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National Bureau of Economic Research

Volume Title: Measuring Business Cycles

Volume Author/Editor: Arthur F. Burns and Wesley C. Mitchell

Volume Publisher: NBER

Volume ISBN: 0-870-14085-X

Volume URL: <http://www.nber.org/books/burn46-1>

Publication Date: 1946

Chapter Title: Preliminary Sketch of the Statistical Analysis

Chapter Author: Arthur F. Burns, Wesley C. Mitchell

Chapter URL: <http://www.nber.org/chapters/c2981>

Chapter pages in book: (p. 23 - 36)

## CHAPTER 2

# Preliminary Sketch of the Statistical Analysis

**O**UR PLAN of measuring the numerous features of cyclical behavior is so much of one piece that some understanding of the whole is needed for full comprehension of the parts. Hence we preface the detailed exposition of the method by a preliminary sketch designed to bring out its general characteristics. Even the most careful reader will save time by viewing the plan as a whole before plunging into the detailed explanations of its parts.

### I Basic Features of the Analysis

The scheme of analysis has two parts. Primarily we seek to determine how each important economic process of which we possess a statistical record behaves during the periods occupied by business cycles. For that purpose we break every time series into segments that coincide with the cycles in general business activity in the country to which the series refers. By analyzing on this basis over one thousand time series representing a wide variety of economic processes in four countries, we are able to get a fairly comprehensive picture of what happens during a business cycle.

While our interest centers in this picture, it cannot stop there. Almost all our series exhibit cyclical fluctuations, but these fluctuations bear widely different relationships in time to business cycles. The dates on which a series makes its cyclical turns may differ little from the turning dates in general business, may usually lead or lag behind them by brief or by considerable intervals, or may have no regular relationship to them. Now the course of business cycles is influenced by cyclical fluctuations in every economic process, whether or not they synchronize with the general tides of activity. Hence, to lay a satisfactory foundation for a theory of business cycles, our analysis based upon business-cycle periods must be supplemented by studies of the cycles peculiar to each series.

The following sections explain and illustrate most of the measures we make, define the technical terms we use, and give a sample of the standard charts upon which we rely heavily in presenting results.

## II Reference Dates, Reference Cycles and Specific Cycles

To learn how different economic processes behave in respect of business cycles, their movements must be observed during the revivals, expansions, recessions, and contractions in general business activity. Before we can begin observing we must mark off these periods. To that end we have made for each of the four countries a table of 'reference dates', showing the months and years when business cycles reached troughs and peaks. These tables were based first upon the business annals compiled for the National Bureau by Willard L. Thorp; then we refined, tested, and at need amended the dates by studying statistical series. The turning points of the cyclical movements in general business activity can be made more precise as the field covered by economic statistics expands. Hence we have more confidence in the reference dates for the United States than in those for the three European countries, and more confidence in the later than in the earlier dates.

After eliminating the seasonal variations of a series, we break it into segments marked off by the reference troughs of the country to which the series relates. Since each segment spans an interval between successive reference troughs, we call it a 'reference-cycle segment', or 'reference cycle' for brevity.<sup>1</sup> Next we compute the average of the monthly values during each 'reference cycle' and convert the data into percentages of this base; these percentages are called 'reference-cycle relatives'. The application of a uniform set of dates to all series for a given country, and the reduction of the original data expressed in diverse units to relatives of their average values during the periods thus marked off, put all the materials into comparable form and enable us to see how different processes behave during successive business cycles.

Next, we look in every series for wave-like movements, the duration of which is of the same order as that of business cycles. We call the cyclical movements peculiar to a series its 'specific cycles'. In most series the dates of the troughs and peaks of the specific cycles are fairly clear, but in some series they are obscured by erratic fluctuations. We mark off the specific cycles by the dates of their turning points as well as we can; compute the average value of the monthly data during each cycle, and convert the monthly data into 'specific-cycle relatives' which correspond in every

<sup>1</sup> We find it convenient to use the term 'reference cycle' in two senses: first, to denote the section of a time series between the dates of successive reference troughs (or peaks), second, to denote the interval between successive troughs (or peaks). The meaning intended should be obvious from the context.

TABLE 4  
Coke Production, United States, 1914-1933  
(Thousands of short tons)

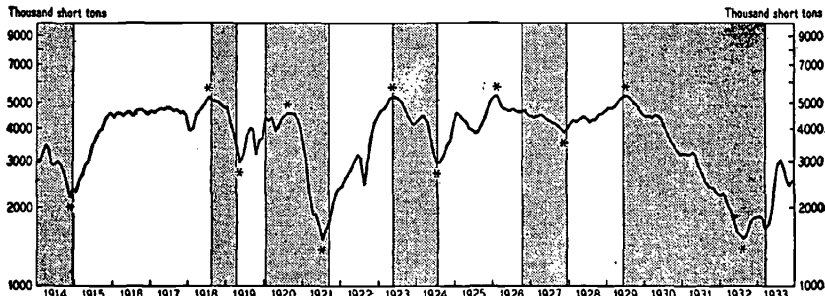
Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
1914	2973	3147	3476	3364	2940	2897	2991	2927	2797	2531	2193	2348
1915	2281	2555	2675	2897	2990	3410	3613	3873	3959	4320	4475	4553
1916	4381	4564	4554	4425	4581	4581	4392	4667	4684	4655	4593	4499
1917	4664	4523	4672	4720	4693	4778	4731	4611	4693	4542	4577	4452
1918	3855	3957	4415	4639	4801	4941	5228	5067	5033	5017	4844	4730
1919	4763	4126	3773	3335	2977	3173	3777	3987	3943	3157	3600	3624
1920	4329	4261	4360	3885	4031	4299	4412	4536	4520	4496	4284	3971
1921	3314	2886	2203	1855	1860	1679	1497	1637	1719	2076	2231	2338
1922	2391	2512	2658	2798	2979	3180	3038	2413	2927	3638	4145	4342
1923	4650	4695	4853	5174	5250	5216	5076	4901	4641	4362	4132	4107
1924	4278	4493	4386	4199	3581	3108	2923	2936	3132	3466	3596	4182
1925	4599	4458	4259	4204	3950	3900	3804	3838	4102	4333	4836	5087
1926	5244	5280	4746	4719	4643	4635	4721	4606	4578	4604	4665	4495
1927	4471	4426	4521	4553	4389	4320	4219	4219	4112	4027	3887	3991
1928	4249	4348	4276	4365	4450	4413	4286	4344	4332	4524	4569	4688
1929	4822	4798	4889	5005	5250	5311	5361	5295	5000	4961	4761	4502
1930	4441	4480	4387	4562	4460	4316	4041	3817	3579	3480	3280	3193
1931	3195	3193	3187	3266	3167	2870	2682	2522	2396	2403	2356	2277
1932	2150	2174	2037	1948	1761	1619	1586	1522	1598	1741	1817	1846
1933	1853	1819	1664	1720	1948	2363	2928	3029	2803	2553	2443	2523

Adjusted for seasonal variations. The original data come from the Bureau of Mines, *Mineral Resources of the United States*, 1925, Part II, p. 545, and later annual numbers (now called *Minerals Yearbook*).

respect to the reference-cycle relatives, except that they show movements during specific cycles.

To exemplify these steps: Table 4 shows by months the seasonally adjusted figures of coke production in the United States from 1914 through 1933, a series chosen because it is relatively short and presents few of the complications we ordinarily encounter. These figures are plotted on Chart 1, which shows also the turning points of business cycles and of the specific cycles in coke production. The average monthly production of coke during the first complete specific cycle (November 1914

CHART 1  
Coke Production, United States, 1914-1933



Adjusted for seasonal variations. Shaded areas represent reference contractions, white areas represent reference expansions. Asterisks identify peaks and troughs of specific cycles. See Table 4.

Logarithmic vertical scale

to May 1919) was 4,246,000 short tons. With that figure as a base, the monthly values in Table 4 for the months covered by this cycle are converted into specific-cycle relatives. The first reference cycle covered by this series runs from December 1914 to April 1919. Average monthly output was 4,305,000 short tons, which is the base upon which the first set of reference-cycle relatives is computed. During the second specific cycle (May 1919 to July 1921) average monthly output was 3,565,000 short tons; during the second reference cycle (April 1919 to September 1921) it was 3,417,000 short tons. These figures are the bases upon which relatives are computed for the second specific and the second reference cycle. The turning points shown on Chart 1 mark off three more specific and three more reference cycles for each of which we compute cycle relatives.

From the specific-cycle relatives we make what we call 'S' tables, of which there are five. Samples of these tables interspersed with explanatory text appear as Tables 5-9. Similarly, from the reference-cycle relatives we make three or four 'R' tables, samples of which appear as Tables 10-12. Important details of our procedure are omitted in the sketch that follows. A full description of the methods underlying the tables is presented in Chapter 5.

### III Timing, Duration and Amplitude of Specific Cycles

The reference dates yield measures of the duration of business cycles and of their expansions and contractions. The dates of the cyclical turns in a given series yield similar measures of the duration of its specific cycles. By comparing the turning dates of specific cycles with the reference dates,

TABLE 5  
Sample of Table S1: Timing and Duration of Specific Cycles  
Coke Production, United States, 1914-1932

Dates of specific cycles  Trough - Peak - Trough (1)	Timing at reference peak		Timing at reference trough		Duration of cyclical movements (mos.)						Per cent of duration of specific cycles		
	No. of months lead (-) or lag (+) (2)	Date of reference peak (3)	No. of months lead (-) or lag (+) (4)	Date of reference trough (5)	Specific cycles			Excess over reference cycles			Ex-pan-sion (12)	Con-traction (13)	
					Ex-pan-sion (6)	Con-traction (7)	Full cycle (8)	Ex-pan-sion (9)	Con-traction (10)	Full cycle (11)			
Nov. 14			-1	12/14									
Nov. 14 - July 18 - May 19	-1	8/18	+1	4/19	44	10	54	0	+2	+2	81	19	
May 19 - Aug. 20 - July 21	+7	1/20	-2	9/21	15	11	26	+6	-9	-3	58	42	
July 21 - May 23 - July 24	0	5/23	0	7/24	22	14	36	+2	0	+2	61	39	
July 24 - Feb. 26 - Nov. 27	-8	10/26	-1	12/27	19	21	40	-8	+7	-1	48	52	
Nov. 27 - July 29 - Aug. 32	+1	6/29	-7	3/33	20	37	57	+2	-8	-6	35	65	
Average.....	-0.2		-1.7		24.0	18.6	42.6	+0.4	-1.6	-1.2	57	43	
Average deviation.....	3.4		1.9		8.0	8.3	10.3	3.5	5.5	2.6	12	12	

we determine the number of months by which the troughs and peaks in the series precede or follow the reference troughs and peaks.

These procedures are illustrated by the sample of Table S1. After the specific cycles have been marked off, the dates of the turning points are entered in column (1). The reference dates with which the specific-cycle turns are compared are entered in columns (3) and (5). The differences in months between the turning dates of the specific cycles and the reference dates are then shown in columns (2) and (4). The durations of the specific cycles and their phases are shown in columns (6) to (8). The differences between the durations of specific cycles and corresponding business cycles are shown in columns (9) to (11). Finally, in columns (12) and (13) the lengths of the specific-cycle phases are expressed as percentages of the durations of full cycles.

Next, the amplitude of the cyclical swings is ascertained by measuring the rise of the specific-cycle relatives from the initial trough of a cycle to the peak, and the fall from the peak to the terminal trough. In order to diminish the influence of random factors upon the amplitudes, we use three-month averages centered on the troughs and peaks. Of course, the results show the rise and fall in percentages of the average value of the series during each specific cycle.

Table S2 gives these amplitude measures in three forms. Columns (2) to (4) show the three-month averages of the specific-cycle relatives cen-

TABLE 6  
Sample of Table S2: Amplitude of Specific Cycles  
Coke Production, United States, 1914-1932

Dates of specific cycles Trough - Peak - Trough (1)	3-month average in specific-cycle relatives centered on			Amplitude of			Per month amplitude of		
	Initial trough (2)	Peak (3)	Terminal trough (4)	Rise (5)	Fall (6)	Rise & fall (7)	Rise (8)	Fall (9)	Rise & fall (10)
Nov.14-July 18-May19	55.5	119.6	74.5	64.1	45.1	109.2	1.5	4.5	2.0
May19-Aug.20-July 21	88.7	125.9	45.0	37.2	80.9	118.1	2.5	7.4	4.5
July 21-May23-July 24	44.3	144.0	82.6	99.7	61.4	161.1	4.5	4.4	4.5
July 24-Feb. 26-Nov.27	69.4	118.2	92.1	48.8	26.1	74.9	2.6	1.2	1.9
Nov.27-July 29-Aug.32	105.5	141.6	41.7	36.1	99.9	136.0	1.8	2.7	2.4
Average . . . . .	72.7	129.9	67.2	57.2	62.7	119.9	2.6	4.0	3.1
Average deviation . . . . .	19.5	10.4	19.1	19.8	22.2	23.0	0.8	1.7	1.2

tered on the initial trough, peak, and terminal trough. The second set of measures, columns (5) to (7), shows the rise from trough to peak, the fall from peak to trough, and the total rise and fall; these figures are obtained from the entries in columns (2) to (4). The third set of measures, columns (8) to (10), shows the amplitudes per month; they are obtained

by dividing the figures in columns (5) to (7) of Table S2 by the corresponding duration figures in columns (6) to (8) of Table S1.

#### IV Measures of Secular Movements

Our method of computing cycle relatives as percentages of the average value during a specific or a reference cycle eliminates from the original data what we call the 'inter-cycle' portion of the secular trend. The 'intra-cycle' portion of the trend we make no effort to eliminate, because we wish to reproduce as faithfully as may be the 'cyclical units' of actual economic experience.

Table S3 throws into relief the secular component of the specific cycles. Columns (2) and (3) show the average value of the seasonally

TABLE 7  
Sample of Table S3: Secular Movements  
Coke Production, United States, 1914-1932

Dates of specific cycles  Trough - Peak - Trough (1)	Average monthly standing (thous. short tons)			Per cent change from preceding phase		Per cent change from preceding cycle on base of			
	Expansion (2)	Contraction (3)	Full cycle (4)	Contraction to expansion (5)	Expansion to contraction (6)	Preceding cycle		Average of given and preceding cycle	
						Total (7)	Per month (8)	Total (9)	Per month (10)
Nov. 14-July 18-May 19	4193	4479	4246	...	+7	...	...	...	...
May 19-Aug. 20-July 21	3906	3099	3565	-13	-21	-16	-0.40	-17	-0.42
July 21-May 23-July 24	3171	4326	3620	+2	+36	+2	+0.06	+2	+0.06
July 24-Feb. 26-Nov. 27	4107	4488	4307	-5	+9	+19	+0.50	+17	+0.45
Nov. 27-July 29-Aug. 32	4577	3319	3760	+2	-27	-13	-0.27	-14	-0.29
Average.....	...	...	...	...	...	...	...	-3.0	-0.05
Average deviation.....	...	...	...	...	...	...	...	12.5	0.30
Weighted average.....	...	...	...	...	...	...	...	...	-0.08

adjusted data during the phases of specific cycles. Column (4) shows the average value during full specific cycles; these values are the bases on which the specific-cycle relatives are computed. Column (5) shows the percentage change from the average standing during a contraction to the average during the following expansion, and column (6) shows the percentage change from the average standing during an expansion to the average during the following contraction. Column (7) shows the percentage change from the average standing during a full specific cycle to that during the next. And column (8) reduces the measures in column (7) to a per month basis, the divisor being the number of months from the midpoint of one cycle to the midpoint of the next.

Column (9) is the same as column (7) except that the percentages are computed on the base of the average of the two cycles being compared,

instead of on the base of the first of the two cycles. This shift of method frees the percentages from secular 'bias' and permits us to strike averages over all cycles. Column (10) bears the same relation to column (9) as column (8) bears to column (7). The weighted average at the bottom of column (10) is obtained by weighting the entries in this column by the intervals between the midpoints of successive cycles.

## V Cyclical Patterns

To show the behavior of a series during the course of its specific cycles in greater detail than in Table S2, each specific cycle is broken into nine stages. Stage I covers the three months centered on the initial trough, stage V covers the three months centered on the peak, and stage IX the three months centered on the terminal trough. Stages II to IV cover successive thirds of the length of the expansion, and stages VI to VIII cover successive thirds of the contraction. By averaging the specific-cycle relatives for the months included in each of these stages, we get 'specific-cycle patterns'. These computations are illustrated in the sample of Table S4.

TABLE 8  
Sample of Table S4: Specific-cycle Patterns  
Coke Production, United States, 1914-1932

Dates of specific cycles	Average in specific-cycle relatives at stage								
	I	II	III	IV	V	VI	VII	VIII	IX
	3 mos. cen- tered on initial trough	Expansion			3 mos. cen- tered on peak	Contraction			3 mos. cen- tered on ter- minal trough
		First third	Middle third	Last third		First third	Middle third	Last third	
Trough - Peak - Trough (1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Nov. 14 - July 18 - May 19	55.5	81.3	108.0	107.1	119.6	118.7	112.6	88.2	74.5
May 19 - Aug. 20 - July 21	88.7	101.2	110.9	117.8	125.9	124.4	86.8	50.4	45.0
July 21 - May 23 - July 24	44.3	58.8	78.9	124.3	144.0	137.0	118.1	105.5	82.6
July 24 - Feb. 26 - Nov. 27	69.4	84.8	95.1	106.2	118.2	108.3	105.2	99.0	92.1
Nov. 27 - July 29 - Aug. 32	105.5	113.8	118.4	133.3	141.6	122.4	86.2	55.9	41.7
Average . . . . .	72.7	88.0	102.3	117.7	129.9	122.2	101.8	79.8	67.2
Average deviation . . . . .	19.5	15.6	12.2	8.9	10.4	6.9	12.2	21.3	19.1

We make 'reference-cycle patterns' on a similar plan; but here the nine stages are marked off on the basis of the cyclical turning dates of general business. By breaking each reference cycle into nine segments, we show the behavior of different economic processes from stage to stage of business cycles and put both our concept of business cycles and the schedule of reference dates to a critical test. Table R1 presents the reference-cycle patterns. It differs from Table S4 in only two respects: the



troughs and peaks are taken from the standard list of reference dates instead of from the turning points of specific cycles, and the entries are expressed in units of reference-cycle relatives instead of specific-cycle relatives.

Additional information concerning cyclical patterns is supplied by Tables S5 and R2. Table S5 shows the rate of change from one stage of specific cycles to the next, and Table R2 shows the rate of change from one stage of reference cycles to the next. The entries in Table S5 are obtained by dividing the differences between successive figures on each

TABLE 9  
Sample of Table S5: Rate of Change from Stage to Stage of Specific Cycles  
Coke Production, United States, 1914-1932

Dates of specific cycles  Trough - Peak - Trough (1)	Average change per month in specific-cycle relatives between stages							
	I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX
	Expansion				Contraction			
	Trough to first third (2)	First to middle third (3)	Middle to last third (4)	Last third to peak (5)	Peak to first third (6)	First to middle third (7)	Middle to last third (8)	Last third to trough (9)
Nov. 14 - July 18 - May 19	+3.4	+1.8	-0.1	+1.7	-0.4	-2.0	-8.1	-6.8
May 19 - Aug. 20 - July 21	+4.2	+2.2	+1.5	+2.7	-0.8	-10.7	-10.4	-2.7
July 21 - May 23 - July 24	+3.6	+2.9	+6.5	+4.9	-2.8	-4.2	-2.8	-9.2
July 24 - Feb. 26 - Nov. 27	+4.4	+1.7	+1.8	+3.4	-2.5	-0.5	-1.0	-1.7
Nov. 27 - July 29 - Aug. 32	+2.4	+0.7	+2.3	+2.4	-3.0	-3.0	-2.5	-2.2
Average.....	+3.6	+1.9	+2.4	+3.0	-1.9	-4.1	-5.0	-4.5
Average deviation.....	0.6	0.6	1.6	0.9	1.0	2.7	3.4	2.8
Average interval (mos.).....	4.3	7.7	7.7	4.3	3.4	5.9	5.9	3.4

TABLE 10  
Sample of Table R1: Reference-cycle Patterns  
Coke Production, United States, 1914-1933

Dates of reference cycles  Trough - Peak - Trough (1)	Average in reference-cycle relatives at stage								
	I	II	III	IV	V	VI	VII	VIII	IX
	3 mos. centered on initial trough (2)	Expansion			3 mos. centered on peak (6)	Contraction			3 mos. centered on terminal trough (10)
		First third (3)	Middle third (4)	Last third (5)		First third (7)	Middle third (8)	Last third (9)	
Dec. 14 - Aug. 18 - Apr. 19	52.8	83.9	106.7	106.6	118.7	116.7	111.0	91.7	78.1
Apr. 19 - Jan. 20 - Sep. 21	98.4	96.9	116.1	101.3	119.2	123.2	117.1	52.3	53.0
Sep. 21 - May 23 - July 24	48.4	63.3	80.2	124.2	139.5	132.7	114.4	102.2	80.0
July 24 - Oct. 26 - Dec. 27	69.5	90.0	98.4	111.6	107.4	105.0	102.4	94.5	94.0
Dec. 27 - June 29 - Mar. 33	114.4	123.1	124.8	138.9	150.2	128.2	83.3	52.3	49.1
Average.....	76.7	91.4	105.2	116.5	127.0	121.2	105.6	78.6	70.8
Average deviation.....	23.8	14.8	12.8	12.0	14.3	8.2	10.2	21.0	15.8

TABLE 11

Sample of Table R2: Rate of Change from Stage to Stage of Reference Cycles  
Coke Production, United States, 1914-1933

Dates of reference cycles  Trough - Peak - Trough (1)	Average change per month in reference-cycle relatives between stages							
	I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX
	Expansion				Contraction			
	Trough to first third (2)	First to middle third (3)	Middle to last third (4)	Last third to peak (5)	Peak to first third (6)	First to middle third (7)	Middle to last third (8)	Last third to trough (9)
Dec. 14 - Aug. 18 - Apr. 19	+4.1	+1.6	0.0	+1.6	-1.3	-2.3	-7.7	-9.1
Apr. 19 - Jan. 20 - Sep. 21	-0.8	+7.7	-5.9	+9.0	+1.1	-0.9	-10.0	+0.2
Sep. 21 - May 23 - July 24	+4.3	+2.6	+6.8	+4.4	-2.7	-4.1	-2.7	-8.9
July 24 - Oct. 26 - Dec. 27	+4.1	+1.0	+1.6	-0.8	-1.0	-0.6	-1.8	-0.2
Dec. 27 - June 29 - Mar. 33	+2.5	+0.3	+2.6	+3.2	-2.8	-3.1	-2.1	-0.4
Average.....	+2.8	+2.6	+1.0	+3.5	-1.3	-2.2	-4.9	-3.7
Average deviation.....	1.6	2.0	3.2	2.6	1.1	1.2	3.2	4.3
Average interval (mos.).....	4.3	7.5	7.5	4.3	3.6	6.5	6.5	3.6

line in Table S4 by the number of months from the middle of one specific-cycle stage to the middle of the next stage. Table R2 is made from Table R1 just as Table S5 is made from Table S4.

### VI Measures of Conformity to Business Cycles

The comparisons in Table S1 between specific and reference cycles show roughly how the wave-like movements in a given series conform to the waves in general business activity. Further light is shed upon this matter by the similarity or the difference between the average specific-cycle and the average reference-cycle patterns of Tables S4 and R1. But it is desirable to measure explicitly the varying degrees of conformity.

Table R3 gives the measures we seek. Column (4), which is derived from columns (2) and (3), supplies essential information on the conformity of the series to business-cycle expansions. That is, the entries in column (4) show the average rise or fall per month during successive reference expansions, while the average near the bottom of this column shows the average rate of change during all the reference expansions covered by the series. Column (7) supplies similar information concerning the behavior of the series during reference contractions. Finally, column (8) expresses the difference between the rates of change during reference expansion and contraction; this measure is needed because some series with rapidly rising trends continue to advance even during reference contractions, and we wish to know how much, if at all, the rate of rise is intensified during expansions in general business and diminished during contractions.

TABLE 12  
Sample of Table R3: Conformity to Business Cycles  
Coke Production, United States, 1914-1933

Dates of reference cycles  Trough-Peak-Trough  (1)	Change in reference-cycle relatives during						Av. change per month during reference contraction minus that during	
	Reference expansion			Reference contraction			Preceding reference expansion (8)	Succeeding reference expansion* (9)
	Total change (2)	Interval in months (3)	Average change per month (4)	Total change (5)	Interval in months (6)	Average change per month (7)		
Dec.14-Aug.18-Apr. 19	+65.9	44.0	+1.50	-40.6	8.0	-5.08	-6.58	-
Apr.19-Jan. 20-Sep. 21	+20.8	9.0	+2.31	-66.2	20.0	-3.31	-5.62	-
Sep.21-May23-July 24	+91.1	20.0	+4.56	-59.5	14.0	-4.25	-8.81	-
July 24-Oct. 26-Dec. 27	+37.9	27.0	+1.40	-13.4	14.0	-0.96	-2.36	-
Dec.27-June 29-Mar.33	+35.8	18.0	+1.99	-101.1	45.0	-2.25	-4.24	...
Average .....	+50.3	...	+2.35	-56.2	...	-3.17	-5.52	...
Average deviation .....	...	...	0.88	...	...	1.25	1.78	...

Index of conformity to reference			
Expansions .....	+100		
Contractions .....	+100		
Cycles, trough to trough .....	+100		
Cycles, peak to peak .....	+100		
Cycles, both ways .....	+100		

\* Only the sign of the difference is entered.

While the averages near the bottom of columns (4), (7) and (8) are useful measures of conformity, they do not indicate the *regularity* with which a series 'responds' to the stimuli of general business expansion and contraction. To bring out this feature of cyclical behavior we make a second set of conformity measures, that is, 'indexes of conformity' which take account of the direction of movements but not their magnitude. When a series rises during a reference expansion we mark it +100; when the series remains unchanged we mark it 0; when it falls we mark it -100. By casting up the algebraic sum of these entries for all cycles and dividing by their number, we get an index of conformity to reference expansions. This result, entered at the bottom of column (4), may vary between +100 (positive conformity to all the reference expansions covered) and -100 (inverse conformity to all the expansions). An equal number of positive and inverse movements produces an index of 0.<sup>2</sup> To measure conformity to reference contractions we proceed in a similar way, but a decline in column (7) is now marked +100, and a rise -100; for a decline means positive conformity to reference contractions and a rise means inverse conformity.

Finally, we make indexes of conformity to business cycles as wholes.

<sup>2</sup> Again, an index of +50 means positive conformity in 3 and inverse conformity in 1 case out of 4; an index of +33 means positive conformity in 2 and inverse conformity in 1 case out of 3.

Here we wish to take account of the fact that some series rise or decline throughout reference cycles, but at a different rate during reference expansions and contractions. A preliminary index is obtained by crediting each difference in column (8) with +100 when the difference is minus, with -100 when it is plus, and then striking an arithmetic mean of these entries. This index shows merely the conformity to business cycles marked off by troughs; hence it is supplemented in column (9) by a similar index showing conformity to business cycles marked off by peaks. A weighted average of the two preliminary indexes gives our final index of conformity to business cycles taken as wholes. A value of +100 for this index means that the rate of change per month during a reference contraction is without exception algebraically lower than the rate of change during the next preceding and following reference expansions.

The sample Table R3 for coke production illustrates these computations. The 'expansion index' comes out +100 because all entries in column (4) are plus. The 'contraction index' comes out +100 because all signs in column (7) are negative. The preliminary 'full-cycle index', taken on a trough-to-trough basis, comes out +100, and so too does the index of conformity to full cycles on a peak-to-peak basis, since all signs in columns (8) and (9) are negative. The final full-cycle index is obviously +100, since it is an average of two preliminary indexes each of which is +100.

Coke production typically rises from stage I to stage V and declines from stage V to stage IX of the reference cycles. Although many series follow this simple pattern, some show pronounced leads or lags at the reference turns. When a series conforms to business cycles with a characteristic lead or lag, we take this fact into account in Table R4. This table is similar to Table R3 in every respect, except that it recognizes differences in timing between the cyclical movements of a given activity and those of business at large. A sample of Table R4 is given in Chapter 5, where our measures of cyclical behavior are described more fully.

## VII Averages and Average Deviations

Most of the measures described above are made for every reference and for every specific cycle covered by a series, and are then averaged for each set of cycles. When averages are struck for all the cycles covered by a series, features peculiar to single cycles tend to fade away, while features common to all or most of the cycles tend to stand out prominently.

In general, the more cycles a series covers, the greater is our confidence that the average discloses faithfully what cyclical behavior is typical of the process represented. But in analyzing price and value series, we usually exclude cycles affected by grave monetary disturbances from the averages. We also make exclusions when some exceptionally powerful random factor, such as a great strike, has warped an individual cycle out of resem-

blance to other cycles in the array. When the cyclical behavior of a long series gives definite indications of having undergone a secular or structural change, we break the series into relatively homogeneous segments, and strike separate averages for each segment.

Our attempt to find what cyclical behavior is characteristic of different economic processes does not end in the contemplation of average measures; for one of the leading features of specific and of business cycles is that they vary in duration, intensity, and other respects. To keep this feature prominently before our minds and to provide materials for studying it at a later stage of the investigation, we compute average deviations from the averages. These deviations are simple measures of the degree to which the figures for individual cycles in a series are clustered about the arithmetic means that we use to represent 'central tendencies'.

### VIII Charts of Cyclical Patterns

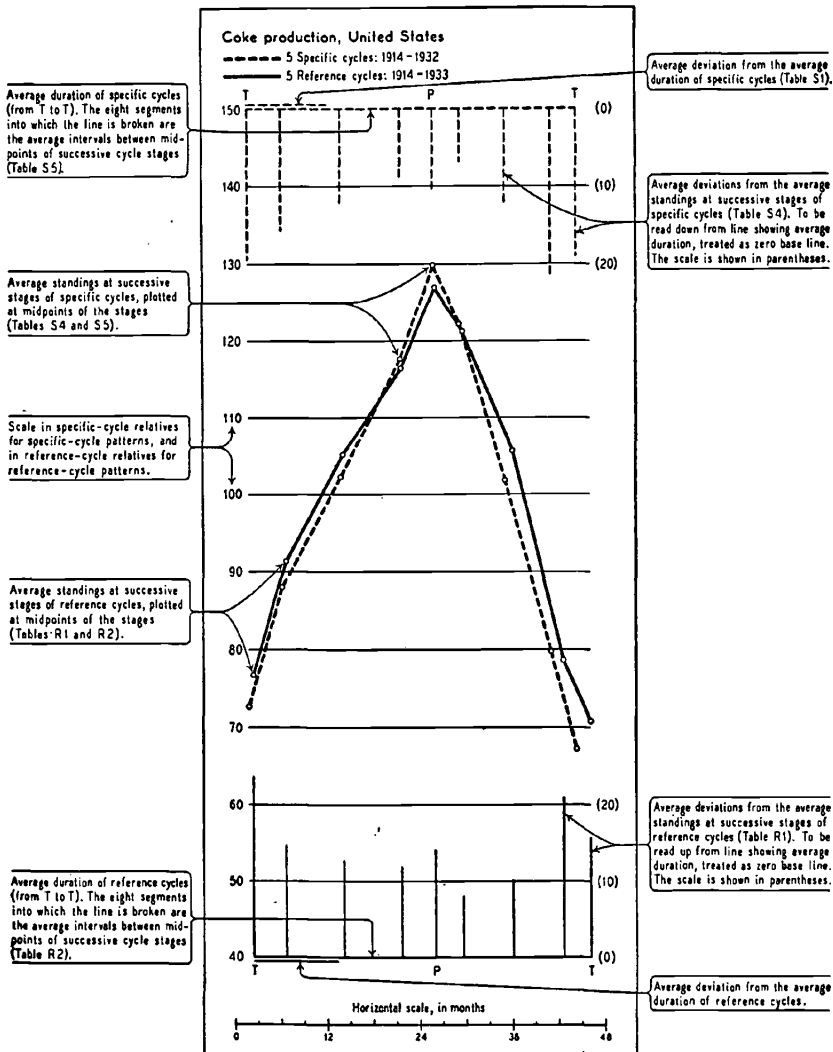
Several results of our statistical analysis that lend themselves readily to graphic presentation are embodied in charts of cyclical patterns. The sample for coke production, shown in Chart 2, pictures the averages and average deviations in Tables S4 and R1, and certain additional measures from Tables S1, S5, and R2.

The curves in the chart trace out the specific-cycle and reference-cycle patterns made by averaging the standings of the individual cycles at each of the nine stages used in Tables S4 and R1. Since coke production corresponds closely in time to business cycles, its two patterns are nearly identical. The more irregular the timing of a series in relation to business cycles, the smaller will be the amplitude of the reference-cycle pattern relative to that of the specific-cycle pattern. The representative value of the two patterns is indicated by the lengths of the vertical lines, which show the average deviations of the individual cycles from their average standings at the nine stages.

The long horizontal lines above and below the cyclical patterns represent the average durations of the specific and reference cycles. We refer to them as 'duration lines'. The vertical lines representing the average deviations from the average standings are dropped from or erected at the midpoints of the cycle stages. The short horizontal lines above and below the duration lines represent the average deviations from the average durations. The ruler at the bottom of the chart defines the time scale; with its aid all durations can be approximated.

When, as in coke production, the specific and reference cycles correspond to one another, the two duration lines are placed so that they show average leads or lags. When the specific and reference cycles of a series do not correspond throughout, the duration lines are so placed that the peak standings of the two patterns are aligned vertically.

CHART 2  
Sample Chart of Cyclical Patterns  
(Drawn to twice the standard scales)



T represents the trough stage (I or IX), P the peak stage (V). For explanation of how the line representing the average duration of specific cycles is placed in relation to the line representing the average duration of reference cycles, see Ch. 5, Sec. VIII.

The charts of cyclical patterns presented later in this volume have been drawn to a strictly uniform set of scales. However, in order to help the reader learn our method, the scales in Chart 2 are twice the standard scales. The explanatory comments on this chart are not repeated in later charts; nor are the scale numbers for average deviations of the standings in successive cycle stages.

## IX Comparison with Customary Techniques

Despite its relative simplicity, our statistical technique may strike the reader as formidable. This impression may be due to the numerous measures we require to reveal the cyclical behavior of a series. But the impression may be due also to the novel features of the technique. It may help the reader to become oriented in our analysis, if our method of finding what cyclical behavior is characteristic of a series is contrasted with the method that has become customary since the publication of Warren M. Persons' basic memoirs on time series analysis.<sup>3</sup>

The customary way of showing the cyclical fluctuations of a time series is to begin by measuring the secular trend and the seasonal variations. These two elements are then combined, and the original data expressed as percentage deviations from corresponding ordinates of the 'secular-seasonal' composite. The percentages are supposed to show cyclical and erratic movements in combination. Sometimes the percentages are expressed in standard deviation units, at other times this step is omitted. In either case the results are presented in a chart where the 'secular-seasonal' composite is reduced to a horizontal line about which oscillate the 'cyclical-erratic' values, smoothed or not as the case may be. What we should call the 'specific cycles' of the series in trend-adjusted form can then be distinguished, though the turning dates may be obscured by erratic movements. The behavior of the specific cycles in relation to business cycles can be made out after a fashion by marking the time scale with the cyclical turning dates in general business activity or by plotting a curve that purports to represent business cycles. Cyclical analysis of economic time series frequently does not go further; although average amplitudes of specific cycles, average leads or lags at the turns of selected 'indicators of business', and coefficients of correlation between the series and some 'indicator of business cycles' are sometimes computed.

Our method follows precedent in eliminating seasonal variations at the outset. It then diverges from the usual technique: first, in breaking the data adjusted for seasonal variations into reference-cycle segments and into specific-cycle segments, which are treated as units of economic experience subject to comparison and averaging; second, in eliminating the inter-cycle portions of the secular trend; third, in preserving the intra-cycle portions of the secular trend in the measures of cyclical behavior, but arranging the measures so as to bring out the secular movements; fourth, in striking averages for a group of cycles to show what cyclical behavior is characteristic of the series; fifth, in charting, not successive specific cycles as a continuum, but average behavior during a group of specific cycles and during the group of contemporaneous business cycles.

<sup>3</sup> *Indices of Business Conditions, Review of Economic Statistics*. Jan. 1919: An Index of General Business Conditions, *ibid.*, April 1919.