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Note 3

Estimation of the labor input increment and the productivity increment

The problem is that of separating an increment to output into a portion associated with an increase in labor input, and a portion associated with a productivity gain. (There are three other cases, representing other combinations of plus and minus changes in the labor input and productivity factors, but the principle involved is the same.) Since there is interaction between the two factors, which are related in a multiplicative way, there can be no definitive solution, but useful approximations to the two components of the increment may be obtained.

In brief, the procedure is as follows:

a) Estimate the increase that would have occurred in total output as a result of the given increase in labor input, but with no change in productivity. This gives what we may call Component *A* of the increment to gross national product.

b) Estimate the increase that would have occurred in total output as a result of the given increase in productivity, but with no change in labor input. This gives what we may call Component *B*.

c) Estimate the interaction component, the portion of the gain in gross national product that represents the combined result of an increment to labor input and an increment to manhour output. This gives what we may call Component *C*.

It is justifiable to assign Component *A* to the labor input factor, Component *B* to the productivity factor. *A* will vary directly with labor input, *B* with productivity. *C*, however, will vary with both factors. *C* is therefore arbitrarily divided, half being assigned to the labor input increment, half to the productivity increment.

This method may be illustrated with reference to decade aggregates for 1901-10 and 1911-20.

Decade	*Gross national product* (billions of 1929 dollars)	*Labor input in total manhours* (relative)	*Output per manhour* (relative)
1901-10	455	100.0	100.0
1911-20	603	111.4	118.9

The increment to gross national product (148 billions of dollars) is the sum of:

Component A (455 × .114):	52 billions
Component B (455 × .189):	86 billions
Component C (148 — 52 — 86):	10 billions

The final estimate of the labor input increment for 1911-20 is 57 billions ($A + \frac{C}{2}$, or 52 + 5); the final estimate of the productivity increment is 91 billions ($B + \frac{C}{2}$, or 86 + 5).

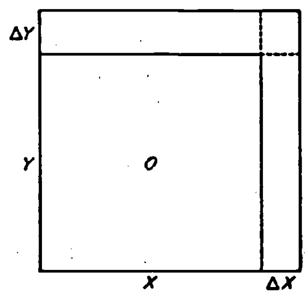
As I have suggested at an earlier point, neither of these increments is to be regarded as the specific product or the marginal product of any of the conventional factors of production. Changes in the productivity ratio $\frac{Q}{MH}$, from which estimates of the productivity increment are derived, are the net result of a complex of movements, all involving relations between output and factor input that are defined conceptually by the traditional laws of return (operating over time). Thus if a change in labor input alters the ratio of effort input to natural resources or to instruments used and, through the play of diminishing returns, reduces average output per manhour of work done, these related changes will be reflected in both the labor input increment and the productivity increment (how their results will be divided between the two increments will depend on the relative magnitudes of the changes in labor input and in average return per manhour of labor input).

The method described is the mathematical equivalent of the following: The labor input component of the increment to product between two periods is the increase in output that would have resulted from the given increase in labor input, had the employed labor force been working at a productivity level equal to the average of manhour output in the two periods compared.

The productivity component of the increment to product is the increase in output that would have resulted from the given gain in output per manhour, had labor input been equal to its average during the two periods compared.

It may be demonstrated that the sum of the two components, as thus established, is equal to the total increment to product.

Let X = Manhours of work input, period 1
$X + \Delta X$ = Manhours of work input, period 2
Y = Output per manhour, period 1
$Y + \Delta Y$ = Output per manhour, period 2
$O = XY$ = Total output, period 1
$O + \Delta O = (X + \Delta X)\ (Y + \Delta Y)$
= Total output, period 2



If we assume that X and Y change linearly between periods 1 and 2, then:

$$\bar{X} = X + \frac{\Delta X}{2}$$

$$\bar{Y} = Y + \frac{\Delta Y}{2}$$

The increment associated with the change in X is given by $\Delta X \left(Y + \frac{\Delta Y}{2}\right)$.

The increment associated with a change in Y is given by $\Delta Y \left(X + \frac{\Delta X}{2}\right)$.

As the sum of these two increments we have:

$$\Delta X \left(Y + \frac{\Delta Y}{2}\right) + \Delta Y \left(X + \frac{\Delta X}{2}\right) = Y\Delta X + X\Delta Y + \Delta X\Delta Y = \Delta O$$

(I am indebted to my colleague Henry Scheffé for this mode of viewing the decomposition of the increment to product.)

Beyond the formal equality thus established, the procedure has logical justification. In the limiting case in which there is no change in productivity, the entire increment to product is assigned to the change in labor input; at the other limit in which there is no change in labor input, the entire increment to product is assigned to the change in productivity. In cases falling between these limits, as we have seen, half of the small rectangle corresponding to the product of ΔX and ΔY (this corresponds to C of the procedure first noted) is assigned to each of the two factors.

The actual values, by decades, of the several components of the increments to product are given below. Component A, it will be noted, is equivalent to $Y\Delta X$, Component B to $X\Delta Y$, and Component C to $\Delta X \Delta Y$.

Decade	*Increment to gross national product*	*Component A*	*Component B*	*Joint component C*
	(billions of 1929 dollars)			
1901-10 (change from 1891-1900)	+161	+77	+67	+17
1911-20 (change from 1901-10)	+148	+52	+86	+10
1921-30 (change from 1911-20)	+235	+20	+209	+6
1931-40 (change from 1921-30)	+5	—129	+158	—24
1941-50 (change from 1931-40)	+650	+396	+173	+81

In deriving final estimates C was divided equally, for each decade, between the labor input component and the productivity component. In presenting these estimates it is recognized, of course, that labor input and productivity have changed together, and have interacted as they changed. Neither would have had the value actually recorded for a given decade had the other not been present as an active factor.

If the changes of the five decades are aggregated, we obtain from the above table the following summary of shifts between the decades 1891-1900 and 1941-1950:

Total increment to gross national product 1891-1900 to 1941-1950	*Component* A	*Component* B	*Joint component* C
1,199	416	693	90

Splitting the joint component C in half, and assigning half to each of the two factors, we have for the half century of growth a labor input increment of 461 billions (of 1929 dollars), a productivity increment of 738 billions. These are, respectively, 38.4 per cent and 61.6 per cent of the total increment of 1,199 billions.

Since the estimated magnitudes of the several components of an increment to national product are affected by the time unit employed, it is of interest to compare the preceding division of the half-century increment to national product with the division we should obtain by treating the half-century increment as a single lump. Relevant measures are given below:

Decade	*Gross national product* (billions of 1929 dollars)	*Labor input in manhours* (relative)	*Output per manhour* (relative)
1891-1900	294	100.0	100.0
1941-1950	1,493	180.5	281.3

Applying to this half-century increment the method just described, we have:

Increment to gross national product	*Component A*	*Component B*	*Joint component C*
1,199	237	533	429

If we split the joint component, assigning half to each of the two factors, we have for the half century a labor input increment of 451.5 billions, a productivity increment of 747.5 billions. These are, respectively, 37.7 per cent and 62.3 per cent of the total increment of 1,199 billions.

Chief interest attaches to the difference between the two values of the joint component, 90 when we move by decade steps, 429 when we move by a single half-century jump. In the latter case a much larger quantity is allocated on the somewhat arbitrary half-and-half division. Yet this division gives final values for the two components that are very close to those obtained from the presumably more accurate decade intervals.

The close agreement is in part fortuitous. Results obtained in the one case are not a check upon the other. The method of division here employed, applied to a single time period and then to subdivisions of that time period, would give identical results only where:

a) the relations between the two variables (labor input and output per manhour) are linear;

b) the separate subperiod movements mark out equal areas above and below the straight line defining the net movements of the two variables between terminal dates of the whole time interval.

(The line defining the relation between labor input and output per manhour for each subperiod — here a decade — connects the appropriate corners of a rectangle similar to that represented by $\Delta X \Delta Y$ in the diagram on page 33. The limiting case, for these conditions, is that in which the movements between subperiods and between terminal dates of the whole time interval are defined by the same straight line.) In the present instance the related movements of labor input and manhour output during the last two decades of the half century departed sharply from the direction of changes during the first three decades. For the half century as a whole there was virtual equality of the deviations above and below the line marking the net fifty-year movements of the two variables; close agreement of the derived measures results. Where there is failure to agree, estimates based upon the shorter intervals would be preferred.