industrial activity, 1944 revision	1900-1938	U	9.3	6.0	5.2	6.0	7.5	6.0	4.5	9.0	9.1	6.7
		A	9.9	6.5	5.2	6.0	7.9	6.2	4.8	8.9	8.7	6.8

Table 28 (concl.)

8 Axe-Houghton index of trade & industrial activity	1879-1927	U	5.8	4.6	4.3	3.2	3.2	3.4	4.4	7.7	8.5	4.7
		A	5.9	4.6	4.4	3.7	3.3	3.0	5.6	7.6	8.1	4.9
9 R.R. bond yields	1858-1933	U	5.1	4.0	3.3	2.6	3.1	2.5	3.2	3.8	4.7	3.4
		A	2.5	1.9	1.6	1.6	2.1	1.8	2.6	2.2	2.1	2.0
No. of cases in which trend adjustment												
Reduces av. deviation			5	4	4	1	3	4	6	9	8	44
Does not change av. deviation			..	2	1	2	..	..	..	..	..	5
Raises av. deviation			4	3	4	6	6	5	3	0	1	32

[a] For fuller identification of series, see Appendix B.

[b] U=unadjusted data; A=trend-adjusted data.

[c] Expressed in percentages of reference-cycle bases.

[d] Average deviations of standings at stages I and IX receive a weight of one-half each.

Table 29

Effect of Shift from T-T to P-P Analysis upon Average Deviations from Average Reference-Cycle Patterns of Seven Series

Series[e]	Period covered	No. of cycles	Type of analysis[c]	Average deviation[f] at reference-cycle stages: I	II	III	IV	V	VI	VII	VIII
Clearings outside N.Y.C., deflated	1882-1929	14	T-T	5.4[a]	5.4	3.5	3.2	3.7	4.3	4.6	5.0
			P-P	2.6	2.6	2.2	3.3	5.4[b]	4.2	2.6	2.7
Pig iron production	1882-1929	14	T-T	15.2[a]	11.5	11.2	9.5	7.7	5.8	12.4	14.6
			P-P	11.1	8.2	6.4	11.6	11.9[b]	12.0	11.1	11.3
Freight car orders	1873-1929[d]	12	T-T	46.5[a]	21.3	25.5	28.5	39.7	22.5	21.0	24.7
			P-P	37.2	19.5	35.5	40.0	55.3[b]	17.7	25.9	23.4
Shares traded, N.Y. Stock Exchange	1882-1929	14	T-T	30.8[a]	26.6	12.5	13.9	19.1	9.5	14.4	14.4
			P-P	18.6	19.9	17.5	14.5	27.8[b]	25.0	15.4	14.9
R.R. stock prices	1860-1929	18	T-T	11.9[a]	11.2	7.6	7.3	4.7	5.1	7.5	8.5
			P-P	6.3	5.0	6.2	11.1	11.2[b]	9.5	8.5	8.6
Call money rates	1860-1929	18	T-T	24.4[a]	23.5	18.8	27.6	57.4	28.7	27.6	22.0
			P-P	18.0	21.2	16.9	25.0	59.8[b]	29.6	36.4	18.3
R.R. bond yields	1860-1929	18	T-T	5.0[a]	4.2	3.3	2.6	3.1	2.6	3.0	3.7
			P-P	2.6	2.0	3.5	4.6	4.9[b]	4.1	3.4	2.5
No. of series in which shift from T-T to P-P analysis											
Reduces av. deviations				7	7	4	1	0	2	2	5
Raises av. deviations				0	0	3	6	7	5	5	2

[a] Average of the two entries for troughs.
[b] Average of the two entries for peaks.
[c] T-T means trough-to-trough, P-P means peak-to-peak.
[d] 1913-23 omitted.
[e] The sources are the same as for Table 24.
[f] Expressed in percentages of reference-cycle bases.

plained and the results of applying it to seven test series are summarized in the body of the chapter (Sec. IIC). Table 29 gives full details of the test and needs no further explanation.

5 Effects of Reference-Cycle Bases upon Average Standings and Deviations at Stages I and IX

The relations between average standings and deviations from them at stages I and IX are peculiar because stage IX of one reference cycle is also stage I of its successor. To illustrate these relations I have chosen a series in which sharp changes in reference-cycle bases are not uncommon: namely, the number of shares sold monthly on the New York Stock Exchange. Column (1) of Table 30 shows our reference dates for five business-cycle troughs, and column (2) the average sales during the three months centered on each of these dates. The three months centered on April 1888

Table 30

ILLUSTRATIONS OF RELATIONS BETWEEN STANDINGS AT STAGES IX AND I OF ADJACENT REFERENCE CYCLES IN SHARES TRADED ON NEW YORK STOCK EXCHANGE

REFERENCE TROUGH	MILLION SHARES TRADED PER MONTH DURING Trough Stage[a]	Cycle in Which Trough Is *Stage IX*	*Stage I*	REFERENCE-CYCLE STANDING AT Stage IX	Stage I	DEVIATION FROM AV. STANDING AT Stage IX	Stage I
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
April 1888	6.65	7.55	5.82	88.1	114.2	−7.5	+30.9
June 1897	6.29	4.73	11.23	133.0	56.0	+37.4	−27.3
June 1908	16.15	20.05	14.53	80.6	111.2	−15.0	+27.9
Jan. 1912	8.98	14.53	8.33	61.8	107.8	−33.8	+24.5
April 1919	24.94	15.25	21.28	163.6	117.2	+68.0	+33.9
Average, 16 cycles, 1879–1938				95.6	83.3	37.0	23.6

[a] Includes three months centered on reference date.

constitute stage IX of the reference cycle dated May 1885–April 1888 and also stage I of the cycle dated April 1888–May 1891. In the earlier cycle, average monthly sales were 7.55 million shares; in the later cycle 5.82 million. Average sales during the three months centered on April 1888 were 6.65 million per month. This figure is 88.1 percent of 7.55 million (the earlier cycle base) and 114.2 percent of 5.82 million (the later base). The analysis of shares sold covers 16 reference cycles from 1879 to 1938. On the average, reference-cycle standings were 95.6 percent of the cycle bases at stage IX and 83.3 percent at stage I. Hence the deviation of the standing at the April 1888 trough considered as stage IX of the 1885–88 cycle was $88.1 - 95.6 = -7.5$; but the deviation at this trough considered as stage I of the 1888–91 cycle was $114.2 - 83.3 = +30.9$. The entries at other troughs are to be read similarly.

The point of present moment is that our use of the same seasonally-adjusted data in computing reference-cycle standings at stages IX and I of adjacent cycles, though establishing a relationship between standings and between deviations at these stages, does not preclude the occurrence of considerable differences when the cycle bases fluctuate in the irregular fashion characteristic of pig iron production, shares sold, and, I may add, many other series.

Technical Note III

Average Deviations of Measures Based upon Differences between Reference-Cycle Standings

The relation suggested in the text, and explored in Table 24, between the average deviation from the mean reference-cycle amplitude of a series and the average deviations from its average standings at the peak and trough stages indicated by its timing variety is predicated upon two basic assumptions:

a) that variations in peak and trough standings are uncorrelated in combinations of successive peaks and troughs

b) that the average deviation tends to be a constant multiple of the standard deviation calculated from the same data, the multiple being about 4/5. Call this constant $1/c$.

Now, let us denote the *population* standard deviation of a chance variable X by σ_x, and the *sample* standard deviation based on any finite sample of observations on X by s_x. Denote the corresponding sample average deviation by $(a.d.)_x$; and designate peak and trough standings by p and t, respectively. Then, given assumption (a), we know that

$$\sigma^2_{p-t} = \sigma^2_p + \sigma^2_t$$

Assumption (b) implies that the following relation *tends* to be fulfilled

$$c^2(a.d.)^2_{p-t} = c^2(a.d.)^2_p + c^2(a.d.)^2_t$$

Factoring out c^2 and taking square roots of both sides, we get the formula

$$(a.d.)_{p-t} = \sqrt{(a.d.)_p^2 + (a.d.)_t^2}$$

A similar argument leads to the following formula for the average deviation of full-cycle amplitudes

$$(a.d.)_{2p-t_1-t_2} = \sqrt{4(a.d.)_p^2 + (a.d.)_{t_1}^2 + (a.d.)_{t_2}^2}$$

where t_1 denotes initial trough and t_2 denotes terminal trough. Here there is the additional assumption that the standings in t_1 and t_2 are uncorrelated.

It should be noted that the truth of assumption (a) does not guarantee even that

$$s_{p-t} = \sqrt{s_p^2 + s_t^2}$$

for *finite* samples. Thus we must be prepared to see the formula

$$(a.d.)_{p-t} = \sqrt{(a.d.)_p^2 + (a.d.)_t^2}$$

hold only approximately even when assumptions (a) and (b) are fulfilled.

A test of the reasonableness of assumption (a) has been made by correlating the standings at turning points in successive cycles of the seven test series employed in Table 24. Few of the correlations

COEFFICIENTS OF CORRELATION BETWEEN REFERENCE-CYCLE STANDINGS AT CHARACTERISTIC TURNING POINTS WITHIN REFERENCE CYCLES

SERIES	NO. OF CYCLES	CORRELATION BETWEEN Initial trough and peak	Terminal trough and peak	Initial and terminal trough
Deflated clearings	16	+.25	−.27	−.72
Pig iron production	16	+.03	−.57	−.57
Freight car orders	17	+.04	−.17	−.43
R.R. stock prices	19	+.15	−.72	−.63
Shares traded	16	+.47	−.37	−.53
Call money rates	20	+.17	−.05	+.06
R.R. bond yields	19	−.82	+.70	−.68

are large, but they reveal a tendency to be positive for the turning points of expansions, and negative for the turning points of contractions and full cycles.

Of course, assumption (b) is strictly true only for a limited class of distributions, the chief of which is the Gaussian distribution; but it is well established empirically also for symmetrical distributions that do not depart widely from the Gaussian form. That assumption (b) does not hold precisely in our seven series is clear from the fact that we do not always get the discrepancies between directly and indirectly computed average deviations of amplitudes which we should expect from the above correlations.

Some idea of the variation in the relative size of average and standard deviations can be had from the accompanying table, which gives ratios of average to standard deviations for each class of turning points in our seven test series. Considering the small number of cyclical observations (16 to 20) in each series, the variation in these ratios does not seem excessive.

RATIOS OF AVERAGE DEVIATIONS OF TURNING-POINT STANDINGS TO CORRESPONDING STANDARD DEVIATIONS

SERIES	RATIOS RELATING TO Initial troughs	Peaks	Terminal troughs
Deflated clearings	.76	.87	.75
Pig iron production	.51	.74	.85
Freight car orders	.83	.80	.79
R.R. stock prices	.87	.65	.62
Shares traded	.81	.84	.71
Call money rates	.75	.60	.84
R.R. bond yields	.69	.73	.82

There is thus a case for working with our assumptions when the problem is to make rough estimates of the average deviation of amplitudes. However, before taking any of these results too seriously, it is necessary to investigate how typical they are of other series and to track down the correlations found between standings at successive turning points. Preliminary investigation indicates that these correlations cannot be dismissed as technical byproducts of our method of manipulating time series.

I am indebted to Millard Hastay for giving this argument its mathematical dress.