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Chapter V

COSTS AND RATE OF OUTPUT: EMPIRICAL STUDIES OF COST FUNCTIONS

THE preceding chapter explored the problems of designating, for any single accounting period, those costs arising from the use of resources and equipment which contribute to the output of more than one period. Such a discussion is a necessary introduction to measurement of the cost function in particular enterprises. The present chapter will examine some of the other problems which arise in an empirical study of cost functions, survey the investigations that have been made thus far, discuss the techniques utilized, and attempt to appraise the significance of the results that have been obtained.

How costs are influenced by variations in the rate of utilization of plant has been the subject of two principal types of inquiry. The first depicts a relationship existing over preceding accounting periods; the second, while necessarily based on the experience of the firm, attempts to show the probable relationship in the next period. Studies in the first category consist largely of investigations undertaken by econometricians with the purpose of "approximating" the cost functions of economic theory; they usually include break-even charts constructed for investment analysis. Those in the second group are developed by accountants, engineers and business managers; these are designed to provide constructive information for the formulation of decisions concerning output, costs and prices.

1. *Problems in the Measurement of Cost Functions*

Since controlled experiments relating to costs have not been undertaken on any appreciable scale,[1] it has been necessary for investigators to rely almost entirely on the accounting records of particular firms. These data show in accounting form not only the components of cost but the total cost for successive accounting periods. The variation in these costs from one period to the next is ordinarily the result of a large number of influences. Any statistical approximation of the cost function in specific instances must, therefore, involve these two steps: (1) elimination of the effects of those factors, other than variations in output, which impinge on costs, and (2) measurement of this residual relationship between costs and the rate of utilization.

The task of approximating a cost function might appear superficially to be simply a question of choosing the appropriate statistical technique whereby the influence of one variable may be isolated from among many. But before any particular statistical methods can be used, a large number of questions of definition and interpretation must be recognized and possible sources of confusion clarified. Issues of this sort arise when one attempts to translate accounting data into the simplified conceptual scheme of a cost function and to adapt them to statistical analysis. In the pages immediately following, these difficulties are discussed and an endeavor is made to indicate the limitations of

[1] A number of experiments in the field of agriculture have measured the relations between such input factors as seed, labor, and fertilizer on the one hand and product on the other. See U. S. Department of Agriculture, *Input-Output Relationships in Milk Production*, Technical Bulletin 815 (May 1942). From input-output functions with given factor prices, total cost functions can be derived. For a summary of some of these studies, see E. H. Phelps Brown, "The Marginal Efficacy of a Productive Factor—First Report of the Econometrica Committee on Source Materials for Quantitative Production Studies," *Econometrica*, IV (April 1936), pp. 123-37. It must be remembered that studies of input-output relations are not equivalent to inquiries concerning cost functions.

possible solutions in order that particular studies may be evaluated critically in the next section.[2] At the outset, cost studies are presumed to refer to a single enterprise as the unit of decision formation in order to avoid arbitrary allocations of cost among component departments.

(1) The familiar cost function of economic theory applies only to one point in time and under equilibrium conditions is strictly timeless. All statistical approximations to such relationships must, however, be based upon observations in different periods. Consequently statistical cost functions, like statistical demand curves,[3] must proceed on the assumption that the theoretical relations being approximated are either unchanged during the period of observation or change in a definable and measurable way. If the theoretical cost function does vary during a period, but no correction for a particular pattern of change is made in the statistical analysis, the statistical cost function must be an average for the period. The longer the period of observation (i.e., the greater the number of values), the more confidence can be placed in the derived statistical relationship, but the less relevant are the results for a picture of the cost-output relation to be met in a particular decision of the enterprise. The tendency on the part of some interpreters of statistical demand and cost curves to regard the derived results as valid for every instant in a period is an unfortunate one. For a statistically derived cost function must be either an average relationship for a period, a relationship constant through a period, or one that changes according to a specified pattern.

[2] The issues discussed here have been treated in a study by Joel Dean, "The Relation of Cost to Output for a Leather Belt Shop," *Technical Paper 2* (National Bureau of Economic Research, 1941). See also, Hans Staehle, "Statistical Cost Functions: Appraisal of Recent Contributions," *American Economic Review*, XXXII (June 1942), pp. 321-33.

[3] Many of the difficulties and limitations encountered in the derivation of statistical cost functions are common to any attempt to approximate statistically the concepts of comparative statics. Others are peculiar to empirical cost studies. See Henry Schultz, *The Theory and Measurement of Demand* (University of Chicago Press, 1938), pp. 3-152, for a discussion of the measurement of demand curves.

(2) Related to the problem just described is a second one—the choice of the period of observation—which arises in the attempt to determine an empirical counterpart to a cost function. The period should be long enough to permit a number of observations covering a wide variation of output; but it is true also that the longer the period, the greater is the possibility of technical change, affecting both processes and products, for which existing statistical methods do not make adequate allowance. Furthermore the relationship between costs and the rate of utilization of plant for an increase in output may differ from that prevailing when output is decreased. Such a cost function would be said to be irreversible. If the interpretation of statistical cost functions is to be clarified, both the length of the period of observation and the reversibility of the function require careful study. One may then check the reliability of the results by taking different periods of observation and comparing the effects of expanding and contracting output in particular periods.

(3) Certain restrictions on the interpretation of statistically derived cost functions arise from the fact that observations are not continuous. The accounting data apply to discrete intervals, which in some instances may be as long as a year, in others as short as a fortnight. Values for a single observation (e.g., a year) which sum the cost and output from day to day cannot reveal the relationship existing at any time within the year unless the assumption is made that the cost function remains constant or changes in a definable pattern during the interval embraced by a single statistical observation. As was analogously argued above in (1) for the total period of observation, the practical result of these alternatives is to make the derived relationship an average for the discrete interval. The statistical function must ignore the problem of the extent to which the value for any discrete interval is determined by the rate of change of output *within* the interval. It begs the question of the behavior of total costs at differing

rates of output *within* the interval of observation.[4] Two periods may have exactly the same output but the costs may differ simply because of the rate of expansion or contraction of output during the period. The longer this interval the more serious the limitation.

(4) A related problem encountered in the transition from the cost function of economic theory to its statistical approximation is the difficulty introduced by variation in the sizes of orders.[5] The output of two periods might be the same, except that in one case it consisted of a large number of small "runs" which might increase costs. Or output might be increased from less than "capacity" to "capacity" by gradual steps or by a single step; in either case the cost-output relations could hardly be expected to remain unchanged. It is possible to recognize the influence of this factor by making the size of orders or the rate of change of output explicit determinants of costs along with the level of output itself.

(5) A rather simple, but sometimes neglected, limitation to the interpretation of any statistical cost function is the fact that it can apply only to the actual range of output. It is probably even more dangerous to project cost functions to volumes of output that have not been achieved than to project trends into the future. Furthermore, since extremes in the volume of output seldom occur as frequently as more "normal" values, and hence do not offer as many points of observation, less confidence can be placed in the shape of the derived cost function at its extremes. This conclusion is of particular importance, because the costs for extreme values of output attract the

[4] To a certain extent the effect on costs of the rate of change of output—calculated from outputs in successive periods—may be accounted for and eliminated from the cost function, but the available techniques are not adequate to deal with all the variations which can and do take place within the accounting period.

On some of the effects of change of output within the interval, see Caleb A. Smith, "Cost-Output Relation for the United States Steel Corporation," *Review of Economic Statistics* (November 1942).

[5] This point is listed separately because output is ordinarily varied in increments larger than a single "unit" of product.

most comment from business executives and persons interested in the consequences of business decisions.

(6) It is often a rather difficult task to make certain that the observed values of output and costs apply to the same period. A cost function in which the costs of one period were related to the output of another would ordinarily be meaningless. The longer the time during which goods are in process, the greater is the likelihood of this type of distortion. Expenditures may be recorded most heavily while goods are being processed, whereas output may not be recorded until the goods have undergone finishing operations on which expenditures are smaller. Thus costs may be accounted in one period and the output to which they contributed in another. When the volume of output is being varied rapidly, this lack of synchronization in the cost and output records may introduce serious distortions unless lag corrections are adopted. Again, lack of comparability may arise because of variations in the time between the *incurrence* of expenses or completion of output and the *recording* of these events.[6]

(7) One of the most perplexing aspects of the attempt to approximate cost functions is the question of the units in which output is to be measured. If each firm produced a single, homogeneous product, as is so frequently assumed in simplifications of economic theory, the measurement of output would be only a matter of counting these units. But (as was noted in Chapter III) in reality firms or even plants produce many different styles, grades, models, and sizes of a "product" as well as many different "products."[7] When output is heterogeneous and composed of items which pass through substantially the same production process, so that their individual costs cannot be isolated by separate study of various divisions of the plant, a com-

[6] The fundamental difficulty of calculating costs (and income) for a period short of the life of an enterprise, discussed in detail in the previous chapter, is again relevant.

[7] The problems of calculating the costs of specific "products" are discussed in Chapter VIII.

mon denominator must be found if the relation between cost and "output" is to be examined.[8]

The major alternatives in the measurement of output for comparison with costs can be briefly surveyed. (a) The units of input factors may be an acceptable index of the volume of output in some instances. In oil refining, for example, the input of crude oil is fairly homogeneous, while the output is composed of highly heterogeneous joint products produced in varying proportions. If the rate of activity of the plant is satisfactorily reflected by the rate of input of the major homogeneous raw material, and if the range of variation in the proportion of different products is not wide, the use of input factors may be taken as an index of output. (b) Since it may be impossible to reduce the measurement of output to a single dimension, more than one measure of output may be used as separate independent variables in multiple correlation analysis. For example, in a study of the relationship between cost and railway traffic volume, it might be necessary to measure "output" in terms of passenger miles, freight ton miles, movement of mail and express, etc. (c) A composite index of output necessarily involves the determination of the relative importance or weight to be attached to the various products making up the composite unit. The weights suggested in particular cost studies include: the relative prices of the products, the amount of raw materials entering into the different products, and their variable or total cost.

[8] For a general discussion of production index problems, see Wassily Leontief, "Composite Commodities and the Problem of Index Numbers," *Econometrica*, IV (January 1936), pp. 39-59; Arthur F. Burns, "The Measurement of the Physical Volume of Production," *Quarterly Journal of Economics*, XLIV (February 1930), pp. 242-62; Edwin Frickey, "Some Aspects of the Problem of Measuring Historical Changes in the Physical Volume of Production," *Explorations in Economics, Notes and Essays contributed in honor of F. W. Taussig* (McGraw-Hill, 1936), pp. 477-86, and "The Theory of Index-Number Bias," *Review of Economic Statistics*, XIX (November 1937), pp. 161-73; O. N. Anderson, "On the Question of the Construction of an Internationally Comparable Index of Industrial Production," *Publications of the Statistical Institute for Economic Research*, No. 1 (State University of Sofia, 1937), pp. 121-31.

In addition to empirical studies of cost functions, output indexes have been made to measure "productivity" and industrial activity, and to serve as a basis for some national income estimates. A system of weights adapted to one of these types of study may be quite misleading if applied to another. For instance, output weighted by the relative prices of constituent items may be useful for national income purposes but inadequate for cost purposes. If relative prices differ because of variation in the character of competition, national income as a flow of services which satisfy given tastes can be regarded as affected by this fact; but it makes little sense to argue that output is any different for purposes of calculating a total cost function. The best weights for individual products, in the construction of composite indexes of output for cost functions, would usually appear to be their direct costs. Only if relative prices were a reflection of difference in relative costs would price weight be theoretically valid for output in the calculation of cost functions.

Regardless of the system of weighting used for indexes of output, the assumption must be made that the weight of each individual element in the composite unit remains unchanged throughout the period of observation. The longer the period of observation the less likelihood there is that any proportion of component products will remain unchanged.

When indexes of output for all the products of a firm are utilized in the calculation of cost functions, the results are not directly applicable to individual products. If an attempt is made to analyze the pricing policy for a single commodity, it is necessary to investigate the extent to which the composite output unit is typical of the fluctuations in output of that commodity. Since the index of output is a weighted average, it is possible that it will not be typical of any particular commodity. This may be a serious limitation where the relative proportion of various commodities in total output fluctuates a great deal.

(8) If the influence upon costs of variations in the rate

of plant utilization is to be isolated statistically, other influences must be measured and eliminated. The designation of the other influences that may be expected to affect costs substantially, and the choice of the methods for eliminating their influence are both vital steps. The former is important because the residual relationship is assumed to be the cost function. If the impact of certain factors is not removed, the cost-output function is distorted. Strictly speaking, it is impossible to be certain that the effects of all other factors have been eliminated. After corrections have been made, the residual variations in cost are assumed to be solely the result of variations in output. The effect of "technical changes" can be expected to present particular difficulties; one cannot be certain that plant, equipment, supervision and products have remained exactly the same even over short periods,[9] for in many industries they are subject to frequent and constant modification. As for the choice of methods for measuring and deflating the effects of factors other than output, it is important because of the different shapes of cost functions that may result. If, for instance, the influence of factor price movements is to be eliminated from costs, the investigator will have to decide on the extent of detail in the classification of factors, and on what constitutes a change in factor prices.

(9) The reliability of a measurement of a cost function must depend also upon the selection of the equation to summarize the cost-output relationship. Since marginal cost is the slope of the total function, the adoption of a linear regression necessarily results in constant marginal costs. Extreme care must be utilized, therefore, in the choice of the mathematical function or the freehand curve used to summarize the observed relation between costs and output. While the adoption of higher order equations will improve the statistical fit, the crucial question is whether the greater number of constants results in a "significantly" improved summary relationship. Statistical tests can be used to weigh the relative merits of proposed equations.

[9] See Chapter X.

Perhaps the simplest of these tests would plot the paired differences between successive observations of costs and output. Some clue as to the shape of the marginal cost function is then derived independently of deductions from a mathematically fitted total function. Or the "no hypothesis test" can be applied; this is a device for calculating the probability of obtaining the observed results from the several suggested functions. If the observed results could have been obtained only with small probabilities from a fitted function, the particular hypothesis can be discarded. Finally, the analysis of variance technique permits a judgment on the relative "goodness of fit" of several functions.[10]

The preceding points indicate most of the important questions involved in an empirical approximation to a cost function. These problems arise in the *process of attempting to approximate the cost function of economic theory*. They have been enumerated, not in order to suggest that it is futile to try to derive exactly the cost function of the economist, but rather to focus the issues on which empirical studies must be appraised. One study may be adjudged superior to another, or an investigation in one industry more feasible than in another, according to the way in which the issues outlined above are treated.

Since the complexities of experience cannot be compressed into the simplified models of economic theory, some way must be found to evaluate statistical results. The acceptable tests of the significance of such results fall into two categories. First, there are the statistical tests of the extent to which the derived relation between output and costs fits or "explains" the observed costs. In the period of observation the cost function permits a "prediction" of costs from output. Tests of this sort indicate the degree of reliability that may be credited to such "predictions." The second type of test is much less definite, though equally

[10] For further discussion and illustration of this technique, see Joel Dean, *Statistical Cost Functions of a Hosiery Mill* (University of Chicago Press, 1941), pp. 37-59.

important; it gauges the significance of empirical results in terms of consistency with other types of information—the impressions and opinions of business executives, and logical deductions from information that can be accepted with greater confidence.

2. *Empirical Studies of Cost Functions*

Two distinct types of investigations have attempted to determine the cost function of economic theory; they are to be differentiated by the methods employed. One group of studies has applied to the cost records of preceding periods statistical techniques of varying refinement in order to isolate the effects of variations in the rate of output from other factors presumed to influence costs. Kurt Ehrke[11] in Germany, Theodore O. Yntema,[12] Joel Dean[13] and Roswell H. Whitman[14] in the United States have derived a number of these empirical cost functions from the accounting records of particular firms. Such cost functions will be designated as "statistically derived" to distinguish them from another group of empirical cost-output studies. The second type of inquiry lies between that just indicated

[11] *Die Übererzeugung in der Zementindustrie von 1858–1913* (Gustav Fischer, Jena, 1933).

[12] *United States Steel Corporation T.N.E.C. Papers, Comprising the Pamphlets and Charts Submitted by United States Steel Corporation to the Temporary National Economic Committee* (United States Steel Corporation, 1940), Vol. I, pp. 223-301. See also citation in footnote 19, below.

[13] *Statistical Determination of Costs, with Special Reference to Marginal Costs* (University of Chicago Press, 1936). Chapter II contains a summary of cost studies concerned with the effects of variations in output. See also *Statistical Cost Functions of a Hosiery Mill* and "The Relation of Cost to Output for a Leather Belt Shop" and "Department-Store Cost Functions," *Studies in Mathematical Economics and Econometrics*, edited by Oscar Lange, Francis McIntyre, and Theodore O. Yntema (University of Chicago Press, 1942), pp. 222-54.

[14] "Cost Functions in the Department Store" (in manuscript). See also "Round Table on Cost Functions and their Relation to Imperfect Competition," *American Economic Review*, Supplement, XXX (March 1940), pp. 400-02.

and break-even charts, discussed in Section 3. Studies of this sort are estimates by engineers and accountants, presented as cost functions and focused on past experience. A number of these inquiries have been made in Germany by E. Schmalenbach,[15] Reinhard Hildebrandt,[16] Herbert Peiser[17] and others.[18] Cost functions derived in this fashion will be labeled "estimated" in order to distinguish them from those "statistically derived"; the terms are in no wise intended to reflect upon the relative accuracy of the empirical cost functions.

Attention will be directed first to the "statistically derived" relationships between cost and output, for which Professor Yntema's study of the United States Steel Corporation will be used as an illustration. This choice follows from the fact that the Steel Study[19] is the largest and one of the most complex yet undertaken and has received wide attention through the hearings of the Temporary National Economic Committee. If a problem as perplexing as the cost function for the entire United States Steel Corporation can be solved satisfactorily, the same methods may well serve in simpler cases. The use of Professor Yntema's study will also permit comparisons with two other inquiries based only upon published quarterly and annual statements of the United States Steel Corporation: the investigation

[15] *Selbstkostenrechnung und Preispolitik* (Gloeckner, Leipzig, 1934).

[16] "Geschäftspolitik auf Mathematischer Grundlage," *Technik und Wirtschaft,* XXIV (1931).

[17] *Der Einfluss des Beschäftigungsgrades auf die industrielle Kostenentwicklung* (Gesellschaft für Betriebsforschung, Berlin, 1929).

[18] H. Müller-Bernhardt, *Industrielle Selbstkosten bei schwankendem Beschäftigungsgrad* (Berlin, 1925). A Danish author, Ivar Jantzen, has made several studies, among which "Voxende Udbytee i Industrien," *Nationaløkonomisk Tidsskrift,* LXII (1924) appears to be the most important. Also see the study by Hans Bolza, "Kostenstudien mit Erfahrungs-Zahlen aus der Praxis," *Nordisk Tidsskrift for Teknisk Økonomie* (June 1937), pp. 97-109.

[19] "Steel Study" will refer to the investigation presented by the United States Steel Corporation to the T.N.E.C. For this reference see, *Hearings Before the Temporary National Economic Committee, 76th Congress, 3rd Session, Part 26, Iron and Steel Industry, January 23, 24, and 25, 1940.*

of Kathryn H. Wylie and Mordecai Ezekiel,[20] which "statistically derives" a cost function, and the break-even chart adopted by Walter Rautenstrauch.[21] The present discussion deals with the Steel Study as an example of techniques that may be used to meet the difficulties (inherent in any empirical study) mentioned in the preceding section, and does not attempt to be a comprehensive critique of this particular investigation.

The procedure followed in the Steel Study can be summarized in three major steps:

(1) From the annual profit and loss statements of the corporation, summarized for the Federal Trade Commission (February 17, 1939), annual total cost figures for the period 1927–38 were obtained by the summing of the following items:

Operating Costs, including cost of goods sold, operating expenses of transportation, etc., selling and administration, taxes (other than income and profit),[22] depreciation and depletion
Idle Plant Expense
Interest on Mortgages and Bonds

The total cost figure [23] was corrected for the duplication involved in inter-company transactions between subsidiaries of the United States Steel Corporation. This was a necessary adjustment, since "the inter-company sales of any one company constitute the costs of the other, and since inter-company profits are eliminated from inventory valua-

[20] "The Cost Curve for Steel Production," *Journal of Political Economy*, XLVIII (December 1940), pp. 777-821.

[21] *The Economics of Business Enterprise* (John Wiley and Sons, 1939), pp. 303-09.

[22] Income and profit taxes are not considered "costs"; they vary with profits rather than in any independent way with output. In this section the term "taxes" will always refer to nonincome and profit taxes.

[23] Discounts from purchases were first subtracted from the above total. Accounting practice charges the corporation or its subsidiaries the list price of purchased materials and supplies, regarding discounts as a debit item. Costs for the present purpose required the correction, providing a net figure on purchases involving discounts.

tions in making inventory adjustments, both costs and sales and revenue are inflated, from a consolidated viewpoint, by the amount of the inter-company items . . ."[24]

(2) Since the output of the United States Steel Corporation consists of at least fifty thousand steel "products" and a multitude of nonsteel items such as cement, coal and iron ore, and transportation services, a major step in the study was the construction of a measure of output with which to compare costs. This was accomplished by correcting the actual tons of each product shipped in each year by the ratio of its average mill costs to the average mill costs of all rolled and finished steel products in the period 1933–37. Each ton of a product whose 1933–37 average mill cost was less than the average of all rolled and finished products was weighted proportionately less than a full ton, while products whose average mill cost was above average counted proportionately more than a full ton. The number of tons of all products was converted into equivalent tons of "average cost rolled and finished steel products." In similar fashion the output of rolled and finished products was adjusted to include "the equivalent tons of steel represented by the products other than steel which are sold on a tonnage basis by the Corporation's subsidiaries."[25] One further aspect of the measurement of output must be noted; the data used in constructing this composite index refer to tons *shipped* (rather than *produced*) in the year.[26] This gap was "bridged" in the Steel Study by the statement that "the amount of inventory fluctuation is relatively small . . . and the effect on the ultimate cost computation is negligible."[27]

(3) Since variations in these total costs were obviously due in part to fluctuations in the prices and efficiencies of input factors, a series of corrections had to be used to re-

[24] United States Steel Corporation, *T.N.E.C. Papers*, I, pp. 237-38.

[25] *Ibid.*, p. 239. Cement products are apparently excluded.

[26] That such a problem would arise was noted in Section 2; the cost data refer to the goods produced in the year.

[27] *Ibid.*, p. 241.

move the effects of these influences on total costs. To this end 1938 was adopted as a base, and costs were reconstructed on the assumption that factor prices and efficiencies then prevailing had been constant through the whole period. Four corrections were made: (a) Total payrolls were adjusted to the 1938 level of average hourly earnings.[28] (b) "Other expenses" than payrolls and (nonprofit) taxes were presumed to be largely purchased supplies and materials, and one half of this total was corrected by the Bureau of Labor Statistics index of wholesale commodity prices other than farm and food products related to a 1938 base.[29] (c) To correct tax costs for variations in tax rates during the period, the 1932–38 relation between taxes and "output"[30] was projected to the 1927–31 period. Two linear relationships between taxes and output were found in the data if the period was split at the year 1931. The 1932–38 relationship was projected backward to the earlier period to eliminate the consequences of tax rate changes. Actual values were used in the 1932–38 period. (d) Since numerous changes in the techniques of production took place in the period, it was important to eliminate their influence. Total costs, corrected as just indicated, were plotted against "output" and the deviations from a least-squares straight line were plotted with time as the independent variable. A straight-line fit through these deviations was presumed to measure the influence of changes in technique. Elimination of this effect on cost resulted in the final relationship between costs and output. A straight line summarized the relation with only small deviations, as can be

[28] Throughout the discussion wage rates and average hourly earnings are identified. The correction is made for variations in wage rates for which an index of average hourly earnings is tacitly presumed to be a suitable measure. For instance, see *ibid.*, p. 246 and Table 12, entitled "Adjustment of Payroll to 1938 Wage Rates." The caption in the table is "average hourly earnings."

[29] No argument for this procedure was given except, "in order not to overadjust, only one half of the 'other expense' items have been adjusted." *Ibid.*, p. 249.

[30] The measure of output was discussed under (2) above.

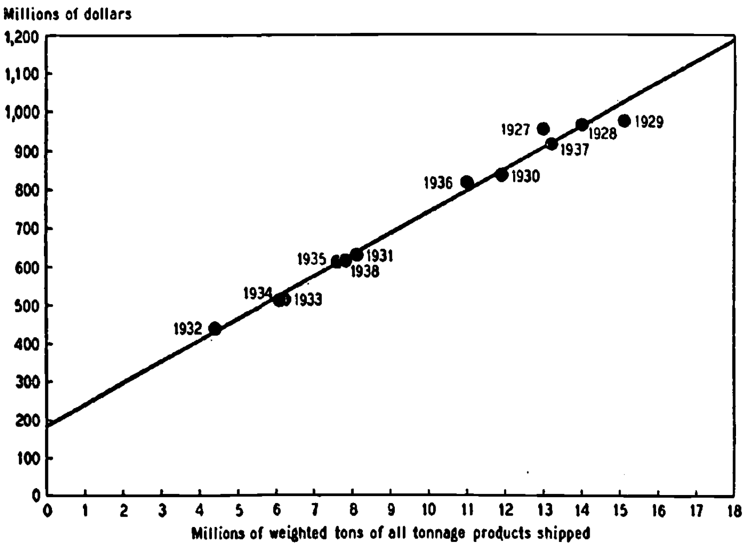
seen from Chart 1, indicating a linear empirical cost function or a horizontal empirical marginal cost function, within the range of observation.[31]

Substantially similar results were found by Kathryn H. Wylie and Mordecai Ezekiel. "Even with the more accu-

Chart 1

RELATIONSHIP BETWEEN TOTAL COSTS OF OPERATION AND VOLUME OF BUSINESS – 1938 CONDITIONS

U. S. Steel Corporation and Subsidiaries



Note: Total costs adjusted to 1938 interest, tax, pension, and wage rates; to 1938 price level; and to 1938 efficiency.

Reproduced from TNEC Hearings, Part 26, Iron and Steel Industry (January 23-25, 1940), p. 13773

rate data . . . the results secured by the Corporation [Yntema's study] were very similar to those secured by our study." Perhaps the only significant difference is the

[31] It is important to note that the effect of all these corrections did not substantially alter the shape of the cost function calculated from unadjusted data. In the form $y = a + bx$, the latter was \$120,530,000 + \$54.51x, where x is a "weighted ton" of output. The corrected function was \$182,000,000 + \$55.73x. The corrected function fits the observed values better, but the difference in the fit between the two functions is not striking. Compare Chart 1 (*ibid.*, p. 240) with Chart 7 (p. 255), which is reproduced above.

fact that the Wylie-Ezekiel investigation found "there was evidence of a sharp downward trend in labor requirements per ton, for given rates of output from 1920 to 1934, with little change thereafter." Even this difference must be viewed as tentative since the regressions of costs on wage rates and efficiency in the Wylie-Ezekiel study were subject to a wide range of error.[32, 33]

While a comprehensive critique of the procedure of the Steel Study is beyond the scope of the present survey,[34] it is important to form some judgment as to the reliability of the results. The present concern is with the degree of confidence that can be placed in the shape of the statistically derived cost function. There is evidence that the techniques used in the Steel Study to meet the problems which arise in every such inquiry (Section 1) have introduced some linear bias into the cost function. Hence, the methods of investigation alone may account for a linear cost function in place of the conventionally assumed, inverted ogive. The possible sources of this sort of bias can be examined briefly.

[32] Two of the Wylie-Ezekiel analyses (based on annual data) show slightly declining marginal costs throughout the range of observed output; the quarterly data reveal a linear total cost function.

The study of Bernard H. Topkis and H. O. Rogers, "Man-Hours of Labor per Unit of Output in Steel Manufacture," *Monthly Labor Review* (May 1935), p. 1161, is interesting in this connection. Manhours plotted against percentage of capacity in the range of 20-60 percent yield a negative average relationship but a rising incremental (marginal) relationship.

[33] The United States Coal Commission developed a formula in 1923 for the determination of "incremental" costs. Although apparently no elaborate calculations are made, there is reason to believe that variations in the rate of output in certain ranges would materially affect costs because of the importance of pumping charges and ventilation. See W. C. Trapnell and Ralph Ilsley, *The Bituminous Coal Industry with a Survey of Competing Fuels,* Federal Emergency Relief Administration (multigraphed, December 1935), pp. 84-86.

[34] For a detailed discussion of the techniques used in the study, see the testimony of Melvin G. de Chazeau, Louis H. Bean, Mordecai Ezekiel and Martin Taitel before the Temporary National Economic Committee, January 23, 1940. See also Caleb A. Smith, *loc. cit.*, where the linearity bias is discussed at length.

(a) In order to correct costs for changes in the efficiencies of input factors, a straight line was fitted to the deviations from a cost-output regression line. Employment of this device is equivalent to assuming that changes in "technology" affect costs in a constant fashion through all phases of the business cycle.[35] Chapter VII will demonstrate that this is probably not the case; technical changes can be expected to reduce costs by a larger extent at some phases of the cycle than at others. If these changes reduce costs most when output is largest, as there seems some reason to believe, then the observed cost values at high levels of output are "too low" compared to what they would have been without the technical change. To make larger corrections for technical change at higher levels of output would introduce an increased slope into the cost function.

(b) The influence of variations in wage rates on payrolls was removed by an index of average hourly earnings. This presumes that both series show equivalent patterns with respect to variations in output. A consistent tendency for average hourly earnings to increase more than rates with increases in output (and vice versa for decreases in output) would result in an overcorrection of payrolls and an underestimate of costs at the higher output levels. That such a relative pattern of earnings and rates is probable for the United States Steel Corporation follows from the discussion in Chapter VI. Earnings may be expected to rise faster than rates because of better coordination between piece workers at higher output levels, graduated bonus payments for increasing output (important in the steel industry), greater effort of wage earners when large backlogs of orders exist, and technical changes which can operate directly on earnings under piece rates. Variations in the composition of the working force between different

[35] The scatter in Chart 6, *ibid.*, p. 253, where the technical change correction is made, fits the regression line very poorly. The regression line drawn in the chart is markedly influenced by the single year 1927.

pay levels at different rates of output are certain to affect payrolls. But no simple pattern can be deduced.[36]

(c) The effects of changes in tax rates were eliminated by extending the linear relationship for the period 1932–38 to the years 1927–31. Such a technique really assumes a linear function for at least this single element of cost and tends to introduce linearity into the total cost function by restricting the segment of costs in which any nonlinear relationships can be found. If the same methods had been used to deflate all "factor" prices, the total function would necessarily have to be linear.[37] The aggregate effect of this method of eliminating tax rate variations is probably not large in the Steel Study.

(d) The acceptance of the Steel Corporation's allocation of overhead over time can also be expected to introduce a bias into the cost function, and in general a linear bias. As was noted in Chapter II, any specification of "fixed" costs to accounting periods shorter than the life of the enterprise must involve estimates of the future performance of the firm, and hence rule-of-thumb methods in the calculation of costs. The conventional methods of

[36] It is commonly supposed that as output expands the occupational composition of the working force contains a smaller *proportion* of higher paid positions, an idea probably derived from the notion of a skeleton work force that is not discharged at low levels of output. But this presumption overlooks the extent to which a large plant can lay off men in each pay level and still maintain skeleton crews without changing the relative proportion of men at each pay interval. Furthermore, account must be taken of promotion and "degrading" policies, seniority or division of work provisions of trade union agreements, etc. A comparison of the proportion of wage earners in various occupational groups reveals no clear cyclical pattern for the industry as a whole. See Bureau of Labor Statistics, "Wages and Hours of Labor in the Iron and Steel Industry," 1929 and 1931, *Bulletin 513* and *567*.

Chart E-17, United States Steel Corporation, *op. cit.*, II, p. 187, shows a slightly negative relation between average hourly earnings and output. The payrolls data, however, include the salaries of management. At low levels of output the weight of management salaries is higher than at capacity output. Average hourly earnings consequently show the slightly negative relation to output.

[37] Except under the circumstances that any element of cost was zero or negative before the common lower limit of output.

straight line allocation tend to underestimate total costs in periods of large output and to overestimate them in times of low output. This bias may be balanced partially by the fact that on some occasions of large output the corporation has written off larger amounts as reserves for amortization of investments. Only half of the usual depreciation was typically charged to completely idle plant. But such corrections are partial and sporadic and make allowances in one period that should be spread over a longer time. The cumulative effect of these four factors is to suggest a more conventional cost function than that derived statistically by Professor Yntema. On the other hand it is impossible to say how much curvature these factors would introduce into the marginal cost function. In view of the similarity between the adjusted and unadjusted functions, the first three of the limitations to the Steel Study just noted probably would not greatly change the derived function, although they do detract somewhat from the confidence to be placed in the empirical results.

Further difficulties of interpretation arise in the Steel Study from the way in which other problems listed in Section 1 were handled. Crucial among these are the measurement of output, the number and frequency of observations, the selection of the form of the cost function, and the comparability between periods of cost and output data. The output of all of the diversified products of the Steel Corporation was measured in terms of tons, weighted by the ratio of average mill costs of any product to the average mill costs of all rolled and finished products. Because of the change in the relative importance of such products as "sheets" through the period 1927–39, this measure of output is not entirely satisfactory. The independent variation in the output of such nonsteel products as coal, transportation, and iron ore raise essentially similar difficulties, although the direction of the impact of these ambiguities upon "output" is not readily apparent.

The Steel Study does not reveal whether any tests were made of the reliability of the selection of a linear regression between the twelve annual values for output and total costs. A cubic or higher order equation might have been more appropriate. The fact that a higher order equation must necessarily fit the observed values of cost and output within narrower limits does not, however, render the linear regression invalid. The difference in "closeness of fit" of the linear and higher order equations must be large enough to be statistically "significant." Perhaps of even greater importance, the difference must be sufficiently large so that the higher order equation indicates more accurately the view of cost behavior which figures in decision formation.

On the basis of the foregoing critical appraisal, it would appear that while a linear function may be the best single estimate of the cost function, the degree of confidence that may be placed in the results leaves quite a range for possible curvature in the total cost function and, *a fortiori*, in the marginal cost function.[38] The statistical function may give a reasonably accurate estimate of the effect on costs of a variation in output from year to year on the average. It should not be presumed, of course, that the derived function is necessarily relevant to decisions regarding output and price for particular products at specific times.

Other studies of cost functions have yielded, in many cases, very similar results.[39] Although methods analogous to those utilized in the Steel Study have been adopted, the peculiar features of each firm and industry have substantially influenced the character of the available data and the techniques employed in each case. For instance, the study of the cement industry by Kurt Ehrke and E. Schneider[40] had a fairly homogeneous product with which

[38] Richard Ruggles, "Linear Total Cost-Output Regression," *American Economic Review*, XXXI (June 1941), pp. 332-34.

[39] J. M. Clark, "Basing Point Method of Price Quoting," *Canadian Journal of Economics and Political Science*, IV (November 1938), p. 479 n., remarks that in the cement industry marginal cost "does not appear to change much with changing rates of utilization."

[40] Kurt Ehrke, *op. cit.*, pp. 275-310. Dr. Hans Staehle reports that calculations based on these cement data reveal rising marginal costs at levels of output above rated capacity.

to deal, whereas Joel Dean's investigations covering firms in the furniture, leather belting, and hosiery industries and a firm in the department store field[41] presented problems in output measurement more complex than those in cement but less involved than in the case of the Steel Corporation. Roswell Whitman's study of a department store is one of the few to show other than a completely linear cost function; he found marginal costs rising sharply at the Christmas season peak in output.[42] Throughout the rest of the range of fluctuation in output there appeared to be a virtually linear cost function.

In addition to "statistically derived" cost-output relations, as was noted at the beginning of this section, a number of studies of "estimated" cost functions have been made, particularly by German writers. These inquiries differ from those typified by the Steel Study in that they utilize cost estimates rather than the direct accounting experience of a firm. The estimates must rely, of course, on the actual cost experience of the enterprise, but not in any simple and automatic fashion. In many ways "estimated" cost functions resemble break-even charts, and for this reason much of the discussion in the next section (3) will apply to them. Both characteristically measure output in terms of percent of "capacity," identifying, at least implicitly, "economic" and "physical" capacity; both frequently require the collaboration of engineers and accountants. The estimated functions are to be distinguished, however, from break-even charts, since they are cast in the form of a cost function, usually are unaccompanied by estimates of revenue so that they cannot indicate a break-even point, and are strictly historical.

The studies of estimated cost functions invariably arrive at a linear function.[43] Reinhard Hildebrandt, for example,

[41] *Loc. cit.*

[42] *American Economic Review*, Supplement, XXX (March 1940), p. 401.

[43] See estimates of the way in which costs for power stations vary with rate of utilization of plant, in Walter Rautenstrauch, *op. cit.*, pp. 109-11.

regards this relationship as typical: "The repeatedly observed phenomenon that, in addition to materials and labor costs, common indirect costs also show themselves as a linear function of rate of plant utilization, and always at similar rates and accompanied by similar values, cannot be regarded as an isolated phenomenon characteristic of only a few plants."[44] The linearity conclusion is somewhat surprising in that the estimated functions, like break-even charts, cover the range of output from zero to full capacity. It is difficult to tell whether this is simply a matter of projecting the relationship expected in the "normal" range of fluctuation to upper and lower limits, or a result of careful calculation. In the absence of extensive experience with levels of output at either extreme, the former possibility appears the more plausible.

3. Relationships Between Sales and Costs

The relationship between sales and costs is frequently presented in break-even charts. They are a relatively recent addition to the tools of business analysts, engineers and executives, and are widely used as an aid to many different sorts of decision. Such charts are of interest here because they contain estimates of the way in which costs vary (or are expected to vary) with sales at constant prices and hence with output. The X-axis shows output (usually measured in percentage of capacity); the Y-axis is labeled in dollars. A line indicating total costs and another indicating revenue at various rates of output are plotted according to this system of coordinates. This kind of chart takes its name from the fact that the lines intersect where costs equal revenue, that is, the "break-even" point of plant utilization. Total cost is sometimes broken down into its component elements, at least into fixed and variable costs. The Steel Study discussed in the preceding section is presented in

[44] *Op. cit.*, p. 127.

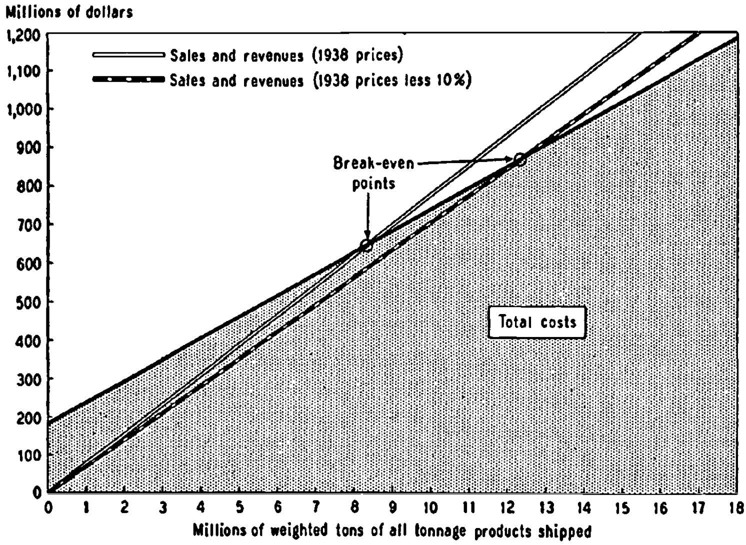
the form of a break-even chart by the addition of two revenue lines as is shown in Chart 2.[45]

At least two types of break-even charts are constructed;[46]

Chart 2

RELATIONSHIP BETWEEN SALES AND COSTS: EFFECT OF REDUCTION FROM AVERAGE 1938 PRICES

U. S. Steel Corporation and Subsidiaries



Note: Costs are based on 1927 - 1938 experience, adjusted to 1938 conditions.
Reproduced from TNEC Hearings, Part 26, Iron and Steel Industry (January 23-25, 1940), p. 13777

one, the historical, is based on the income and expense of an enterprise over a period of years as shown in the profit

[45] United States Steel Corporation, *op. cit.*, Vol. II, p. 57.

[46] In addition, some firms draw up detailed budgets showing costs and realizations for different outputs, and consequently the break-even point, without any chart being drawn. See, for instance, Samuel H. Selman, "Going to Make a Profit?", *Factory Management and Maintenance*, 96 (1938), p. 92. Similarly, the rate of utilization of a particular machine is recognized as relevant to the decision whether or not to buy new equipment. See, for example, E. M. Richards, "To Buy or Not to Buy Equipment," *Factory Management and Maintenance*, 91 (December 1933), p. 499.

and loss statement.[47] The other, the budgeting break-even chart, is based on an estimate of the relationship which may be expected to prevail for an ensuing period. The historical break-even chart is used by investors in choosing between alternative issues, as is evidenced by the fact that the term "break-even point" has become a commonplace in investment analysis.[48] Adjustments for factor price changes and certain problems of measuring output are skirted by the use of dollar sales to measure output. Break-even charts of this type must be employed with extreme caution. In particular, long periods of observation should be avoided, especially when there is a rather steady increase in the volume of output, as the chart may become in effect a curve showing how costs behave under conditions of expanding plant rather than of the fluctuations of output in a relatively constant plant. A break-even chart of this sort is most valuable when the enterprise has not introduced any fundamental changes in the scale of operations in a period during which there has been sufficient variation in output to establish the expense line clearly. The two following break-even charts, one for the First National Stores (Chart 3),[49] and the second for the Granite City Steel Company (Chart 4),[50] illustrate respectively a case in which output was expanded virtually throughout the period, presumably through an expansion in the number of operating units, and a case of apparently unaltered plant until the last year covered.

The budgeting type of break-even chart based on esti-

[47] Break-even charts summarizing profit and loss statements do not make explicit corrections for changes in factor prices. The use of dollar sales as a measure of output serves as a partial corrective. For illustrations of historical break-even charts, see Walter Rautenstrauch, *op. cit.*, Ch. 7.

[48] The term is used at least four times in the Standard Trade and Securities Analysis of *Steel and Iron Companies*, Basic Survey, Part II (March 31, 1939), Vol. 91, No. 26, Sec. 3.

[49] Rautenstrauch, *op. cit.*, p. 311.

[50] *Ibid.*, p. 308. The most significant change was the installation of a continuous strip mill in 1936. The point for this year is the farthest from the regression line.

mates is used by executives in making decisions. Such a chart provides a simple and convenient way in which to estimate the effects on profits of changes in wage rates or material prices; the cost line need only be moved appropriately. One set of cost and revenue lines must be drawn for each level of wage rates, material prices, and price realizations from goods for sale. It has been suggested that these charts are useful also for comparisons of the performance of plants of different companies.[51] When various price structures are being considered, break-even charts provide a convenient way of presenting the effects on profits of different revenue lines (arising from different prices).[52] Such charts purport to provide direct answers to a question like the following: With given factor prices, what must sales return to provide a profit of a specified amount? Charts relating output and specific elements in costs may afford important help when budgets are drawn up for a subsequent period, in view of expected levels of output.[53]

The present interest in break-even charts is centered on the shape of the cost lines—the way in which costs vary with different amounts of output. Almost all break-even charts, whether historical or budgeting, appear with linear total cost-output relations. The historical break-even charts show straight line relationships partly because of the bias of investigators in looking for a simple relation, such as a fixed cost plus a constant variable cost per unit of output, but also because the points plotted on the chart for a period

[51] For example, a chart comparing Chrysler, General Motors and Packard is presented in Rautenstrauch, *op. cit.*, p. 303.

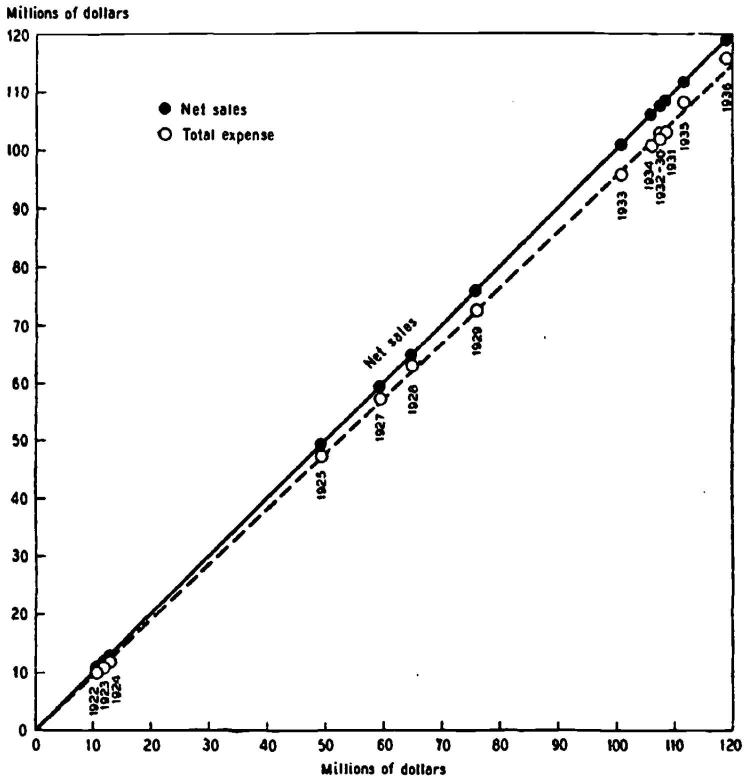
[52] Chart 2 (*supra*) from the Steel Study is drawn to show the effect of a 10 percent price change. It should be noted that revenue lines are ordinarily drawn as if the price of goods sold were independent of the amount sold, in spite of the fact that interest in the break-even point implies a realization that the market will not automatically take any output the firm can produce.

[53] Charles Reitell, "Merchandising Operations," in *Corporate Finance Statements, Proceedings of the Accounting Institute, 1940* (Columbia University Press, 1940), pp. 31-32, illustrates the estimation of delivery costs from a scatter of costs and volume.

during which the plant has not been fundamentally changed usually reveal an essentially linear relationship. In the case of budgeting break-even charts this result would appear

Chart 3

THE BREAK-EVEN CHART
OF FIRST NATIONAL STORES, INC.



Reference: Moody's financial manuals.
Note: The solid line represents sales plotted against sales for reference purposes. The broken line shows expenses on the vertical axis and sales on the horizontal axis.
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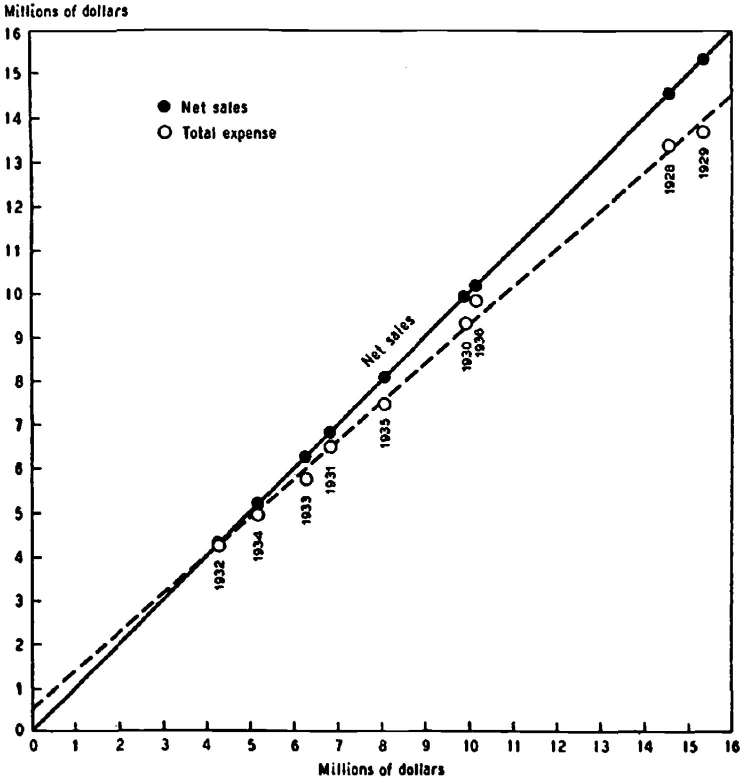
to be determined in large part by the way in which they are constructed. Total costs (either estimated or historical) are very sharply divided between the fixed and the variable. The former are presumed to be absolutely invariable with respect to output, whereas the variable costs, in the absence of more specific and detailed information to the contrary, are held to be directly proportional to output. As a critic of

Chart 4
THE BREAK-EVEN CHART
OF THE GRANITE CITY STEEL COMPANY



Reference: Moody's financial manuals.
Note: The solid line represents sales plotted against sales for reference purposes. The broken line shows expenses on the vertical axis and sales on the horizontal axis.
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this linearity points out, "The conventional break-even chart is based on the assumption that all variable expense has a straight line relationship or is in direct proportion to activity, and that all the items classed as fixed expense remain as constant fixed expense for the whole range of plant activity." [54]

The budgeting break-even chart in the hands of a careful cost executive becomes a delicate tool which must be adjusted frequently for greatest accuracy.

> The modified break-even chart constitutes a variable budget. As labor rates change and methods are improved, costs change so that the chart should be revised at least twice a year at six-month intervals, and more frequently if necessary. During the period of time for which the chart is effective, the relationship between expense, income, and activity should be as shown on the modified break-even chart, provided the average of selling prices remains the same. When the average of selling prices is changed, it is a simple matter to superimpose a new income line on the break-even chart.[55]

It would be very helpful to the study of cost-price relationships if the part played in pricing decisions by these charts could be appraised in a number of firms. Unfortunately such charts are extremely confidential business documents and are not as readily available as historical break-even charts.

The fact that linearity is implicit in the customary methods of making budgeting break-even charts is in itself significant to the purposes of this study. If decisions are made on the assumption of a linear relationship between costs and output, this fact alone is basic to any explanation of such decisions as pricing, quite apart from the "real"

[54] H. R. Mallory, "A Silk Mill," *Mechanical Engineering* (August 1933). According to this executive, variable costs start out at a certain rate and then decrease at a declining rate, approaching a constant per unit of output figure. Rautenstrauch, who uses straight lines, points out (*op. cit.*, pp. 268-69) that the results obtained by this varied variable cost are substantially linear for a range of output from about 25 percent of capacity to capacity. In the chart for the United States Steel Corporation given on page 305 of his study the points for the years 1928–33 clearly present a slightly curved line for the cost relationship, with variable costs decreasing as output increases. Rautenstrauch draws no line for these years alone.

[55] *Ibid.*, p. 271.

shape of the cost function. Although the actual cost function of an enterprise at a given time may be of the inverted ogive shape, as is customarily assumed by economists, unless the curvature is sufficient not only to be recognized in the experience of the business but also to alter a decision substantially, the linearity assumption on the part of the business executives may be entirely valid. The implicit assumption in most break-even charts that the cost function is linear may be simply a reflection of the fact that for all practical purposes of decision formation such an assumption is adequate. In view of the uncertainties and range of indeterminacy surrounding decisions, it would require a rather large curvature in the total cost functions to modify any decision. Except for extreme levels of output, linearity may be an adequate working conception of the cost function for the business executive.

4. *Significance of the Empirical Studies*

Empirical studies of the relation between variations in output and costs, of the types just examined—statistical cost functions, estimated functions, and cost-revenue charts—indicate with few exceptions a linear covariation, that is, a linear cost-output relationship. These results are of particular interest, since economic theory has usually presumed an inverted ogive for total costs and marginal costs that are markedly U-shaped. This view has been supported by rather common-sense propositions. As variable factors are applied first to a fixed factor, the incremental cost may be expected to decline with improvement in the combination of factors; after a great many units of one factor have been applied the combination is bound to worsen and the incremental costs will increase. The belief that marginal costs between these extremes of output describe a smooth U-shaped curve reflects a bias in economic theory. If marginal costs approach the horizontal over large ranges of output, small changes in demand make for larger changes in output. An economic system with linear cost functions is more unstable than one with markedly U-shaped marginal cost functions.[56]

The marginal cost curve was generally held to be U-shaped throughout the whole range of output on the ground that the conditions implicit in discussions of the "law of diminishing returns" were typical of industrial production. (a) That is, the fixed factor or combination of fixed factors were assumed to be entirely indivisible; plant and equipment had to be operated as a whole or not at all. (b) Within the framework of the Marshallian "short run," as output varied plant and equipment (fixed factors) were thought to be invariable except for adaptations to insure the optimum combination of all factors. The "form" of the fixed factors was envisaged as changing to permit the technical optimum combination, but the "quantity" of the "fixed" factors was unchanged. With given factor prices, marginal costs would be U-shaped with variations in output under these two conditions.[57]

The examination of empirical studies in Sections 2 and 3 concluded that in the cases explored a linear cost function was the most probable relation within the observed range of fluctuations of output, although basic difficulties in the

[56] "Economists who make use of the competitive analysis of value have a strong unconscious bias in favor of rising and falling supply price, simply because, if supply price is constant, their analysis has nothing interesting to say." Joan Robinson, *Economics of Imperfect Competition* (Macmillan, London, 1933), p. 118.

[57] The general acceptance of U-shaped *cost curves* was supported by empirical studies of *production functions* in agriculture. E. H. Phelps Brown, *op. cit., Econometrica,* IV (April 1936), pp. 123-27. Marshall (*Principles,* Book IV, Ch. III) refers to a study of the Arkansas experiment station comparing yields per acre with the amount of ploughing and harrowing. In this connection see Bernard F. Haley, "A Preliminary Study of the Laws of Varying Costs" (Harvard Ph.D. thesis, November 1932).

It is also probably true that this view of a U-shaped marginal cost curve has been supported by arguments not strictly applicable to the cost function. For instance, some writers have pointed to the importance of overtime rates, poorer workers, etc., at high levels of output as proof of the U-shape of the curve. These may be induced changes in cost, but they are not precisely related to the cost function.

method and techniques made it impossible to place sufficient confidence in the solution to preclude marginal cost curves with considerable curvature. This conclusion certainly does not justify the statement that all cost functions are linear, but it does suggest that the conditions underlying discussions of "diminishing returns" not only need to be re-examined, but may not be as typical as presumed. Indeed, even so cautious a conclusion as this must be qualified. The results of the several types of empirical studies designed to measure cost-output relations may be substantially influenced by the accounting conventions for allocating costs over different periods (see Chapter IV).

It is quite possible, however, to construct a simplified model of production that is consistent with linear cost functions. In fact, the ordinary discussion of the conditions affecting diminishing returns may be so generalized and modified as to include both linear and U-shaped marginal cost functions as special cases.[58] Only empirical research, of course, can indicate the relative representativeness of the various possible cases.[59] One element in the explanation of linear cost functions may be the fact that fixed "factors" are more or less divisible. The more divisible the plant and equipment, the more it is likely that the variable costs may be linear. When a small volume of output is being produced, it is possible in many firms to shut down completely parts of the plant or groups of machines.[60] Within wide limits the fixed factors may be compared to small units whose combined costs are simply proportional. But divisibility may be achieved in yet another way with fixed

[58] George Stigler, "Production and Distribution in the Short Run," *Journal of Political Economy*, XLVII (June 1939), pp. 305-27. The discussion uses the terminology suggested in this article. See also Joel Dean, *Statistical Cost Functions of a Hosiery Mill*, Ch. I.

[59] Such reformulation of theoretical tools and designation of the typicality of special cases illustrate the importance and usefulness of the coordination of empirical studies and theoretical analysis.

[60] For political or humanitarian reasons the firm may refrain from a complete shutdown, particularly when plants are in a one-factory town.

plant and equipment that cannot be operated in "units." Where starting or stopping costs are not large it may be possible to operate a plant more or fewer hours per week, with one or more shifts. As a matter of practice this factor is very important, although it really involves a cost function with a meaning different from that implied in the timeless functions of comparative statics.

A second element in an explanation of linear cost functions has been suggested most clearly by George Stigler, but like the second type of divisibility just discussed this analysis involves a concept of a cost function different from that implied in comparative statics. Plant and equipment are frequently constructed to produce either a fluctuating output or different types of output rather than a single specified volume or product. "Were it not for the flexibility built into plants, outputs in excess of optimum would involve prohibitive marginal costs, while those at less-than-optimum outputs would be very unprofitable." [61] As output changes, minor variations are made in the plant and its equipment. With the expansion of output in a shoe factory, for instance, it may be possible within wide limits to add identical machines.[62] Or, in a paper mill, instead of making only one width of paper, a machine may be *flexible* to the extent of making several widths, thicknesses and types, or may run at two or three different speeds. A lathe may be used to turn out material of varying lengths. If a single type of product or a specified amount were certain to be produced, no flexibility would be required. It is the possibility of other outputs or types of products which introduces the need for flexibility. The flexible equipment could not produce the designated output or product as cheaply as equipment entirely specialized, but for other outputs or products costs would be lower. The

[61] *Op. cit.*, p. 316.

[62] An electric power generating system may be so built as to combine hydroelectric power and steam, permitting relatively more constant costs as output varies. See A. H. Markwart, "Aspects of Steam Power in Relation to a Hydro Supply," *Transactions, American Society of Mechanical Engineers* (1926), p. 187.

expectation of variations in output introduces flexible (and less specialized) fixed factors and hence relatively more linear cost functions than could be anticipated for more specialized plant and equipment.

Several of the explanations just offered with reference to possible linearity in the cost function involve a concept of a cost function different from that presumed by comparative statics. This traditional function of economic theory shows the way in which costs vary with output, when all other relations are assumed to be unchanged. One could formulate still another concept, which would simply be the way in which business executives at any one time expected costs to vary with output, regardless of the source of the impact on costs. The concept would include changes in cost due to factor price changes arising from output variation, the variation in costs attributable to flexible plant, minor technical changes, etc. In any system of determinants of cost, the concept would include the effects on costs of cross-derivatives with respect to output. It could be argued that such a function would be more relevant to actual decisions. But there can be little doubt that this concept would also be difficult to measure statistically, because the part of the change in costs which would be correlated with output would have to be separated from any total change in costs.

For the present it is important to note that the statistical linearity of cost functions could be explained by the divisibility and flexibility of fixed plant and equipment. This would mean, strictly speaking, that the U-shaped marginal cost function of economic theory is not invalid. However, if empirical studies should show that conditions of divisibility and flexibility were typical, only a linear function could be both pertinent and useful for the interpretation of economic phenomena.[63]

[63] The significance of a marginal cost function of this shape for pricing in an individual firm and in various types of market structures is discussed in Chapter XI.

5. *Research Possibilities*

Since only a limited number of studies have been made of the effects of variations in the rate of plant utilization on costs, and because of the importance ascribed in economic theory to this relationship, the field appears to be a very important and rewarding area for research. The survey of difficulties inherent in the problem, and the appraisal of the Steel Study of Professor Yntema in Sections 1 and 2, support this impression. The sample of studies that have been made so far depends largely on the chance of personal contacts, the cooperation of particular firms, and the convenient availability of data. A large number of useful inquiries could be made by investigators with some statistical sophistication in a wider and more representative group of enterprises. There is need for additional studies to test the present meager sample in firms confronted by different types of circumstances. For instance, the new studies should include firms with wider fluctuations in output, those allowing more frequent observations, those in service and mining industries; and more attention should be paid to smaller scaled industries and to areas of industry with less technical change.

As research projects these individual studies of cost functions would have the advantage of narrow scope. A number of such inquiries might profitably be undertaken by university candidates for degrees where business contacts are available, or indeed by business enterprises themselves. In the latter case, there would certainly be an occasion for more cooperative research between universities and business firms.

It is appropriate here to make a suggestion with respect to the method of research which will be applicable to all the topics of succeeding chapters. Since costs and prices are both influences on, and results of, business decisions, much more than we now know can be learned about the process of reaching these decisions. Explanations could be pitched on various levels of abstraction, from economic theory to psychoanalysis. But in view of the present state of knowledge of the motivation and direction of conduct, it appears most advisable that studies of business decisions proceed along the following lines: (a) Case studies of particular firms should combine qualitative material from interviews with statistical techniques. (b) Attempts should be made to ascertain what information is available to those who make decisions. (c) In every enterprise certain persistent patterns of policy or working rules-of-thumb can be identified. (d) Individual instances of action should be examined, and not merely issues of broad "policy." While such studies cannot give definite answers to all questions, or "explain" all actions of the firm, they can provide a useful and almost indispensable counterpart to statistical inquiries.

In conclusion, we may venture several suggestions which are even more specific. Since the measurement of output presents one of the most difficult tasks in cost function studies, those types of industries in which firms produce relatively homogeneous products that are not altered substantially through time offer the simplest cases for investigation. Although it is true that a firm seldom, if ever, produces for any considerable period a product completely homogeneous with respect to size, model, style or orders, there are enterprises in industries like cement, electric power, gold mining, grey goods, bread baking, etc., which present much simpler cases than firms producing agricultural implements or women's dresses. It would be valuable, too, to extend such studies to nonmanufacturing industries such as mining, the service trades, wholesaling and retailing. In some cases attempts to measure output would encounter insuperable obstacles, but in mining or trucking, for instance, the problems ought not to be too difficult. In this field a rather useful study would be the collection and analysis of a group of engineering estimates of the way in which costs vary with output. The assumptions underlying such calculations might prove significant for explanations of business behavior.