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CHAPTER 7

Effects of Trend Adjustments on Cyclical Measures

As SHOWN in an earlier volume, business cycles are a development of late modern times. They emerged with the intensification of technical changes, the vast expansion of commercial and industrial activity, and the widening organization of economic life on the basis of making and spending money incomes.[1] Cyclical fluctuations are so closely interwoven with these secular changes in economic life that important clues to the understanding of the former may be lost by mechanically eliminating the latter. It is primarily for this reason that we take as our basic unit of analysis a business cycle that includes the portion of secular trend falling within its boundaries. In this way we prepare materials that we consider more useful for the explanation of business cycles than similar materials based upon trend-adjusted data.

It is desirable, however, to ascertain as definitely as possible how our various measures of cyclical behavior would be affected by the elimination of intra-cycle trends. The method is obvious. We must compare in detail the results obtained by applying our technique to sample series before and after their secular trends are eliminated. That step will aid in interpreting our results at large. In particular it will aid in comparing the results we get from series that come to us with trends eliminated by their compilers with those we get from the far more numerous series not so adjusted.[2] And it will clarify our reasons for retaining the portion of secular trend that falls within the limits of single cycles.

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[1] See Mitchell, *Business Cycles: The Problem and Its Setting,* Ch. II, Sec. I.

[2] The adjustment for secular trend is sometimes explicit, sometimes implicit. An example of the latter is a long series showing the percentage of trade-union workers or insured workers out of employment.

I Materials Used in the Tests

Our tests cover the unadjusted and trend-adjusted forms of six monthly American series. To economize effort we have utilized series whose secular trend has already been calculated by other investigators. Three of the series analyzed in the preceding chapter—pig iron production, 'deflated' bank clearings outside New York, and railroad bond yields—are taken up also in this chapter. For these series we have used materials worked up by F. R. Macaulay for the National Bureau.[3] Our fourth series consists of the index of business conditions prepared by the American Telephone and Telegraph Company, both with and without trend adjustment, for the period since 1899.[4] The fifth consists of Frickey's composite of bank clearings in seven important cities exclusive of New York, covering 1875–1914, before and after trend adjustment.[5] The sixth series is the production of electric power since 1919, from which we have ourselves eliminated the secular trend.[6]

In each series the trend-adjusted data are relatives of the 'unadjusted' data to the corresponding ordinates of secular trend; that is, each monthly figure is expressed as a percentage of the corresponding trend value. But we also investigate how the results would be affected if the 'adjusted' data were taken in the form of absolute deviations from trend. Both the 'unadjusted' and 'adjusted' data are corrected for seasonal variations, except railroad bond yields, where we find no seasonal movement. The adjusted data correspond to the unadjusted data in every respect except that they are freed from their secular trends. As an illustration, both forms of the data for pig iron production are presented in Chart 35.

Our sample of series represents a fair variety of secular movements. Five series have rising trends (Table 84). The sixth, railroad bond yields, has an oscillatory trend; in this series we make separate comparisons for

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[3] For the sources of the unadjusted forms of these series, see Ch. 6, note 7. The trend-adjusted figures are not given in Macaulay's *Interest Rates, Bond Yields and Stock Prices.* But they are shown graphically (with seasonal unremoved in pig iron production) on pp. 223-6 of this source; see also Macaulay's Appendix, Table 11.

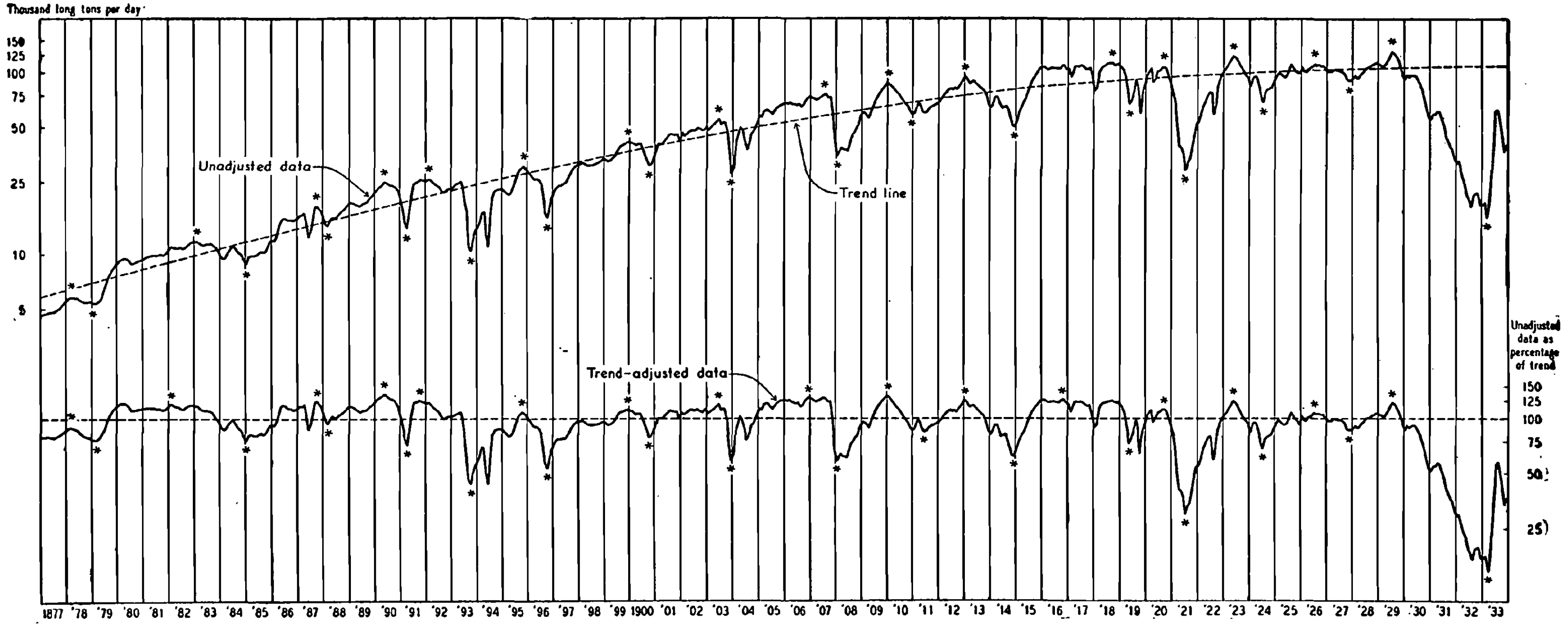
[4] Through July 1932, both adjusted and unadjusted figures come from *Index of Industrial Activity in the United States* (a confidential report of the American Telephone and Telegraph Company, Oct. 20, 1932). Since Aug. 1932, the adjusted figures come from the company's *Summary of Business Conditions in the United States;* the unadjusted figures and the equation of trend applied to them, from the Chief Statistician's Division of the company. These series have recently been revised extensively, according to later confidential releases by the Chief Statistician's Division. Since the revisions do not affect the methodological issues treated in this chapter, we have not recomputed our results.

[5] Edwin Frickey, Bank Clearings outside New York City, 1875–1914, *Review of Economic Statistics,* Oct. 1925, pp. 260-1. We are indebted to Frickey for sending us the seasonal indexes applied to his composite over the period 1875–1902.

[6] The figures for 1919 come from a paper on The Nature of Cyclical Fluctuations in Electric Power Production Data (University of Illinois, Bureau of Business Research, 1927, *Bulletin 16*). After 1919, *Survey of Current Business,* Nov. 1927, p. 26; *ibid.,* 1932 Supplement, pp. 142-3, and 1936 Supplement, p. 85. Slight revisions of these figures back to 1920 have been made by the Federal Power Commission; our computations are based on the unrevised figures. The trend is a straight line fitted by 'least squares' to annual averages of seasonally corrected monthly data in 1919–32.

CHART 35

Pig Iron Production, United States, 1877–1933
Unadjusted and Trend-adjusted



Corrected for seasonal variations. Asterisks identify peaks and troughs of specific cycles. For sources of data, see Sec. I.

Logarithmic vertical scales

TABLE 84

Change per Decade of Monthly Ordinates of Secular Trend

Six American Series

Series and measure	Unit	1860–1870	1870–1880	1880–1890	1890–1900	1900–1910	1910–1920	1920–1930
RAILROAD BOND YIELDS								
Absolute change	Per cent	0.2	−1.9	−0.8	−0.5	0.6	1.2	−0.6
Percentage change	....	*3*	*−30*	*−18*	*−14*	*20*	*31*	*−13*
DEFLATED CLEARINGS								
Absolute change	Mill. 1913 $[a]	...	...	29.5	44.3	66.1	99.4	148.1
Percentage change	....	...	...	*66*	*60*	*56*	*54*	*52*
FRICKEY'S CLEARINGS								
Absolute change	Mill. $	...	...	303	668	1,055	...	...
Percentage change	....	...	...	*61*	*83*	*72*	...	...
A.T.&T. INDEX								
Absolute change	Av. 1899 = 100	...	...	...	...	60.4	60.4	60.4
Percentage change	....	...	...	...	...	*59*	*37*	*27*
PIG IRON PRODUCTION								
Absolute change	Th. long tons[a]	...	...	10.4	19.8	29.0	27.0	13.0
Percentage change	....	...	...	*132*	*108*	*76*	*40*	*14*
ELECTRICITY OUTPUT								
Absolute change	Bill. kil.-hrs.	...	...	...	...	...	...	4.0
Percentage change	....	...	...	...	...	...	...	*113*

This table shows the decennial absolute and percentage increments of *monthly* ordinates of secular trend centered at June 30 of the decennial dates. The increments of deflated clearings from 1910 to 1930 are computed from 'deflated bank debits outside New York' adjusted to the level of 'deflated clearings outside New York' in Jan. 1919.

[a] Per day.

segments of rising and declining trends as well as for the full period covered by the data. The trends vary considerably in steepness and curvature. The percentage rate of growth in pig iron production during the 1880's is somewhat larger than that of electric power production during the 1920's, and is several times its own advance during the 1920's.[7] What is a declining rate of growth in percentage units is a uniform rate of growth in the units of the A.T.&T. business index, an increasing rate of growth in 'deflated' dollars of bank clearings, and first a rising and then a falling rate of growth in tons of iron produced. Regrettably, our sample is less satisfactory in representing the cyclical movements encountered in experience than in representing the secular movements. All the series we treat bear a positive and rather close relation to business cycles. However, in the course of analysis we shall attempt to take account of these deficiencies of the sample.

II The Number of Specific Cycles

It seems reasonable to expect that the elimination of secular trends will tend to increase the number of specific cycles as we count them; for any sharp retardation of growth in the unadjusted data will tend to be con-

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[7] Table 84 records the decennial rates of change shown by the lines of secular trend. These may differ appreciably from rates of change computed from the original data for separate decades.

verted into an actual decline once the secular trend is removed. So also will any sharp retardation of decline tend to be converted into a rise. But the removal of trends will not add to the number of specific cycles if the amplitude of the 'cyclical component' of a time series is large compared with the amplitude of the 'secular component'. Indeed, under certain circumstances, a specific cycle will be lost when the secular trend is removed. Thus, if the trend is upward, a cycle will disappear in the adjusted data whenever the rate of rise during a cyclical expansion of the unadjusted data is no greater than the rate of rise of the fitted trend line during the corresponding period.[8] Similarly, if the trend is downward, a specific cycle will disappear whenever the rate of decline of a cyclical contraction in the unadjusted data is no greater than the rate of decline of the trend.

In our present sample the elimination of trends has slight influence on the number of specific cycles (Table 85). In pig iron production the list of specific cycles is the same in the adjusted as in the unadjusted data. The like is true of the A.T.&T. index and the short series on electric power production. In both clearings series the specific cycles in the adjusted and unadjusted data agree in number, yet fail to correspond throughout. On the one hand, the expansion in the unadjusted data during 1878 to 1881 is matched by three phases in the adjusted data; on the other, the cyclical rise of 1882–83 in the unadjusted data is wiped out in the adjusted data. In bond yields, our longest series, the differences between the two lists of specific cycles are more numerous. Here we find two extra cyclical movements in the adjusted data—one during the expansion

TABLE 85

List of Specific Cycles in Unadjusted and Trend-adjusted Data

Six American Series

Series	Direction of trend	All cycles			Corresponding cycles		Noncorresponding cycles		
		Period covered	Number in		Period covered	No. in unadj. & adj. data	Period covered	Number in	
			Unadj. data	Adj. data				Unadj. data	Adj. data
Deflated clearings	Upward	1878–1933	15	15	1884–1933	13	1878–1884	2	2
Frickey's clearings	Upward	1878–1914	11	11	1884–1914	9	1878–1884	2	2
A.T.&T. index	Upward	1900–1933	9	9	1900–1933	9	....	..	..
Pig iron production	Upward	1879–1933	15	15	1879–1933	15	....	..	..
Electricity output	Upward	1921–1933	2	2	1921–1933	2	....	..	..
Railroad bond yields	Oscillatory	1860–1931	20	21	[a]	16	[b]	4	5
Railroad bond yields	Downward	1868–1899	8	8	1868–1899	8	....	..	..
Railroad bond yields	Upward	1899–1918	5	7	[c]	3	[d]	2	4

[a] 1860–64, 1868–99, 1905–09, 1914–31.
[b] 1864–68, 1899–1905, 1909–14.
[c] 1905–09, 1914–18.
[d] 1899–1905, 1909–14.

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[8] The two rates of rise may be expressed in units of the original data or their logarithms. It is mathematically possible for a cycle to disappear when the rates of rise are expressed in units of the original data but not when they are expressed in logarithms, and vice versa.

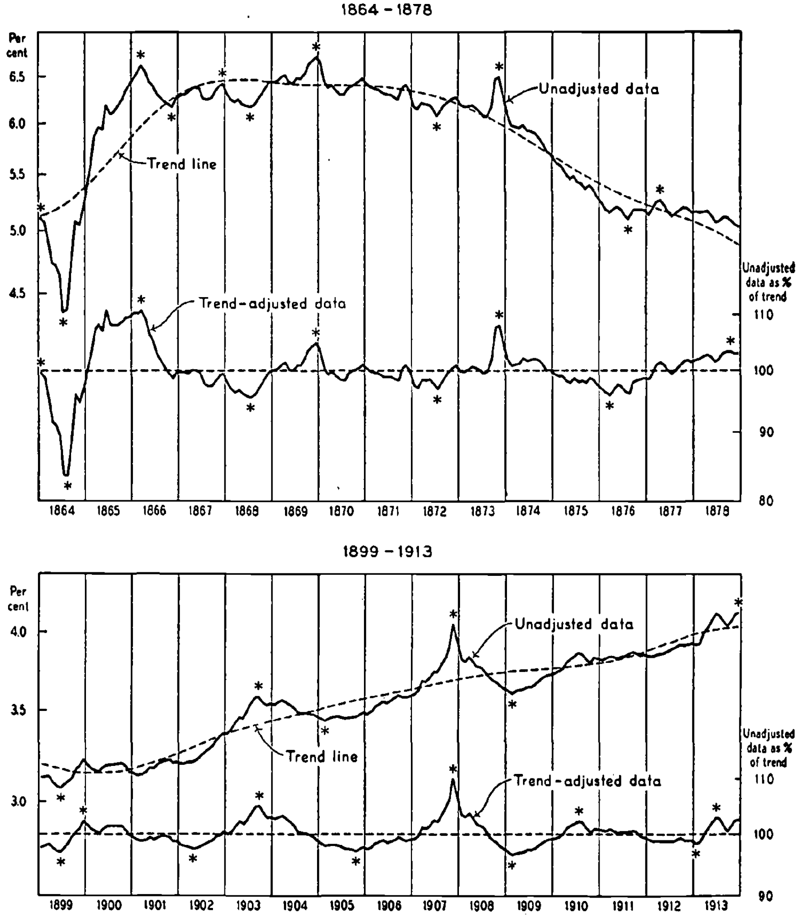
of 1899–1903 and another during the expansion of 1909–13; but these additions are partly offset by the virtual disappearance of the rise in 1866–67 which we treat as a specific-cycle expansion in the unadjusted data. The manner in which specific cycles are gained or lost in this series is displayed by Chart 36. In summary, we recognize 72 specific cycles in the unadjusted data of our six series and 73 cycles in the adjusted data; the net difference between the two totals is accounted for by bond yields.

Of course, a judgment factor enters into the present comparisons. The representation of the secular trend of a series by a mathematical curve or

CHART 36

Railroad Bond Yields, United States

Unadjusted and Trend-adjusted



Corrected for seasonal variations. Asterisks identify peaks and troughs of specific cycles. For sources of data, see Sec. I.

Logarithmic vertical scales

moving average involves judgment. So too does the decision to eliminate trends by division or by subtraction. Finally, the boundary line between a movement that is cyclical under our rules and one that is not becomes uncomfortably vague at times. But plausible variations in the form of trend lines are not likely to influence appreciably our count of specific cycles, since we do not limit the count to full swings about the trend line. If we consider a fluctuation large enough to qualify as a specific cycle, we recognize it as such irrespective whether it falls on one or both sides of the trend line. The removal of trend by division instead of subtraction rarely changes the cyclical turns, and is hardly likely to change the count of cycles of the particular duration in which our interest centers. True, our method of marking off specific cycles leaves margins of uncertainty. But the cyclical movements in our test series are for the most part very clearly defined, and our decisions have been checked with care. We therefore trust the general character of the results for the series covered by the present tests. And since these series seem fairly representative of the bulk of economic time series,[9] we feel justified in concluding that the removal of secular trends will rarely increase much the number of specific cycles as we count them.

This judgment is limited to monthly, or at most to quarterly data; it definitely excludes annual data. Trend adjustments have a substantial influence on the number of specific cycles in annual data, but we postpone analysis of this effect until Section VII.

III Timing and Duration of Specific Cycles

The effect of removing trends upon the turning points of specific cycles depends on the direction of the 'trend', its slope, and the slope of the 'cyclical component'. Consider a theoretical time series built up by adding a 'cyclical component' to a linear 'trend', the former having sufficient amplitude to stamp its movements on the composite series. If the cyclical component is triangular, both its rising and declining phases being linear, the cyclical turns of the composite should coincide with the turns of the cyclical component; in other words, removal of the 'true' trend will leave the cyclical timing of trend-adjusted data the same as that of unadjusted data. On the other hand, if the cyclical component is gently rounded at tops and bottoms, as for example in a sine curve, the turns of the composite will tend to deviate from those of the cyclical component. If the trend is rising the peaks of the composite will tend to come later and the troughs earlier than corresponding turns of the cyclical component; if the trend is declining, peaks will tend to come earlier and troughs later.

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[9] Except in length. The addition of a cycle or two is more disturbing in short than in long series; see bond yields in Table 85.

These relations are based on special assumptions, but they may be generalized and put in a form suited to empirical time series. Assume that a rising trend is removed from a time series. This operation will either leave the dating of cyclical peaks unchanged or make the peaks come earlier than in unadjusted data. It will do the former if the rate of rise towards the close of a cyclical expansion in the unadjusted data is greater than the rate of rise of the secular trend during the same period; it will do the latter if the first rate is smaller than the second. In other words, whether the cyclical peak is pushed back or left unmolested by eliminating the trend depends on how steep is the rise in the closing stages of the cyclical expansion compared with the rise of the secular trend. Similarly, if the rate of rise in the early stages of a cyclical expansion in the unadjusted data is smaller than the rise of the secular trend, the removal of trend will make the cyclical trough come later; while if the first rate is larger than the second the dating will be unaffected. The effects of a declining trend on cyclical timing are opposite to those of a rising trend; that is, the removal of a declining trend tends to make cyclical peaks come later and troughs earlier than in unadjusted data, but in order that this tendency become effective the rate of fall towards the beginning and close of a cyclical contraction in the unadjusted data must be smaller than the rate of decline of the secular trend during the corresponding periods.

Table 86 shows the effects of removing trends in practice. The most interesting result is that slightly more than two-thirds of the cyclical turns

TABLE 86

Size and Frequency of Leads or Lags of Specific-cycle Turns in Trend-adjusted Data at Corresponding Turns of Unadjusted Data

Five American Series with Upward Trends

Lead (–) or lag (+) of trend-adj. at turn of unadj. data (mos.)	Number of leads or lags in											
	Deflated clearings		Frickey's clearings		A.T.&T. index		Pig iron production		Electricity output		Five series	
	P	T	P	T	P	T	P	T	P	T	P	T
Below –12	1	..	..	..	..	..	1	..	..	..	2	..
–7 to –12	..	..	..	..	1	..	2	..	1	..	4	..
–1 to –6	5	..	1	..	2	..	2	..	1	..	11	..
0	8	10	9[a]	6	7	6	11	14	..	3	35	39
+1 to +6	..	2	..	2	..	4	..	2	..	..	..	10
+7 to +12	..	3	..	2	..	..	..	..	..	..	..	5
Over +12	..	..	..	1	..	..	..	..	..	..	..	1
Total	14	15	10	11	10	10	16	16	2	3	52	55
Leads	6	..	1	..	3	..	5	..	2	..	17	..
Coincidences	8	10	9[a]	6	7	6	11	14	..	3	35	39
Lags	..	5	..	5	..	4	..	2	..	..	..	16
Total	14	15	10	11	10	10	16	16	2	3	52	55

P stands for peak, T for trough. See Table 87 for the periods covered.

[a] A one-month computed lag included here, because it arises from a slight error in the published figures from which we dated the specific cycles of the adjusted data; that is, we dated a peak in Sept. 1881, whereas the correct month is August.

in our five series with upward trends are unaffected by the trend adjustment.[10] The reason must be that the march of secular trends is usually less vigorous than that of the unadjusted data during early and late stages of cyclical expansions.[11] In those instances in which the elimination of trends alters the dating of cyclical turns, the direction of the shifts conforms to expectations. Most shifts are small, but some exceed six months and a few exceed a whole year. It is plain, therefore, that if cyclical turns were dated from trend-adjusted data, the apparent succession of revivals or recessions of different economic activities would at times be seriously modified.[12] Of course, Table 86 merely indicates the general character and size of the shifts produced by trend adjustments. The specific results are bound to vary with the circumstances surrounding each series, the type of trend used, the manner in which the trend is fitted to the data, and the manner in which the trend is removed. In the present experiments the trends have been removed by division; that is, the unadjusted figures have been expressed as percentages of the corresponding ordinates of secular trend. If our rising trends had been removed by subtraction, the tendency of adjusted data to lag at the troughs of unadjusted data would be slightly stronger, while the tendency to lead at the peaks would be slightly weaker.[13] On the other hand, if our rising trend lines were replaced by others rising still faster, both tendencies would be more prominent than in the present measurements.

The average effect of trend adjustments is larger in our sample at peaks than at troughs. The effect varies appreciably from series to series, and without close regard to the steepness of the secular trend. The average shift is less than two months in 9 out of 16 comparisons (Table 87); it exceeds four months at the cyclical peaks in electric power production, a short series rising with exceptional swiftness. Our one sample of a declining trend, railroad bond yields during 1868–99, shows a shift in average timing at peaks and troughs opposite to that of the several series with

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[10] Bond yields show a similar result. Of the 41 corresponding turns in this series, 28 coincide in the unadjusted and adjusted data. In all six series, 102 out of 148 corresponding turns coincide (49 out of 73 peaks and 53 out of 75 troughs).

[11] Or what comes to the same thing, the rate of fall in the late and early stages of cyclical contractions in the adjusted data is, usually, absolutely larger than the rate of rise of the secular trend during corresponding periods.

[12] If trends of different series are not only converted into horizontal lines, but the fluctuations about the trends are expressed in standard deviation units, as is still common practice, about all that can be learned is the relation in time of the cyclical movements. And even this limited knowledge may be false since the timing relations of experience may be distorted.

[13] The absolute deviations are, of course, equivalent to percentage deviations multiplied by corresponding ordinates of trend. Upon comparing the adjusted data in the form of absolute deviations from trend with the percentage deviations from trend, we found their troughs coinciding in every instance, but three peaks in the percentage deviations preceded corresponding peaks in the absolute deviations. In two of these three instances the peaks in the absolute deviations coincided with the peaks in the unadjusted data.

rising trends. The rough coincidence of average timing in the two forms of railroad bond yields during the full period 1857–1932 is an instructive example of how the effects of a rising and declining trend neutralize each other.

TABLE 87

Average Timing of Specific Cycles
in Trend-adjusted Data at Corresponding Turns of Unadjusted Data
Six American Series

Series and trend	Period covered	Number of corresponding turns		Average lead (–) or lag (+) of adjusted at turns of unadjusted data (mos.)	
		Peak	Trough	Peak	Trough
UPWARD					
Deflated clearings....	1878–1933	14	15	–3.2	+2.4
Frickey's clearings....	1878–1914	10	11	–0.2	+3.7
A.T.&T. index......	1899–1933	10	10	–1.4	+0.5
Pig iron production...	1878–1933	16	16	–2.9	+0.5
Electricity output....	1921–1933	2	3	–5.5	0.0
Railroad bond yields..	1899–1918	5	6	–3.0	+1.5
DOWNWARD					
Railroad bond yields..	1868–1899	8	9	+3.2	–0.6
OSCILLATORY					
Railroad bond yields..	1857–1932	21	20	+0.9	+0.2

The periods run from the year of the first to the year of the last corresponding turn, whether peak or trough, covered by our analysis of monthly data. They are longer in a few instances than the all-cycle periods listed in Table 85, since the latter start and end with a trough.

The turning points in iron production (both the unadjusted and trend-adjusted data) are shown in Chart 35. The turning points of deflated clearings and railroad bond yields (unadjusted data) are shown in Chart 53 and Appendix Table B3; but see the note introducing Appendix B.

The shifts in cyclical turns produced by trend adjustment must modify the durations of the phases of specific cycles. Every time the removal of a rising trend makes a cyclical peak come earlier, expansions are shortened and contractions lengthened. The effect is similar when a cyclical trough is pushed forward; and is likely to be similar, as far as the relation between the average durations of expansions and contractions is concerned, when the number of specific cycles is increased. The removal of a declining trend has opposite effects. The changes in the cyclical durations of our test series produced by trend adjustments are shown in Table 88. The largest effect appears in the rising segment of railroad bond yields —a result that follows from the gain of two specific cycles. It is interesting to observe that even after the trend is removed, cyclical expansions run longer than contractions in every series except bond yields.

Since secular trends have slight influence on the number of specific cycles in our test series, the average duration of full specific cycles is virtually the same in the adjusted and unadjusted data. The one appreciable discrepancy comes in the rising segment of bond yields.

TABLE 88

Average Duration of Specific Cycles in Unadjusted and Trend-adjusted Data
Six American Series

Series and group	Direction of trend	No. of specific cycles		Average duration in months						Average per cent of duration of specific cycles			
				Expansion		Contraction		Full cycle		Expansion		Contraction	
		Un	Ad	Un	Ad	Un	Ad	Un	Ad	Un	Ad	Un	Ad
CORRESPONDING CYCLES													
Deflated clearings	Up	13	13	33.3	27.7	11.3	16.9	44.6	44.6	76	61	24	39
Frickey's clearings	Up	9	9	30.6	26.4	9.4	13.6	40.0	40.0	76	66	24	34
A.T.&T. index[a]	Up	9	9	25.9	23.8	17.3	19.4	43.2	43.2	60	56	40	44
Pig iron production[a]	Up	15	15	28.8	25.1	14.5	18.1	43.3	43.2	67	59	33	41
Electricity output[a]	Up	2	2	48.5	43.0	22.5	28.0	71.0	71.0	75	62	25	38
Railroad bond yields	Up	3	3	22.7	17.0	11.0	14.3	33.7	31.3	69	53	31	47
Railroad bond yields[a]	Down	8	8	16.8	20.6	29.6	25.8	46.4	46.4	38	44	62	56
Railroad bond yields	Oscil.	16	16	17.4	18.8	24.4	22.6	41.8	41.4	46	46	54	54
ALL CYCLES													
Deflated clearings	Up	15	15	32.6	25.6	11.4	17.9	44.0	43.5	75	59	25	41
Frickey's clearings	Up	11	11	29.8	24.1	9.9	15.1	39.7	39.2	74	63	26	37
Railroad bond yields	Up	5	7	35.4	13.7	11.2	19.7	46.6	33.4	75	40	25	60
Railroad bond yields	Oscil.	20	21	21.0	17.4	21.4	23.1	42.4	40.5	52	43	48	57

'Un' stands for unadjusted, 'Ad' for trend-adjusted data. See Table 85 for the periods covered.

[a] All cycles correspond in the unadjusted and adjusted data.

IV Amplitude of Specific Cycles

When an upward trend is removed from a time series the cyclical rise in the original units must be reduced and the cyclical fall increased. When a downward trend is removed the cyclical rise must be increased and the fall reduced. Table 89 indicates the magnitude of the changes in absolute cyclical amplitudes that result from the removal of the secular trend from pig iron production.[14] The changes vary considerably from phase to phase and from cycle to cycle, as may be expected from the differences in the durations, amplitudes, patterns, and intra-cycle trends of the specific cycles. We also find that the amplitudes of full specific cycles are usually changed much less than their expansions and contractions. This result reflects the opposite effects of the removal of trend on the rise and on the fall. But these opposite effects are not evenly balanced: the amplitude of full specific cycles is reduced in 12 and increased in only 3 instances. Similar reactions appear in other series, whatever the direction of their trends (Table 90). Of the 64 corresponding cycles in the adjusted and unadjusted data of our six series, the absolute amplitude of full specific cycles is smaller in the adjusted data in 50 and larger in only 14. Apparently, if we removed secular trends before taking cyclical measures, the amplitudes of full specific cycles, expressed in the original units, would as a rule be reduced.

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[14] Of course, the trends are removed by subtraction. The relative amplitudes in Table 89 are discussed later.

TABLE 89

Absolute Amplitude of Specific Cycles in Unadjusted and Trend-adjusted Data Pig Iron Production, United States, 1879–1933

Specific cycle[a]	Amplitude in thousand long tons per day						Excess of adjusted over unadjusted data					
	Rise		Fall		Rise & fall		In thousand long tons per day			In percentages[b]		
	Unadj.	Adj.	Unadj.	Adj.	Unadj.	Adj.	Rise	Fall	Rise & fall	Rise	Fall	Rise & fall
1879–1885	6.18	3.35	2.37	4.19	8.55	7.54	−2.83	+1.82	−1.01	−59.4	+55.5	−12.6
1885–1888	8.88	5.87	3.50	4.01	12.38	9.88	−3.01	+0.51	−2.50	−40.8	+13.6	−22.5
1888–1891	9.89	6.98	9.34	10.70	19.23	17.68	−2.91	+1.36	−1.55	−34.5	+13.6	−8.4
1891–1893	10.19	9.36	14.28	17.60	24.47	26.96	−0.83	+3.32	+2.49	−8.5	+20.8	+9.7
1893–1896	18.69	14.72	13.03	14.92	31.72	29.64	−3.97	+1.89	−2.08	−23.8	+13.5	−6.8
1896–1900	24.27	17.03	9.74	11.82	34.01	28.85	−7.24	+2.08	−5.16	−35.1	+19.3	−16.4
1900–1903	22.14	15.03	22.59	23.99	44.73	39.02	−7.11	+1.40	−5.71	−38.3	+6.0	−13.6
1903–1908	44.28	33.69	37.73	39.27	82.01	72.96	−10.59	+1.54	−9.05	−27.2	+4.0	−11.7
1908–1910	49.06	42.82	26.01	30.63	75.07	73.45	−6.24	+4.62	−1.62	−13.6	+16.3	−2.2
1910–1914	32.82	27.99	41.44	47.40	74.26	75.39	−4.83	+5.96	+1.13	−15.9	+13.4	+1.5
1914–1919	60.98	52.12	39.77	41.64	100.75	93.76	−8.86	+1.87	−6.99	−15.7	+4.6	−7.2
1919–1921	32.89	29.35	73.52	74.28	106.41	103.63	−3.54	+0.76	−2.78	−11.4	+1.0	−2.6
1921–1924	89.55	86.54	48.53	50.31	138.08	136.85	−3.01	+1.78	−1.23	−3.4	+3.6	−0.9
1924–1927	36.45	33.89	18.70	20.17	55.15	54.06	−2.56	+1.47	−1.09	−7.3	+7.6	−2.0
1927–1933	36.93	35.50	110.00	112.23	146.93	147.73	−1.43	+2.23	+0.80	−3.9	+2.0	+0.5

The trend was removed by subtraction.

[a] Years of the initial and terminal troughs of the specific cycles in monthly data, unadjusted for trend.

[b] The base of the percentages is the mean of each pair of amplitudes being compared; e.g., the rise for the 1879–85 cycle is $-59.4 = 100\left(\frac{-2.83}{\frac{6.18 + 3.35}{2}}\right)$. This method equalizes the theoretical scale of plus and minus percentages.

TABLE 90

Frequency Distribution of the Differences between Amplitudes of Corresponding Specific Cycles in Unadjusted and Trend-adjusted Data

Deviation of amplitude of adj. from amplitude of unadj. data[a] (per cent)	Number of differences between adjusted and unadjusted data							
	Five series with upward trends						Six series	
	Absolute amplitude			Relative amplitude[b]			Absolute rise & fall	Relative rise & fall[b]
	Rise	Fall	Rise & fall	Rise	Fall	Rise & fall		
Below −70.0	6	..	1	7	..	1	1	1
−70.0 to −50.0	11	..	..	11	..	1	1	2
−50.0 to −30.0	15	..	5	14	..	4	5	4
−30.0 to −20.0	4	..	6	4	..	8	10	12
−20.0 to −10.0	8	..	13	8	..	12	16	16
−10.0 to 0.0	4	..	13	4	..	13	17	16
0.0 to +10.0	..	13	10	..	13	9	11	10
+10.0 to +20.0	..	11	..	..	11	..	3	3
+20.0 to +30.0	..	5	..	..	5	..	..	..
+30.0 to +50.0	..	9	..	..	11	..	..	..
+50.0 to +70.0	..	7	..	..	5	..	..	..
Over +70.0	..	3	..	..	3	..	..	..
Total.........	48	48	48	48	48	48	64	64

See Table 85 for the series included and the periods covered by their corresponding cycles.

[a] See note to preceding table for the method used in computing the percentage deviations. Two items at the stated class limits were distributed after carrying the percentages to an extra decimal.

[b] See pp. 284-5 concerning the measures of relative amplitude of the adjusted data (same as method C in Table 92).

This expectation may be readily tested with the aid of a few symbols. Assume that a_t, b_p, c_t, d_p, etc. are ordinates of secular trend at dates of successive troughs and peaks of the specific cycles in unadjusted data, the subscript t indicating a trough date and the subscript p a peak date. Assume further that the unadjusted data at corresponding turns are $(a_t - m)$, $(b_p + n)$, $(c_t - o)$, $(d_p + p)$, etc., and that the removal of the trend leaves the dating of the cyclical turns unchanged. Then the absolute amplitude of a full cycle running from trough to trough in the unadjusted data is $(b_p + n) - (a_t - m) + (b_p + n) - (c_t - o)$, or $2b_p - (a_t + c_t) + (m + 2n + o)$. But $(m + 2n + o)$ is the amplitude of a full cycle running from trough to trough in the adjusted data. Hence the absolute amplitude of the unadjusted data will be larger or smaller than that of the adjusted data according as $2b_p - (a_t + c_t)$ is plus or minus.[15] It may be shown in a similar manner that when the cycles are taken on an inverted basis, the amplitude of a full cycle in the unadjusted data is larger or smaller than that of the adjusted data according as $(b_p + d_p) - 2c_t$ is plus or minus. Table 91 makes explicit some of the relations implicit in this and the preceding expression.[16] It appears that the removal of secular trends may increase the amplitudes of full cycles, decrease them, or leave them unchanged: the result depends upon the direction of the trend, its curvature, the relative duration of expansions and contractions, and the treatment of the cycles as positive or inverted.[17] But the critical point is that when the trend is rising and expansions are longer than contractions, or when the trend is declining and contractions are longer than expansions, it appears that the amplitude of full cycles of adjusted data is more likely to fall short of than to exceed the amplitude of unadjusted data. These are the conditions we usually encounter in time series. They characterize also our present sample, and thus explain the tendency of the

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[15] To simplify the argument, the use of three-month averages to represent standings at peaks and troughs is disregarded.

[16] The relations are obvious to common sense. If, for example, the trend is upward and linear, and the cycle phases are of equal duration, the amplitude of rise is increased by the trend component to the same degree that the amplitude of fall is decreased; for the cumulation of the trend component in one phase is the same as in the other, the two being of the same length. But if the expansion is longer than the contraction, the amplitude of rise is increased more by a linear trend than the amplitude of fall is reduced; for the trend component is now greater for the longer of the two phases. And so on from line to line of the table.

[17] The result depends also on the extent to which cyclical turns are shifted by removing the trend. Table 91 is made on the assumption that the trend changes none of the cyclical turns—a valid assumption only two-thirds of the time according to our sample. If this assumption is dropped, the relations between the absolute amplitudes of unadjusted and adjusted data are modified as follows: (1) If removal of the trend affects the dating of peaks but not of troughs, and the cycles are taken positively, $A = m + 2z + o$, where $z > n$. (See the text and Table 91 for the meaning of the symbols.) Hence $U - A$ is algebraically smaller than it would be if the dating of the peaks were unaffected. Similar relations obtain between U and A when the cycles are inverted, if the trend affects the dating of the troughs but not of the peaks. (2) Other things equal, A is increased more than in the preceding case if the dating of both troughs and peaks is affected by removing the trend.

TABLE 91

Relation between Amplitudes of Full Specific Cycles in Unadjusted and Trend-adjusted Data under Different Conditions

Curvature of trend	Upward trend		Downward trend	
	Cycles taken from trough to trough	Cycles taken from peak to peak	Cycles taken from trough to trough	Cycles taken from peak to peak
Duration of expansion equals that of contraction				
Linear	$U = A$	$U = A$	$U = A$	$U = A$
Concave	$U > A$	$U < A$	$U > A$	$U < A$
Convex	$U < A$	$U > A$	$U < A$	$U > A$
Duration of expansion longer than that of contraction				
Linear	$U > A$	$U > A$	$U < A$	$U < A$
Concave	$U > A$	$U \gtreqless A$	$U \gtreqless A$	$U < A$
Convex	$U \gtreqless A$	$U > A$	$U < A$	$U \gtreqless A$
Duration of expansion shorter than that of contraction				
Linear	$U < A$	$U < A$	$U > A$	$U > A$
Concave	$U \gtreqless A$	$U < A$	$U > A$	$U \gtreqless A$
Convex	$U < A$	$U \gtreqless A$	$U \gtreqless A$	$U > A$

U stands for the amplitude of a full specific cycle (rise and fall) in unadjusted data, A for the amplitude of a full cycle in adjusted data. The comparisons are made on the assumption that the turning points of the specific cycles coincide in the adjusted and unadjusted data. The table applies to relative as well as to absolute amplitudes; provided the relative amplitude of adjusted data is envisaged as in 'method C', described on p. 285.

The troublesome terms 'convex' and 'concave' are used as follows: the trend is said to be convex if its second derivative with respect to time is positive, concave if its second derivative is negative.

amplitudes of full specific cycles to run lower in adjusted than in unadjusted data.

The preceding analysis is based on amplitudes expressed in the original units, whereas our standard practice is to work with amplitudes expressed in cycle relatives. In the unadjusted data the trend affects our amplitude measures in two ways. First, it influences the amplitude expressed in the original units. Second, it influences the final result *via* the cycle bases; for example, if the trend is upward, a set of cyclical rises divided by a set of cycle bases yields smaller quotients if the bases relate to positive than if they relate to inverted cycles. Since the removal of trend lines frees the data from 'intra-cycle' as well as 'inter-cycle' trends, cyclical rises and declines must be roughly equal on the average in relative as in absolute measures of amplitude, providing our general plan of measuring amplitudes is followed. In the unadjusted data, on the other hand, a trend factor must remain in the amplitude measures, whether expressed in units of the original data or in cycle relatives. True, our standard method, which converts the absolute amplitude of a specific cycle into a percentage of the average value of the series during the cycle, involves in effect an adjustment for secular trend. But since the 'trend' is a step-line of cyclical averages, a line that is horizontal within each specific cycle, a trend component remains within each cycle and each cyclical phase, and its impact on any given phase must vary according as the cycles have been marked off by peaks or by troughs. We may expect therefore the removal of secular trends not only to yield measures of the relative amplitude of specific cycles that differ from our standard measures, but to modify these measures in different ways according as the cycles have been analyzed on a positive or on an inverted basis.

For the moment, we concentrate on positive analysis, which is our typical method of handling specific cycles. Table 90 shows the direction and degree in which the relative amplitude of single specific cycles is changed by eliminating trends. In this table we have followed our standard method in computing the relative amplitude of the specific cycles in the unadjusted data, while in the adjusted data we converted the absolute amplitude of each specific cycle into a percentage of the average value of the *unadjusted* data during the period occupied by that cycle. It appears from Table 90 that the effects of eliminating secular trends on the relative amplitudes of expansions, contractions and full cycles are distributed in virtually the same way as are the effects on the absolute amplitudes. The results could hardly be otherwise. For the base used in computing the relative amplitude of a specific cycle is the same in the unadjusted and adjusted data whenever their cyclical troughs coincide, and the difference between the bases is usually slight even in the absence of coincidence. It may be recalled that the effects of trend adjustments analyzed in Table 91 assume coincidence of cyclical turns. Since on this assumption the cycle bases of adjusted and unadjusted data are identical, the removal of trend will have exactly the same effect on the relative as on the absolute amplitudes.[18] Hence the reasons previously advanced for the tendency of amplitude measures of full specific cycles to run lower in adjusted than in unadjusted data apply to the relative amplitudes no less than to the absolute amplitudes.

Of course, this argument is based on a particular method of measuring the relative amplitude of specific cycles in trend-adjusted data—a difficulty that should be faced explicitly. At least four plausible methods may be distinguished. (A) If the adjusted data are expressed as trend relatives, we can simply apply our standard technique to these data. (B) The relative amplitude may also be measured directly from the trend relatives, that is, without adjusting the relatives for differences in their average

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[18] See above, note 17, for the effects of trends on absolute amplitudes of full cycles when the trend shifts cyclical turns. Point (1) of that note applies also to relative amplitudes, for under the assumed conditions the cycle bases are the same in adjusted and unadjusted data. So too does point (2) with this additional comment: the increase in the absolute amplitude tends to be counteracted or reinforced in the relative amplitude according as the cycles are positive or inverted, because (regardless of the direction of the trend) the cycle base tends to be raised in the former case and lowered in the latter.

TABLE 92

Average Amplitude of Corresponding Specific Cycles
in Unadjusted and Trend-adjusted Data Computed by Different Methods
Six American Series

Series and trend	No. of corresponding specific cycles	Average relative amplitude of											
		Unadjusted data treated according to our standard technique			Adjusted data treated in the form of								
					Trend relatives						Absolute deviations from trend, but 'correcting' for the level of cycles (*Method C*)		
					According to our standard technique (*Method A*)			As in Method A, but not 'correcting' for the level of cycles (*Method B*)					
		Rise	Fall	Rise & fall	Rise	Fall	Rise & fall	Rise	Fall	Rise & fall	Rise	Fall	Rise & fall
UPWARD													
Deflated clearings. .	13	25.9	13.7	39.7	14.8	17.6	32.4	14.8	17.5	32.3	14.4	19.0	33.4
Frickey's clearings. .	9	30.2	13.6	43.8	17.4	17.3	34.7	17.2	17.3	34.5	16.5	18.4	34.8
A.T.&T. index. . . .	9	30.5	26.1	56.5	23.9	29.6	53.6	23.5	28.3	51.8	23.4	30.8	54.2
Pig iron production.	15	62.1	54.8	116.8	52.2	57.7	109.9	49.9	54.0	103.9	49.8	60.6	110.5
Electricity output. .	2	44.2	13.8	58.0	18.0	22.2	40.1	17.6	22.2	39.8	16.4	26.0	42.3
Railroad bond yields	3	12.9	8.4	21.3	8.7	10.5	19.2	8.7	10.5	19.2	8.6	10.7	19.3
DOWNWARD													
Railroad bond yields	8	6.3	14.6	20.9	8.6	8.4	17.0	8.6	8.4	16.9	8.8	8.3	17.1
OSCILLATORY													
Railroad bond yields	16	9.4	14.7	24.1	10.2	11.1	21.2	10.1	11.0	21.2	10.3	10.9	21.3

Methods A, B and C are identified more fully in the text. See Table 85 for the periods covered.

level during successive specific cycles. (C) When trend-adjusted data are expressed as absolute deviations from trend—and that is the form in which they must be taken to show the effect of trend adjustments on amplitudes in the original units—a different procedure is necessary. Here the absolute amplitude of each cycle may be expressed as a percentage of the average value of the *unadjusted* data during the period occupied by the cycle, or (D) as a percentage of the average value of the ordinates of secular trend during the period occupied by the cycle. It seems that method A should be better than method B whenever the trend is so fitted that some full cycles are sunk below or raised above the line of trend, for in such cases method A supplies what is in effect a supplementary trend adjustment. For similar reasons method C seems better than method D. But it is more difficult to choose between methods A and C either on theoretical or practical grounds.[19]

Table 92 compares the average amplitudes of unadjusted data with the average amplitudes of adjusted data measured by methods A, B and

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[19] If the amplitudes of rise and fall in the adjusted data are equal in the original units, the rise of a specific cycle will still equal the fall if computed by method C but not by method A or B: while if the amplitudes of specific cycles of trend relatives are constant, the rise will equal the fall if ascertained by method A or B but not by method C. Since the amplitude of percentage deviations from trend is apt to vary less than the amplitude of absolute deviations from trend, these considerations seem to argue in favor of methods A and B. So too does the analysis of positive vs. inverted measures, later in this section.

C. The three methods of computing the relative amplitude of trend-adjusted data yield closely similar results, although there are numerous differences of detail. The most prominent is that the average cyclical rise is smaller and the average fall larger in method C than in method A or B in every sample of an upward trend, while the average rise is larger and the average fall smaller in method C than in method A or B in the one sample of a declining trend.[20] But whatever the method, the removal of an upward trend reduces the average cyclical rise and increases the average cyclical fall, the removal of a downward trend produces opposite effects, and the removal of an upward or downward trend reduces the joint rise and fall.

TABLE 93

Comparison of Relative Amplitudes of Corresponding Specific Cycles in Unadjusted and Trend-adjusted Data

Amplitude of adjusted data determined by method[a]	Number of instances in which the relative amplitude of adjusted data					
	Is larger than that of unadjusted data			Is smaller than that of unadjusted data		
	Rise	Fall	Rise & fall	Rise	Fall	Rise & fall
A.	. .	44	8	48	4	40
B.	2	43	8	46	5	40
C.	. .	48	9	48	. .	39

Based on 48 corresponding cycles in the unadjusted and trend-adjusted data of five American series with upward trends. See Table 85 for the series and periods covered, and Table 90 for more detailed comparisons involving method C.

[a] The several methods are identified in Table 92, and more fully on pp. 284-5.

Theoretically, these effects are not strictly necessary in any of the methods. The removal of an upward trend must reduce the cyclical rise and increase the cyclical fall when the amplitudes are measured in the original units. But strange as it may seem, the removal of an upward trend

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[20] Assume that the ordinates of secular trend at the dates of initial trough, peak, and terminal trough of a specific cycle in *adjusted* data are, successively, a, b, and c; that the values of the cyclical component in the original units are $-m$, $+n$, and $-o$ at the corresponding dates; and that these dates are the same no matter how the secular trend is eliminated. Then, according to method B, the cyclical rise is $100\left(\frac{n}{b}+\frac{m}{a}\right)$ and the cyclical fall is $100\left(\frac{n}{b}+\frac{o}{c}\right)$; while according to method C, the rise is $100\left(\frac{n}{k}+\frac{m}{k}\right)$ and the fall is $100\left(\frac{n}{k}+\frac{o}{k}\right)$, where k is the average value of the unadjusted data during the cycle. The cyclical rise will be smaller in method C than in method B, provided k is larger than the weighted harmonic mean of a and b; their respective weights being m and n. And the cyclical fall will be larger in method C than in method B, provided k is smaller than the weighted harmonic of b and c, their respective weights being n and o. It seems reasonable to expect that k will rarely be below the first harmonic or above the second, so long as the secular trend is upward and the specific cycles are taken positively.

If specific cycles are taken invertedly, it may be expected that the average value of the unadjusted data during the inverted cycle running from the date of b to the next peak will usually be above the weighted harmonic of b and c, while the average value of the unadjusted data during the inverted cycle ending with the date of b will usually be below the weighted harmonic of a and b. Hence inverted analysis should tend to produce differentials between methods B and C that are opposite in sign to those produced by positive analysis. See Table 94, where positive and inverted measures are contrasted.

The preceding remarks may be readily rephrased for the case of a declining trend.

may increase the relative amplitude of the cyclical rise or reduce the relative amplitude of the cyclical fall. For example, if a cyclical contraction is exceptionally violent, the average value of the original data during the cycle including this phase may lie considerably below the trend; hence the amplitude of the fall in the adjusted data, if ascertained by method B, may be considerably smaller than the amplitude of the fall in the unadjusted data. The summary of cycle-by-cycle comparisons in Table 93 demonstrates that such curious results are rare in practice. Nevertheless, Table 92 discloses an instance in which the *average* fall of adjusted data covering fifteen specific cycles is smaller than the average fall of unadjusted data, despite the upward trend of the series.[21]

Table 94 compares amplitude measures of positive and inverted cycles on the plan of Table 92. We have already shown in Chapter 5 how measures of amplitude of unadjusted data depend on the decision to analyze specific cycles on a positive or on an inverted basis. In brief, if the secular trend of a series is upward, the amplitude of cyclical rises is likely to be smaller and of cyclical falls larger when the specific cycles are taken positively than when they are taken invertedly; but if the secular trend is downward, rises are likely to be larger and falls smaller in positive than in inverted cycles. The influence of positive versus inverted treatment is similar in the adjusted data if the amplitudes are measured by method C, because cycle bases affect these measures in much the same way as they affect the measures of unadjusted data. Method B, on the other hand, avoids the use of cycle bases; hence there can be no difference between amplitude measures for positive and inverted cycles, provided, of course, they cover the same period. Method A may show differences, but

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[21] If the cyclical turns in unadjusted and trend-adjusted data are coincident, the cyclical fall in the adjusted data, according to method B, is $100\left(\frac{n}{b} + \frac{o}{c}\right)$, and the cyclical fall in the unadjusted data is $100\left(\frac{b-c}{k} + \frac{n+o}{k}\right)$. (The symbols are defined in the preceding note.) If a specific cycle in the unadjusted data has a mild expansion followed by an exceptionally sharp contraction, and the trend line during this cycle moves upward at a gentle pace, k may be much lower than b or c. If that happens $\frac{n+o}{k}$ will be considerably larger than $\frac{n}{b} + \frac{o}{c}$, and this excess will be only slightly offset by $\frac{b-c}{k}$; in other words, the amplitude of fall in the unadjusted data will be larger than in the adjusted data. An example that approximates this hypothetical case is the decline of iron production from 1929 to 1933; the amplitude of which is 148.9 in the unadjusted data, but only 103.8 in the adjusted data treated by method B. This extreme discrepancy is the main reason for the paradoxical result in Table 92, to which we refer in the text. (See Chart 35.)

Method A also may produce curious results. For example, if the cyclical fall in the adjusted data analyzed by method B is only slightly larger than in the unadjusted data, while the average of the trend relatives during the cycle is well above 100, the fall in the adjusted data treated by method A will be smaller than in the unadjusted data.

Nor is method C devoid of this difficulty. The absolute cyclical fall after an upward trend is removed must exceed the absolute fall of unadjusted data. But the base on which the former is expressed may be higher than the base on which the latter is expressed. Hence the relative amplitude of the fall may be smaller in the adjusted than in the unadjusted data, though the likelihood of such a result is slight.

These remarks may be readily extended to cyclical rises.

TABLE 94

Average Amplitude of Corresponding Specific Cycles in Unadjusted and Trend-adjusted Data on Positive and Inverted Plans Five American Series

Series, trend, and measure	Average relative amplitude								Excess of average of adjusted data over average of unadjusted data					
	Unadjusted data		Adjusted data											
	Pos.	Inv.	Method A		Method B		Method C		Method A		Method B		Method C	
			Pos.	Inv.	Pos.	Inv.	Pos.	Inv.	Pos.	Inv.	Pos.	Inv.	Pos.	Inv.
UPWARD TREND														
Deflated clearings														
Rise	25.0	25.7	14.0	14.0	14.0	14.0	13.6	14.5	−11.0	−11.7	−11.0	−11.7	−11.4	−11.2
Fall	10.3	9.6	14.1	14.4	14.4	14.4	14.7	13.6	+3.8	+4.8	+4.1	+4.8	+4.4	+4.0
Rise & fall	35.4	35.3	28.1	28.5	28.5	28.5	28.3	28.1	−7.3	−6.8	−6.9	−6.8	−7.1	−7.2
Frickey's clearings														
Rise	29.4	30.5	16.4	16.7	16.2	16.2	15.5	16.9	−13.0	−13.8	−13.2	−14.3	−13.9	−13.6
Fall	13.8	12.7	17.0	17.7	17.1	17.1	18.2	16.7	+3.2	+5.0	+3.3	+4.4	+4.4	+4.0
Rise & fall	43.2	43.2	33.4	34.4	33.3	33.3	33.7	33.6	−9.8	−8.8	−9.9	−9.9	−9.5	−9.6
A.T.&T. index														
Rise	30.7	31.4	25.1	24.8	24.6	24.6	24.5	25.2	−5.6	−6.6	−6.1	−6.8	−6.2	−6.2
Fall	20.2	19.1	23.5	23.8	23.6	23.6	24.4	23.1	+3.3	+4.7	+3.4	+4.5	+4.2	+4.0
Rise & fall	51.0	50.6	48.6	48.6	48.1	48.1	48.9	48.3	−2.4	−2.0	−2.9	−2.5	−2.1	−2.3
Pig iron production														
Rise	61.9	64.5	53.1	52.1	50.5	50.5	50.9	53.1	−8.8	−12.4	−11.4	−14.0	−11.0	−11.4
Fall	48.0	43.9	51.2	51.9	50.4	50.4	54.1	49.4	+3.2	+8.0	+2.4	+6.5	+6.1	+5.5
Rise & fall	110.0	108.4	104.3	104.1	100.9	100.9	105.1	102.6	−5.7	−4.3	−9.1	−7.5	−4.9	−5.8
DOWNWARD TREND														
Railroad bond yields														
Rise	6.0	5.8	8.6	8.6	8.6	8.6	8.8	8.4	+2.6	+2.8	+2.6	+2.8	+2.8	+2.6
Fall	14.4	15.0	8.4	8.4	8.4	8.4	8.3	8.6	−6.0	−6.6	−6.0	−6.6	−6.1	−6.4
Rise & fall	20.4	20.8	17.0	17.0	16.9	16.9	17.1	17.0	−3.4	−3.8	−3.5	−3.9	−3.3	−3.8
OSCILLATORY TREND														
Railroad bond yields														
Rise	9.3	9.1	10.1	10.1	10.0	10.0	10.3	10.0	+0.8	+1.0	+0.7	+0.9	+1.0	+0.9
Fall	15.4	16.0	10.8	11.0	10.9	10.9	10.7	11.2	−4.6	−5.0	−4.5	−5.1	−4.7	−4.8
Rise & fall	24.7	25.1	21.0	21.1	20.9	20.9	21.0	21.1	−3.7	−4.0	−3.8	−4.2	−3.7	−4.0

Methods A, B and C are identified in Table 92, and more fully on pp. 284-5. With one exception, the number of corresponding cycles is one less for each sample than in Table 92, the expansion of the first and the contraction of the last cycle being dropped. In railroad bond yields (oscillatory trend) the number of cycles is 12 in this table, but 16 in Table 92; the expansion of the first and the contraction of the last cycle in each of the four clusters of corresponding cycles shown in Table 85 (note 'a') was dropped. The brief series on electricity and the rising segment of bond yields are omitted because of fewness of cycles.

they are bound to be slight and erratic, since the cycle bases are usually close to 100 and in any case deviate more or less erratically from this value. We can therefore say that in methods A and B the amplitude measures do not depend upon whether the specific cycles are treated positively or invertedly. We also know that the average rise will tend to equal the average fall in methods A and B, while the presence of a trend makes the rise and fall unequal in the case of unadjusted data. It follows that if method A or B is applied to adjusted data, the average rise and fall, taken separately, will be closer to corresponding averages of unadjusted data when their specific cycles are treated positively than when they are treated invertedly—a conclusion that is equally valid for series with upward or downward trends. Or to put the same thing in different words, if method A or B is used, the removal of secular trends will alter our standard measures of amplitude of rise and fall, taken separately, more when the specific cycles have been handled invertedly than when they have been handled positively. No such systematic difference will appear if method C is used. However, as Table 94 shows, the differences on account of positive versus inverted treatment are as a rule very small in relation to the size of the amplitudes of expansions and contractions, taken separately, and they are practically of no consequence whatsoever in the amplitudes of full cycles.[22]

TABLE 95

Average Amplitude of Corresponding and All Specific Cycles
in Unadjusted and Trend-adjusted Data
Six American Series

Series and group	Direction of trend	No. of specific cycles		Average amplitude in specific-cycle relatives						Average of adj. data as relative of average of unadj. data		
				Rise		Fall		Rise & fall		Rise	Fall	Rise & fall
		Un	Ad	Un	Ad	Un	Ad	Un	Ad			
CORRESPONDING CYCLES												
Deflated clearings	Upward	13	13	25.9	14.8	13.7	17.6	39.7	32.4	57	128	82
Frickey's clearings	Upward	9	9	30.2	17.4	13.6	17.3	43.8	34.7	58	127	79
A.T.&T. index[a]	Upward	9	9	30.5	23.9	26.1	29.6	56.5	53.6	78	113	95
Pig iron production[a]	Upward	15	15	62.1	52.2	54.8	57.7	116.8	109.9	84	105	94
Electricity output[a]	Upward	2	2	44.2	18.0	13.8	22.2	58.0	40.1	41	161	69
Railroad bond yields	Upward	3	3	12.9	8.7	8.4	10.5	21.3	19.2	67	125	90
Railroad bond yields[a]	Downward	8	8	6.3	8.6	14.6	8.4	20.9	17.0	137	58	81
Railroad bond yields	Oscillatory	16	16	9.4	10.2	14.7	11.1	24.1	21.2	109	76	88
ALL CYCLES												
Deflated clearings	Upward	15	15	26.9	15.6	13.4	17.4	40.2	33.0	58	130	82
Frickey's clearings	Upward	11	11	31.2	19.7	13.7	18.3	44.8	38.0	63	134	85
Railroad bond yields	Upward	5	7	13.3	6.7	6.3	7.1	19.6	13.8	50	113	70
Railroad bond yields	Oscillatory	20	21	10.8	9.9	12.5	10.0	23.3	19.9	92	80	85

'Un' stands for unadjusted, 'Ad' for trend-adjusted data. The amplitudes of the adjusted data were computed by our standard technique from trend relatives. See Table 85 for the periods covered.

[a] All cycles correspond in the adjusted and unadjusted data.

It may be well to pause at this point and sum up the main findings, which are simple enough in essence. Four conclusions stand out. (1) The removal of an upward trend tends to reduce the amplitude of cyclical rise and increase the amplitude of cyclical fall; the removal of a declining trend has opposite effects. These effects must register in amplitudes expressed in the original units, and they are very likely to do so in amplitudes expressed in cycle relatives. (2) The removal of any definite trend tends to reduce the amplitude of full specific cycles, whether expressed in the original units or in cycle relatives. This effect is not necessary mathematically, but it is likely to dominate in the time series with which we deal. (3) The average effect produced by trend adjustments (Table 95) is considerable in the case of amplitudes of expansions and contractions

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[22] Cf. pp. 135-6.

taken separately, and is appreciable even in the amplitudes of full cycles.[23] (4) The effects of trend adjustments on measures of cyclical amplitude depend partly upon 'real' factors, that is, the duration, amplitude, pattern, and intra-cycle trend of different specific cycles; and partly upon 'technical' factors, such as the particular trend line used, the method used to eliminate the trend, the method used to measure the amplitude of trend-adjusted data, and whether the specific cycles are analyzed positively or invertedly. The technical factors are more important in the amplitudes of cyclical phases than in the amplitudes of full cycles; but, in general, if the trend line is at all plausible, the 'technical' effects are reasonably sure to be swamped by the 'real' effects.

To complete the present analysis, we show in Tables 96 and 97 how the removal of secular trends affects the per month amplitudes. It appears that the per month amplitude of full specific cycles is affected by trend adjustments in about the same ratio as is the total amplitude proper. This result reflects the slight influence of trend adjustments upon the duration of full cycles.[24] The effects of trends on the *rates* of rise and fall, taken separately, are relatively smaller and less uniform than the effects on the amounts of rise and fall. For the removal of an upward trend tends not only to reduce cyclical rises and increase cyclical falls, but also to shorten expansions and lengthen contractions. Likewise the removal of a downward trend tends to reduce both the amplitude and the duration of contractions, and to increase the amplitude and duration of expansions. Whether the removal of trend increases or diminishes the per month amplitude depends therefore upon three factors: the direction of the trend, its influence on the amplitude of a phase, and its influence on the corresponding duration. The main line of cleavage is between cyclical movements that are in the same direction as the trend and the cyclical movements that oppose the trend. Assume that the removal of trend affects the amplitude of cyclical phases in greater proportion than their duration. Then the rate of cyclical rise will be reduced when an upward trend is eliminated and the rate of cyclical fall will be reduced when a downward trend is eliminated. The dominance of these tendencies in our sample appears clearly in the distribution of single cycles in Table 96, as well as in the averages of Table 97. But in order that the removal of an upward trend reduce the rate of cyclical fall or the removal of a downward trend reduce the rate of cyclical rise, the trend adjustment must increase the amplitudes in smaller proportion than the durations. These tendencies are not prominent in the distribution of single cycles, though they

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[23] So far as the removal of an upward (downward) trend tends to increase the number of specific cycles, it will tend to intensify the reduction of the average cyclical rise (fall) and, though to a lesser extent, offset the increase of the average cyclical fall (rise); hence it will tend to intensify the reduction of the average amplitude of full cycles. But the noncorresponding cycles in our sample are too few to cut an appreciable figure. See Table 95, and Sec. II of this chapter.

[24] But see Ch. 8, note 18.

TABLE 96

Rates of Rise and Fall of Specific Cycles in Unadjusted Data Compared with Corresponding Measures of Trend-adjusted Data

Six American Series

Series	Number of corresponding specific cycles	Number of instances in which			
		Rise per month in adjusted data is		Fall per month in adjusted data is	
		Larger than in unadjusted data	Smaller than in unadjusted data	Larger than in unadjusted data	Smaller than in unadjusted data
			Upward trend		
Deflated clearings......	13	2	11	7	6
Frickey's clearings......	9	..	9	4	5
A.T.&T. index.........	9	1	8	6	3
Pig iron production.....	15	3	12	8	7
Electricity output.......	2	..	2	1	1
Railroad bond yields....	3	1	2	2	1
Total................	51	7	44	28	23
			Downward trend		
Railroad bond yields....	8	6	2	..	8

The rates of rise or fall in specific-cycle relatives were computed by our standard technique, to as many places as was necessary to establish a difference between the adjusted and unadjusted data. See Table 85 for the periods covered.

TABLE 97

Average Per Month Amplitude of Corresponding and All Specific Cycles in Unadjusted and Trend-adjusted Data

Six American Series

Series and group	Direction of trend	No. of specific cycles		Average per month amplitude in specific-cycle relatives						Average of adj. data as relative of average of unadj. data		
				Rise		Fall		Rise & fall		Rise	Fall	Rise & fall
		Un	Ad	Un	Ad	Un	Ad	Un	Ad			
CORRESPONDING CYCLES												
Deflated clearings	Upward	13	13	0.81	0.64	2.05	1.93	0.92	0.74	79	94	80
Frickey's clearings	Upward	9	9	1.00	0.69	1.78	1.65	1.13	0.88	69	93	78
A.T.&T. index[a]	Upward	9	9	1.30	1.12	1.87	1.79	1.35	1.28	86	96	95
Pig iron production[a]	Upward	15	15	2.43	2.49	4.68	4.02	2.89	2.72	102	86	94
Electricity output[a]	Upward	2	2	0.97	0.48	0.80	0.69	0.92	0.56	49	86	61
Railroad bond yields	Upward	3	3	0.58	0.58	1.73	0.74	0.71	0.65	100	43	92
Railroad bond yields[a]	Downward	8	8	0.39	0.49	0.50	0.35	0.46	0.38	126	70	83
Railroad bond yields	Oscillatory	16	16	0.56	0.62	0.94	0.65	0.66	0.60	111	69	91
ALL CYCLES												
Deflated clearings	Upward	15	15	0.83	0.80	1.92	1.77	0.94	0.79	96	92	84
Frickey's clearings	Upward	11	11	1.02	0.98	1.65	1.59	1.15	1.01	96	96	88
Railroad bond yields	Upward	5	7	0.45	0.57	1.16	0.44	0.53	0.45	127	38	85
Railroad bond yields	Oscillatory	20	21	0.57	0.64	0.84	0.56	0.64	0.55	112	67	86

The per month averages are unweighted. 'Un' stands for unadjusted, 'Ad' for trend-adjusted data. The amplitudes of the adjusted data were computed by our standard technique from trend relatives. See Table 85 for the periods covered.

[a] All cycles correspond in the adjusted and unadjusted data.

leave their impress on the averages. In our sample the effect of trend adjustments is generally greater on the amplitudes and smaller on the durations, relatively to the size of the figures, when the cyclical movement is in the same direction as the trend than when it opposes the trend.[25] The tendency of trend adjustments to reduce the per month amplitudes in the different circumstances represented in Table 96 reflects this fact, although it does not follow inevitably from it.

TABLE 98

Average Specific-cycle Patterns of Unadjusted and Trend-adjusted Data
Six American Series

Series and form of data	Direction of trend	No. of specific cycles	Average in specific-cycle relatives at stage								
			I	II	III	IV	V	VI	VII	VIII	IX
			Initial trough (3 mos.)	Expansion			Peak (3 mos.)	Contraction			Terminal trough (3 mos.)
				First third	Middle third	Last third		First third	Middle third	Last third	
DEFLATED CLEARINGS											
Unadjusted	Up	15	85.7	90.5	99.2	106.7	112.6	108.7	106.0	101.9	99.2
Adjusted	....	15	92.7	96.6	100.8	104.7	108.3	103.7	100.0	94.1	90.9
FRICKEY'S CLEARINGS											
Unadjusted	Up	11	84.2	88.2	98.3	106.6	115.4	113.5	111.1	105.8	101.7
Adjusted	....	11	89.4	95.1	99.2	104.9	109.1	105.0	101.5	96.3	90.7
A.T.&T. INDEX											
Unadjusted	Up	9	84.7	90.4	101.4	109.6	115.1	111.0	104.5	94.0	89.1
Adjusted	....	9	89.7	94.6	103.1	109.4	113.7	108.8	100.9	88.7	84.0
PIG IRON PRODUCTION											
Unadjusted	Up	15	67.3	82.5	103.7	116.5	129.3	122.6	108.2	88.4	74.6
Adjusted	....	15	73.5	86.6	106.9	115.3	125.7	118.0	101.5	84.2	67.9
ELECTRICITY OUTPUT											
Unadjusted	Up	2	74.0	81.0	96.6	111.0	118.2	116.2	113.3	107.1	104.3
Adjusted	....	2	92.0	94.8	99.9	106.6	110.0	106.2	100.2	93.2	87.8
RAILROAD BOND YIELDS											
Unadjusted	Up	5	93.4	95.7	99.3	102.8	106.7	104.3	103.4	102.0	100.5
Adjusted	....	7	97.0	97.6	99.9	101.9	103.8	101.7	100.3	98.3	96.7
RAILROAD BOND YIELDS											
Unadjusted	Down	8	100.4	101.8	102.8	104.2	106.7	102.8	99.0	94.9	92.1
Adjusted	....	8	96.3	98.1	100.0	101.8	104.8	101.8	100.1	98.1	96.5
RAILROAD BOND YIELDS											
Unadjusted	Oscil.	20	96.1	98.4	101.5	104.0	106.9	103.7	100.6	96.9	94.3
Adjusted	....	21	95.2	97.4	100.5	102.7	105.1	102.1	100.0	97.1	95.1

The patterns of the adjusted data were computed by our standard technique from trend relatives. See Chart 37 for the periods covered.

Chart 37 and Table 98 compare the patterns of the specific cycles of the adjusted and unadjusted data, traced out by the average standings in successive stages of the cycles. In the main the chart recapitulates the differences in specific-cycle behavior shown by preceding tables. The largest difference between any two patterns is in electric power production, as is to be expected. The two patterns of bond yields differ notably in periods

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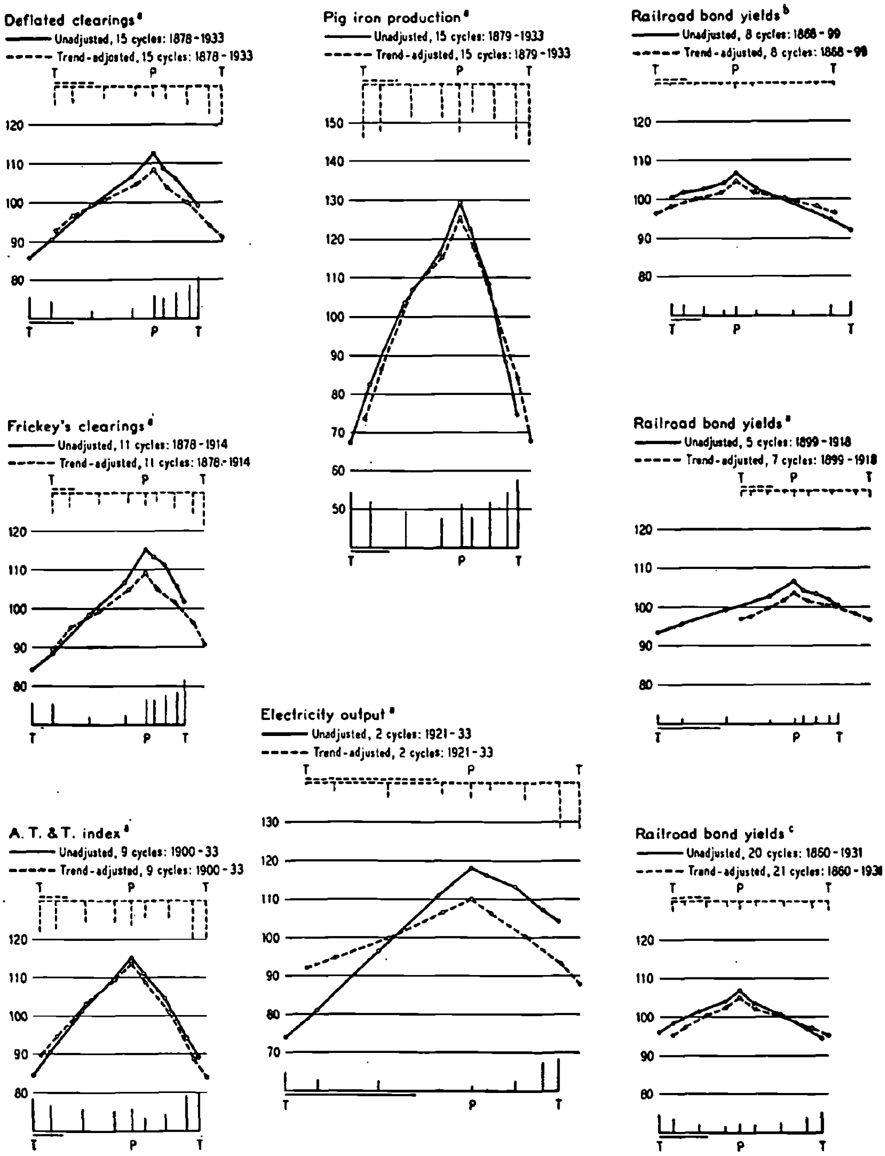
[25] See Tables 88 and 95.

CHART 37

Average Specific-cycle Patterns of Unadjusted and Trend-adjusted Data
Six American Series



Horizontal scale, in months 0 12 24 36 48 60

See Table 98. For explanation of chart, see Ch. 5, Sec. VI.

[a] Upward trend

[b] Downward trend

[c] Oscillatory trend

over which the trend moves in a single direction, but are fairly similar when the period includes opposing trends. It is worth observing that the patterns of the adjusted data of our sample look no more like sine curves than do the patterns of the unadjusted data.

V Reference-cycle Measures

When we break series on the basis of the turning points in general business activity instead of the turning points peculiar to each series, the shift has scarcely any effect on the trend component. On the other hand, the amplitudes of the cycles are reduced in varying degree, and leads or lags emerge. As a result the trend component of the unadjusted data appears more prominent in the reference-cycle patterns of Chart 38 than in the specific-cycle patterns of Chart 37. The trend obscures the response of bond yields to business cycles if we take periods of rising and falling trends separately, and we must look closely at the figures in Table 99 to detect it.

TABLE 99

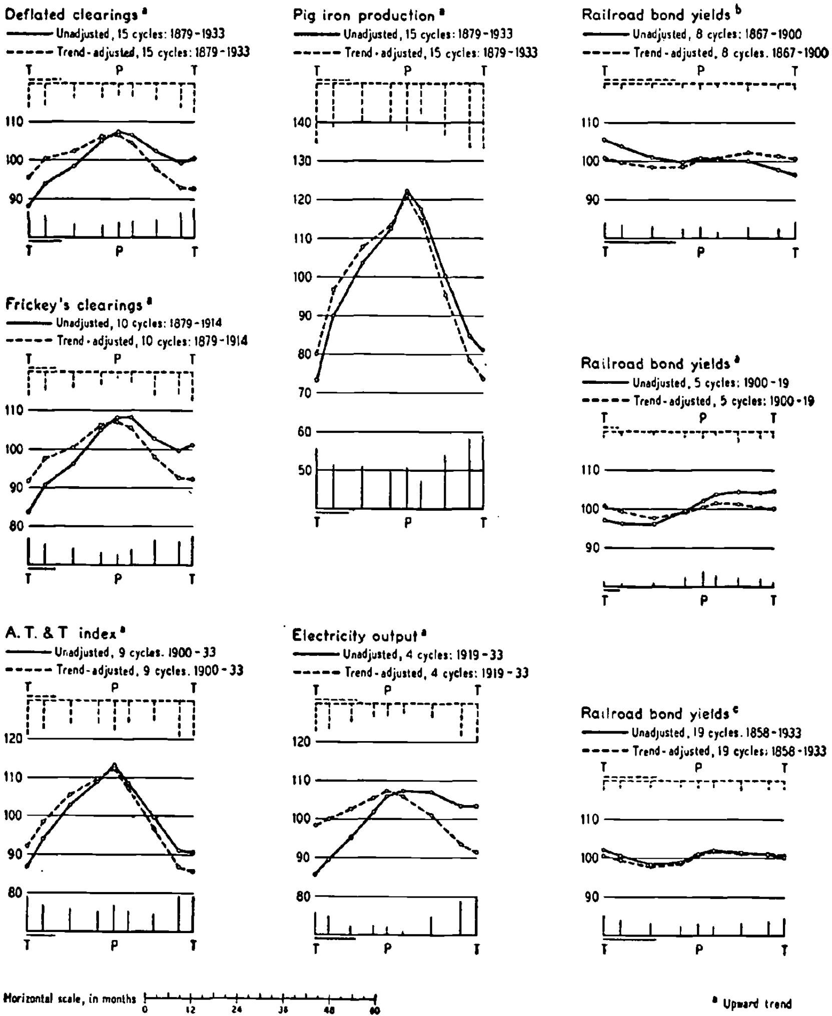
Average Reference-cycle Patterns of Unadjusted and Trend-adjusted Data
Six American Series

Series and form of data	Direction of trend	No. of reference cycles	Average in reference-cycle relatives at stage								
			I	II	III	IV	V	VI	VII	VIII	IX
			Initial trough (3 mos.)	Expansion			Peak (3 mos.)	Contraction			Terminal trough (3 mos.)
				First third	Middle third	Last third		First third	Middle third	Last third	
DEFLATED CLEARINGS											
Unadjusted	Up	15	88.1	94.0	98.4	105.2	107.5	106.7	102.3	99.5	100.6
Adjusted	....	15	95.6	100.4	102.2	106.2	106.7	104.5	97.9	93.0	92.9
FRICKEY'S CLEARINGS											
Unadjusted	Up	10	83.5	90.7	96.1	104.9	108.1	108.2	102.7	99.7	101.1
Adjusted	....	10	91.6	97.6	100.2	106.0	107.0	105.6	97.9	92.5	92.2
A.T.&T. INDEX											
Unadjusted	Up	9	86.8	94.1	102.7	109.1	113.2	108.8	99.8	91.1	90.8
Adjusted	....	9	92.1	98.5	105.5	109.8	112.7	107.3	96.7	86.8	85.7
PIG IRON PRODUCTION											
Unadjusted	Up	15	73.3	90.0	103.5	112.5	122.2	117.6	100.4	84.8	81.1
Adjusted	....	15	80.0	96.7	107.6	113.3	121.0	114.8	95.4	78.4	73.7
ELECTRICITY OUTPUT											
Unadjusted	Up	4	85.7	89.5	95.3	101.8	105.9	107.3	107.0	103.4	103.4
Adjusted	....	4	98.2	100.0	102.5	105.3	107.2	105.9	101.0	93.6	91.5
RAILROAD BOND YIELDS											
Unadjusted	Up	5	97.0	96.3	96.1	99.7	102.0	103.9	104.3	104.2	104.7
Adjusted	....	5	100.8	99.4	97.8	99.5	100.4	101.7	101.3	100.2	100.1
RAILROAD BOND YIELDS											
Unadjusted	Down	8	105.4	103.7	100.9	99.7	100.6	100.2	100.1	97.9	96.7
Adjusted	....	8	100.6	99.8	98.4	98.7	100.2	100.7	102.1	101.3	100.8
RAILROAD BOND YIELDS											
Unadjusted	Oscil.	19	102.0	100.5	98.3	98.9	101.0	102.0	101.5	101.1	100.2
Adjusted	....	19	100.3	99.3	97.9	98.8	100.8	101.9	101.4	101.4	100.8

The patterns of the adjusted data were computed by our standard technique from trend relatives. See Chart 38 for the periods covered.

CHART 38

Average Reference-cycle Patterns of Unadjusted and Trend-adjusted Data
Six American Series


Deflated clearings[a]
Pig iron production[a]
Railroad bond yields[b]
Frickey's clearings[a]
Railroad bond yields[a]
A. T. & T index[a]
Electricity output[a]
Railroad bond yields[c]

See Table 99. For explanation of chart, see Ch. 5, Sec. VII.

[a] Upward trend
[b] Downward trend
[c] Oscillatory trend

However, when the periods of rising and falling trends in bond yields are combined, the response is as clear in the unadjusted as in the adjusted data.[26]

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[26] In several series the standing of the cyclical patterns of the adjusted data is lower in stage IX than in stage I. This can be explained partly by the tendency of the trend lines to exaggerate the contraction of 1929–33. But the drift of the cyclical patterns is only a rough guide to the average intra-cycle trend. If, for example, the average value of a series is the same in successive cycles, the average standings in stages I and IX will still differ, except when the standing at the terminal trough of the last cycle is the same as the standing at the initial trough of the first cycle.

TABLE 100

Conformity to Business Cycles of Unadjusted and Trend-adjusted Data
Six American Series

Series and form of data	Direction of trend	No. of reference cycles[a]	Stages matched with reference expansion[b]	Av. change per month in reference-cycle relatives during stages matched with reference			Index of conformity to reference				
									Cycles taken		
				Expansions	Contractions	Cycles[c]	Expansions	Contractions	From trough to trough	From peak to peak	Both ways[d]
DEFLATED CLEARINGS											
Unadjusted	Up	15	VIII-V	+0.78	-0.50	-1.28	+100	+73	+87	+86	+86
Adjusted	....	15	VIII-V	+0.43	-0.85	-1.29	+100	+87	+87	+86	+86
FRICKEY'S CLEARINGS											
Unadjusted	Up	10	VIII-V	+0.94	-0.63	-1.57	+100	+60	+100	+100	+100
Adjusted	....	10	VIII-V	+0.55	-1.01	-1.56	+100	+100	+100	+100	+100
A.T.&T. INDEX											
Unadjusted	Up	9*	I-V	+1.33	-1.17	-2.50	+100	+80	+100	+100	+100
Adjusted	....	9*	I-V	+1.08	-1.39	-2.47	+100	+100	+100	+100	+100
PIG IRON PRODUCTION											
Unadjusted	Up	15	I-V	+2.26	-2.27	-4.53	+100	+100	+100	+100	+100
Adjusted	....	15	I-V	+1.93	-2.57	-4.50	+100	+100	+100	+100	+100
ELECTRICITY OUTPUT											
Unadjusted	Up	4	I-VI	+0.98	0.00	-0.98	+100	0	+100	+100	+100
Adjusted	....	4	I-VI	+0.31	-0.62	-0.93	+100	+50	+100	+100	+100
RAILROAD BOND YIELDS											
Unadjusted	Up	5	III-VI	+0.45	+0.03	-0.42	+100	-40	+100	+100	+100
Adjusted	....	5	III-VI	+0.24	-0.17	-0.41	+100	+100	+100	+100	+100
RAILROAD BOND YIELDS											
Unadjusted	Down	6	III-VI	0.00	-0.25	-0.24	0	+100	+100	+50	+80
Adjusted	....	6	III-VI	+0.19	-0.07	-0.26	+100	+67	+100	+100	+100
RAILROAD BOND YIELDS											
Unadjusted	Oscil.	19*	III-VI	+0.24	-0.17	-0.41	+47	+30	+79	+58	+68
Adjusted	....	19*	III-VI	+0.26	-0.14	-0.40	+74	+70	+79	+68	+74

[a] An asterisk means that an additional reference contraction at the beginning of the series is covered by the contraction and full-cycle indexes. For reasons stated in note 29, only 6 cycles are covered in the period of secular decline of bond yields, whereas 8 cycles are covered in Chart 38. Subject to these exceptions, the periods covered by this table are shown in Chart 38.

[b] See note 31.

[c] Difference between reference contraction and reference expansion (see Table 47, col. 8).

[d] Described in the text as the 'final full-cycle index'.

As explained in Chapter 5, we measure the conformity of each time series to reference expansions, to reference contractions, and to full cycles. The 'expansion' measures report merely the rate at which, or the regularity with which, a series rose or fell during reference expansions, or during the reference-cycle stages matched with reference expansions. The 'contraction' measures provide similar information for reference contractions. It is obvious from their nature that the removal of trend must modify the rates of change during reference expansions and contractions, and that it is likely to change also the conformity indexes for these phases. But the measures of full-cycle conformity contain an automatic adjustment for secular trends, since a decline in the rate of increase from reference expansion to reference contraction is treated the same way as a rise during reference expansion followed by an actual decline during reference contraction. We should find, therefore, that formal removal of secular trends from the original data has slight influence on the measures of conformity to full business cycles.

Imagine a series built up by adding ordinates of a linear trend to a series of monthly cyclical values. This trend must increase the *rate* of change, in the original units, both during the stages matched with reference expansions and during the stages matched with reference contractions. Further, the rate for 'expansion' segments must be increased by exactly the same amount as the rate for 'contraction' segments, since the slope of the trend is assumed to be constant. If the trend is downward instead of upward, the two rates must be reduced the same amount. It follows that the influence of the trend can be wiped out by striking a difference between the two rates. We record this difference, expressed in reference-cycle relatives, in column (8) of our standard Tables R3 and R4, and treat it as an indicator of conformity to full reference cycles.[27] Of course, if the trend is nonlinear this indicator is not completely free from the influence of trend. But since the average slope of the secular trend of the original data is unlikely to differ appreciably during the two segments into which reference cycles are broken, the differential between the two rates of change should be substantially, if not entirely, free from trend.

Table 100 shows the influence of secular trends on average rates of change during the reference cycles covered by our test series. The adjusted data are analyzed in the form of trend relatives, and therefore do not correspond precisely to the theoretical model in the preceding paragraph. Nevertheless, in every comparison an upward trend increases the average rise per month during the stages matched with reference expansions to about the same degree as it reduces the fall per month during the stages matched with reference contractions.[28] A declining trend has similar effects in the opposite direction.[29] Hence the influence of the secular trend tends to cancel out in the average change per month referring

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[27] See Ch. 5, Sec. IX-X.

[28] If a series bears an inverted relation to business cycles, an upward trend will reduce the average fall per month during the stages matched with reference expansions to about the same degree as it increases the rise per month during the stages matched with reference contractions.

[29] Only 6 cycles are included in the period of secular decline of bond yields, whereas 8 cycles are covered in Table 99 and Chart 38. The reason for the difference is that Macaulay's trend line is somewhat undulatory even within this period. The rise of the trend line from 1888 to 1892 and the resumption of the rise early in 1900 have a negligible influence on the cyclical measures previously analyzed, but they confuse the conformity analysis. Hence we limit the conformity measures to cycles during which the trend declined unequivocally, that is, the four cycles from 1867 to 1888 and the two from 1891 to 1897. Since stages III-VI are matched with reference expansions, the periods actually analyzed are shifted half a phase forward from the standard reference cycles.

to full cycles, whether the trend rises, falls, or changes direction within the period covered by a series.

In making the index of conformity to full reference cycles, we take severer precautions to control the influence of secular trends. The method is fully described in Sections IX and X of Chapter 5, but a few additional remarks may help to clarify the exact mathematical nature of this index. Assume that absolute deviations from trend show zero conformity to one or more business cycles; that is, that the average change per month is some constant c during both reference expansion and contraction. Then the addition of a linear trend must leave the conformity zero, whether the reference cycles are taken from trough to trough or from peak to peak; for if the slope of the trend is m, the change per month becomes $c + m$ during both expansion and contraction. A concave trend in relation to the axis of time will make the conformity of the trend-cycle composite positive if reference cycles are marked off by troughs, and inverted if the cycles are marked off by peaks; for the change per month during successive reference phases is now $c + d_1$, $c + d_2$, $c + d_3$, etc., where the d's are successively smaller algebraically. Similarly, a convex trend will make the conformity inverted for reference cycles marked off by troughs, and positive for cycles marked off by peaks. But these opposite 'biases' tend to offset one another in the final full-cycle index, since we take the cycles both from trough to trough and from peak to peak in making this index.

The result is similar when the cyclical and trend components are combined by multiplication, instead of addition. If the standings at the three stages from which conformity is measured fall on a straight line, with the equation $a_1 + b_1x$, the insertion of a linear trend, with the equation $a_2 + b_2x$, will make the conformity of the trend-cycle composite positive or inverted for cycles marked off by troughs, and inverted or positive for cycles marked off by peaks, according as b_1b_2 is minus or plus. The insertion of convex or concave trends will now produce one result, now another. But in these instances as when the trend is linear, excepting occasional shifts from concavity to convexity or vice versa in the trend-cycle composite,[30] the 'bias' for reference cycles marked off by troughs must oppose the 'bias' for reference cycles marked off by peaks. Hence the final full-cycle index of unadjusted data should be practically independent of the trend.

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[30] That is one reason why the index for cycles taken by troughs and the index for cycles by peaks may differ in practice. But a difference might arise merely from rounding numbers, when the indexes are computed from reference-cycle relatives instead of the original data; or from the fact that the index for reference cycles taken by troughs cannot cover exactly the same period as the index for cycles by peaks.

Still another factor may be illustrated by a hypothetical example. Assume that a series has a horizontal 'secular trend'; that each of its specific cycles starts at a reference trough, rises throughout one reference cycle, and falls throughout the next reference cycle; and that the second differences of the monthly values of each specific cycle, considered as a discrete unit, are uniformly minus. In this case the conformity to reference cycles taken by troughs will be invariably positive, while the

In our test series the final full-cycle index is actually the same or almost the same in the unadjusted as in the adjusted data (Table 100). On the other hand, the indexes of conformity to reference expansions and contractions bear clearly the stamp of secular trends. In the one sample of a declining trend the expansion index is lower and the contraction index higher for unadjusted than for adjusted data. In the six samples of a rising trend the contraction index made from unadjusted data is lower than the index made from adjusted data in five series and the same in one. Since the expansion indexes of our several samples of a rising trend are all +100 in the adjusted data, they cannot be higher in the unadjusted data. Barring such limiting cases, secular trends must impart opposite biases to the expansion and contraction indexes. A rising trend tends to increase positive conformity to reference expansions and to diminish positive conformity to reference contractions. A declining trend tends to have opposite effects.[31]

VI Variability of Cyclical Measures

A striking feature of Charts 37-38 is that the cyclical patterns of different series are more alike when made from adjusted data than when made from unadjusted data. The same is true of the separate segments of bond yields, our one series subject to different trends. Thus, in the adjusted data, the reference-cycle pattern of deflated clearings does not differ much from that of electric power production. But the unadjusted data indicate that, in the periods represented, power production rose more vigorously on the average during reference expansions than the volume of clearings;

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conformity to cycles taken by peaks will be alternately inverted and positive. The difference between the two conformity indexes cannot be ascribed to the secular trend, since the trend is assumed to be horizontal. Of course, a change of assumptions will change the conclusion. Thus if we regard the long specific cycles as the 'secular trend' and assume that the 'short-run' cyclical component is zero, we must say that the 'trend' is solely responsible for the difference between the two indexes; also, that the 'bias' of the index on a peak basis opposes only in part the 'bias' of the index on a trough basis, the reason being that the timing of the oscillatory 'trend' is correlated with the timing of business cycles.

[31] The comparisons between conformity measures of adjusted and unadjusted data in Table 100 are based on the division of reference cycles that seemed most appropriate for the unadjusted data of each series. Consequently, the comparisons reflect the influence of the trend factor alone. But they do not necessarily reflect the *full* influence of the trend, since the trend may modify the stages characteristic of expansion and contraction. If the conformity measures of the adjusted data were made on that division of reference cycles which seemed best for these data, the comparison would reflect the full influence of the trend; but it would reflect also nonsecular factors whenever the division of the reference cycles of adjusted data differed from that of unadjusted data.

As we have seen in Sec. III, the shifts in cyclical timing produced by secular trends are, usually, not very large in our sample. We should expect therefore the division of reference cycles in our standard Table R4 to be similar for the adjusted and unadjusted data. In fact, a difference arises only in Frickey's clearings and electricity output. In Frickey's clearings the expansion stages of the adjusted data are I-V; the average rates of change on this basis are + 0.65, − 0.83, and − 1.48, and the conformity indexes are all + 100. In electricity the expansion stages of the adjusted data are also I-V; the average rates of change are + 0.44, − 0.56, − 1.00; the conformity indexes are + 100 except the contraction index, which is + 50.

also, that its declines during reference contractions were milder. Again, the reference-cycle pattern of the adjusted data on bond yields when the trend is upward is a rough duplicate of the pattern when the trend is downward; but in the unadjusted data the former pattern shows virtually no decline during the stages matched with reference contractions, while the latter shows virtually no rise during the stages matched with reference expansions.

The vertical lines on the charts representing average deviations of the cyclical patterns are in some instances longer in the unadjusted data, in other instances shorter. But when averages are struck, it appears that the elimination of secular trends usually reduces the differences among the successive cycles. In eight out of ten comparisons the average deviations of the patterns of the adjusted data are smaller than those of the unadjusted data (Table 101). Also, the removal of trends reduces the differences among the durations of specific cycles in every series covered by our tests, and among the amplitudes in every series except one.

It seems, therefore, that if we removed secular trends completely from the original data at the start of the analysis we would find that the variability of cyclical measures within a series is usually reduced. And this is likely to mean that the scope and frequency of secular changes in cyclical measures would also be reduced. Table 102 illustrates the point for bond yields. For the present purpose we may consider a shift in average cyclical

TABLE 101

Average Deviations from Average Measures of Cyclical Behavior
Five American Series, Unadjusted and Trend-adjusted

Series and form of data	Number of		Average deviation		Average of average deviations from the nine average standings of	
	Specific cycles	Reference cycles	Duration of specific cycles	Rise & fall of specific cycles	Specific-cycle patterns	Reference-cycle patterns
DEFLATED CLEARINGS						
Unadjusted	15	15	11.6	13.8	5.7	5.2
Adjusted	15	15	10.2	7.9	4.7	4.9
FRICKEY'S CLEARINGS						
Unadjusted	11	10	6.1	10.0	6.3	5.2
Adjusted	11	10	5.9	8.6	4.1	4.6
A.T.&T. INDEX						
Unadjusted	9	9	7.6	15.4	6.6	7.1
Adjusted	9	9	7.4	14.9	6.8	7.2
PIG IRON PRODUCTION						
Unadjusted	15	15	9.9	26.8	11.9	12.9
Adjusted	15	15	9.1	29.5	11.3	12.4
RAILROAD BOND YIELDS						
Unadjusted	20	19	12.6	7.2	3.2	3.6
Adjusted	21	19	9.7	6.7	1.6	2.0

The average deviations of the durations are expressed in months; the others in cycle relatives. See Table 85 for the periods covered by the specific cycles, Chart 38 by the reference cycles. Electricity output is omitted because of its brief statistical record.

TABLE 102

Average Measures of Specific Cycles in Railroad Bond Yields, United States
Unadjusted and Trend-adjusted Data, 1868–1899 and 1899–1918

Measure and form of data (1)	Downward trend 1868–1899		Upward trend 1899–1918		Excess of (5) over (3) (6)
	Number of specific cycles (2)	Average (3)	Number of specific cycles (4)	Average (5)	
DURATION OF EXPANSIONS					
Unadjusted	8	16.8	5	35.4	+18.6
Adjusted	8	20.6	7	13.7	−6.9
DURATION OF CONTRACTIONS					
Unadjusted	8	29.6	5	11.2	−18.4
Adjusted	8	25.8	7	19.7	−6.1
AMPLITUDE OF RISE					
Unadjusted	8	6.3	5	13.3	+7.0
Adjusted	8	8.6	7	6.7	−1.9
AMPLITUDE OF FALL					
Unadjusted	8	14.6	5	6.3	−8.3
Adjusted	8	8.4	7	7.1	−1.3
LAG AT REFERENCE PEAKS					
Unadjusted	7	8.6	4	7.5	−1.1
Adjusted	7	9.7	6	4.5	−5.2
LAG AT REFERENCE TROUGHS					
Unadjusted	7	15.6	4	0.8	−14.8
Adjusted	7	15.6	6	6.7	−8.9

The lags and durations are expressed in months, the amplitudes in specific-cycle relatives. The entries for timing in col. (2) and (4) show the number of observations included in the averages. These are not consistent with the timing averages in Table 87. The averages in Table 87 are based on turns that correspond in the unadjusted and adjusted data, whether or not they correspond to the reference dates; the averages in this table include turns that correspond to the reference dates, whether or not they correspond in the unadjusted and adjusted data; also, the timing of the specific-cycle trough of June 1899 (both unadjusted and adjusted data) is included in the averages of both periods in Table 87, but excluded from both in this table.

behavior from 1868–99, when the trend of bond yields was downward, to 1899–1918, when the trend was upward, as indicating a secular change in cyclical behavior. The table shows that the differences between average cyclical measures of the two periods run smaller in the adjusted than in the unadjusted data; in other words, secular changes in the cyclical measures are less pronounced in the trend-adjusted data.[32] But the measures of timing at reference peaks convey a warning that the elimination of secular trends from the original data will not always make the cyclical measures for different periods more alike.

This warning is important. A fitted trend line may segregate the secular from the cyclical component in such fashion that cyclical measures made from the trend-adjusted data are virtually free from secular change. When this happens, as in the amplitudes of Table 102, we may say that the secular changes which appear in the cyclical measures made from the unadjusted data are due to the trend. But the trend line that effects this

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[32] Of course, the secular changes may or may not be 'statistically significant'; cf. Ch. 10. Further, they may be interpreted as indicating what happens upon passing from a contraction phase to an expansion phase of a long cycle; on this, see Ch. 11, especially Sec. III.

segregation with respect to some features of cyclical-secular change may be less successful with respect to other features. That is, a secular change may still appear in certain cyclical measures of the trend-adjusted data, and it may be smaller or larger than in the corresponding measures of the unadjusted data. For example, there is a decline from the first to the second period in the duration of contractions in the adjusted data; the declining trend of the unadjusted data during the first period tends to lengthen, and the rising trend during the second period tends to shorten, the contractions; hence the decline in the duration of contractions is larger in the unadjusted than in the adjusted data. A rather different result appears in the measures of timing at reference peaks. According to the adjusted data, the average lag at peaks declines from 9.7 months in the first period to 4.5 in the second; this change is counteracted by the tendency of the declining trend in the unadjusted data during the first period to reduce lags (or increase leads) and of the rising trend during the second period to increase lags (or reduce leads); the net result is that the timing of the unadjusted data is nearly the same in the second period as in the first.

VII The Time Unit and Trend Adjustments

In the preceding chapter we have shown how cyclical measures depend upon the time unit in which the observations are given. In this chapter we have shown how cyclical measures depend upon the retention or elimination of secular trends. These influences cross one another. Other things being equal, the steeper the trend the greater are the changes in cyclical measures induced by a shift from monthly to annual data. And the coarser the time unit the greater are the changes in cyclical measures induced by the elimination of a given trend. If we had carried through the analysis of the preceding chapter on the basis of trend-adjusted data, we would have found less startling differences between cyclical measures made from monthly and annual data. On the other hand, if the analysis of this chapter had been based on annual data, we would have established larger effects of secular trends.

Assume that the secular trend is removed from a monthly time series characterized by a rising trend. This operation is not likely to alter the number of specific cycles appreciably, since the amplitude of the 'cyclical component' is usually large compared with the amplitude of the 'trend component'. True, a retardation of increase in the original data is now and then converted into an actual decline that we must count as a cyclical movement, but this tendency is offset by the conversion under certain circumstances of a cyclical rise in the original data into a retarded decline in the adjusted data.[33] If, now, the original and trend-adjusted monthly series are put into annual form, some specific cycles are likely to be lost

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[33] See above, pp. 273-5.

by each. But since the process of trend adjustment tends to lengthen and intensify cyclical contractions, fewer cycles are likely to be lost in the annual summations of the trend-adjusted data than in the annual summations of the original data. The brief and mild contractions that cannot survive in annual summations of the original data stand a good chance of survival in annual summations of the trend-adjusted data.

The dependence of the number of specific cycles upon the form of a time series may be explained in another way. A shift from monthly to annual data has no effect on the 'secular component' of a time series, but it dampens the 'cyclical component', so that a considerable fraction of the cyclical movements are converted into mere retardations of growth if the trend is upward or into mere retardations of decline if the trend is downward. These hidden cycles stand an excellent chance of coming to the surface again when secular trends are removed. For example, three cyclical contractions in pig iron production disappear when monthly figures unadjusted for trend are converted into calendar-year sums, but two of the three contractions are recovered when the trend is removed from the annual sums. Similar results are obtained in other series (Table 103). The number of specific cycles in the unadjusted forms of the test series used in this chapter is 72 in monthly but only 53 in annual data, while the corresponding numbers in the trend-adjusted data are 73 and 67.[34]

TABLE 103

Number of Specific Cycles in Unadjusted and Trend-adjusted Data
Six American Series, Monthly and Annual

Series	Number of specific cycles			
	Monthly data		Annual data	
	Unadjusted	Adjusted	Unadjusted	Adjusted
Deflated clearings	15	15	10	13
Frickey's clearings	11	11	7	10
A.T.&T. index	9	9	9	9
Pig iron production	15	15	12	14
Electricity output	2	2	1	2
Railroad bond yields	20	21	14	19
Total	72	73	53	67

The numbers represent the full cycles within the periods covered by the monthly data, as shown in Table 85. In pig iron production there is no trough in either annual series corresponding to the monthly trough in 1879; hence the count of annual cycles starts in 1884 and ends in 1932.

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[34] The trends removed from the annual (calendar-year) and from the monthly data are, of course, the same.

The proper way to obtain annual trend-adjusted figures is to remove the trend from annual data; or to convert monthly figures adjusted for trend, but not for seasonal, into annual sums or averages. (The last statement implies a 'relative' seasonal, the type we have used.) We followed the latter practice in iron production, deflated clearings, and bond yields. In the other series we took annual sums of monthly figures adjusted for both trend and seasonal; but it is practically certain that this change in method has no influence on the count of their specific cycles.

TABLE 104

Cyclical Measures of Unadjusted and Trend-adjusted Data
Two American Series, Monthly and Annual

Measure	Deflated clearings				Pig iron production			
	Monthly		Annual		Monthly		Annual	
	Unadj.	Adj.	Unadj.	Adj.	Unadj.	Adj.	Unadj.	Adj.
NO. OF CYCLES								
Specific	15	15	10	13	15	15	12	14
Reference	15	15	15	15	15	15	15	15
AV. DURATION OF SPECIFIC CYCLES[a]								
Expansion	32.6	25.6	49.2	25.8	28.8	25.1	31.0	22.3
Contraction	11.4	17.9	16.8	24.9	14.5	18.1	17.0	18.0
Full cycle	44.0	43.5	66.0	50.8	43.3	43.2	48.0	40.3
AV. AMPLITUDE OF SPECIFIC CYCLES[b]								
Rise	26.9	15.6	29.7	11.4	62.1	52.2	41.1	27.2
Fall	13.4	17.4	9.8	13.8	54.8	57.7	32.4	32.8
Rise & fall	40.2	33.0	39.5	25.1	116.8	109.9	73.5	60.0
Rise per month[c]	0.8	0.8	0.7	0.5	2.4	2.5	1.5	1.2
Fall per month[c]	1.9	1.8	0.4	0.5	4.7	4.0	2.0	1.8
Rise & fall per month[c]	0.9	0.8	0.6	0.5	2.9	2.7	1.7	1.6
AV. LEAD (−) OR LAG (+)[a]								
At reference peaks	+3.2	−1.7	+1.8	−5.1	+1.9	−1.3	+0.3	−2.4
At reference troughs	−5.8	−3.4	−3.8	−1.9	−3.4	−2.9	−3.7	−3.1
CONFORMITY TO BUSINESS CYCLES								
Expansion stages[d]	VIII-V	VIII-V	I-V	I-V	I-V	I-V	I-V	I-V
Av. change per month[e] during stages matched with reference								
Expansions	+0.78	+0.43	+0.65	+0.29	+2.26	+1.93	+1.46	+1.08
Contractions	−0.50	−0.85	−0.17	−0.53	−2.27	−2.57	−1.65	−1.99
Index of conformity to reference								
Expansions	+100	+100	+100	+73	+100	+100	+100	+100
Contractions	+73	+87	+7	+73	+100	+100	+73	+100
Cycles	+86	+86	+86	+93	+100	+100	+100	+100

For the periods covered, see Charts 39-42. The average timing measures in this table include turns corresponding to reference dates; hence the apparent inconsistency with the averages in Table 87.

[a] In months.

[b] In specific-cycle relatives.

[c] Unweighted average.

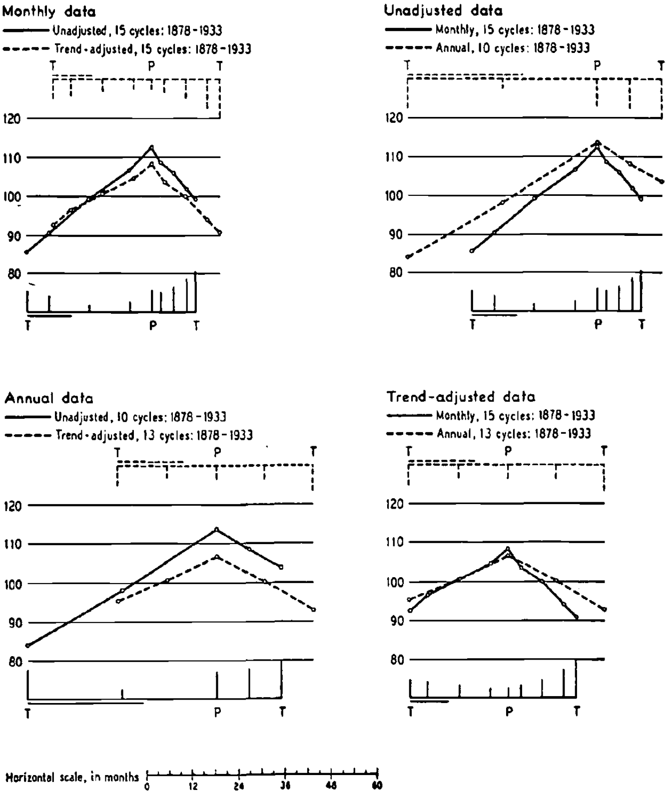
[d] Matched in every instance with reference expansion.

[e] In reference-cycle relatives.

In Table 104 and Charts 39-42 we compare the cyclical measures of different forms of the data on deflated clearings and iron production. Although these series are subject to peculiarities that blur some theoretically interesting effects, they help to round out the preceding argument. Whether a series is monthly or annual, the removal of trends has similar effects on the timing of specific cycles, the duration of expansions and contractions, their amplitudes, and the conformity to reference expansions and contractions. But since trend adjustments influence the number of specific cycles in annual data much more than the number in monthly data, the effects of trend adjustments are likely to be greater on annual than on monthly cyclical measures, especially on the average duration of specific cycles and the indexes of conformity to reference expansions and contractions. In general, the removal of trends will tend

CHART 39

Average Specific-cycle Patterns
Bank Clearings outside New York City, Deflated
Unadjusted and Trend-adjusted Data, Monthly and Annual



For explanation of chart, see Ch. 5, Sec. VI.

to alter these measures of annual data in the direction of the measures of monthly data unadjusted for trend. Consequently, an investigator restricted to annual data is likely to make better estimates of the number of specific cycles in the underlying monthly data, and of their conformity to reference expansions or contractions,[35] by adjusting the annual figures for trend than by using them as they come. Further, an investigator is likely to have somewhat better success in estimating cyclical measures of monthly trend-adjusted data from annual trend-adjusted data than in estimating cyclical measures of monthly unadjusted data from annual unadjusted data. But no device on this plane can counteract the coarseness of annual figures in measuring cyclical timing, their dampening effect on cyclical amplitudes, or their obfuscation of cyclical patterns.

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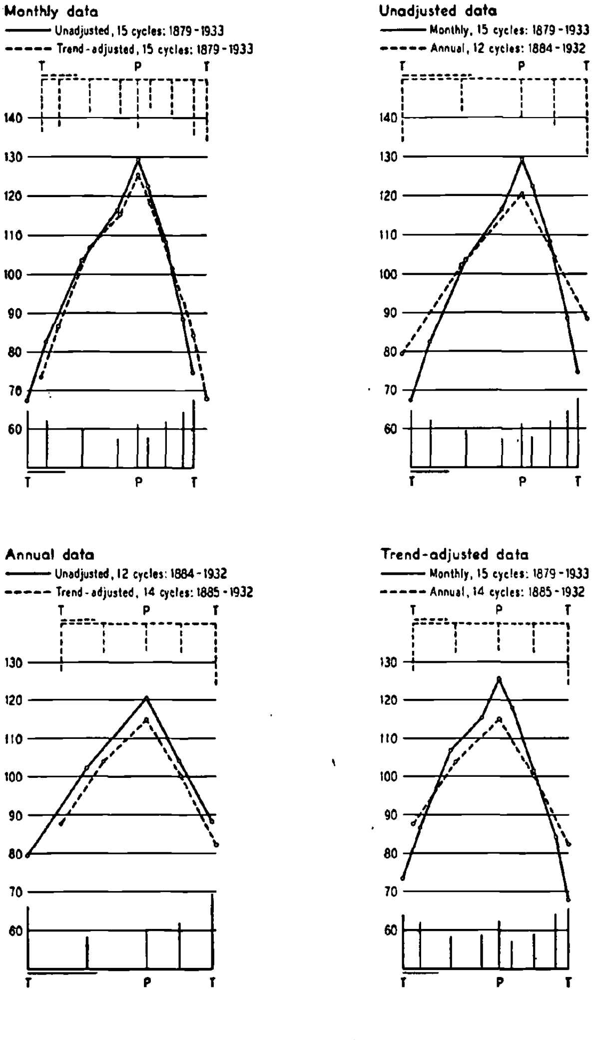
[35] To reference expansions if the trend is downward, to reference contractions if the trend is upward.

CHART 40

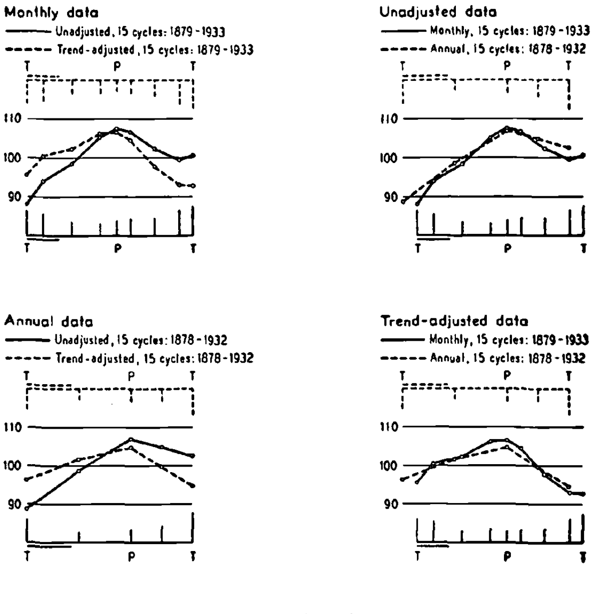
Average Specific-cycle Patterns
Pig Iron Production, United States
Unadjusted and Trend-adjusted Data, Monthly and Annual



For explanation of chart, see Ch. 5, Sec. VI.

CHART 41

Average Reference-cycle Patterns
Bank Clearings outside New York City, Deflated
Unadjusted and Trend-adjusted Data, Monthly and Annual



For explanation of chart, see Ch. 5, Sec. VII.

VIII Conclusions

We have seen that cyclical measures of different series, as well as cyclical measures of the same series, tend to be more alike when made from trend-adjusted data than when made from unadjusted data. To us that is a disadvantage of trend adjustments. The variations of cyclical behavior among and within series count in the interplay of forces that produce the business cycles of experience, and we therefore wish to preserve them. An investigator who seeks to gauge the role played by railroad construction, government spending, installment credit, or agricultural production in past business cycles and their probable role in future business cycles cannot remove secular trends without sacrificing the main part of his problem. It may be legitimate for students concerned with secular trends to put cyclical fluctuations out of sight, but students of cyclical behavior cannot take similar liberty with secular trends. If the trends characteristic of different business activities are set aside, inquiry is apt to be limited to the tendency of economic processes to fluctuate in unison. Our aims, as indicated in Chapter 1, are more ambitious.[36]

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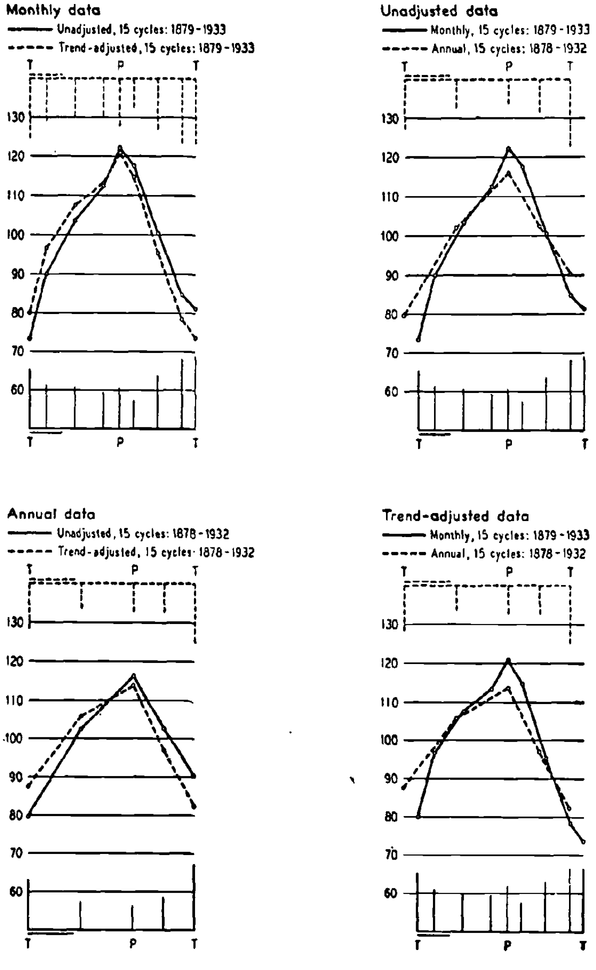
[36] See also Ch. 3, Sec. I-III, and Ch. 10, Sec. VIII.

CHART 42

Average Reference-cycle Patterns
Pig Iron Production, United States
Unadjusted and Trend-adjusted Data, Monthly and Annual



Monthly data

Unadjusted data

Annual data

Trend-adjusted data

For explanation of chart, see Ch. 5, Sec. VII.

At the same time, as we have argued in Chapter 3, the retention of intra-cycle trends is a disadvantage when the task is chiefly to describe how business cycles manifest themselves in different activities. But this chapter has demonstrated that the disadvantage is less serious than might be supposed. Since secular trends exercise a systematic influence on nearly all of our cyclical measures, we can take rough account of their influence whenever desirable. Sometimes, as in indexes of conformity to full business cycles, no qualification is necessary. In most measures, the nature of the allowance depends on the direction of the trend; while the magnitude of the allowance depends partly upon the steepness of the trend, partly on other factors which may be of equal or greater importance. For instance, in judging the influence of trends on average measures of timing or duration of expansions and contractions, it is necessary to note the shapes of the specific cycles in the neighborhood of turning points. Again, in judging the influence of trends on amplitude measures, note must be taken of the curvature of the trend, as well as its direction and steepness; also the duration of expansions relatively to contractions, and several other factors. The main considerations that are relevant to each cyclical measure have been set out in the body of this chapter. Of course, judgments of the influence of trends are bound to be rough. But they would not be highly precise even if trend lines were fitted and removed by formal methods. As every statistician knows, secular trends of time series are rarely, if ever, susceptible of precise and objective determination. There is an arbitrary element not only in the choice of the trend line, but in every other step of trend adjustment: the period used in fitting the trend, the time unit in which the data are expressed for this purpose, the method used to fit the selected trend, and the method used to remove the trend.

Our standard Table S3 supplies the essential facts concerning the secular movements of each series. By studying this table in conjunction with other measures, we can usually judge roughly what contribution secular trends make to our measures, and thus allow for the deficiencies of our method in describing the scope of cyclical fluctuations. In annual series this check is less effective than in monthly. But annual data at best are very crude approximations for our purposes. Hence the reasons for trend adjustment, although stronger in annual than in monthly data, do not seem to us sufficient to justify the additional cost. We repeat, however, that if the resources at our disposal permitted it, we would analyze all series presented in the following monographs, or at least the more important ones, in both unadjusted and trend-adjusted forms. And we would feel still better equipped for the work ahead if we could supplement analyses of data adjusted and unadjusted for trend by analyses of data freed from erratic flutterings. That we cannot do. As a substitute we present in the following chapter sample measurements of the effects that 'erratic' movements exercise on our averages.