Volume Title: National Economic Planning

Volume Author/Editor: Max F. Millikan, editor

Volume Publisher: NBER

Volume ISBN: 0-87014-310-7

Volume URL: http://www.nber.org/books/mill67-1

Conference Date:

Publication Date: 1967

Chapter Title: Locational Choices in Planning

Chapter Author(s): Thomas Vietorisz

Chapter URL: http://www.nber.org/chapters/c1420

Chapter pages in book: (p. 39 - 130)

Locational Choices in Planning

THOMAS VIETORISZ

NEW SCHOOL FOR SOCIAL RESEARCH

Introduction

OBJECTIVE AND PROSPECT

This paper deals with the application of mathematical programing resource allocation models to the problems of economic development planning by geographical locations: local areas or regions within a country or countries within a partially integrated supranational economic community. Planning decisions in this field are politically highly sensitive, and the quantitative information that can now be provided to policymakers as a background for these decisions is far from satisfactory.

The models mentioned are among the most up-to-date tools for the quantitative study of planning problems. Such models can be formulated to represent the major developmental choices of economic systems; at the same time, they also furnish a frame of reference for the evaluation of individual projects and branches of economic activity, thus pointing the way toward the eventual consolidation of policy choices at different levels of detail into a unified decision system of balances and priorities. While the application of these models is subject to limitations —mathematical problems in dealing with economies of scale and other nonconvexities, time lags, probability distributions—their potential contributions to the conceptual understanding and empirical definition of planning problems, especially in regard to locational choices, are far from being fully exploited.

The bulk of the paper is dedicated to the formulation and analysis of multiperiod locational models, both in aggregated and disaggregated form. Since economic development is such a thoroughly dynamic phenomenon, the results of a purely static analysis are inherently to be distrusted; for this reason, it has been regarded as indispensable to deal with multiperiod models even if this places a considerable formal burden on the analysis. Since multiperiod models with locational and interindustry detail have not been thoroughly studied before, it was thought worthwhile to present and interpret in terms of such a comprehensive model a number of results derivable from partial models of different kinds, i.e., models without either locational, multiperiod, or interindustry detail.

A key question of planning in regard to locational choices is the extent to which development should be geographically balanced or unbalanced. While it has been far from possible to clarify this matter conclusively, the extent to which planning models formulated in different ways tend to lead to a greater or lesser degree of geographical concentration of economic activities has been a persistent concern throughout the paper.

The argument in favor of *unbalanced* growth asserts that the concentration of resources into limited areas will permit these areas to grow sufficiently fast to acquire a certain momentum of growth that will eventually be transmitted to the lagging areas, while a dispersal of the former resources over all areas would deny the possibility of a successful "take-off" to any area. The argument in favor of *balanced* growth points out that a development process limited to some points will lead to excessively narrow markets in many lines of production, thus leading to a failure to achieve adequate economies of scale, and that it will deprive the system as a whole of the potential contribution of savings, skills, and other resources that would be forthcoming from the lagging areas if their economic development and cultural transformation were not held back by the draining off of resources to other areas.

There seems to exist a widespread notion at present that mathematical programing models can be expected to yield optimal growth for a system of regions as a whole when growth is unevenly distributed among the regions. Thus it is often postulated that the maximization of national income without constraints on the regional distribution of this income will lead to socially and politically unacceptable results. Therefore, such constraints have to be introduced in explicit recognition that they will lead to a certain sacrifice of national income. This sacrifice, the reasoning goes, is the price that has to be paid for the social or political benefits to be won.

The foregoing notion is based on qualitative considerations rather than on solid empirical evidence or a careful analysis of the structure and behavior of regional allocation models. In point of fact, there is only one highly aggregated analytical model that explicitly arrives at a conclusion concerning the benefit of concentrated investments within a multiregional system, and the limitations of the approach that has been used are even in this case clearly recognized by the authors (Rahman, 1963, and comment by Dorfman). The origins of the notion are therefore to be sought in the great difficulty, well known to any person with practical planning experience, of finding economic activities in regard to which backward regions have a clear-cut advantage under customary criteria of project evaluation. In practice backward regions often come out poorly in regard to almost all conceivable activities, the classical theory of comparative advantage notwithstanding.

The argument in favor of balanced regional growth hinges on the presence of economies of scale—a consideration that has never been brought adequately within the purview of economic theory or practical criteria of project evaluation. It also hinges in part on qualitative and extraeconomic factors having to do with psychological motivations and cultural change. It is thus possible to hold an opinion in favor of the long-term optimality of balanced regional growth on the basis of such general considerations and still to subscribe to the notion that mathematical programing models will yield optimal growth data, under conditions of regional concentration of investments. The considerations regarding balanced regional growth can then be regarded as "background information" that is to be relied upon to "modify" the results of the incomplete mathematical analysis for purposes of policy decisions.

It will be shown that the over-all picture that emerges from the analysis of regional resource allocation models depends to a significant degree on the assumptions that are built into these models. It will also be shown that possibilities of reformulating these models exist which suggest that institutional arrangements involving planning can probably be created under which conflicts between over-all system growth and the geographical dispersion of this growth are reduced and perhaps eliminated.

The discussion is introduced by a survey of the principal areas in which locational choices arise in planning and the main analytical difficulties that are still unresolved, and is followed by an appraisal of methods for reconciliation of multiple objectives. Thereafter, a general locational model is formulated and its features—particularly the connections between optimal solutions of the model and standard social accounting concepts such as national or regional income and the relationship between production possibilities and preferences—are explored. Next, simplified versions of the general model, including aggregated and one-commodity formulations, are analyzed both by individual periods and by long-run behavior features, followed by a sketch of generalization possibilities to multicommodity models. Finally the relationship among preferences at different policy levels, incentives, and autonomous growth trends is discussed, and certain ways of reformulating the model are explored. The latter are illustrated by some simple numerical examples at a highly aggregated level.

SURVEY OF LOCATIONAL PROBLEMS IN ECONOMIC PLANNING

Problems involving locational choice occur at several levels in economic planning. National development plans are generally formulated without a spatial dimension in the first instance: Such plans, whether they are in global or interindustry terms, have to be *broken down* by major regions of the country in order to check their implications for regional growth; the general objectives set out in national-level plans, furthermore, have to be translated into specific development projects involving in each case the choice of particular locations. Conversely, in a given country a variety of regional or municipal development and promotion programs based on different assumptions and using widely varying planning methods may initially be formulated in relative independence: Such plans have to be *integrated* into a consistent national plan, or, if a national plan has been formulated independently, the inconsistencies between the set of area plans and the national plan have to be resolved. This problem, which arises in the context of planning for a single country, has a supranational counterpart involving the coordination of development plans of independent countries within the framework of common markets or industrial development communities.

The geographical breakdown of a single plan versus the integration of separate area plans are clearly complementary and call for an interplay between two or more planning levels which are organized, at least partially, in a hierarchical fashion, with geographical location acting as the organizing principle.

The problem of breakdown versus integration of plans arises also in a different context, namely, in the relation between central and sectoral (or industrial) planning. While the central plan itself may set out industrial targets in considerable detail, it is practically always found expedient to relegate detailed feasibility studies, project planning, and the execution of *sectoral plans* to lower-level planning organs. Locational analyses are often carried out at this level in connection with the feasibility and project studies. Their results subsequently are incorporated in sectoral development programs which, suitably summarized and abstracted, are communicated to the upper-level planning center. The choice of location thus also enters the decision-making process through a second hierarchical system whose organizing principle is the subdivision of the economy by sectors or industries rather than by geographical areas.

Finally, locational choices arise in connection with *city planning* and its extensions. The focus here has historically been on rational land use, the efficient layout of transportation arteries and terminals, and social problems arising in connection with urban life; in recent years, however, there has been an increasing extension of emphasis from physical to economic planning problems.

The first requirement to be met by any plan is that it should be free from major contradictions and should, in regard to secondary detail, also be as free as possible from inconsistencies. Locational choice does not offer a marked increase of analytical difficulties in comparison with other planning problems as long as this limited objective is at the center of planning efforts; there is, however, a heavy expansion of statistical requirements. Locational detail in data is not easy to come by when relying on traditional statistics the same way as, for example, industrial-process detail is hard to find. When the criterion of efficiency is introduced into locational planning several sources of analytical difficulty have to be faced, difficulties that are not peculiar to locational choices alone but are particularly troublesome in this field. They include, first of all, the setting of development goals, which will be one of the concerns of the present paper. Secondly, economies of scale and other sources of nonconvexity acquire a key importance since they are essential in the delineation of market areas and thus in the interaction between regions; the implications of nonconvexity with regard to the existence of multiple equilibriums and the breakdown of the price system in achieving an over-all optimum are of great practical consequence. Next, the problems of efficient allocation of resources in a double hierarchical planning system, organized both by sector and by geographical area, are far from being adequately understood even in the absence of nonconvexities and a fortiori in their presence. Finally, any adaptive system—whether of a free-market variety or involving organized planning decisions—when operating in the field of locational forces is subject to the effects of long-term lags in adjustments, due to the long lifetime of plant and equipment and of transport arteries and terminals. During the lifetimes of such investments they can be largely regarded as fixed parts of the economic environment, and a great variety of secondary adaptations will take place on the basis of their existing locations. These adaptations are generally of the kind which reinforce the original choice of location of the long-term investment; thus socially undesirable locational patterns, once established, acquire a life and momentum of their own which become exceedingly hard and costly to modify. For this reason locational choices have to be undertaken with a time perspective that is disproportionately long in comparison with the accustomed planning periods: It is often reasonable to consider a time span of fifty years or even more.

No attempt is being made in this paper to present a comprehensive study of current planning practices in relation to locational choices or a complete analysis of the theoretical and practical issues involved. A number of studies on regional planning in different countries have recently appeared, and a compilation and analysis of this material and other pertinent information is at present underway elsewhere. An encyclopedic summary of research in regional and locational problems in Western countries is available, together with a survey article of recent data covering the field of regional economies. References to this material are in the Bibliographical Note at the end of this paper.

The Setting of Development Goals

THE WELFARE IMPLICATIONS OF LOCATIONAL CHOICES

The problem of efficient choice between available alternatives involves the consideration of objectives in relation to instruments. The selection of proper objectives is, however, not obvious in regard to the locational choices that arise in planning, since there is generally more than one entity whose welfare has to be taken into account and which may participate in a more or less autonomous fashion in the process of goal setting. Thus in a country with several regions the question arises of the proper objective of development: Is it the advance of the country as a whole without regard to the regional distribution of this advance, or are the interests of the individual regions to be incorporated in the definition of a national goal and, if so, in what form?

There are very few explicit locational choices in which this dilemma does not enter in one way or another. Perhaps a plant location problem involving only a marginal part of the economy and considered against a background of a satisfactory regional balance might be thought of as being devoid of this multiplicity of goal units, in that a choice which contributes most to the system as a whole and does not disturb the balance of its parts could be regarded as advancing equally the welfare of all parts. Such a formulation, while apparently in accord with common sense, presupposes an understanding of what constitutes a "satisfactory" balance, and offers no clue as to how such a balance is to be established from a starting position that is unbalanced. Moreover, it offers no help in regard to the many important locational decisions in planning that are far from being marginal in their effects upon the economy. None the less, the formulation has some merit in a negative way, since it throws light upon investment decisions in planning that are *not regarded as having a locational dimension,* even though any investment obviously has to be located physically at some point in space. For example, when the question arises whether a store should be located at a given street corner or three blocks further away, the interests of groups of persons associated with either location may be influenced to some extent by the decision. Yet it is not customary for planners to think about such a decision as involving a principle of locational welfare balancing. While the example mentioned is trivial, the same lack of concern for locational welfare balancing often extends to much more important decisions involving more extended geographical areas. It would appear that efficiency decisions taken without overt concern for locational welfare balancing are based on some implicit assumption about an underlying "satisfactory" balance.

This problem is, of course, not peculiar to locational choice, since it arises in goal setting for any collection of individuals. It has been discussed extensively in the economic literature under such headings as interpersonal welfare comparisons and the derivation of community indifference maps or community welfare functions. In locational choices involving the regions of a country, however, the problem assumes particular political importance because political pressures, under many kinds of existing institutional arrangements, are relatively easy to organize on a geographical basis. A situation which is in many ways analogous to the political balance of regions within a country is the balance of sovereign nations within a common market or an economic development community. In both cases the different geographical units have common as well as contrary interests. Institutional rationalizations, however, tend to stress the common interests in the case of regions within a country, at the same time blunting the demands for an immediate and equal geographical sharing of over-all benefits, while the same rationalizations tend to stress the vigorous defense of "fair" shares in the supranational case, with a more reserved admission of common benefits. For this reason, the flexibility of supranational planning is greatly reduced, as witnessed by the problems of several contemporary attempts at common market formation. These difficulties are accentuated by an emphasis on joint investment decisions as against a more conservative approach largely restricted to trade liberalization and labor exchange.

The analytical approach to the problem of locational choices will differ according to whether (1) a single decision-making center can be assumed to exist which, if necessary, reconciles conflicts between the welfare objectives of the different units of the system (and, by implication, between any unit and the system as a whole); or whether (2) an interplay exists between several partially independent decision-making units. For the former case, a maximizing model can be constructed whose solutions are studied in conjunction with function of the welfare objectives adopted for different regions or locations. These solutions have a normative value, provided that the underlying welfare objectives are accepted. In the second case, the interplay between the different units has the nature of a strategic game whose outcome depends on the elements of strength possessed and the strategies followed by each participant.

Throughout the discussion that follows, attention will be centered on the first alternative. In particular, the question will be posed: "To what extent do necessary conflicts exist between the welfare objectives of individual geographical units within a larger system? To the extent that current formulations of allocation models overstate these conflicts, the interests of the separate subunits will be recognized as being more complementary than conflicting, and cooperation and the delegation of powers to a common decision-making center may often become the best strategy for these independent units. If the common interests do not dominate the divergent interests quite to the same extent, the analysis of alternative strategies cannot be avoided; the latter situation, however, will no be studied further in the present paper.

MULTIPLE OBJECTIVES AND PROGRAMING MODELS

The problem of efficient choices in planning can be analyzed by means of linear programing models and their nonlinear extensions. In such models an objective, defined in terms of activity scales, can (without loss of generality) be assumed to be maximized, subject to constraints imposed by technological possibilities and institutional limitations.[1] When there are several objectives which are to be observed simultaneously, as in the case of the development of a system of regions where

[1] A minimization problem can be converted into a maximization problem and a minimal constraint can be converted into a maximal constraint by a change of signs. For standard discussions of linear programing, see Dantzig (1963), Gass (1958), Hadley (1961), and Simonnard (1962).

the advance of each region is desirable for its own sake, these multiple goals must be reduced to a single objective by one of the following two techniques:

1. Maximization of a weighted sum of the several objectives
2. Maximization of a single objective while the remaining objectives are treated as constraints, in the sense that admissible solutions to the problem are required to attain or exceed prescribed levels of the latter objectives

For example, in an interregional development model where the maximization of the net products of two individual regions constitutes independent welfare objectives, the first technique assigns a weight to the net product of each region and maximizes the weighted sum of the net regional products, while the second technique maximizes the net regional product of one region subject to the subsidiary constraint that the net product of the other region has to exceed a certain minimum.

In national development models of the linear programing type the measure used for quantifying the national welfare objective is generally additive between geographical regions. Thus the objective may be to maximize national product or total consumption. If the regional distribution of development is to be treated as an independent welfare objective in such models, it is natural to incorporate it by means of the second technique discussed above, i.e., in the form of additional constraints imposed on the model that specify the absolute or relative levels of development to be attained in the individual regions. For example, if the over-all objective is the maximization of national product, the percentage of the total product to be generated in each individual region may also be prescribed. In this formulation the additivity of the measure of development between regions is preserved, i.e., the development of each region is given equal weight. In accordance with the previous discussion, however, there is an alternative formulation, corresponding to the first technique mentioned. In the objective function of the latter, the product (or other criterion of development) of each region is summed with *unequal* weights, so as to channel into selected regions more development than would result from a maximization undertaken with equal weights in the absence of prescribed levels of regional development. In this formulation the optimal value of the objective function no longer represents net national product (or other additive national measure), although the latter can of course be derived easily by means of a side calculation.

In these models, constraints on the interregional distribution of national product are generally imposed at the cost of a decrease in national product for the system as a whole. At best the constraints will leave the latter unchanged if they are either not binding in the optimal solution or if they are just on the margin of having become binding. While this inescapable fact is often interpreted to mean that some national income has to be sacrificed for the sake of attaining a greater equality between regions, it should be noted that the only conclusion that logically follows from what has been said above is that *any prescribed deviation* from the previous optimal solution, be it in the direction of greater equality or greater inequality between regions, will generally imply a sacrifice in national income. It is therefore essential to understand the behavior of models without built-in regional distribution constraints, because such models, if not adequately formulated, can easily point to incorrect policy conclusions.

The two ways of reducing multiple objectives to a single objective that have been cited earlier correspond to the use of price-type and quantity-type control instruments in planning. In general a separate control instrument is needed for setting the value of each separate policy objective.[2] In optimizing models the prescribed search for an optimum replaces one control instrument; with this understanding, the reduction of the multiplicity of regional-locational welfare objectives to a single objective reveals itself as a special case of the application of this principle. When using the first of the two techniques for effecting the reduction, the $n - 1$ relative weights assigned to the regional welfare goals of n regions act as price-type control instruments; with the second technique, the prescribed levels of welfare objectives in $n - 1$ regions act as quantity-type control instruments. Mixed formulations are also possible, i.e., maximizing the weighted sum of one group of regional objectives while the remaining ones are imposed as constraints.

The following important question arises in connection with the two alternative formulations of a problem in terms of price-type or quantity-type control instruments: Given one of the two formulations, is it possible to switch to the other formulation by appropriately choosing the weighting or constraint parameters of the latter in such a way that the optimal solutions will coincide? If it is possible to guarantee this in a given set of circumstances then the two formulations can be said to be equivalent.

Price- and quantity-type control instruments lead to equivalent re-

[2] On the use of control instruments in planning, see Tinbergen (1956), Sec. 3.3. On price-type versus quantity-type control instruments, see Chenery (1958).

sults in this sense only when the optimal solutions of the models are unique; in the case of multiple optimums there is only a limited correspondence. Unique solutions can be guaranteed when the models are *strictly convex* from a mathematical point of view; unfortunately, linear models are convex only in a weaker sense, and they will generally lead to multiple optimums in the course of the above reformulation. These multiple optimums are not of the kind familiar from the use of graphic techniques in economic analysis that result when two curves intersect at several points. They have the appearance of a mountaintop plateau rather than the unique tip of a sugarloaf-shaped mountain. When such multiple optimums occur only one thing can be guaranteed: By an appropriate choice of parameters the problem can be reformulated so that the two alternative formulations by the two techniques cited above will have *at least one optimal solution in common.* A more detailed inspection of this limited correspondence shows, moreover, that apart from the optimal solution that is shared between the two formulations many optimal solutions can exist under one formulation that are nonoptimal or even infeasible under the other formulation. When the model is not convex, i.e., when it embodies economies of scale or indivisibilities, it is no longer possible to make even the above limited assertion of correspondence.

These issues are discussed in more detail in Appendix 1.

Formulation of Resource Allocation Models for Locational Choices

A GENERAL MODEL: PRINCIPAL FEATURES

In order to offer a concrete basis for the subsequent discussion concerning the structure of allocation models as customarily formulated, a linear programing model with a threefold breakdown of detailed information (time periods, locations, industries) is presented in Table 1. While the empirical realization of such a model would overstrain the statistical resources of all but the most advanced economies and would seldom be useful unless a great emphasis was given to central planning decisions, it has the advantage of allowing the discussion of all the relevant factors in a unified way. In practice, unwanted detail can be eliminated by aggregation: thus, the model can be made static (single-period); it can be left dynamic and can instead by aggregated by regions while maintaining the interindustry detail; or it can be aggregated by

TABLE 1 An Illustrative Locational Model

PRIMAL VARIABLES		Exo-genous (=1)	PERIOD 1 Production			PERIOD 1 Transport		PERIOD 1 Stock-Holding		PERIOD 2 Production			PERIOD 2 Transport		PERIOD 2 Stock-Holding		PERIOD 3 Production			PERIOD 3 Transport		PERIOD 3 Stock-Holding			
Activity Levels →	Resource Surpluses ↓	X_*0	XX^1_{*1}	XX^1_{*2}	XX^1_{*3}	XZ^1_{*1}	XZ^1_{*2}	XH^1_{*1}	XH^1_{*2}	XX^2_{*1}	XX^2_{*2}	XX^2_{*3}	XZ^2_{*1}	XZ^2_{*2}	XH^2_{*1}	XH^2_{*2}	XX^3_{*1}	XX^3_{*2}	XX^3_{*3}	XZ^3_{*1}	XZ^3_{*2}	XH^3_{*1}	XH^3_{*2}		
PERIOD 1	SB^1_1 =	H^0_1	$-B^1_{11}$	$-B^1_{12}$	$-B^1_{13}$	$-L^1_{11}$	$-L^1_{12}$																	$*YR^1_1$	Commodity Stocks
	SB^1_2 =	H^0_2	$-B^1_{21}$	$-B^1_{22}$	$-B^1_{23}$	$-L^1_{21}$	$-L^1_{22}$																	$*YR^1_2$	
	SF^1_1 =	Q^1_1	$-F^1_{11}$	$-F^1_{12}$	$-F^1_{13}$	$-M^1_{11}$	$-M^1_{12}$																	$*YW^1_1$	Primary Factor Flows
	SF^1_2 =	Q^1_2	$-F^1_{21}$	$-F^1_{22}$	$-F^1_{23}$	$-M^1_{21}$	$-M^1_{22}$																	$*YW^1_2$	
	SA^1_1 =	$(H^0_1-C^1_1)$	A^1_{11}	A^1_{12}	A^1_{13}	$(T^1_1-N^1_{11})$	$-N^1_{12}$	-1																$*YP^1_1$	Commodity Flows
	SA^1_2 =	$(H^0_2-C^1_2)$	A^1_{21}	A^1_{22}	A^1_{23}	$-N^1_{21}$	$(T^1_2-N^1_{22})$		-1															$*YP^1_2$	
PERIOD 2	SB^2_1 =							1		$-B^2_{11}$	$-B^2_{12}$	$-B^2_{13}$	$-L^2_{11}$	$-L^2_{12}$										$*YR^2_1$	Commodity Stocks
	SB^2_2 =								1	$-B^2_{21}$	$-B^2_{22}$	$-B^2_{23}$	$-L^2_{21}$	$-L^2_{22}$										$*YR^2_2$	
	SF^2_1 =	Q^2_1								$-F^2_{11}$	$-F^2_{12}$	$-F^2_{13}$	$-M^2_{11}$	$-M^2_{12}$										$*YW^2_1$	Primary Factor Flows
	SF^2_2 =	Q^2_2								$-F^2_{21}$	$-F^2_{22}$	$-F^2_{23}$	$-M^2_{21}$	$-M^2_{22}$										$*YW^2_2$	
	SA^2_1 =	$-C^2_1$						1		A^2_{11}	A^2_{12}	A^2_{13}	$(T^2_1-N^2_{11})$	$-N^2_{12}$	-1									$*YP^2_1$	Commodity Flows
	SA^2_2 =	$-C^2_2$							1	A^2_{21}	A^2_{23}	A^2_{23}	$-N^2_{21}$	$(T^2_2-N^2_{22})$		-1								$*YP^2_2$	
PERIOD 3	SB^3_1 =														1		$-B^3_{11}$	$-B^3_{12}$	$-B^3_{13}$	$-L^3_{11}$	$-L^3_{12}$			$*YR^3_1$	Commodity Stocks
	SB^3_2 =															1	$-B^3_{21}$	$-B^3_{22}$	$-B^3_{23}$	$-L^3_{21}$	$-L^3_{22}$			$*YR^3_2$	
	SF^3_1 =	Q^3_1															$-F^3_{11}$	$-F^3_{12}$	$-F^3_{13}$	$-M^3_{11}$	$-M^3_{12}$			$*YW^3_1$	Primary Factor Flows
	SF^3_2 =	Q^3_2															$-F^3_{21}$	$-F^3_{22}$	$-F^3_{23}$	$-M^3_{21}$	$-M^3_{22}$			$*YW^3_2$	
	SA^3_1 =	$-C^3_1$													1		A^3_{11}	A^3_{12}	A^3_{13}	$(T^3_1-N^3_{11})$	$-N^3_{12}$	-1		$*YP^3_1$	Commodity Flows
	SA^3_2 =	$-C^3_2$														1	A^3_{21}	A^3_{22}	A^3_{23}	$-N^3_{21}$	$(T^3_2-N^3_{22})$		-1	$*YP^3_2$	
PERIOD 4	SO = MAX!																					$(P^4_1+R^4_1)$	$(P^4_2+R^4_2)$	$*Y0$	Terminal Stocks
		= -D0 MIN!	= $-DX^1_1$	= $-DX^1_2$	= $-DX^1_3$	= $-DZ^1_1$	= $-DZ^1_2$	= $-DH^1_1$	= $-DH^1_2$	= $-DX^2_1$	= $-DX^2_2$	= $-DX^2_3$	= $-DZ^2_1$	= $-DZ^2_2$	= $-DH^2_1$	= $-DH^2_2$	= $-DX^3_1$	= $-DX^3_2$	= $-DX^3_3$	= $-DZ^3_1$	= $-DZ^3_2$	= $-DH^3_1$	= $-DH^3_2$		↑ Shadow Prices ← Profits

DUAL VARIABLES

industries while maintaining the interregional structure. Examples of each of these alternative aggregated formulations are available.[8]

The dynamic features of the model in Table 1 are set out in the simplest possible form in order to concentrate attention on the locational-interregional structure. Thus, consumptions of all commodities and supplies of primary resources are treated as exogenously given parameters while terminal stocks, with a prescribed weighting, are treated as the maximand. The presentation of the model follows Tucker's condensed linear programing format (Graves and Wolfe, 1963); the details of notation and formal interpretation will be found in Appendix 2.

The principal characteristics of the model are the following. The unused surpluses of all resources are expressed as linear combinations of the activity levels; the coefficients of the balances involving these resources appear as rows. There are three kinds of resources: commodity stocks, primary factors, and commodity flows. Each of these resources is distinguished by time periods and by locations. (The terms "location" and "region" will be used interchangeably.) The time periods appear in Table 1 in explicit form while the locations are left implicit by the use of matrix notation; in this notation the model appears very much as though only a single location existed. The model shows two commodities and two primary factors.

There are three kinds of activities: production, transport, and stock holding. These activities appear as columns of coefficients. The coefficients denote resource requirements or demands if negative, and outputs or supplies if positive. The levels of the activities are variable and appear as algebraic unknowns heading each column. *Production* activities have commodity outputs (A; if a given A is negative, it denotes an intermediate input); they also have stock requirements ($-B$) and primary factor requirements ($-F$); their unknown level is designated by the compound symbol XX. Production activities, like resources, are distinguished by time period and by location; the latter distinction is implicit in the matrix notation used. Three production activities are given. *Transport* activities show net regional imports of commodities (T), while transport costs are broken down by detailed stock requirements (L), factor inputs (M), and commodity inputs (N). The model includes a separate transport activity for each origin-destination pair, as can be seen in the detailed interpretation of the matrices in Appendix 2. The unknown levels of these activities are denoted by the compound

[8] See Bibliographical Note.

symbol XZ. *Stock-holding* activities transfer stocks from one time period to the next; they can be interpreted as the purchase of a unit of stock at the end of period t, its rental for productive purposes in period $(t + 1)$, and its resale at the end of period $(t + 1)$. The coefficients I refer to a unit of a commodity or stock in a given time period and at a given location. The unknown levels of these activities are denoted by XH. In addition to the foregoing activities, there is also a dummy activity of *exogenous supplies and demands* whose level (XO) is fixed at unity; it specifies the initial level of stocks (H); the time profile of the consumption of all goods (C) and the supply of all primary factors (Q) in all regions.[4]

Resource balances follow the format:

$$\begin{pmatrix}\text{surplus}\\ \text{of a}\\ \text{resource}\end{pmatrix} = \begin{pmatrix}\text{all}\\ \text{supplies or}\\ \text{outputs}\end{pmatrix} - \begin{pmatrix}\text{all}\\ \text{demands or}\\ \text{requirements}\end{pmatrix}.$$

Surpluses are treated as algebraic unknowns, in the same way as activity levels; in the model they are denoted by compound symbols whose first letter is S.

For example, the balance of the flow of the first commodity in the first time period is interpreted as follows:

$$\underbrace{SA_1^1}_{\substack{\text{surplus of}\\ \text{first commod-}\\ \text{ity in first}\\ \text{time period}}} = \underbrace{H_1^0}_{\substack{\text{initial}\\ \text{stock}\\ \text{inherited}\\ \text{from}\\ \text{zero time}\\ \text{period}}} - \underbrace{C_1^1}_{\substack{\text{consump-}\\ \text{tion in}\\ \text{first time}\\ \text{period}}} +$$

$$\underbrace{(A_{11}^1 \cdot XX_1^1 + A_{12}^1 \cdot X_2^1 + A_{13}^1 \cdot XX_3^1)}_{\substack{\text{net output by the three production activities}\\ \text{in the first time period, after deduction of}\\ \text{intermediate inputs}}} +$$

$$\underbrace{T_1^1 \cdot XZ_1^1}_{\substack{\text{net import}\\ \text{into given}\\ \text{region in}\\ \text{first period}}} - \underbrace{(N_{11}^1 \cdot XZ_1^1 + N_{12}^1 \cdot XZ_2^1)}_{\substack{\text{amount used directly in all}\\ \text{transport activities in}\\ \text{first time period}}} - \underbrace{I \cdot XH_1^1}_{\substack{\text{amount held}\\ \text{as stock in}\\ \text{first time}\\ \text{period}}}$$

As can be seen, the initial stock and the consumption are exogenously given constants, while the other quantities are derived as the products of the respective coefficients by the proper activity levels indicated in the

[4] Migration can be handled by adjusting the exogenous labor supplies for the periods in question.

column headings. Since this entire balance is in matrix notation, it is valid simultaneously for each separate region. In particular, in this notation multiplication by I corresponds to multiplication by 1 in ordinary algebra; thus the quantity

$$I \cdot XH_1^1 - H_1^0$$

is the increase in stocks between period zero and period 1, i.e., the investment in stocks of commodity 1 in each region.

Likewise, stock balances can be interpreted as follows: Surpluses of stocks in period t equal the amounts required for production and transport one period later (there is one-period time lag between investment and its utilization) minus the amount actually held in period t. The interpretation of primary factor balances is analogous but involves no time lag.

In this formulation the maximand is an ordinary resource surplus defined like any other. In the model, terminal stocks (in the fourth period) with a prescribed weighting are chosen as the maximand. The problem consists in programing the unknown activity levels and resource surpluses in such a way that terminal stocks (the last surplus) be a maximum, while no other resource surplus is negative (i.e., there are no overdrawn resources) and no activity level is negative (i.e., no activity runs in reverse).

The maximization of terminal stocks is a proper objective since it is equivalent to maximizing the growth potential of the system after the necessarily limited planning period. The composition of these terminal stocks by commodity and by region is determined as part of the solution of the problem; it will, however, depend strongly on the weighting that is exogenously assigned to the terminal stocks. The weights chosen summarize the assumptions on the nature of the growth process beyond the planning horizon of the model. Rather than assigning weights to the terminal stocks, the planning horizon can also be taken into account by assuming constant proportional growth for the system as a whole or prescribed rates of growth for individual parts of the system beyond the horizon. The advantage of the maximand presented in Table 1 is, however, that it is not only particularly simple, but also convenient for the analysis of problems of locational choice, since it is additive between regions.

Every resource allocation problem formulated by means of mathematical programing explicitly contains (or, in the case of linear models, implies) a resource valuation problem. For the present model the unknowns of this valuation problem are listed in the right-hand and bottom margins of Table 1 and are designated by compound symbols whose first letter is Y or D. The Y-type symbols are interpreted as forming a "shadow" price system based on a unit valuation of terminal stocks (which thus play the role of the *numeraire*). YR, YW, and YP designate stock rents of commodities, flow prices of primary factors (i.e., wages, etc.), and flow prices of commodities, respectively. The product of each input coefficient for a given activity by the corresponding shadow price of the right-hand margin represents a revenue (if positive) or a cost (if negative). The algebraic sum of these terms for a given activity represents profits (negative losses) computed at shadow prices. These shadow profits, which are algebraic unknowns, appear in the bottom margin. For example, the shadow profit on the first productive activity in period 1 is obtained as follows:

$(-B_{11}^1 \cdot YR_{11}^1 - B_{21}^1 \cdot YR_2^1)$	$(-)$	rental cost on stocks of commodities 1 and 2 used in the production activity
$+ (-F_{11}^1 \cdot YW_{11}^1 - F_{21}^1 \cdot YW_{21}^1)$	$(-)$	wage cost and other primary factor payments on primary factors used in the production activity
$+ (A_{11}^1 \cdot YP_{11}^1 + A_{21}^1 \cdot YP_{21}^1)$	$(+)$	net revenue on commodity output after deduction of payments for intermediate-input commodities
$= -DX_1^1$	(equals)	profit on activity performed at unit level, where DX is a shadow loss, $-DX$ is a shadow profit

Due to mathematical reasons the optimal solution to the allocation problem simultaneously yields the shadow prices and shadow losses of the valuation problem. The latter are always nonnegative; i.e., shadow prices are positive or zero, and no activity ever shows profits at these prices but at best breaks even. At the same time the system of shadow prices is such that it minimizes profits on the exogenous supply-demand activity; in fact, this minimum is numerically equal to the maximum of the allocation problem.[5] Due to this equality the correspondence of

[5] Provided both the maximum of the allocation problem and the minimum of the valuation problem exist, Max (SO) = Min $(-DO)$.

the allocation problem and the valuation problem in Table 1 can be stated in the following form: The optimal value of terminal stocks is imputed by the model to the scarce exogenous supplies minus exogenous demands. In particular, if factor supplies and commodity demands are zero at all time periods, the model imputes the entire value of terminal stocks to the initial stocks; moreover, it can be shown that the imputed value of stocks for intermediate time periods is also held constant (see Appendix 3). With given nonzero consumption profiles the model imputes to initial stocks as well as to stocks in all other periods a value that is larger than the value of terminal stocks by an amount exactly sufficient to finance consumption in periods subsequent to the period in question. If, in addition, nonzero factor supplies are also included in the model, their effect on the time profile of imputed stock values is the opposite of the effect of consumption.

The shadow prices utilized in the definition of aggregate social accounting concepts above, it should be noted, are *not* current prices for each period. This is clear from the fact that the revenues and costs of stock-holding activities are summed over two successive time periods; concepts based on current prices could not be summed in this way without appropriate discounting operations. The shadow prices are therefore seen to form a price system which has the properties of a set of current prices to which discounting operations have already been applied. It is, in fact, readily possible to define a set of current prices together with an appropriate discount rate once the shadow price system is given. The system of current prices is then anchored in one of the time periods by assuming that in this period current and shadow prices coincide; it is further assumed that an arbitrarily given commodity serves as the value standard between the base time period and another time period, in the sense that its current price remains constant between the two periods. The interest rate for discounting purposes on these assumptions turns out to be the current rent of the stock of the value-standard commodity; the current rents of the stocks of other commodities have to be adjusted for value changes of the stock between periods in order to arrive at the same interest rate (see Appendix 3). Instead of a single commodity, a weighted average can serve equally well as the value standard; the latter can, moveover, change between time periods.

The system of relative shadow prices is thus a more fundamental property of the model than the period-to-period interest rate associated with the choice of an arbitrary value standard. The arbitrariness of the latter can be reduced in practice by tying it to the structure of consumption from period to period or by some similar means, but in any event the shadow prices contain all the information necessary for defining any given system of current prices and the corresponding period-to-period interest rates.

The model lends itself readily to interpretation by means of standard social accounting concepts. The value aggregates embodying these concepts are built up from "accounting values" corresponding to each coefficient in the optimal solution of the model. These accounting values are obtained by multiplying each coefficient both by the corresponding activity level of the optimal solution (giving total physical resource amount) and by the proper shadow price (giving the value of the former). Accounting values have the mathematical property of summing to zero both by rows and by columns, since (by rows) any resource that may have a nonzero surplus will be a free good with a zero shadow price, yielding a zero *value* for the surplus, while (by columns) any activity that may have a nonzero loss will not be used in the optimal solution, yielding a zero *total* loss. This property is highly convenient for accounting purposes since it permits the definition of aggregate concepts based exclusively on activity scales and shadow prices, without reference to resource surpluses and losses on activities.

National income and product (jointly for all regions) can be derived for any time period from the accounting values of rows having the index of the given time period. Accounting values are first summed by rows to zero; then these equalities are themselves summed; finally, the terms corresponding to production and transport activities are canceled, since these sum to zero vertically for the given time period (see Appendix 3). In this way the following relation is obtained, for example, for the first period:

$$\begin{aligned}(H_1{}^0 \cdot YR_1{}^1 + H_2{}^0 \cdot YR_2{}^1) + (Q_1{}^1 \cdot YW_1{}^1 + Q_2{}^1 \cdot YW_2{}^1) = \\ (C_1{}^1 \cdot YP_1{}^1 + C_2{}^1 \cdot YP_2{}^1) + \\ [(XH_1{}^1 - H_1{}^0) \cdot YP_1{}^1 + (XH_2{}^1 - H_2{}^0) \cdot YP_2{}^1]\end{aligned}$$

or in a more condensed notation:

$$\underbrace{H^0 \cdot YR^1}_{\text{stock rental income}} + \underbrace{Q^1 \cdot YW^1}_{\text{wage and other primary factor income}} = \underbrace{C^1 \cdot YP^1}_{\text{consumption}} + \underbrace{(XH^1 - H^0) \cdot YP^1}_{\text{investment}}$$

This is the well-known identity between national income at factor cost and national product, with all aggregates defined at shadow prices.

If this expression is transformed into current prices, stock rental income is replaced by the *difference* between interest income and the net increase in stock valuations (see Appendix 3). Alternately, national income and investment could both be redefined by adding to each side of the identity the net increase in stock valuations. Then national income becomes the sum of wage and interest incomes while investment is obtained as the difference of the current stock values in the two periods. Note, however, that these now become dependent on the choice of the value-standard commodity stock.

Corresponding expressions for regional income and product can also be derived from the subset of rows for one individual region in a given time period (see Appendix 3). When the operations are performed as indicated above for the national concepts, it is found that the expression for regional income at factor cost now equals the sum of consumption, investment, net regional exports having the nature of pure transfers (without regard to transport costs), plus all cost-type commodity and factor inputs of the region into transport activities. In other words, the model treats commodity and factor inputs into transport as part of the final product of each region, a somewhat surprising result in view of the fact that transport is thought of as an intermediate commodity. Since, however, intermediate commodity and factor inputs into *exports* are customarily treated as part of final product, it is clear that the model makes no distinction between commodities and factors leaving the region that actually arrive at other regions, and commodities and factors leaving the region that are utilized for running the transport activities themselves. Thus, it is convenient to redefine net regional exports to include all transport-cost-type commodity and factor inputs. Since in the derivation of national product for the system as a whole net exports so defined cancel out (see the definition of national product, above), it can be concluded that commodity and factor inputs into transport activities show up in national product in the form of consumption and investment totals *at the required locations.*

In sum, the model is characterized by the following key features:

1. It maximizes for the economy as a whole an over-all development criterion (value of terminal stocks) that is additive between regions. There are no constraints on the regional distribution of this criterion; i.e., the accumulation of terminal stocks may be realized by means of any technically feasible regional distribution of productive activities.

2. Consumption demands and factor supplies are prescribed in physical terms for all commodities and factors in all regions and all time periods. This feature of the model has several implications:

a. The structure of production exhibits a far-going independence from the structure of consumption; as an extreme, it could happen that a given region does not develop its productive structures at all while participating in consumption in the prescribed manner by means of a steady stream of interregional transfers.

b. All decisions with regard to time preference between present and future consumption and effort are prejudged in the formulation of the model. Thus, in particular, no *ex ante* relationship is prescribed for the division of national and regional product between consumption and saving, even though, of course, the respective ratios can be readily calculated *ex post* once the optimal solution to the model has been obtained.

c. All relations with regard to the price elasticities of commodity demands and factor supplies are likewise prejudged in the formulation of the model: Since the latter are given in physical quantities that are constant regardless of the corresponding shadow prices in the optimal solution of the model, all price elasticities are in fact taken to be equal to zero.

3. The treatment of dynamic features is the simplest possible in a multiperiod model. In particular:

a. All stocks are treated as completely liquid at the end of each accounting period. Thus, no distinction is made between fixed capital and inventories, and no limits are placed on the reduction of the levels of stocks between time periods.

b. The transfer of stocks from one time period to the next is treated as costless; no storage charges of any kind are included in the model, and thus no joint storage activities occur.

c. There are no time lags in the model, apart from the one-period lag between investment and the availability of stock capacity for production. In particular, the inputs and outputs of all production and transport activities are restricted to one given time period.

4. The treatment of transport is also kept comparatively simple. In particular:

a. There is only one transport activity connecting each pair of locations. Thus alternative regional inputs for running a given transport activity are excluded, and problems such as the carriage of shipments in the bottoms of one or the other region with corresponding income generation for one or the other region are prejudged. Likewise, the optimal means of transport, i.e., water or overland carriage, is also prejudged.

b. Joint carriage of different commodities by a single transport activity is excluded.

c. Joint service by a single transport activity to and from different locations, like for example a cargo ship touching a series of ports, is excluded.

5. All fixed costs and other elements of nonconvexity in production and transport are ignored. This feature of the model places a sharp restriction on its degree of realism, but it cannot be avoided without opening up a host of major analytical problems that fall outside the limits of the present paper.

6. The optimal solution to the model traces out, on the primal side, the time path of production activities and investments in all regions, as well as the evolution of interregional transport flows. On the dual side, it yields the time profile of commodity shadow prices, shadow wages for primary factors, and shadow rents of scarce stock capacities in all regions. Shadow prices, wages, and rents are expressed in units of terminal wealth (stock valuation) which acts as the numeraire resource of the shadow price system.

TECHNOLOGICAL CHOICES VS. PREFERENCES

This particular form of the model was chosen as the point of departure because it is a summary of technological choices open to the system of regions as a whole embracing alternatives in regard to production, transport, and stock holding, while the representation of preference functions is excluded. Thus the prescribed magnitudes of the parameters of the model and the optimal solution values of the variables satisfy purely technological relationships in the most efficient way possible, but the same constellation of parameters and optimal values of variables need not, and generally will not, be preferred to other possible constellations that can be obtained by prescribing different parameter values. In short, the optimal solutions to the model are Pareto-optimal for any prescribed set of parameters; by varying the parameters systematically, all trade-offs between parameters, i.e., the entire hypersurface of production possibilities, can be traced out.[6]

[6] It may be objected that the maximization of terminal wealth with prescribed weighting amounts to the inclusion of a preference function in the model. This, however, need be true only in a purely formal sense. The weights if desired can be regarded as completely arbitrary, having the sole purpose of defining a tangent that will allow the construction, one portion at a time, of the Pareto-optimal production-possibility surface. A genuine commodity- and time-preference function as envisaged by neoclassical economics could rarely be regarded as linear over extended ranges of the variables. A preference function with the required curvature, technically a concave function, if empirically derivable at all, could be satisfactorily approximated within a linear model in a piecewise linear fashion, in the

From the point of view of practical planning applications, some form of this general strategy is often attractive since, given the production possibility surface, the policymakers can apply to the latter an implicit set of preferences in the process of selecting a particular constellation as the most preferred from among the ones available. This strategy avoids the great difficulties of constructing a reliable explicit representation of the structure of preferences. At the same time, the effort required for tracing out the complete production-possibility hypersurface is overwhelming in almost any practical task, since the number of point solutions required for characterizing with any accuracy a function in a large number of dimensions is enormous. It is thus highly advantageous to be able to add to the model sufficient information with regard to the structure of preferences to permit the approximate identification of a "relevant range" for decision-making; the detailed description of production possibilities can then be restricted to this range.

This additional information regarding preferences can be included in the model in two forms:

1. The choice of the magnitudes of particular parameters in the numerical formulation of the model can be based on the approximate anticipation of the optimal solution; thus the parameter values chosen are such as are believed to result in a "reasonable" solution from the point of view of preferences. To the extent that this attempt is successful the first parameter-solution constellation falls within the "relevant range" for decision-making, and the exploration of production possibilities can thereafter be restricted to small parameter changes around the initial values. For example, with regard to regional growth preferences, i.e., the balancing of growth rates in individual regions against the growth of the system as a whole (in so far as a conflict exists), the initial parameter choice can well consist in an equal weighting of terminal stocks in different regions.

2. Certain parameters can be unlocked and made explicitly variable

same way that production functions are approximated. The linear pieces would then obey relationships of the type:

$$x_1/g_{i,1} + x_2/g_{i,2} + \cdots + x_n/g_{i,n} \geqq V;\ j = 1, \ldots, m,$$

where the $g_{i,j}$ are the parameters of the ith linear piece and V is a new variable used as an index of the preference level that is being maximized.

Unless the weighting of terminal stocks is therefore explicitly interpreted as one local portion of the piecewise linearized concave preference function, it is entirely justified to regard the model as being restricted to the parametric representation of technological choices open to the planner.

within the formulation of the model, and specific relationships can be prescribed between this new variable and the other variables.

The most obvious candidate for such treatment is the set of consumption parameters included in the model, since the effect of time preferences connecting present and future consumptions can be reasonably approximated by prescribing savings in relation to final product. This can be done either for the system as a whole or for individual regions, on the assumption that savings equal investments. Preferences with regard to the interregional distribution of consumption can be added independently of production, since net production can be redistributed by means of uncompensated transfers; thus, for example, it can be prescribed that per capita consumption of all commodities be equalized in all regions. When the above relationships are prescribed in aggregate terms, it is generally also necessary to approximate the structure of preferences between different consumption goods in the same time period by means of constraints that specify ratios or other simple relationships between the physical consumptions of individual commodities; otherwise the process of optimization might tend to channel all consumption into one or a few goods.

The formal introduction of such constraints into the model of Table 1 is straightforward so long as the weights used in defining aggregate concepts are *constant*. In fact, if aggregate concepts are defined in terms of historical prices and their simple projections into future time periods, the condition of constancy for the weighting parameters will be fulfilled; however, this procedure is analytically questionable for two reasons: (1) Future prices depend on the structural changes introduced into the economy planning, and cannot be taken as simple projections of past prices. (2) The purpose of formulating mathematical programing models is to get away from the irrationalities of past prices observed in imperfect markets or under administrative control procedures; it is thus a flaw of the analysis to bring these back into the model by way of the definition of aggregate concepts.

While these drawbacks are undeniable it is recalled that the purpose of the relationships referred to in the foregoing paragraphs is *not* to arrive at the exact most-preferred solution, but only to identify the relevant decision range for further detailed exploration; the accuracy requirements in regard to preferences are therefore substantially relaxed. The pocedure is nevertheless inelegant. Then could not past and projected prices be replaced by the shadow prices themselves in the formulation of the aggregate concepts?

In considering the latter possibility, the first problem to be faced is that the prices to be used as weights in the aggregate constraints become unknowns; in other words, the aggregate constraints have terms that contain the product of a primal variable (activity scale) and a dual variable (shadow price). Such a model is no longer linear, as its "primal" side becomes inseparably fused with its "dual" side; the analytical problems it raises are largely unexplored, even though a clear resemblance to ordinary nonlinear programing problems is evident that can in all probability be exploited to obtain efficient solution algorithms. One intuitively obvious strategy, which may or may not be computationally efficient, consists in starting with a set of trial values of the shadow prices, solving the linear model formulated in terms of these, checking the trial values against the results, and iterating with revised trial values. In this manner, the circularity of formulating a model in terms of its own solution is broken, while the linearity of each trial model is preserved.

The use of aggregate concepts defined in terms of shadow prices is aesthetically appealing but raises the further problem that the aggregate magnitudes might be highly sensitive to the optimal values of the shadow prices. In so far as these are quite different from historical prices, the historical relationship between aggregate magnitudes will no longer furnish a reliable guide for approximating the structure of preferences. Thus the historical savings rate applied to the shadow-priced savings and income concepts may well be inappropriate for representing time preferences, and recourse to a savings rate expressed in stable prices may become the better choice after all in the absence of empirical observations on savings rates at near-equilibrium prices.

Behavior of Resource Allocation Models Involving Locational Choices

SIMPLIFIED VERSIONS OF THE GENERAL MODEL

Having formulated a general resource allocation model the question may now be posed: What does this model reveal about the problem of locational choices, in particular the choice between geographically balanced or unbalanced growth? In order to answer this question it is necessary to analyze the behavior of the model under different assumptions. Due to the complexity of the model it will, however, be convenient to analyze primarily the behavior of simplified versions and to generalize this analysis qualitatively by reference to the fully detailed model.

An aggregated multiperiod interregional growth model has been described in a recent article by M. A. Rahman (1963); the same model has been presented in a slightly different form and some of the results have been derived in a simpler way in a comment on the foregoing article by Robert Dorfman. This work, to be referred to as the Rahman-Dorfman model, will be used in the following discussion unless specifically noted to the contrary; it will become apparent that it constitutes a special case of the general model of Table 1. The discussion of the properties of this model is thus a convenient take-off point for subsequent generalization.

The Rahman-Dorfman model is formulated in terms of the aggregate capital stocks $K_{i,t}$ in each region i and each time period t.[7] Aggregate investment in a region is the difference between capital stock in the region in two subsequent time periods. Reinvestible surplus in each region is related to capital stock by means of constant reinvestment coefficients s_i that represent the ratio between the ordinary savings rate σ_i and the marginal capital-output ratio k_i:

$$s_i = \sigma_i / k_i$$

The Rahman-Dorfman model, like the general model of Table 1, maximizes terminal wealth, i.e., the valuation of terminal stocks at prescribed weighting parameters:

$$\text{Max! } \Sigma K_{iT} c_{iT}$$

where the c_{iT} coefficients correspond to the $(P^4 + R^4)$ parameters in the model of Table 1, except for the fact that the c_{iT} coefficients are aggregated by commodities. T is the time index of the last planning period. The maximization is subject to the constraint that total reinvestible

[7] While Rahman formulates an interregional model Dorfman's model is interpreted in terms of sectors rather than regions; the formal analogy between the two models is, however, very close; and Rahman makes explicit use of some of Dorfman's results in an interregional context. Thus where Dorfman refers to sector i I shall refer to region i in the subsequent exposition. Moreover, where Rahman explicitly uses aggregate concepts, such as income, consumption, investment and savings, Dorfman circumvents this by postulating that the capital stock in each of his sectors can be meaningfully measured in physical units chosen so as to make the price of a physical unit equal unity; on this assumption, he sums the capital stocks for different sectors. I shall drop this disguise and treat capital as a frankly aggregate concept. If this is done, Dorfman's s_i coefficient, the amount of reinvestible surplus generated per unit of physical capital in sector i, becomes identified with the ratio of the savings rate to the ordinary capital-output ratio in aggregate terms, as indicated in the text below.

surplus summed for all regions must be sufficient to cover all investments in each individual time period t:

$$\sum_i (K_{i,t+1} - K_{i,t}) \leqq \sum_i s_i K_{it}$$

The solution to this problem hinges on working backward in the determination of investments period by period. If there are no further constraints, the solution is trivial: Shift *all* existing capital in the last period to the region where it has the highest valuation $\bar{c}_{iT}$, since this will maximize terminal wealth; in previous time periods, maintain *all* capital in the region where it has the highest reinvestment ratio s_i, since this will lead to the fastest buildup of the capital stock. The solution becomes more interesting when decumulation constraints are added on the capital stock invested in each region: Since it is now no longer possible to shift existing capital stock at will, but only to redirect further investment from one region to another, there are opposed attractions for investment; on the one hand, toward regions where the terminal stock valuation is high, and on the other, toward regions where the reinvestment ratio is favorable. In general, the effect of a high reinvestment ratio which cumulates at compound interest will outweigh the effect of an adverse terminal valuation if the planning period is chosen long enough. In any event, if in a given period *some* investment goes to a given region, *all* investment has to go there.

The reinvestment ratio, it is recalled, is high in a region to the extent that the savings rate is high and the capital-output ratio is low. If stocks are equally valued in all regions at the end of the planning period, the former criterion will channel all investment into the region with highest reinvestible surplus; this can be counteracted only by slanting terminal stock valuations in favor of the low-reinvestment regions. Since in underdeveloped regions the savings rate is low, the criterion under equal weighting will not channel investment into these regions unless they have an unusually favorable marginal capital-output ratio. As social overhead investments in underdeveloped regions are likely to be deficient, favorable capital-output ratios cannot occur in these regions unless the productivity of capital in directly productive activities is unusually high. The latter productivity depends, however, on several elements: the inherent technological relationships, the supply of labor, and the supply of other potentially scarce factors, primarily land and natural resources. Since capital-output ratios and savings are measured in aggregate terms at market prices that are known to be disequilibrium prices—at least as far as labor is concerned, but very probably also in many or most other respects—the aggregate ratios cover up a tangle of diverse elements and leave the application of the suggested regional investment criterion on very shaky grounds. The tangle is further compounded if an attempt is made to value terminal stocks at projected market prices within each region.[8]

The analysis is largely unchanged when the maximization of terminal national product replaces the maximization of terminal wealth. The usual additive definition of system product (national product for all regions) implies equal weighting; the terminal consideration is now however no longer the reinvestment rate but just the capital-output ratio, since it is immaterial, from the point of view of the terminal national product, to what use—investment or consumption—that product is put. Thus productivity alone takes on the role of terminal weights, while savings and productivity both play a role, in the form of the reinvestment ratio, in all earlier periods.

The net policy conclusions of this analysis are highly prejudicial to underdeveloped regions unless the latter exhibit unrealistically favorable marginal capital-output ratios. These conclusions will, however, be greatly modified by a more detailed analysis of aggregation problems and the relaxation of the constancy of certain parameters.

In terms of the comprehensive linear programing model of Table 1, the Rahman-Dorfman model can be represented as a special case corresponding to the following assumptions:

1. A single commodity
2. Zero transport costs

A single-commodity, two-regional model with no transport costs is presented in Table 2. The nomenclature follows that of Table 1 except that in the present case all symbols refer to single scalars rather than to vectors or matrices as was the case in Table 1.[9] It can be seen that

[8] It should be clearly understood that the Rahman-Dorfman model has never been put forward by its authors as anything but a suggestive exercise in the consideration of certain resource allocation problems; in particular, the extreme solutions it gives by channeling all investment into a single activity in each period are explicitly regarded as wholly unrealistic. It is recalled that Dorfman in his reformulation of Rahman's model entirely abandoned the regional interpretation of the analytical structure of the model in favor of a sectoral interpretation within which the aggregation problems can to some extent be skirted by means of the measurement of the capital stock of a sector in terms of engineering units.

[9] Correspondingly, the stock, factor, and product coefficients B, F, and A, are carried in the present table in lower case. In order to simplify the subscripting of these coefficients, they have been subscripted 1 . . . 4, rather than being identified with regard to region (A or B), or activity (1 or 2).

TABLE 2

PRIMAL VARIABLES		Exogenous (=1)	PERIOD 1 Production Stocks					
	Activity Levels →	X0	XX^1_{A1}	XX^1_{A2}	XX^1_{B1}	XX^1_{B2}	XH^1_A	XH^1_B
	Resource Surpluses ↓	*	*	*	*	*	*	*
PERIOD 1	$SB^1 =$	H^0	$-b_1$	$-b_2$	$-b_3$	$-b_4$		
	$SF^1_A =$	Q^1_A	$-f_1$	$-f_2$				
	$SF^1_B =$	Q^1_B			$-f_3$	$-f_4$		
	$SA^1 =$	$(H^0 - C^1)$	a_1	a_2	a_3	a_4	-1	-1
PERIOD 2	$SB^2 =$						1	1
	$SF^2_A =$	Q^2_A						
	$SF^2_B =$	Q^2_B						
	$SA^2 =$	$-C^2$					1	1
PERIOD 3	$SB^3 =$							
	$SF^3_A =$	Q^3_A						
	$SF^3_B =$	Q^3_B						
	$SA^3 =$	$-C^3$						
PERIOD 4	MAX! S0 =							
		=	=	=	=	=	=	=
		-D0	$-DX^1_{A1}$	$-DX^1_{A2}$	$-DX^1_{B1}$	$-DX^1_{B2}$	$-DH^1_A$	$-DH^1_B$
		MIN!						

Simplified One-Commodity Model

PERIOD 2						PERIOD 3							
Production Stocks						Production Stocks							
XX^2_{A1}	XX^2_{A2}	XX^2_{B1}	XX^2_{B2}	XH^2_A	XH^2_B	XX^3_{A1}	XX^3_{A2}	XX^3_{B1}	XX^3_{B2}	XH^3_A	XH^3_B		
*	*	*	*	*	*	*	*	*	*	*	*		
												* YR^1	Stock (Rent)
												* YW^1_A	Factors (Wages)
												* YW^1_B	in Reg. A, B
												* YP^1	Flow (Price)
$-b_1$	$-b_2$	$-b_3$	$-b_4$									* YR^2	Stock (Rent)
$-f_1$	$-f_2$											* YW^2_A	Factors (Wages)
		$-f_3$	$-f_4$									* YW^2_B	in Reg. A, B
a_1	a_2	a_3	a_4	-1	-1							* YP^2	Flow (Price)
				1	1	$-b_1$	$-b_2$	$-b_3$	$-b_4$			* YR^3	Stock (Rent)
						$-f_1$	$-f_2$					* YW^3_A	Factors (Wages)
								$-f_3$	$-f_4$			* YW^3_B	in Reg. A, B
				1	1	a_1	a_2	a_3	a_4	-1	-1	* YP^3	Flow (Price)
										$(P^4_A+R^4_A)$	$(P^4_B+R^4_B)$	* Y0 (=1)	Term. Stock Value
=	=	=	=	=	=	=	=	=	=	=	=		Shadow Prices
$-DX^2_{A1}$	$-DX^2_{A2}$	$-DX^2_{B1}$	$-DX^2_{B2}$	$-DH^2_A$	$-DH^2_B$	$-DX^3_{A1}$	$-DX^3_{A2}$	$-DX^3_{B1}$	$-DX^3_{B2}$	$-DH^3_A$	$-DH^3_B$		– Profits

DUAL VARIABLES

while primary factor flows are distinguished between regions A and B, the single commodity is balanced jointly rather than separately for the two regions. This procedure is justified by the fact that transport costs on this commodity between the two regions are assumed to be zero. Thus the two separate regional balances of the commodity can be merged, and transport activities can be dropped entirely.

Stock levels are identified as separate activities, but in the present model they are not balanced separately for each of the two regions, since all stocks are liquidated at the end of each period and thus can be transferred from one region to another at will. Since the model always works with the sums of stock levels in the two regions in any period, these two activities could have been merged into a single one. The present form has been retained to call attention to the possibility of imposing stock decumulation limits in each region, in a manner analogous to the Rahman-Dorfman model.

The primary factors in the two regions need not be the same ones; in fact, the structure of the model indicates that any interregional comparison of the absolute levels of factor inputs that are immobile between regions is meaningless: For example, there is no operational significance to the comparison of land-area inputs into analogous activities in the two regions, since the qualities of land (soil, climate, topography, etc.) are inherently different and thus a pure area measure means nothing; a weighting by means of prices simply begs the question; and no referral to a common standard is possible since land is immobile. Thus the only meaningful question is the scarcity of any immobile factor relative to its own total supply. An interregional comparison and weighting emerges only *after* an optimal solution and its corresponding shadow prices are obtained.

The virtue of the model in its present form is that it yields an over-all optimum whenever period-to-period transitions are optimized. The reason for this is that there is only a single connecting link—the combined level of stocks in region A and B, without distinction as to its regional structure—between successive time periods. When more than one link is introduced—for example, when more than one commodity is included, or when the unitary nature of the single commodity between the two regions is destroyed by introducing nonzero transport costs—then period-to-period optimization can no longer automatically achieve an over-all optimum, and substitutions between activities involving separate time periods have to be considered in addition to the substitutions between contemporaneous activities that suffice for solving the simpler problem (see below).

The model of Table 2 lends itself to two kinds of analysis: An investigation of investment decisions in each time period, and a study of the growth properties of the chain of one-period solutions. It should be noted that the main points of difference between this model and the Rahman-Dorfman model consist in the following:

1. The explicit inclusion of regional factor constraints in the present model;
2. The omission, for the time being, of capital decumulation constraints in the individual regions. It is recalled that in the Rahman-Dorfman model in the absence of regional capital decumulation constraints the effect of terminal stock-weighting coefficients upon the optimal solution is restricted to the last time period, instead of spreading into preceding time periods. Thus the advantage given to regions with high reinvestment coefficients is exaggerated when capital decumulation constraints are absent. Consequently any conclusion tending to soften the strong polarization of regional investment decisions that is reached in the absence of decumulation constraints will necessarily be strengthened when the latter are reimposed.
3. The exogenous treatment of consumption. This feature of the model will be subject to alteration later.

ALLOCATIONS IN A SINGLE TIME PERIOD

A convenient way of analyzing the nature of regional investment decisions in a single period is to maximize the generation of reinvestible surplus at the end of the period, on the assumption that reinvestible surplus inherited from the preceding period is fixed at a succession of constant levels that show a systematic increase. A single-period optimizing model is shown in Table 3-A. For simplicity, only one activity per region is shown. Let this be the one with the better capital productivity, i.e., a higher ratio a_i/b_i for each region. We also assume that region A is the one with a better capital productivity, i.e., the most productive activity in region A has a higher a_i/b_i ratio than the most productive activity in B. The period shown in Table 3-A is the first period, but it could be any period t: The level of stock inherited from the previous period $t - 1$ is always regarded as constant. The maximand, $X\Pi^t$, is total final product defined as

$$X\Pi^t = SA^t + XH_A{}^t + XH_B{}^t - H_A{}^{t-1} - H_B{}^{t-1} + C^1 = a_1XX_{A1}{}^t + a_3XX_{B1}{}^t$$

In other words, the variable surplus of the commodity in the flow balance of period t, SA^t which in any case has to be reduced to zero in an optimal multiperiod solution, is merged with the variable total stock formation, $XH_A{}^t + XH_B{}^t$, and with some constant terms that do not affect the optimal level of the variables. The constants include the inherited stock levels in the two regions (in Table 3-A, the sum of these for the zero period is simply denoted by H^0) and the level of consumption.

Table 3 summarizes what happens as the amount of capital (stock of the commodity) inherited from the previous period is increased from a very low level to progressively higher levels. In each small linear programing problem this amount of capital is treated as a parameter. In Table 3-A we find the indicated solution (which is identical with the original formulation of the problem and specifies that production in both regions is zero: The "nonbasic" variables which are being set to zero appear in the top margin following the unit-level exogenous vector), to be primal-feasible, i.e., the entries in the first column are nonnegative, but dual-infeasible, since the a_1 and a_3 entries in the last row indicate forbidden positive profits. The entire stock H^0 is now in surplus since there is no production; stock rent is zero; and both productive activities are shown to be profitable. Since we assume the capital stock to be very scarce, we will increase the level of production of that activity which gives the largest production a_i per unit of capital stock used, b_i, at the same time, we reduce the slack of capital to zero. This can be accomplished by choosing the coefficient indicated by an asterisk (*) as the pivot and doing a Gaussian elimination (see Appendix 2, Transformation Rules); this leads to Table 3-B.

This table can immediately be seen to be dual-feasible, since the last two entries in the bottom row are negative, as required; it is also primal-feasible as long as the relative scarcity of capital is such that the primary factor in region A is in surplus; i.e., H^0 does not exceed the amount of stock required for the full utilization of available factor supply in A, $b_1Q_A{}^1/f_1$. Under these assumptions, this is an optimal solution: The value of total final product is H^0a_1/b_1; the activity in region A is used at a level H^0/b_1 and the production activity in the other region not at all; stock rent equals a_1/b_1; and factor wages in both regions are zero.

If the amount of capital inherited from the previous period is increased the available factor supply in region A will eventually be exceeded. Under these conditions Table 3-B becomes primal-infeasible: $Q_A{}^1 - (f_1H^0/b_1)$ becomes negative, indicating that the factor in region A is a bottleneck resource and the indicated level of final product, H^0a_1/b_1, cannot be attained. At this point the scale of the productive activity in region A is limited by the factor supply in region A. If another production activity existed in region A it would now be necessary to test which of two alternative choices to follow: To economize on the scarce factor of region A by beginning to use a second activity in region A that is less factor-intensive and more capital-intensive, or to turn to the production activity in region B that does not use the scarce factor of region A at all. Since we have limited ourselves to one activity per region in the present model this choice does not arise: The only available alternative is to start using the production activity in region B. After pivoting on the element designated by an asterisk we arrive at Table 3-C which is again found to be dual-feasible; it is also primal-feasible provided that H^0 is above the lower limit $b_1Q_A{}^1/f_1$ which corresponds to the capital requirement for full utilization of factor A, but below the upper limit $(b_1Q_A{}^1/f_1) + (b_3Q_B{}^1/f_3)$ which corresponds to the capital requirement for the full utilization of both regional factors. If H^0 expands even further the primal feasibility is again violated and has to be restored by making factor B a scarce factor and making capital stock free; after the required pivot-step we get Table 3-D.

By comparison of Tables B, C, and D, each of which may be optimal depending on the relative magnitudes of the parameters, it can be seen that the optimal value of final product is always obtained in the form

$$\text{opt.}\ (X\Pi^1) = H^0 \cdot YR^1 + Q_A{}^1 \cdot YW_A{}^1 + Q_B{}^1 \cdot YW_B{}^1$$

with rents and wages taking the values shown in the table below.

Table	*Rent*	*Wage A*	*Wage B*
3-B	a_1/b_1	free	free
3-C	a_3/b_3	$\dfrac{b_3a_1 - a_3b_1}{f_1b_3}$	free
3-D	free	a_1/f_1	a_3/f_3

With the introduction of additional activities in each region the number of possible optimal configurations increases and the formulas become more involved, but the optimal value of final product continues as a linear expression in terms of inherited stock and factor supplies.

We can now relate these results to the Rahman-Dorfman model. To begin with, we can relax the assumption of prescribed constant consumption and define the latter as a constant fraction of final product. Formally, this amounts to redefining the maximand as the final product net of consumption:

$$\overline{X\Pi}^t \equiv a_1 \cdot XX_{A1}{}^t + a_3 \cdot XX_{B1}{}^t + C^t = a_1 \cdot XX_{A1}{}^t \cdot (1 - c) + a_3 \cdot XX_{B1}{}^t \cdot (1 - c) = (a_1\sigma) \cdot XX_{A1}{}^t + (a_3\sigma) \cdot XX_{B1}{}^t$$

TABLE 3

SINGLE-PERIOD SOLUTIONS

TABLE 3-A

	1^{*}	XX^{1}_{A1}*	XX^{1}_{B1}*		
$SB^{1} =$	H^{0}	$-b_1^{*}$	$-b_3$	$*YR^{1}$	$a_1/b_1 > a_3/b_3$
$SF^{1}_{A} =$	Q^{1}_{A}	$-f_1$	0	$*YW^{1}_{A}$	Primal Feasible
$SF^{1}_{B} =$	Q^{1}_{B}	0	$-f_3$	$*YW^{1}_{B}$	Dual Infeasible
MAX! $XII^{1} =$	0	a_1	a_3	$*YP^{1}(=1)$	
	=	=	=		
	MIN! - D0	$-DX^{1}_{A1}$	$-DX^{1}_{B1}$		

Capital stock H^{0} is relatively scarce. Pivot so as to bring more captial-productive activity into base (star shows pivot).

TABLE 3-B

	1^{*}	SB^{1}*	XX^{1}_{B1}*		
$XX^{1}_{A1} =$	H^{0}/b_1	$-1/b_1$	$-b_3/b_1$	$*DX^{1}_{A1}$	Dual Feas. $(b_3a_1 > a_3b_1)$
$SF^{1}_{A} =$	$Q^{1}_{A} - f_1H^{0}/b_1$	f_1/b_1	f_1b_3/b^{*}_1	$*YW^{1}_{A}$	Primal Feas. if: $H^{0} \leq b_1 Q^{1}_{A}/f_1$
$SF^{1}_{B} =$	Q^{1}_{B}	0	$-f_3$	$*YW^{1}_{B}$	
MAX! $XII^{1} =$	$H^{0}a_1/b_1$	$-a_1/b_1$	$-(b_3a_1 - a_3b_1)/b_1$	$*YP(=1)$	
	=	=	=		
	MIN! - D0	$-YR^{1}$	$-DX^{1}_{B1}$		

If not primal feasible, there is enough capital to make factor in Region A scarce. Pivot so as to bring also production in other region into base.

TABLE 3-C

	1^*	SB^{1*}	SF^1_{*A}		
$XX^1_{A1} =$	Q^1_A/f_1	0	$-1/f_1$	$*DX^1_{A1}$	Dual Feas. (as above)
$XX^1_{B1} =$	$H^0/b_3-(b_1/b)(Q^1_A/f_1)$	$-1/b_3$	b_1/f_1b_3	$*DX^1_{B1}$	Primal Feas. if: $b_1Q^1_A/f_1 \leq H^0 \leq (b_1Q^1_A/f_1 + b_3Q^1_B/f_3)$
$SF^1_B =$	$Q^1_B-f_3(H^0/b_3)-\frac{b_1}{b_3}\frac{Q^1_A}{f_1})$	f_3/b_3^*	$-f_3b_1/f_1b_3$	$*YW^1_B$	
MAX! $XII^1 =$	$H^0a_3/b_3 + Q^1_A\frac{b_3a_1-a_3b_1}{f_1b_3}$	$-a_3/b_3$	$-\frac{b_3a_1-a_3b_1}{f_1b_3}$	$*YP^1(=1)$	
	= MIN!- D0	= $-YR^1$	= $-YW^1_A$		

If not primal feasible, there is enough capital to make factors of both regions scarce. Capital becomes free resource when pivoting on respective slacks.

TABLE 3-D

	1^*	SF^1_{*B}	SF^1_{*A}		
$XX^1_A =$	Q^1_A/f_1	0	$-1/f_1$	$*DX^1_{A1}$	Dual Feas. (as above)
$XX^1_B =$	Q^1_B/f_3	$-1/f_3$	0	$*DX^1_{B1}$	Primal Feas. if: $b_1Q^1_A/f_1+b_3Q^1_B/f_3 \leq H^0$
$SB^1 =$	$H^0-b_1Q^1_A/f_1-b_3Q^1_B/f_3$	b_3/f_3	b_1/f_1	$*YR^1$	
MAX $XII^1 =$	$Q^1_Aa_1/f_1+Q^1_Ba_3/f_3$	$-a_3/f_3$	$-a_1/f_1$	$*YP^1(=1)$	
	= MIN!- D0	= $-YW^1_B$	= $-YW^1_A$		

where $\overline{X\Pi}^1$ is the redefined maximand, while c and σ are the consumption and savings ratios, respectively. It is clear that this change has no effect other than replacing the constant output coefficient of each activity by another constant obtained as a product of the former by the savings ratio; thus the entire analysis is formally unchanged; except for replacing a_i by $a_i' \equiv \sigma_i a_i$.

Next let us examine the conclusion based on the Rahman-Dorfman model that investment should be channeled (in the absence of decumulation constraints) to the region with the highest reinvestment ratio in all but the terminal period. The reinvestment ratio in each region is $\sigma_i(b_i/a_i) = \sigma_i a_i/b_i \equiv a_i'/b_i$. By reference to Table 3 the conclusion is correct as long as there is a surplus of primary factor in region A (and of course also in region B). We then have the situation corresponding to Table 3-B in which production takes place entirely in region A, yielding a stock rent of a_1'/b_1; thus all of the capital stock inherited from the previous time period has to be invested in region A. If investment were channeled instead into region B, stock rent would fall to a_3'/b_3, and activity 1 would show a positive profit, indicating that the solution was not dual-feasible. [These results follow from Table 3-B by symmetry between activities 1 and 3; the only difference is that an interchange of subscripts 1 and 3 will render $-(b_3a_1 - a_3b_1)$ positive.]

If, however, the parameters of the problem are such that the situation of Table 3-C is obtained, i.e., the primary factor in region A is in scarce supply, then investment can no longer be polarized into region A without incurring an inefficiency. Optimality requires a splitting of production (and investment) between A and B; if we insist on investing only in A, then the inherited capital cannot be fully utilized due to the shortage of factor A. Thus the criterion of polarizing investment into the region with highest reinvestible surplus breaks down. This is even more so for the case of Table 3-D, but the situation of a stock surplus has to be excluded in practice as unrealistic. In other words, regions with a less favorable reinvestment ratio obtain a share of total investment to the extent that the regions with more favorable reinvestment ratios run out of local primary factors.

This conclusion is further strengthened when we include additional activities in each region. When region A has a second activity with a less favorable capital productivity than the first, the polarization of investment toward region A would require that when the limit of factor-A supply is reached, the second activity should gradually begin replacing the first activity, in order to substitute the use of stock for the use of factor A. A detailed study of this case indicates that for the latter course to be optimal, it is required not only that the reinvestible surplus of the second activity a_2'/b_2 be higher than that of activity 3 in region B, but the more stringent condition

$$f_1(a_2'b_3 - a_3'b_2) \geqq f_2(a_1'b_3 - a_3'b_1)$$

also has to be satisfied.[10] When it is not, the application of the reinvestible-surplus criterion will result in a misallocation of resources even when the criterion is applied to the marginal investment decisions rather than to the entire investment.

INTERRELATIONS BETWEEN TIME PERIODS

What happens when the regional capital decumulation constraints of the Rahman-Dorfman model are reintroduced? In the former model these constraints have the effect of progressively reallocating investment, as the terminal period is approached, from regions with high reinvestment ratios to regions with high terminal stock valuation. These constraints, if included in the model of Table 2, will destroy the independence of period-by-period optimization from longer-term considerations, since they introduce additional links between time periods. Thus while in the case of a single link the Pareto-optimal frontier for each time period coincides with a single point (the optimal total stock carried forward into the next period), with decumulation constraints added there is a possibility of trading off terminal valuation benefits in a region against a lesser short-term capital generation. Finding an optimal time path then becomes a matter of solving the problem in the large.

The Rahman-Dorfman solution hinges on the application of elementary principles of dynamic programing to the above problem. Dorfman's solution is particularly simple: It consists in a recursive evaluation of the effects of investment in each time period upon the objective function. He determines investment in the last period by selecting the

[10] Lack of space prevents a detailed analysis of the more complete model. The criterion of choice in the text is closely related to the column selection criterion of the dual simplex algorithm. In a slightly modified form,

$$\frac{f_1}{b_3 \dfrac{a_1'}{a_3'} - b_1} \geqq \frac{f_2}{b_3 \dfrac{a_2'}{a_3'} - b_2},$$

it can be interpreted as follows. The denominators represent the amount of capital saved by activities 1 and 2, respectively, relative to activity 3. We prefer activity 2 over activity 3 when, per unit of the former saving, activity 2 is less factor-A intensive than activity 1.

region with the highest terminal stock valuation. Any investment in the previous period will have two effects: (1) It will contribute *directly* to the objective function by means of its own terminal valuation, which cannot be changed once it is committed since investment cannot be transferred to another region; and (2) it will also contribute *indirectly* to the objective function by generating reinvestible surplus that can be freely disposed of in the last period, at a payoff that has already been determined. Thus the summing of the known direct and indirect effects for each region will yield a linear expression of terminal payoff in terms of current investment variables; from among these, the best one can be selected. This, in turn, creates a known payoff for the indirect effects of investment one period further back, and so forth. In this fashion, the recursion runs all the way to the original time period and yields an optimal investment profile.

The feature of the Rahman-Dorfman model that allows the application of such a simple recursion is the fact that no matter how little or how much reinvestible surplus is carried forward from one time period into the next, the allocation decisions remain unaffected in all subsequent time periods. As long as this feature is preserved, the regional polarization of investment inherent in the structure of the model is not essential to the result. Thus if it were possible to guarantee in advance that factor A would be scarce and factor B would be in oversupply in all time periods (or even if it were possible to specify in advance any alternating sequence of patterns chosen from among those of Tables 3-B, 3-C, and 3-D, so that a given pattern could be guaranteed for a given time period in advance of the solution) the simple recursion could still be applied. If, however, the size of the reinvestible surplus carried forward affects the choice of an optimal allocation pattern—as is the general case with the model of Table 2 when decumulation constraints are imposed—the simplicity of the Dorfman solution is lost, and a systematic method of keeping track of the available alternatives is required. In any event the key feature of the Rahman-Dorfman solution is preserved: The effects of terminal valuation are carried backward into previous planning periods but become progressively weaker as the time span between a given period and the terminal period lengthens.

Returning now to the model of Table 2 with decumulation constraints excluded the next question concerns the dynamic behavior of this model over the planning period. This turns out to be a simple generalization of growth at compound interest once the rent and wages for each period have been determined by means of single-period optimizations.

Denoting the previously derived constant values of rent and wages in time period t by r^t, $W_A{}^t$, $W_B{}^t$, we obtain: [11]

$$XH^t = XH^0(1 + r^1) \cdots (1 + r^t) + (Q_A{}^1 \cdot W_A{}^1 + Q_B{}^1 \cdot W_B{}^1 - C^1)(1 + r^2) \cdots (1 + r^t) + \cdots + (Q_A{}^{t-1} \cdot W_A{}^{t-1} + Q_B{}^{t-1} \cdot W_B{}^{t-1} - C^{t-1})(1 + r^t) + (Q_A{}^t \cdot W_A{}^t + Q_B{}^t \cdot W_B{}^t - C^t)$$

On the dual side the flow price of the commodity is the numeraire for the price system in each period; interrelations between periods are created by the stock-holding activities which allow the referral of all prices to the numeraire of the model as a whole, associated with the terminal stock valuation row. If terminal stock valuations at time T in both regions are equal to unity, YP^{T-1} will equal unity, and the commodity flow prices will be:

$$YP^1 = (1 + r^2) \cdots (1 + r^{T-1});\ YP^t = (1 + r^{t+1}) \cdots (1 + r^{T-1})$$

For nonunitary terminal stock valuations, YP^{T-1} will be equal to the highest terminal stock valuation and all prices will be multiplied by this factor.

For the definition of an interest rate in current prices the value-standard good is the single commodity; if its current price is held constant for all periods, the interest rate in each period will coincide with the shadow rent of stock for that period. Growth of the initial stock at compound interest is then a special case that can arise when the a, b, and f

[11] The derivation is as follows:

$$XH_A{}^1 + XH_B{}^1 = XH^1 = XH^0 - C^1 + XH^0 \cdot r^1 + Q_A{}^1 \cdot W_A{}^1 + Q_B{}^1 \cdot W_B{}^1 = XH^0(1 + r^1) + Q_A{}^1 \cdot W_A{}^1 + Q_B{}^1 \cdot W_B{}^1 - C^1$$

Likewise,

$$\begin{aligned} XH^2 &= XH^1(1 + r^2) + Q_A{}^2 \cdot W^2 + Q_B{}^2 \cdot W_B{}^2 - C^2 \\ &= [XH^0(1 + r^1) + Q_A{}^1 \cdot W_A{}^1 + Q_B{}^1 \cdot W_B{}^1 - C^1](1 + r^2) + Q_A{}^2 \cdot W_A{}^2 + Q_B{}^2 \cdot W_B{}^2 - C^2 \\ &= XH^0(1 + r^1)(1 + r^2) + (Q_A{}^1 \cdot W_A{}^1 + Q_B{}^1 \cdot W_B{}^1 - C^1)(1 + r^2) + (Q_A{}^2 \cdot W_A{}^2 + Q_B{}^2 \cdot W_B{}^2 - C^2) \end{aligned}$$

The generalization is immediate.

parameters are constant from period to period and consumption is exactly equal to the shadow wage bill in each period.[12]

In the general case, stock accumulation, i.e., system growth, depends both on the initial stock and on the time profile of consumption demands and factor supplies. In particular, factor supplies make a positive contribution to system growth. None the less, growth is faster when factor supplies are ample enough to leave a surplus and, accordingly, when wages are zero. The apparent contradiction between this statement and the structure of the accumulation formula can be reconciled by a detailed consideration of the transition between situations of unlimited and limited factor supply. By reference to Tables 3-B and 3-C, for example, consider what happens when $H^0 = b_1 Q_A{}^1/f^1$, i.e., when the initial capital is exactly at the margin of exhausting the supply of factor A in period 1. Final product (the maximand) from Table 3-B is $H^0 a_1/b_1$; the same expression can also be obtained from Table 3-C by substituting the value of $Q_A{}^1$ at the margin into the formula for the value of the maximand. This identical final product is, however, distributed between the factors of production in a different manner in the two cases. In Table 3-B it is imputed entirely to stock rent; in Table 3-C it is divided between rent and wages. If now, beginning at the marginal position which has been indicated above, we *reduce* the supply of factor A while leaving H^0 unchanged, final product will be reduced but unit rent and wages will remain constant. In sum, when a factor constraint becomes binding, the wage bill of the factor is taken out of total product, thereby reducing the unit rent and the rent bill; moreover, in all situations except at the margin, the total product itself also falls. Thus a factor shortage cuts into total rent in two ways: (1) It reduces the percentage share of total product imputed to rent, and (2) it reduces the size of total product. A corollary is that the share of final product imputed to a regional factor in any period may at times be increased by restricting the supply of this factor, but only at the expense of a drop in system product that is larger (except at the margin) than the increase in total imputation to the factor.

[12] This simple special case can be interpreted, if desired, as the outcome of an idealized perfectly competitive institutional situation with no savings out of wages and no consumption out of rents. It is recalled, however, that the term "wages" refers to the shadow flow prices of all primary factors that are not themselves produced by any activity, i.e., these factors include natural resources, land, etc., in regard to which the above institutional assumption becomes weak. It will be shown later that the interpretation can break down when more than one link between successive time periods (e.g., more than one commodity stock) is introduced.

An interesting property of the model is that it leaves the accumulation rate of the system as a whole unchanged at the margin while the interest rate undergoes a large discontinuous change. The reason for this is that savings are assumed to be independent of income distribution (at shadow prices) between rent and wages; this assumption holds regardless of whether consumption levels are prescribed exogenously or are determined as fixed proportions of final product in each region. Such an assumption can be realistic when savings decisions are arrived at by political methods and personal consumption is made independent of the shadow incomes imputed to factors, as can be done by the rationing of consumption or by the establishment of transfer payments that are superimposed on the shadow price system. Moreover, when the means of production are under community ownership, shadow rents are not directly related to the consumption or savings of any single group of individuals. The question of who controls the means of production of a given region as a whole can, however, remain open even under these conditions. Thus the issue of the regional distribution of income remains to be clarified.

MULTICOMMODITY GENERALIZATION POSSIBILITIES

Before proceeding to the above task it will be attempted to sketch out the generalization of these results from single-link models that can be optimized period by period to multiple-link models where such a procedure will not necessarily assure long-run optimality. If more than one commodity is present, then during each period a *weighted* combination of stocks, i.e., the capital wealth of this period evaluated at shadow flow prices, must be maximized. Given only the terminal stock weights, the proper shadow prices to be used for the maximization in a given period are not known in advance, thus proving *ex post* that a period-by-period optimization in fact attained within the over-all optimum solution is not the same thing as being able to produce the latter solution by means of a period-to-period optimization.[13] None

[13] On the relation between period-to-period optimums and efficient paths of capital accumulation over the long run, see Dorfman, Samuelson, and Solow (1958), Chap. 12. For the multicommodity case it remains true that with constant *a, b,* and *f* coefficients the system will grow at a compound rate of interest equal to the stock rent (in current prices) of the value-standard commodity whenever the total value of consumption equals the total wage bill. This case, however, can no longer always be interpreted as the outcome of perfect competition with no savings out of wages and no consumption out of rents. While it is true that in a multicommodity system perfect competition will assure that an efficient path of capital accumulation will be followed over any three successive periods and thus

the less, the properties of the single-period optimums can be analyzed on the assumption that constant weights exist for stocks at the end of each period; in addition, the long-term properties of the optimal solution can be studied by reference to von Neumann-type models, in search of regional "turnpike"-like theorems. The present discussion will be restricted entirely to the first type of analysis.

When transport costs are introduced into the single-commodity model of Table 2 with exogenously given consumption levels, they work against the concentration of production into a few regions, since a more dispersed production will reduce transport costs between sites of production and consumption. When consumptions are endogenously given as fractions of regional income including rent remissions, the same result will obtain. Moreover, in the presence of regional stock decumulation constraints transport costs will in all periods work in favor of regions with larger terminal stock weights over regions with a higher capital productivity, since the surplus of the latter will eventually have to be transferred at least in part to the former regions, at the expense of incurring transport costs.

When the number of commodities in the model is increased from one to several, the analysis indicates further tendencies toward the geographical dispersal of production:

1. Between different regions, the *a* and *b* coefficients and thus relative capital productivities can differ in different lines of production, with one region having a productivity advantage in one line, another region in another line. Under these conditions each region will tend to specialize in a given line of production. Thus for example, advanced countries differ in their specializations in regard to the products of high-level technology, thereby creating a basis for commodity interchange.

2. Between advanced and backward regions, however, the relations between *a* and *b* coefficients will tend to run parallel in most industries, as backward regions will tend to have generally poorer technology, i.e., poorer capital stock and intermediate-input coefficients. This will create an absolute disadvantage for backward regions in regard to these in-

over the entire long run—see the dynamic "invisible hand theorem," *ibid.*, p. 319—the perfect competitors are assumed to be aware, in addition to stocks inherited from the previous period and current prices, only of prices in the next period but not beyond. Thus when *terminal* stock valuations are prescribed, as in the models under discussion, the perfectly competitive mechanism assumed is not capable of selecting the required efficient path from among the many efficient paths that lead to different relative stock accumulations in the terminal period. On the other hand, if relative stock valuations are given either for the initial or for the following period, this will enable a perfectly competitive system to lock onto one efficient path and to trace it all the way through to its terminal implication.

puts: The disadvantage can be said to be "absolute," since the possibility of transporting commodities between regions allows a direct comparison of flow and stock inputs. An optimal program will therefore tend to channel investment into the regions that have better productivities with regard to stocks and intermediate-flow inputs, i.e., into the advanced regions.

Productive activities, however, also have inputs of immobile primary factors that are in limited supply in each region; these factor supplies in the more productive regions can be exhausted before all investment resources are fully utilized. At this point, as already indicated in the single-commodity case, two choices exist:

1. The scarce factors in the advanced region can be economized by means of substitution by commodity-stock or intermediate-flow inputs in the same region; i.e., there can be a progressive shift to activities in the same region that use less of the scarce primary factors and more of the stock and intermediate-commodity inputs.
2. The scarce factors in the advanced region can be economized by a shift to the activities of the backward regions that do not use the scarce primary factors of the advanced region at all. To be more precise, if labor is regarded as an immobile primary factor, both the activities in the advanced and the backward regions will use labor, but the backward regions will not use the labor of the advanced region.

A comparison of the two substitution possibilities above leads to the conclusion that the advanced regions will tend to shift to the backward regions those of their activities that are intensive users of the scarce primary factors in the advanced regions yet are not burdened with an excessive absolute disadvantage in the backward regions. This shift and with it the geographical dispersal of production and investment will be more pronounced to the extent that the pool of investible resources for the system as a whole is large in relation to the primary factor supplies of the advanced regions. Thus a larger over-all savings ratio will tend to disperse investments regionally even when the backward regions have an absolute disadvantage in all lines of production.

Preferences, Incentive Effects, and Autonomous Growth Trends

ALTERNATIVE PREFERENCE SYSTEMS

In the foregoing sections consumption was at times treated as exogenously given and at times tied to regional or over-all product by means of a constant savings ratio. The fact has, moreover, been pointed out that consumption and investment can under certain conditions be tied directly to factor shadow incomes by suitable institutional assumptions. The present section will be dedicated to a more systematic discussion of the underlying issues and to the sketching out of certain reformulations of the model that are possible once the customary assumptions are modified.

It is recalled from the section Interrelations Between Time Periods, above, that the general locational model can be regarded as a representation of the technological possibilities open to a society, with preference functions excluded from the model. If so interpreted the model can be used to trace out parametrically the Pareto-optimal production possibility hypersurface. Approximate representations of preference functions can then be adjoined to the former model, e.g., in the form of savings or commodity-by-commodity consumption ratios, in order to locate the relevant decision range. Once this range has been located in an approximate fashion the savings ratios or other approximate preference indicators can be dropped and the relevant range of the Pareto-optimal hypersurface can be explored in detail with a view to matching it with an implicit set of planning and policymaking preferences.

This interpretation rests on the postulate that there exists a single set of over-all planning preferences embracing all commodities, all regions, and all time periods. This set of preferences permits trading off *on the preference side* (independently of the production-possibility side) present against future consumptions, consumptions in one region against consumptions in another, consumptions of one commodity against consumptions of another, as well as any cross-combinations of the former. A match of the complete set of preference trade-offs against the complete set of production trade-offs results in the selection of the over-all grand optimum in a straightforward extension of standard neoclassical notions to a dynamic, multiregional, central planning situation. The only remaining task necessary to complete this picture is to treat primary factor supplies symmetrically with consumption, i.e., to unlock the respective parameters and to include them as variables both in the production and in the preference trade-offs.

This postulate implies an absolute centralization of planning decisions; this is, however, not the only possible way of adjoining preferences to the Pareto-optimal production-possibility surface. That other possibilities exist is indicated by the fact that in the former model there is no relationship between factor incomes at shadow prices and optimal consumption levels except at the level of the system as a whole; in other words both the amounts of commodities consumed and the amounts of primary factors supplied by region and by time period are determined solely by central preferences and over-all scarcities.

The opposite extreme is a complete decentralization of decisions on the basis of the shadow price system. Adjoin to the former model a number of individuals and distribute among these individuals claims to the shadow incomes generated by all primary factors and all *initial* stocks. Define the income of an individual in a time period as the sum of the former claims plus new claims generated by stock accumulation, where claims to newly accumulated stocks in each period are distributed in proportion to savings provided by individuals for investment purposes in the same period. As long as there is only one commodity in the model the meaning both of savings and of stock rents is unambiguous. Then in each time period each individual makes a consumption-savings decision subject to the constraint on his total income for the period as defined above; this decision is based on the matching of his income constraint against his time preference between present and next-period consumption. If we are now willing to make the assumption that period-to-period time preferences are independent of consumption possibilities in later periods (i.e., if we are willing to assume a more restricted set of time preferences than in the central decision model) then it is possible to decentralize both with regard to individual time preferences and with regard to production decisions (e.g., by an institutional assumption of perfect competition) since period-by-period optimization of production will guarantee a long-run Pareto-optimum (as discussed in the preceding section) while period-by-period optimization will likewise guarantee (by the above assumption) a long-run optimum with regard to the structure of preferences. If we are not willing to assume this much with regard to the independence of period-to-period time preferences then decentralization on the side of individual preferences is possible only if the latter can be matched against a Pareto-optimal production possibility hypersurface linking all time periods that is generated by central planning computations.[14] In other words the function of planning in the latter context is reduced to the generation and display of a complete representation of the technological possibilities confronting society over time while leaving perfect autonomy of decision to each individual.

When there is more than one commodity it is necessary to make one

[14] The reason for this is that present consumptions cannot be taken as determined unless future consumption possibilities are already known. Only central planning computations are able to generate and display such consumption possibilities extending into the long-run future.

further adjustment in the analysis on the side of preferences. Single-commodity income, consumption, savings, and investment now give way to aggregate concepts weighted by shadow prices. The income constraint of each period can always be replaced by an equivalent wealth constraint in shadow prices linking two successive periods (see Appendix 3 for the derivation of such period-to-period balances for wealth):

$$\underset{\text{next-period wealth}}{(P^{t+1} + R^{t+1})H^{t}} = \underset{\text{present-period wealth}}{(P^{t} + R^{t})H^{t-1}} + \underset{\text{present-period factor income}}{W^{t}Q^{t}} - \underset{\text{present-period consumption}}{P^{t}C^{t}}$$

Since the consumption of the next period will be taken out of the next period's wealth there is now a trade-off between more consumption in this period and less consumption during the next period; moreover, there is a trade-off between different kinds of consumption during both periods. On the consumption side these trade-offs require knowledge of the same relative commodity valuations that are required on the production side for the determination of the structure of stocks that are accumulated. Decentralization with regard to individual preferences and with regard to production decisions is therefore equally dependent on the availability of these valuations at the time the consumption or production decisions are taken. If we extend the previously mentioned dynamic "invisible hand" theorem to variable consumption and assume that the system is already on an efficient time path of capital accumulation under perfect competition, the theorem will guarantee that the system will stay locked on a single path, but it offers no explanation of how this path came to be selected from among infinitely many possible efficient time paths. We are thus forced to accept one of three possible interpretations:

1. The efficient path has been chosen at random, but this does not matter since all preferences are satisfied at all times; thus even though the system drifts blindly toward a terminal condition the exact nature of this terminal condition is a matter of indifference.

2. The terminal condition is *not* a matter of indifference to individuals; therefore, contrary to assumption, their period-to-period time preferences are not independent over the long run. In this case, we arrive at the same conclusion of having to match individual preference against a Pareto-optimal production-possibility hypersurface generated by central planning computations that we already encountered in the one-commodity case.

3. The terminal condition does not enter individual preferences but is a matter of *social* concern. In this case we have to select a given efficient time path that will lead to the prescribed terminal condition, i.e., the system has to be switched from its historic path to another path at the beginning of the planning period, and the shadow prices corresponding to this path have to be given initially in order to lock the system onto the new path. The required shadow prices, however, can be computed even by central planning methods only if the preference functions of all individuals in all periods are known in advance.

Interpretation 2 appears perhaps somewhat less unrealistic than the others, but it has to be admitted that this entire analysis based on a deterministic view of the future is highly unsatisfactory. Savings-investment decisions inherently have a probabilistic element that is inescapable even in these highly formalized models as soon as we get to the terminal period, since terminal stock valuations are proxies for post-terminal consumption possibilities that extend into the future in an infinite regress. Therefore, unless we wish to follow the road of determinism to its bitter end with the formulation of infinite-period models with finite preference levels, we have to face up to the broader implications of indeterminacy. This task, however, falls entirely outside the limits of the present paper.

In addition to the interesting match between a representation of production possibilities over time that are generated by central planning and perfectly decentralized individual decisions, the above analysis also demonstrates the indifference of the individual to the physical nature of the stocks to which he has a claim, thus naturally leading to the notion of capital in the abstract, without reference to any particular collection of stocks. This can be seen as follows: A change in the consumption of any good can be traded off against the formation of any stock of equal shadow value, P^tH^t, in the same period; no matter what the nature of this stock is, the zero-profit condition on all actually utilized stock-holding activities guarantees that wealth in the next period at shadow prices will exactly equal the shadow value of the stock in this period:

$$(P^{t+1} + R^{t+1})H^t = P^tH^t$$

Thus while the structure of stocks is of paramount importance to the system as a whole it is of no importance to the individual as long as his claims are small in relation to total stock holdings in the system, since under equilibrium the period-to-period payoffs are equalized on all stocks. We can therefore conceive of a decentralized system that operates as follows: All savings decisions (as well as decisions pertaining to the structure of consumption and factor supply) are arrived at in a completely decentralized fashion; all savings are pooled and invested under central management;[15] central planning computations determine and display the entire production-possibility surface in such a manner that individuals can adjust their preferences to a known set of present and future production possibilities. For the system to be determinate rather than taking on the nature of a strategic game the preference functions of individuals have to be independent of changes in the production-possibility surface and the preferences of other individuals, as required by neoclassical theory.

An intermediate stage between complete centralization and complete decentralization of decisions is also possible, namely, decentralization to the regional level. In this case regional incomes and wealth are defined as indicated in the section Interrelations Between Time Periods, above, and Appendix 3, except that an export surplus of given shadow value in any period is treated as a capital export, giving rise to a corresponding claim to the wealth of an import-surplus region in the following period. Unless the doubly unrealistic assumption of a single commodity and no transport costs is insisted upon, the Pareto-optimal production-possibility surface will now, for the reasons discussed above, have to be generated by central planning. Regional decentralization of decisions with regard to consumption, savings, and factor supply can, however, be accomplished as before, with regions taking on the role of individuals in the former analysis. In particular, an important result that carries over from individual to regional decisions under this model is the pooling of savings: Under the assumptions of this model these are always distributed between the regions merely on the basis of capital productivity and factor limitation criteria regardless of their regional origin. It can be demonstrated with the aid of the single-commodity model discussed in the previous section that a region that insisted on investing its savings locally when this was contrary to overall investment criteria would reduce its own accumulation of wealth, since this wealth is composed indifferently of stocks invested at home and abroad. An important difference from full decentralization, however, is the following. The independence of preference functions from

[15] After a long-run solution has been obtained by central planning computations, the execution of production decisions is in theory compatible with an institutional arrangement of perfect competition once the system has been locked onto the desired efficient path of capital accumulation by means of planned and administered first- or second-period shadow prices.

production possibilities can be taken as far less assured in the case of regions than in the case of individuals, because regions often represent a sufficiently large fraction of the supply or demand of a resource to affect noticeably the outcome of the optimal solution following autonomous changes in the corresponding regional supply or demand behavior. Thus restricting the supply of a regional factor can under suitable conditions capture a larger wage bill for this factor than would otherwise be possible, but only at the cost of a generally larger than compensating drop in system income, as discussed in the previous section. Assuming, for example, that the entire capital stock of the region in question was under claim by other regions, this would unequivocally increase regional income.

The above three ways of adjoining preference functions to the Pareto-optimal production-possibility surface are mutually exclusive, since each covers the entire range of available choices and it is known that in general there exists no way of synthesizing composite preference functions from individual ones.[16] There appears to be no reason, however, why different levels of preference functions could not govern different domains of social decision provided only that they either operate in nonoverlapping decision ranges or that they are subject to rules of precedence in case of conflict.

This whole area requires much further study. It is nonetheless clear that the levels at which decisions are exercised imply definite consequences with regard to the structure of incentives. Full decentralization of the decision process creates economic incentives for individual effort and for the supply of individual savings by tying changes in the income and wealth of individuals directly to the shadow prices of their factor supplies and the rent on their capital. Full centralization breaks such links almost completely except for an over-all social constraint imposed on all individuals collectively. Intermediate stages of decentralization can evidently operate effectively by defining the disposable incomes of individuals by means of suitable transfers that are determined on the basis of social preferences. In social systems where the means of production are in public ownership, the link between shadow rents and individual incomes can be completely broken without thereby necessarily eliminating economic incentives for individual effort; in this case, however, savings-investment decisions have to be controlled through political channels. As a general proposition, to the extent that areas of economic decision are exempted from the operation of individual prefer-

[16] See Arrow (1951).

ences they have to be subjected to higher-level preferences, to be determined by political processes that have to be different from the logically prohibited synthesis of composite preference functions.

While the abstract structure of preference is a useful analytical tool for the exploration of issues pertaining to decentralization and to multilevel decisions, it is recalled that preference functions are regarded as empirically all but useless. Therefore, wherever fully decentralized market decisions by individuals are excluded, a realistic planning strategy has to replace them by a two-step process: (1) The relevant decision range is approximately located by means of functional relationships that act as *proxies* for the preference functions, i.e., savings ratios, consumption ratios, supply and demand relationships; and (2) within the relevant decision range, the Pareto-optimal production-possibility surface is explored in detail for the purpose of applying an *implicit* set of planning-policymaking preferences to the alternatives so disclosed. The savings relationships built into the previously discussed Rahman-Dorfman model will now be inspected from this perspective and particularly the issues concerning the structure of incentives will be analyzed in more detail.

The Rahman-Dorfman model makes the assumption that savings are transferred between regions without setting up corresponding claims to future incomes; in other words the surpluses of a region are centrally appropriated without compensation except for such future development as might be channeled toward the region itself as a result of a preferred terminal weighting. The savings ratio in each region is assumed to be a constant fraction of current income. These assumptions imply, first of all, a set of central planning preferences for the system as a whole that subordinate the path of development in each region to the objective of maximal terminal stock accumulation; the development of individual regions is considered as an end in itself only as a terminal condition realized by the assignment of regional weights to terminal stocks. In terms of the behavior of the model that has been previously discussed this amounts to a preference for growth in the short run no matter how this growth is distributed between the regions, tempered only by the objective of arriving in the long run at a regional distribution that is considered equitable. Secondly, the constancy of regional savings coefficients independently of direct immediate benefits to a region resulting from this saving implies either that central preferences are imposed on a region, overriding the preference structure of the latter, or that the interregional political consensus assigns great weight to maximal short-run growth subject to an equitable long-run distribution of benefits.

The contrary assumption with regard to preferences in the context of the Rahman-Dorfman model would be to postulate that regional savings give rise to corresponding claims against future income. This amounts to a decentralization of the savings decisions to the level of the regions. Savings are now no longer exacted by central coercion or contributed under an interregional political consensus but are motivated by the returns that accrue to the region as a result of these savings. Since under this assumption savings generate income for a region no matter in what geographical location they are invested, there is now no longer any reason to prefer investment in regions with high savings rates even though their capital-output ratios might be unfavorable. The regional distribution of investments thus becomes independent of the regional saving rates and is determined by the interaction of the capital-output ratios with terminal weightings; if the latter are equal, the capital-output ratio becomes the sole criterion of regional investment.[17]

The above two versions of the structure of preferences of the Rahman-Dorfman model can be generalized to multifactor, multicommodity models by the techniques already discussed in previous sections, without materially affecting the conclusions except for substantially relaxing the extreme concentration of investments into a single region that is a consequence of operating with a single constraint in every period. A more interesting generalization from the point of view of preferences, incentives, and growth possibilities can be obtained by relaxing the assumption concerning the constancy of saving rates.

SOME REFORMULATIONS

What happens when savings rates in the regions are made variable in response to central investment decisions?

In terms of preferences such a situation can be interpreted as the interaction of two preference systems: the central supraregional preference system and the decentralized regional preference system.

[17] A question can be raised concerning the legitimacy of maintaining terminal weightings in a model where the savings decisions are decentralized to the level of the regions. In a multicommodity model terminal weights assigned to stocks that differ by commodity and by region are required for the representation of posthorizon growth possibilities; thus the terminal stock weights can be regarded as genuine indicators of the intrinsic worth of the stocks to any individual or region that may have a claim on them. In a model aggregated by commodities, such as the Rahman-Dorfman model, the same interpretation can be put on terminal weights if they represent solely future productivities rather than central preferences with regard to long-run regional development.

Assume a central decision-making process such as is postulated in the first version of the Rahman-Dorfman model, i.e., regional savings are appropriated by the central decision-making authority without crediting the region with claims against future income. Savings are now, however, no longer assumed to be a constant fraction of current regional income but are determined in response to an autonomous preference structure operating within each region. In making the central investment allocations the supraregional authority therefore has to take into account the reactions of the regions to these central allocation decisions.

This simple change in the formulation of the model has far-reaching implications since it opens the door to the inclusion of a new set of behavioral relations into regional resource allocation models, with important consequences for the issue of regionally balanced versus unbalanced growth.

Tables 4, 5, and 6 are simple illustrations of the changes in behavior that can be introduced into the Rahman-Dorfman model by allowing regional savings to respond in a specific manner to central allocation decisions or to exhibit an autonomous behavior trend of their own.

Table 4 shows the basic model with constant savings ratios. This model will serve as a standard of comparison for two alternate reformulations presented in Tables 5 and 6. There are two regions, A and B, with constant savings ratios of 20 per cent in either; the regions differ only in regard to their capital-output ratios, which are taken to be 4 and 3, respectively, in all periods. If terminal stock weightings are assumed to be equal in both regions then the maximization of either terminal income or terminal wealth for the two regions requires the transfer of all savings from A to B. In the tables, only incomes are shown; the increase of capital in each region or in the two regions jointly can be obtained by adding successive investments. In Table 4 the time profiles of incomes, savings, and investments are derived for the two regions for five successive periods on the assumption that the savings transferred from A to B equal none, half, or all of the savings in A. In accordance with the previous analysis the highest income growth is obtained when all of A's savings are transferred to B.

In Table 5 the assumption of constant savings ratios is replaced by the assumption that savings in a region in each time period will equal the investments *in the region itself* during the preceding time period. This assumption introduces incentive effects of savings in each region that require a special interpretation.

This interpretation rests on the assumption that savings decisions in each region are arrived at by a process of planning that takes into account so far unquantified effects and is responsive to popular pressures. It is assumed that people in general expect the payoff to sacrifices of present consumption to take the form of development within the region itself. While some awareness of systemwide benefits that accrue to savings channeled to other regions need not be ruled out, it is nevertheless highly reasonable to postulate an asymmetry in the popular recognition of benefits that are near and tangible and reflect themselves in visible material progress within the region, as against benefits that are far off and that have to be accepted on the verbal assurances of political leaders. This interpretation can thus be regarded as in a sense intermediate between the two savings assumptions discussed earlier.

One extreme assumption, it is recalled, was the independence of savings decisions from the regional location of investments. For the latter assumption to have any claim to a degree of realism we have to postulate one of several alternatives.

1. Savings may originate with individuals, who deposit them in a banking system that is under central control and allocates investments purely on the basis of interregional efficiency criteria. In practice, however, a banking system will often show a local bias in investments due to problems of communications and control.

2. Savings may originate in capitalist enterprises: These could in theory exhibit a genuine indifference between the location of reinvestments, but in practice most enterprises, especially the smaller ones, have a strong bias toward local reinvestment due to the problems of cultural differences, communications, and control that often have to be overcome when investing in other regions.

3. Savings may originate with regional government. Now even if the institutional possibility existed of investing these savings in other regions and obtaining corresponding benefits in future time periods in the form of interest receipts from these other regions, the assumption of a systematic long-term trend for the government of a region to accumulate savings and lend them out on interest to other regions is farfetched indeed.

4. Savings might be accumulated in a region by means of an over-all planning and decision-making system within the region that goes beyond the ordinary housekeeping operations of government. Now while the models analyzed earlier indicate that the highest payoff on such savings could be obtained if these were invested purely on the basis of the interregional efficiency criteria that follow from these models, in practice there are a number of considerations that create a bias even in this case toward local as against extraregional development investments.

TABLE 4 Interregional Growth Model 1

Constant Savings Ratios, A: 20%, B: 20%

Constant Cap/Output Ratios, A: 4, B: 3

	Income			Savings Ratio		Savings		Investment		Investment Ratio		Income Change	
	Total	A	B	A	B	A	B	A	B	A	B	A	B
	ΣY	YA	YB	SRA	SRB	SA	SB	IA	IB	IRA	IRB	ΔYA	ΔYB
	No Transfers: IB(t) = SB(t)												
0	200	100	100	20	20	20	20					5	6.667
1	211.667	105.000	106.667	20	20	21.000	21.333	SA = IA		Not		5.250	7.104
2	224.021	110.250	113.771	20	20	22.050	22.754	SB = IB		Required		5.513	7.577
3	237.111	115.763	121.348	20	20	23.153	24.270					5.788	8.082
4	250.981	121.551	129.430	20	20	24.310	25.886					6.078	8.620
5	265.679	127.629	138.050	20	20								

Transfer $\frac{1}{2}A \rightarrow B$: $IB(t) = SB(t) + \frac{1}{2}SA(t)$

0	200	100	100	20	20	20	20	10	30		2.500	9.990
1	212.490	102.500	109.990	20	20	20.5	21.998	10.250	32.248		2.563	10.739
2	225.792	105.063	120.729	20	20	21.013	24.146	10.507	34.653	Not	2.627	11.539
3	239.958	107.690	132.268	20	20	21.538	26.454	10.769	37.223	Required	2.692	12.395
4	255.045	110.382	144.663	20	20	22.076	28.933	11.038	39.971		2.760	13.310
5	271.115	113.142	157.973	20	20							

Transfer All $A \rightarrow B$: $IB(t) = SB(t) + SA(t)$

0	200	100	100	20	20	20	20	0	40		0	13.320
1	213.320	100	113.320	20	20	20	22.664	0	42.664	Not	0	14.207
2	227.527	100	127.527	20	20	20	25.505	0	45.505	Required	0	15.153
3	242.680	100	142.680	20	20	20	28.536	0	48.536		0	16.162
4	258.842	100	158.842	20	20	20	31.768	0	51.768		0	17.239
5	276.081	100	176.081	20	20							

TABLE 5 Interregional Growth Model 2

SYMMETRICAL INCENTIVE EFFECTS ON SAVINGS

SR(t) = IR(t - 1) in Both A and B

Constant Cap/Output Ratios, A: 4, B: 3

	Income			Savings Ratio		Savings		Investment		Investment Ratio		Income Change	
	Total	A	B	A	B	A	B	A	B	A	B	A	B
	ΣY	YA	YB	SRA	SRB	SA	SB	IA	IB	IRA	IRB	ΔYA	ΔYB
A. - No Transfers: IB(t) = SB(t)													
0	200.000	100	100	20	20	20	20			20	20	5	6.667
1	211.667	105	106.667	20	20	21.000	21.333			20	20	5.250	7.104
2	224.021	110.250	113.771	20	20	22.050	22.754	SA = IA		20	20	5.513	7.577
3	237.111	115.763	121.348	20	20	23.153	24.270	SB = IB		20	20	5.788	8.082
4	250.981	121.551	129.430	20	20	24.310	25.886			20	20	6.078	8.620
5	265.679	127.629	138.050	20	20								

B. - Transfer $\frac{1}{2}$A → B: IB(t) = SB(t) + $\frac{1}{2}$SA(t)

0	200.000	100	100	20	20	20	20	10	30	10	30	2.500	9.990
1	212.490	102.500	109.990	10	30	10.250	32.997	5.125	38.122	5.000	34.660	1.281	12.695
2	226.466	103.781	122.685	5	34.660	5.189	42.523	2.595	45.118	2.500	36.775	0.649	15.024
3	242.139	104.430	137.709	2.500	36.775	2.611	50.642	1.306	51.948	1.251	37.723	0.327	17.299
4	259.765	104.757	155.008	1.251	37.723	1.311	58.474	.656	59.130	0.626	38.146	0.164	19.690
5	279.619	104.921	174.698	.626	38.146								

C. - Transfer All A → B: IB(t) = SB(t) + SA(t)

0	200	100	100	20	20	20	20	0	40	0	40	0	13.320
1	213.320	100	113.320	0	40	0	45.328	0	45.328	0	40	0	15.094
2	228.414	100	128.414	0	40	0	51.366	0	51.366	0	40	0	17.105
3	245.519	100	145.519	0	40	0	58.208	0	58.208	0	40	0	19.383
4	264.902	100	164.902	0	40	0	65.961	0	65.961	0	40	0	21.965
5	286.867	100	186.267	0	40								

TABLE 6 Interregional Growth Model 3

AUTONOMOUS GROWTH OF SAVINGS RATIO

TOGETHER WITH INCENTIVE EFFECTS ON SAVINGS

SR(t) = Min (SR(t - 1) + 2, IR(t - 1) + 2, 30) in Both A and B

Constant Cap/Output Ratios, A: 4, B: 3

	Total	Income		Savings Ratio		Savings		Investment		Investment Ratio		Income Change	
		A	B	A	B	A	B	A	B	A	B	A	B
	ΣY	YA	YB	SRA	SRB	SA	SB	IA	IB	IRA	IRB	ΔYA	ΔYB
A-No Transfers: IB(t) = SB(t)													
0	200.000	100	100	20	20	20	20	20	20	20	20	5	6.667
1	211.667	105.000	106.667	22	22	23.100	23.467	23.100	23.467	22	22	5.775	7.815
2	225.257	110.775	114.482	24	24	26.586	27.476	26.586	27.476	24	24	6.647	9.150
3	241.054	117.422	123.632	26	26	30.530	32.144	30.530	32.144	26	26	7.633	10.704
4	259.391	125.055	134.336	28	28	35.015	37.614	35.015	37.614	28	28	8.754	12.525
5	280.670	133.809	146.861	30	30								

B - Transfer $\frac{1}{2}$A → B: $IB(t) = SB(t) + \frac{1}{2}SA(t)$

0	200	100	100	20	20	20	20	10	30	10	30	2.500	9.990
1	212.490	102.500	109.990	12	22	12.300	24.198	6.150	30.348	6.000	27.592	1.538	10.106
2	224.134	104.038	120.096	8	24	8.323	28.823	4.162	32.985	4.000	27.466	1.041	10.984
3	236.159	105.079	131.080	6	26	6.305	34.081	3.153	37.234	3.000	28.406	.788	12.399
4	249.346	105.867	143.479	5	28	5.293	40.174	2.647	42.821	2.500	29.845	.662	14.259
5	264.267	106.529	157.738	2.5	30								

C - Transfer All A → B: $IB(t) = SB(t) + SA(t)$

0	200.000	100	100	20	20	20	20	0	40	0	40	0	13.320
1	213.320	100	113.320	2	22	2	24.930	0	26.930	0	23.765	0	8.968
2	222.288	100	122.288	2	24	2	29.349	0	31.349	0	25.635	0	10.439
3	232.727	100	132.727	2	26	2	34.509	0	36.509	0	27.507	0	12.157
4	244.884	100	144.884	2	28	2	40.568	0	42.568	0	29.381	0	14.175
5	259.059	100	159.059	2	30								

There is the factor of *risk* that is not absent even in the most highly coordinated interregional planning system: Development investments within the region itself cannot be readily removed, but the claims to the proceeds of extraregional investments might have to be surrendered in the face of some systemwide or extraregional emergency. There is also the question of *indirect benefits due to local production activities* that are not quantified within the models so far discussed. Even a high level of consumption does not necessarily create a condition of economic development, since the latter is a cultural phenomenon that depends on the simultaneous presence of many factors, among them a highly organized and well-functioning production apparatus that is capable of generating its own supply of human skills and technological advances.

In sum under almost any kind of institutional assumption it is realistic to postulate a strong bias toward internal as against external investments of regional savings.

The other extreme assumption was that of constant regional savings ratios in the face of supraregional planning decisions that channel these savings to other regions. In practice a greater or lesser loss of motivation toward savings in a region is likely to result whenever these savings are removed from the region. Bank deposits by individuals are probably the least sensitive to this factor, but in the less developed regions these are likely to be small. Surplus captured in a systematic way by the central government in the form of income or turnover taxes, etc., may not be strongly affected in the short run by the regional allocation of this surplus, but the required impositions will meet greater political and administrative (tax evasion) resistance if they are popularly resented as a form of draining off resources from the region in question. The reinvestment of the profits of capitalist enterprises under central planning direction may be bitterly resisted if the enterprises are small and backward. If they are large and rational and if a good basis for cooperation exists between these enterprises and the central planning authorities—a quite realistic assumption for certain European countries or Japan—a favorable basis might exist for escaping from the consequences of the extraregional diversion of savings. To the extent, nevertheless, that regional planning organizations exist that have an influence upon planned savings decisions within the region and that are subject to the considerations mentioned in the previous paragraph, they will again tend to create a negative force in regard to savings if the latter are channeled outside the region. In sum it is reasonable to postulate that whenever savings are channeled outside a region, a negative force will be exerted on these savings. The assumption that regional savings will be limited to the amount of regional investment in the previous period is perhaps an excessively strong representation of this negative force, but it is considered as an acceptable first approximation for the purposes of defining a simple illustrative model.

While the regional savings-investment relationship postulated above may have large components that represent private rather than planned savings, *the role of planning in this postulated relationship is crucial,* since it is indispensable for assuring that there will always exist regional investment opportunities corresponding to regional savings. The relationship is thus assumed to operate in a general atmosphere of capital shortage characterizing accelerated economic development rather than in an atmosphere of effective demand limited by profitable investment opportunities. Thus while under limited effective demand *ex ante* savings schedules are often not filled because of the scarcity of investment opportunities, under the postulated relationship *ex ante* savings schedules for a region as a whole can always be automatically adjusted upward or downward to regional investment allocations. Such an adjustment presupposes a considerable degree of planning.

One might suppose that the incentive effects with regard to regional savings that have been discussed above would lead to a more uniform distribution of interregional investment allocations, since the benefits of channeling savings from regions with lower to those of higher productivities would be counteracted by a drop in the savings rate in the regions whose savings are removed. A comparison of Table 4 with Table 5 embodying the indicated incentive effects shows, however, that just the opposite is the case: The transfer of savings from A to B results in faster over-all system growth in the presence of incentive effects than in their absence. A little reflection will reveal why this should be so. Since incentive effects are symmetrical between regions the loss to the system due to the lowering of A's savings rate is more than offset by the gain due to the *raising* of B's savings rate, which operates on a more favorable capital-output ratio.

Incentive effects can lead to an equalization of regional development only when they are asymmetrical between the regions that participate in capital transfers. The question now arises: Are incentive effects likely to be symmetrical between regions and, if not, under what conditions can asymmetrical incentive effects be expected to occur?

An asymmetrical incentive relationship for savings that may be highly realistic under the conditions characterizing economic development in many parts of the world is obtained when the former savings-investment relationship is supplemented by an autonomous growth trend of the savings rate up to a fixed practical limit. This autonomous growth trend is postulated as the *outcome of conscious planning efforts* in the face of the usual difficulties and resistances encountered in underdeveloped countries that make it practically impossible to introduce abrupt upward changes in the savings rate without excessive political cost. The specific form of the relationship adopted for illustrative purposes is a period-by-period autonomous rise of 2 percentage points up to a limiting rate of 30 per cent.[18] This autonomous trend holds only while no savings are drained off to other regions. If the latter loss of savings occurs, however, the previous disincentive relationship takes over: The savings rate (SR) drops to the investment rate (IR) and thereafter continues its gradual rise from this low point. Such a relationship can be expressed by the equation

$$SR\ (1) = \mathrm{Min}\ [SR(t-1) + 2, IR(t-1) + 2, 30],$$

where the savings rate of a region is expressed as the smallest of three quantities: the autonomous growth trend starting from the savings rate of the previous period, the autonomous growth trend starting from the investment rate of the previous period, and the limiting savings rate. Table 6 shows the development of the two regions as well as of the system as a whole under these conditions, with the usual assumptions of transferring none, half, or all of the savings of A to B.

The case where no savings are transferred from the less productive to the more productive region is now clearly superior to either of the two alternative transfer patterns. Thus whenever it is realistic to postulate the presence of strong asymmetric incentive effects of this kind, the usual conclusions with regard to the advantage of concentrating investment in the more productive regions have to be replaced by a strategy of permitting each region to develop on the basis of its own self-generated capital resources.

Whether the illustrative case of Table 6 can be taken as representative of the relationship of advanced to backward regions in general is open to some question, since less developed areas at times exhibit favorable capital-output ratios.[19] The illustrative case also shows an unrealistically large sudden increase in the investment ratio in region B that is surely inconsistent with the absorptive capacity for investment that might reasonably be expected to exist in almost any region. The purpose of

[18] Both of these magnitudes are chosen for illustrative purposes only without a claim to the representation of realistic orders of magnitude for particular instances.

[19] In Tables 4–7, the *backward* region has been assumed to have the *less favorable* marginal capital-output ratio in order to explore the possibilities of growth (and perhaps eventual income) equalization under the most difficult conditions. To the extent that the reverse is the case, the task is considerably easier.

the illustration is to demonstrate the kind of asymmetry that is required in order to reverse the usual conclusions with regard to the concentration of investment in productive regions, rather than to furnish a realistic representation of all the limiting factors involved in decisions concerning the transfer of savings from one region to another.[20]

The autonomous growth mechanism of the savings rate that has been postulated as one feature of the asymmetrical savings-investment relationship is not only a powerful disincentive to interregional transfers but also a most effective motor of economic development in the backward regions once certain constraints that hold back the development of these regions are eliminated. If the backward region is, for example, held back by its inability to convert savings into investment due to the inadequate growth of its traditional exports on the one hand and scale limitations on the domestic production of investment goods on the other, then the advanced region can easily help to break this constraint and set into motion a self-accelerating process of growth in the backward region by simply *planning* to draw upon the backward region as a source of supply of certain commodities in addition to traditional exports, thereby enabling the backward region to buy more investment goods. This requires no capital transfer of any kind whatsoever from the advanced to the backward region, but of course it presupposes an institutional environment of active planning efforts in the backward region as well as both the ability and intention on the part of the advanced region to include the development problems of the backward region within the compass of its own planning processes. Table 7 furnishes a simple numerical illustration of this kind of assistance to the backward region that will be recognized as an example of the "trade-not-aid" approach to international development problems. Under the latter arrangement the growth rate of the backward region, previously limited to approximately 3 per cent per annum, rapidly rises toward an eventual 7.5 per cent per annum.[21]

[20] For the purpose of this illustration the autonomous advance of the savings rate was taken to be determined solely by the difficulties of appropriating the surplus of the backward region rather than by limitations on the absorptive capacity for investment that generally furnishes a simultaneous constraint. The latter can be as binding or at times more binding than the constraint on the generation of appropriable surplus; in the latter case, accordingly, the advance of the savings rate in a region will be governed by the slow expansion of absorptive capacity for capital investment rather than by resistances to the expansion of reinvestible surplus.

[21] The detailed behavior of this model raises interesting points in regard to the financing of the expansion of nontraditional exports and in regard to the evaluation of development projects related to this export expansion in region A which cannot be discussed further here.

TABLE 7 Interregional Growth Model 4

AUTONOMOUS GROWTH OF SAVINGS RATIO

SR(t) = MIN(SR(t-1) + 2, IR(t-1) + 2, 30)
Constant Cap/Output Ratios, A:4, B:3
Situation 1 - Income Elasticity
of Imports (B) = 0.7

	TOTAL	Income A	Income B	Imports B	Savings Ratio A	Savings Ratio B	Savings A	Savings B	Investment A	Investment B	Income Change A	Income Change B	Income Growth A	Income Growth B	Import Growth B
	ΣY	YA	YB	MB	SRA	SRB	SA	SB	IA	IB	ΔYA	ΔYB	$\frac{\Delta YA}{YA}$	$\frac{\Delta YB}{YB}$	$\frac{\Delta MB}{MB}$
SITUATION 1 - LIMITED TRADITIONAL EXPORTS															
0	200	100	100	10	10	30	10	30	10	30	2.5	10	2.5	10	7
1	212.500	102.500	110.000	10.700	10.439	30	10.700	33.000	10.700	33.000	2.675	11.000	2.610	10	7
2	226.175	105.175	121.000	11.449	10.886	30	11.449	36.300	11.449	36.300	2.862	12.010	2.721	10	7
3	241.137	108.037	133.100	12.250	11.339	30	12.250	39.930	12.250	39.930	3.063	13.310	2.835	10	7
4	257.510	111.100	146.410	13.108	11.798	30	13.108	43.923	13.108	43.923	3.277	14.641	2.950	10	7
5	275.428	114.377	161.051	14.026	12.263	30	14.026								
SITUATION 2 - "TRADE-NOT-AID"															
0	200	100	100	10	10	30	10	30	10	30	2.5	10	2.5	10	23.000
1	212.500	102.500	110.000	12.300	12	30	12.300	33.00	12.300	33.000	3.075	11.000	3.0	10	20.171
2	226.575	105.575	121.000	14.781	14	30	14.781	36.300	14.781	36.300	3.695	12.010	3.5	10	18.280
3	242.370	109.270	133.100	17.483	16	30	17.483	39.930	17.483	39.930	4.371	13.310	4.0	10	16.999
4	260.051	113.641	146.410	20.455	18	30	20.455	43.923	20.455	43.923	5.114	14.641	4.5	10	16.113
5	279.806	118.755	161.051	23.751	20	30	23.751						⋮		
													7.5		

NOTE. Assumptions:
a. No capital transfers; exports = imports
b. Region A produces no capital goods
c. Region A uses all of its foreign exchange to import capital goods
d. Situation 1: limited traditional exports from A to B.
Situation 2: planned imports in B to limit of savings growth in A.

CONCLUSION

The autonomous growth tendency of the savings rate postulated in the foregoing illustrations is not the only possible phenomenon of its kind. Similar autonomous growth tendencies can reasonably be postulated for such factors as the evolution of skills, the development of an indigenous technology capable of generating its own advance, and in general the evolution of an integrated, self-contained cultural pattern that we associate with the notion of a high level of economic development. Likewise, in addition to the incentive effects of the transfer or regional utilization of savings there are many other kinds of possible behavioral relations of a similar nature including the one that is probably the most important, namely, the effect of the level, rate of advance, and acceleration of economic development itself upon the supplies of all factors, the supplies of savings, and the productivity coefficients characterizing the technology of a region.

The simple numerical illustrations that have been given at a highly aggregated level can do no more than open up an area that requires a great deal of further study; they suggest nevertheless that a view of economic development as an essentially autonomous growth process that is frustrated by a series of specific constraints can be made operational by the formulation of precise mathematical models for the investigation of these phenomena. This approach holds out the promise of extending the use of mathematical programing development models from the present formulations that are centered almost exclusively on technological interrelations to formulations that give a considerably wider scope to relationships involving human motivations and behavior.

APPENDIX 1

The Correspondence Between Price-type and Quantity-type Control Instruments in the Reconciliation of Regional Development Objectives

The correspondence between (a) the weighted sum of two objectives and (b) a single objective function with one additional constraint representing a secondary objective, can be demonstrated graphically in two dimensions.

By reference to Figure 1, assume that the space of feasible solutions is OAP_1P_2B, determined by technical constraints. Assume further that the maximization of X_1 and of X_2 are separate objectives. Then any positive weighting of these two objectives will carry the system to an optimal solution along the boundary AP_1P_2B.

If the weighting is such that its slope corresponds to the slope of the line FF, the optimal solution is P_2. It is asserted that FF may be replaced:

a. either by the maximization of X_1 subject to a proper constraint replacing X_2 in the weighted objective function;

b. or by the maximization of X_2 subject to a proper constraint replacing X_1 in the weighted objective function.

In the first case, the proper constraint is $X_2 \geq D$. With this constraint, the feasible region shrinks to P_2DB; maximizing X_1 over this region yields P_2, as required. In the second case, the proper constraint is $X_1 \geq C$. With this constraint, the feasible zone becomes $AP_1 P_2 C$; maximizing X_2 over this zone yields again P_2, as required.

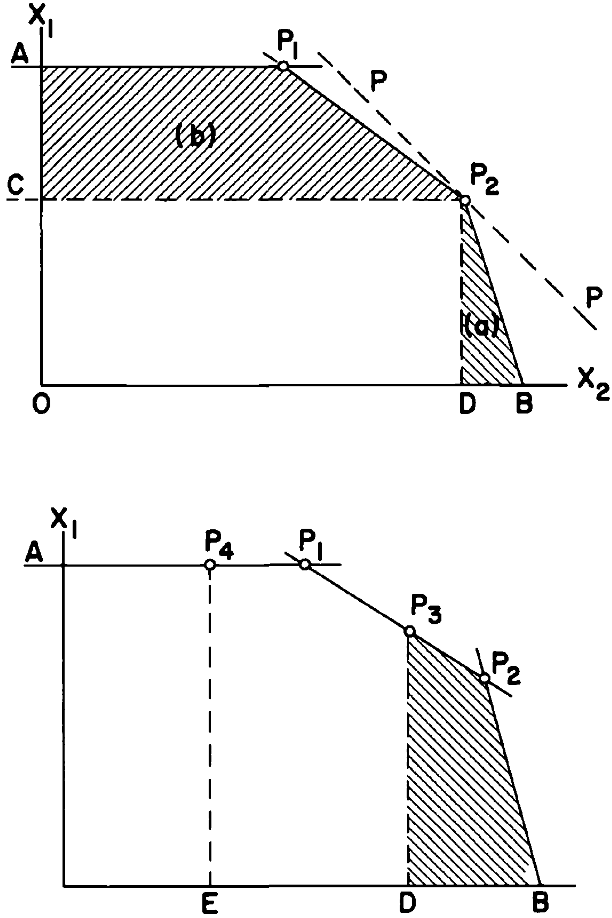

Figure 1 and 2

If the weighting is such that the slope of FF coincides with the slope of P_1P_2, all points of the line segment P_1P_2 will be optimal. By a proper choice of C or D in the inequalities above, any one point of the segment P_1P_2 may be made optimal in the corresponding transformed problem. Thus in Figure 2, P_3 is made optimal by maximizing X_1 over the region DP_3P_2B. Note, however, that in this case of multiple optima C or D cannot be chosen so as to duplicate the entire multiple optimum P_1P_2 in a single equivalent problem, but rather, to each one of the multiple optima there corresponds a transformed problem.

The above demonstrates one side of the limited correspondence between price-type and quantity-type control instruments in the reconciliation of independent objectives, namely the replacement of a weighted optimum by a single objective plus a constraint. Conversely, it can be demonstrated that the optimal solution attainable by a single objective plus a constraint can be replaced by a combined objective with properly chosen weights. By reference again to Figure 2, replace the maximization of X_1 subject to the technical constraints plus the constraint $X_2 \geq D$, by a weighted maximum in X_1 and X_2, after omitting the last constraint. This can evidently be done if the weights are chosen so that the slope of the combined objective function coincides with the slope of the line segment P_1P_2. To P_3, the optimal solution of the original problem, there corresponds now the multiple optimum over the segment P_1P_2 in the transformed problem, which contains the point P_3, as required. It should be noted, however, that this is not a perfect equivalence because the transformed problem has many solutions that are not solutions to the original problem, and some of these are not even feasible in the original problem, e.g., the points of the segment P_1P_3 excluding P_3 itself.

If the original problem is chosen with the constraint $X_2 \geq E$ (See Figure 2) it will have multiple optima along the line segment P_4P_1. The corresponding transformed problem will now also have mutliple optima along this line segment, and also along the line segment AP_4 which is infeasible in the first problem, except for P_4 itself.

In sum when either of the corresponding problems has mutliple optima, there is no perfect equivalence, but in every case, <u>the corresponding problems have at least one optimal solution in common</u>.

The above demonstration can be generalized in the following ways:

1. The positive weighting of the separate objectives is not essential to the correspondence. With negative weightings allowed, the gradient of the combined objective function may point in any direction rather than only into the positive quadrant, as above. The only change required now is to reverse the inequality in the constraint replacing a negatively weighted individual objective.
2. The individual objectives need not be X_1 and X_2 alone, but may be any two linear functions in these variables, with the combined objective being a weighted sum of the two functions. Such a weighted sum is always equivalent to another weighted sum in terms of the two variables X_1 and X_2 alone; thus the correspondence established for the latter will hold for the former.

3. The correspondence also holds for convex nonlinear problems, i. e., convex constraints with concave objective functions, where the latter are equivalent to convex sets of "acceptable" points. The latter are defined as points which are equal or preferred to an auxiliary variable signifying a constant value of the objective function. With the aid of this auxiliary variable any problem having a nonlinear concave objective function can be transformed into an equivalent problem with a linear objective function (which is identical to the auxiliary variable mentioned above); this linear objective function is maximized over a convex point set. When there are two nonlinear objectives, each of these can be subjected independently to the above transformation, and the problem now becomes one of demonstrating the correspondence of the weighted objectives and a single objective with an additional linear constraint, over a convex nonlinear point set. By reference to Figures 1 and 2, the demonstration does not rely on the linearity of the boundary of the feasible point set, only on its convexity; thus the generalization is immediate. Moreover, when the boundary has no linear segments the possibility of multiple optima does not arise and there exists a perfect equivalence rather than a limited correspondence as in the linear case where the possibility of multiple optima has to be taken into account.

4. In more than two dimensions multiple optima may occur not only along line segments but also along planes or hyperplanes. For a generalization to these cases the above argument can be framed in purely algebraic terms which yields an extension to _n_ dimensions.

A straightforward application of the above principles to the reconciliation of regional development objectives might consist in establishing a correspondence between (1) the maximization of a weighted objective, e. g., the sum of regional products for two regions, and (2) the maximization of a single objective, e. g., the product of region _A_, subject to the attainment of a prescribed minimum level of the product of region _B_. It often happens, however, that it is desired to establish another kind of correspondence that is slightly different from the one discussed above, namely a correspondence between (1) a weighted maximization where the weights are _unequal_ and (2) a weighted maximization with _equal weights_, subject to the attainment of a minimum level of the objective of one or the other region. For example, it may be desired to replace (1) the maximization of national wealth (which is an equally-weighted sum of the wealth of several regions), subject to regional distribution constraints, by (2) the maximization of an unequally weighted sum of regional wealth in the several regions. (The problem of assigning terminal weights to capital stock in different regions is encountered in multi-period regional allocation models.)

By reference to Figure 3, P_1 is the optimal solution to the unequally-weighted maximum problem, and P_2 is the optimal solution to the qually-weighted maximum problem. If it is desired

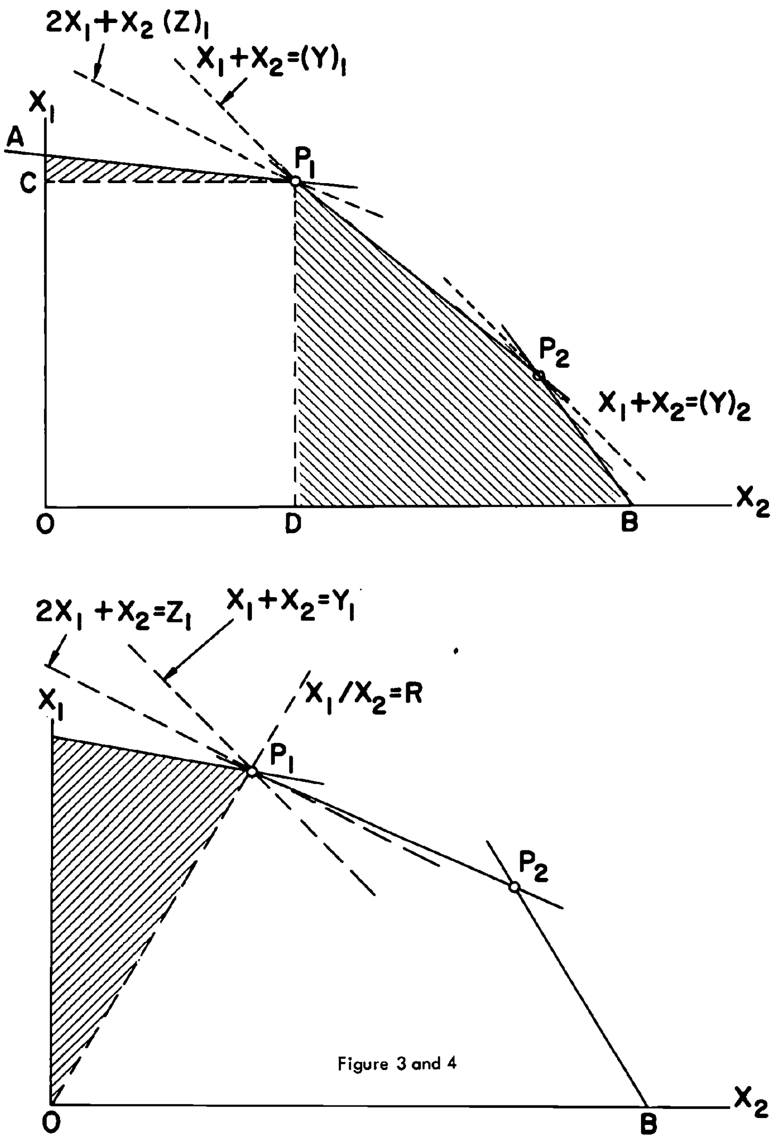

Figure 3 and 4

to reformulate the first problem so as to replace its objective by an equally-weighted objective plus a proper constraint, it is found that the symmetry between the constraints $X_1 \geq C$ and $X_2 \geq D$ no longer exists. Only the first of these constraints will assure that $X_1 + X_2$ attains its maximum at P_1; the other constraint will not prevent the system from going to P_2, the maximum of the equally-weighted objective. In other words, the constraint has to be imposed on the income of the lagging region, X_1, as is intuitively obvious, otherwise the system will go to the maximum P_2 implying an unacceptably low value of X_1.

In Figure 4, another reformulation is shown in which the income of the lagging region, X_1, is constrained to be a minimum proportion of the income of the leading region, X_2. Here again, we encounter an asymmetry in that the constraint must operate in the proper direction, otherwise the system will go to the maximum P_2. Finally, the two incomes may be constrained to the line representing the exact ratio R, in which case the feasible region collapses to points along the segment OP_1, with the optimum again at P_1.

By reference to Figures 3 and 4, the unconstrained, equally-weighted maximum P_2 corresponds to a higher level of (X_1+X_2) than the maximum of the same objective function subject to regional distribution constraints, P_1; in other words, $(Y)_2 > (Y)_1$.

APPENDIX 2

Notation, Formal Interpretation and Transformation Rules for the Model Presented in Table 1

A. Notation

1. Variables

All variables are given two-letter names, to be interpreted as single symbols, not as algebraic products. In Table 1, all variables are vectors, except as indicated below:

Primal principal variables

$XX_j^t \equiv \left[(XX_j^t)_1, \ldots, (XX_j^t)_n \right]$, level of production activity j at time t in each region 1, . . . , n

$XZ_i^t \equiv \left[(XZ_i^t)_{11}, \ldots, (XZ_i^t)_{1n}, (XZ_i^t)_{2i}, \ldots, (XZ_i^t)_{2n}, \ldots, (XZ_i^t)_{n1}, \ldots, (XZ_i^t)_{nn} \right]$, level of transport activity for commodity i at time t from each region 1, . . . , n to each region 1, . . . , n

$XH_i^t \equiv \left[(XH_i^t)_1, \ldots, (XH_i^t)_n \right]$, stock-holding activity for commodity i at time t in each region 1, . . . , n

XO, a scalar, level of exogenous supply/demand activity, pre-set to unity

Primal slack variables

$SB_i^t \equiv \left[(SB_i^t)_1, \ldots, (SB_i^t)_n \right]$, surplus of stock of commodity i at time t in each region 1, . . . , n

$SF_i^t \equiv \left[(SF_k^t)_1, \ldots, (SF_k^t)_n \right]$, surplus of flow of primary factor k at time t in each region 1, . . . , n

$SA_i^t \equiv \left[(SA_i^t)_1, \ldots, (SA_i^t)_n \right]$, surplus of flow of commodity i at at time t in each region 1, . . . , n

SO, a scalar, weighted sum (value) of terminal stocks, to be maximized. May be interpreted as a surplus, since no prescribed demands exist.

Dual principal variables

$YR_i^t \equiv \left[(YR_i^t)_1, \ldots, (YR_i^t)_n \right]$, shadow rent per period on stock of commodity i at time t in each region 1, ..., n

$YW_k^t \equiv \left[(YW_k^t)_1, \ldots, (YW_k^t)_n \right]$, shadow price of primary factor k at time i in each region 1, ..., n

$YP_i^t \equiv \left[(YP_i^t)_1, \ldots, (YP_i^t)_n \right]$, shadow price of commodity i at time t in each region 1, ..., n

YO, a scalar, valuation of terminal stocks; pre-set to unity, it can be interpreted as the numeraire of the shadow price system

Dual slack variables

$DX_j^t \equiv \left[(DX_j^t)_1, \ldots, (DX_j^t)_n \right]$, shadow loss (negative profit) on production activity j at time t in each region 1, ..., n

$DZ_i^t \equiv \left[(DZ_i^t)_{11}, \ldots, (DZ_i^t)_{1n}, (DZ_i^t)_{21}, \ldots, (DZ_i^t)_{2n}, \ldots, (DZ_i^t)_{n'}, \ldots, (DZ_i^t)_{nn} \right]$, shadow loss on transports activity for commodity i at time t from each region 1, ..., n to each region 1, ..., n.

$DH_i^t \equiv \left[(DH_i^t)_1, \ldots, (DH_i^t)_n \right]$, shadow loss on stock-holding activity for commodity i at time t in each region 1, ..., n.

DO, a scalar, shadow loss on exogenous supply/demand activity (see formal interpretation). Dual objective is the minimization of (-DO).

2. Parameters

The parameters enumerated are vectors or matrices.

(a) Parameters which represent initial or terminal values of certain variables (vectors)

$H_i^0 \equiv \left[(H_i^0)_1, \ldots, (H_i^0)_n \right]$, zero-period value of XH_i^t, initial stock endowment of commodity i in each region 1, . . . , n.

$P_i^4 \equiv \left[(P_i^4)_1, \ldots, (P_i^4)_n \right]$, fourth-period value of YP_i^t, post-horizon shadow price of commodity i in each region 1, . . . , n.

$R_i^4 \equiv \left[(R_i^4)_1, \ldots, (R_i^4)_n \right]$, fourth-period value of YR_i^t, post-horizon one-period shadow rent for stock i in each region 1, . . . , n.

(b) exogenous supply and demand parameters (vectors)

$Q_k^t \equiv \left[(Q_k^t)_1, \ldots, (Q_k^t)_n \right]$, exogenous supply of primary factor k at time t in each region 1, . . . , n

$C_i^t \equiv \left[(C_i^t)_1, \ldots, (C_i^t)_n \right]$, exogenous demand of commodity i at time t in each region 1, . . . , n

(c) technical coefficient matrices

$I \equiv \begin{Bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{Bmatrix}$, identity matrix of order n

$B_{ij}^t \equiv \begin{Bmatrix} (B_{ij}^t)_1 & & \\ & \ddots & \\ & & (B_{ij}^t)_n \end{Bmatrix}$, diagonal matrix of stock input coefficients for stock of commodity i used in activity j at time t in each region 1, . . . , n.

$F_{kj}^t \equiv \begin{Bmatrix} (F_{kj}^t)_1 & & \\ & \ddots & \\ & & (F_{kj}^t)_n \end{Bmatrix}$, diagonal matrix of primary factor input coefficients for factor k used in activity j at time t in each region 1, . . . , n.

$$A_{ij}^t \equiv \begin{Bmatrix} (A_{ij}^t)_1 & & \\ & \ddots & \\ & & (A_{ij}^t)_n \end{Bmatrix},$$ diagonal matrix of commodity output (if positive) or intermediate input (if negative) flow coefficients for commodity i produced or used in activity j at time t in each region 1, . . . , n

(d) <u>transfer matrices</u>

T_i^t, the transfer matrix for commodity i at time t, from each region 1, . . . , n to each region 1, . . . , n, has the form shown below:

<u>index of transport activity:</u>

$$\begin{array}{ll} & \begin{array}{cccccccccccc} \underline{11} & \underline{12} & \dots & \underline{1n} & \underline{21} & \underline{22} & \dots & \underline{2n} & \dots & \underline{n1} & \underline{n2} & \dots & \underline{nn} \end{array} \\ \begin{array}{r} \text{i-th} \quad 1) \\ \text{commodity} \quad 2) \\ \text{balance in} \quad \vdots \\ \text{this region} \quad n) \end{array} & \left\{ \begin{array}{cccc|cccc|c|cccc} 0 & -1 & & -1 & 1 & & & & & 1 & & & \\ & 1 & & & -1 & 0 & \dots & -1 & & & 1 & & \\ & & \ddots & & & & \ddots & & & & & \ddots & \\ & & & 1 & & & & 1 & \dots & -1 & -1 & \dots & 0 \end{array} \right\} \end{array}$$

In this matrix a unit of commodity i imported to a region appears as (+1), a unit exported appears as (-1).

(e) <u>transport cost matrices</u>

$$L_{ie}^t \equiv \begin{Bmatrix} (L_{ie}^t)^1_{11}, \dots, (L_{ie}^t)^1_{1n}, \dots, (L_{ie}^t)^4_{2n}, \dots, (L_{ie}^t)^1_{n'}, \dots, (L_{ie}^t)^1_{nn} \\ \vdots \\ (L_{ie}^t)^n_{11}, \dots, (L_{ie}^t)^n_{1n}, (L_{ie}^t)^n_{21}, \dots, (L_{ie}^t)^n_{2n}, \dots, (L_{ie}^t)^n_{n'}, \dots, (L_{ie}^t)^n_{nn} \end{Bmatrix}$$

Matrix of <u>stock requirement</u> coefficients at time t for the stock of commodity i used in the transport of commodity e, the stock belonging to each region 1, . . . , n (vertical dimension of matrix) and the transport activities connecting each region 1, . . . , n with every other region 1, . . . , n (horizontal dimension of matrix).

M^t_{im}	Same indexing as above, M, m substituted for L, e coefficients	primary factor requirement
N^t_{in}	Same indexing as above, N, n substituted for L, I coefficients	commodity flow requirement

B. Formal Interpretation

The linear programming problem in Table 1 is set out by means of Tucker's condensed format as explained below (Tucker in Graves and Wolfe, 1963). Primal variables appear in the top margin, primal slacks at the left, dual variables on the right, dual slacks at the bottom. The equations of the primal problem are derived by setting equal the primal slacks to the matrix products of each row in the table by the variables in the upper margin. The equations of the dual problem are correspondingly derived by setting equal the dual slacks(with their signs reversed) to the matrix products of each column in the table by the dual variables in the right-hand margin. All variables are restricted to non-negative values, with the exceptions noted below. Any row may be chosen as the primal maximand: the corresponding primal slack may be of any sign, and the corresponding dual variable must be set to the fixed value of unity. Likewise, any column may be chosen as the dual minimand: the corresponding dual slack may be of any sign, and the corresponding primal variable must be set to the fixed value of unity.

All outputs in the table are positive, all inputs negative. The rows are interpreted as resource balances, and the primal slacks as resource surpluses. One of these is maximized as activity scales vary, subject to all others being non-negative. This resource, with unit valuation, is the numeraire resource of the system. The columns are interpreted as economic activities, and the dual slacks as losses (with their signs reversed: as profits). One of these profits is minimized as resource valuations vary, while the others are restricted to being nonpositive. The corresponding column is the (unit) fixed-scale activity of the system, e. g., in the present problem the vector of the exogenous parts of the primal resource balances. Note that the above profit minimization is undertaken as prices, not quantities, vary. This implies assigning the smallest possible weighted valuation to the scarce exogenous input resources and the largest possible weighted valuation to the exogenous demands of the system consistent with the no-profit stipulation for the other activities. (The latter stipulation can be thought of as an analogue to perfect competition.) In other words, the prices are to be such that they reduce the scarcity of the scarce resources and raise the reward value of the prescribed consumption as much as possible.

Both activities and resource balances are distinguished by time periods. There are four kinds of activities: production, transport, stock-holding, and an activity specifying exogenous supplies and demands; and three kinds of resources: commodity flows, commodity stock levels, and primary factor flows. Primary factors cannot be produced or transported. For convenience in presentation the model has only two commodities and three productive activities at each location and in each period. All activities and resource balances are distinguished by _n_ locations; locational distinctions are, however, left implicit in the notation of Table 1 by grouping the corresponding marginal entries of the table into vectors and the inner entries into submatrices. For example the A_{ij}^{t} elements denote diagonal submatrices of coefficients for n locations; these are production coefficients of flow inputs (if negative) or outputs (if positive) at time t of the ith resource in the jth activity. The B_{ij}^{t} and F_{ij}^{t} elements likewise denote diagonal submatrices of stock and factor requirement coefficients. The T_{i}^{t} elements denote a complete set of transport connections pairwise between the n locations; as seen in Table 1, there is one such T_{i}^{t} element for each flow-type resource. It can also be seen in Table 1 that the transport activities have coefficients L_{ie}^{t}, M_{im}^{t}, N_{in}^{t} which denote stock factor and flow requirements for performing transport activities: these are thus generalizations of transport costs in terms of explicit resource utilizations.

C. Transformation Rules

In this format all variables in the top and right-hand margins (except the ones fixed as unit level) are regarded as non-basic, i.e., to be set to zero; consequently the first column gives the numerical values of all basic primal variables while the last row gives the numerical values of all basic dual variables (with signs reversed).

In a transformation of the tableau (a Gaussian elimination process) the variables corresponding to one column and one row are interchanged, and all numerical entries of the tableau are recomputed by the following rules:

Definitions. Pivot p: element of the tableau at the intersection of the interchanged row i' and column j': $p \equiv a_{i'j'}$

Pivot - row element	$r \equiv a_{i'j}$, $j \neq j'$
Pivot - column element	$c \equiv a_{i,j'}$, $i \neq i'$
General element	$e \equiv a_{i,j}$, $i \neq i'$, $j \neq j'$.

Transformation Rules

Element	Short notation	Full notation
pivot	$p \rightarrow 1/p$	$a_{i',j'} \rightarrow 1/a_{i',j'}$
pivot-row element	$r \rightarrow -r/p$	$a_{i',j} \rightarrow -a_{i',j}/a_{i',j'}$
pivot-column element	$c \rightarrow c/p$	$a_{i,j'} \rightarrow a_{i,j'}/a_{i',j'}$
general element	$e \rightarrow e - r.c/p$	$a_{i,j} \rightarrow a_{i,j} - a_{i',j} \cdot a_{i,j'}/a_{i',j'}$

Note:

A tableau in Tucker's general format can be rewritten with the negative signs shifted from the bottom margin to some other margin; in the latter case the sign-reversal rule may be interchanged between the pivot-row and pivot-column element transformations.

APPENDIX 3

Accounting Concepts and Current Prices

In this Appendix we shall discuss the definition of accounting concepts based on the model of Table 1 including national and regional product and income; the evolution of stock value (wealth) from period to period; and the relationship between shadow prices, current prices and the rate of interest.

The accounting concepts to be dealt with are based on the "accounting values" of coefficients in the model. To derive the accounting value associated with a coefficient the latter is multiplied both by the corresponding activity scale and by the corresponding shadow price in the optimal solution. Accounting values add to zero both by rows and by columns as a consequence of the law of complementary slacks, well-known in linear programming.

1. Notation

In designating the optimal-solution values of primal and dual variables the "X" or "Y" part of the double symbol is dropped. In addition an expression such as

$$P^1 . A^1 . X^1$$

where the subscripts of the variables are dropped designates the <u>sum of all accounting values</u> involving the A^1 coefficient group, i.e.,

$$P^1_1 . A^1_{11} . X^1_1 + P^1_1 . A^1_{12} . X^1_2 + P^1_1 . A^1_{13} . X^1_3 + P^1_2 . A^1_{21} . X^1_1 +$$
$$+ P^1_2 . A^1_{22} . X^1_2 + P^1_2 . A^1_{23} . X^1_3 \quad .$$

2. National Income and Product

(a) Sum to zero the accounting values of rows having the index of a given time period <u>t</u>:

$$
\begin{array}{llllll}
 & R^t.I.H^{t-1} & -R^t.B^t.X^t & -\ R^t.L^t.Z^t & & =0 \\
W^t.Q^t.1 & & -W^t.F^t.X^t & -\ W^t.M^t.Z^t & & =0 \\
-P^t.C^t.1 & +P^t.I.H^{t-1} & \underbrace{+P^t.A^t.X^t}_{=0} & +\ \underbrace{P^t.(T^t-N^t).Z^t}_{=0} & -\ P^t.I.H^t & =0
\end{array}
$$

(b) Sum the above expression by columns. Since the third and fourth columns above contain all the non zero entries for a complete column in Table 1, they have to sum to zero, as indicated. The other summations give:

$$-P^t.I.(H^t - H^{t-1}) - P^t.C^t.1 + R^t.I.H^{t-1} + W^t.Q^t.1 = 0$$

(c) Dropping I and 1 coefficients and rearranging, we get:

$$P^t.(H^t - H^{t-1}) + P^t.C^t = R^t.H^{t-1} + W^t.Q^t$$

$$\underbrace{\begin{array}{cc}\text{Investment} & \text{Consumption} \\ \text{at time } \underline{t} & \text{at time } \underline{t}\end{array}}_{\text{National product}} \quad \underbrace{\begin{array}{cc}\text{Rentals} & \text{Wages} \\ \text{at time } \underline{t} & \text{at time } \underline{t}\end{array}}_{\text{National income}}$$

The above demonstrates the identity between national income and product. (For the definition of the latter in current prices see sec. 6 below.)

3. Regional Income and Product

Repeat the above operations using only the rows referring to a particular region $\underline{h}$. Now column 4 no longer sums to zero since a transport activity always has coefficients involving other regions than $\underline{h}$ (otherwise there can be no transfer of a commodity from region $\underline{h}$ to $\underline{k}$ or $\underline{k}$ to $\underline{h}$) and the latter are omitted in the summation. In addition there can be stock and factor inputs from regions other than $\underline{h}$ in both transport and regional production activities; these also will have the effects of preventing the respective columns from summing to zero.

Method of dealing with foreign-region coefficients: treat the accounting values of all positive ones as exports from region h, of all negative ones as imports to region h. A little algebra will show that the summation of the accounting values of region h in the columns corresponding to production and transport will yield net exports from region h with sign reversed. Upon rearranging the summation as under heading (c) above, net exports will appear together with consumption and investment as part of regional product. Regional income will be analogous to national income and will consist of payments to regional stocks and primary factors.

4. Period-to-Period Evolution of Stock Value

(a) In the expression for the national income/product identity isolate $P^t.H^t$:

$$P^t.H^t = (P^t + R^t).H^{t-1} + W^t.Q^t - P^t.C^t$$

(b) From the accounting values of the columns of stock holding activities, express $P^t.H^t$:

$$P^t.H^t = (P^{t+1} + R^{t+1}).H^t$$

(c) Substitute into expression under (a):

$$(P^{t+1} + R^{t+1}).H^t = (P^t + R^t).H^{t-1} + W^t.Q^t - P^t.C^t.$$

From this expression it is evident that with $Q^t = C^t = 0$ for all t, stock valuation is constant from period to period, and thus also between the starting (zero) period and the terminal period. By progressive substitution we similarly derive for the general case:

$$\begin{aligned}(P^t+R^t).H^{t-1} &= (P^{t-1}+R^{t-1}).H^{t-2}+W^{t-1}.Q^{t-1}-P^{t-1}.C^{t-1}\\ &= (P^{t-2}+R^{t-2}).H^{t-3}+W^{t-1}.Q^{t-1}+W^{t-2}.Q^{t-2}-\\ &\qquad P^{t-1}.C^{t-1}-P^{t-2}.C^{t-2}\\ &\vdots\\ &= (P^1+R^1).H^0+W^{t-1}.Q^{t-1}+\ldots+W^1.Q^1-P^{t-1}.\\ &\qquad C^{t-1}-\ldots-P^1.C^1,\end{aligned}$$

where t can be the terminal period, T.

5. Current Prices and the Rate of Interest

The relationship between shadow prices, current prices and the rate of interest is derived from stock holding activities connecting two successive periods:

$$(P_i^{t-1})_h \leq (P_i^t)_h + (R_i^t)_h$$

where the equality holds when the stock of commodity i in region h is held in non-zero amounts.

In order to create a system of current prices,

(a) The rate of interest r is defined by an ordinary discounting formula that connects current prices and shadow prices:

$$(\pi_i^{\tau})_h = (P_i^{\tau})_h \left(\frac{1}{1+r}\right)^{t-\tau}$$
$$(\rho_i^{\tau})_h = (R_i^{\tau})_h \left(\frac{1}{1+r}\right)^{t-\tau},$$

where π and ρ are current flow prices and stock rents, τ is the index of any time period, and t is the index of the base time period for the current price system. Substitution into the earlier given shadow price relation yields:

$$(\pi_i^{t-1})_h (1+r) \leq (\pi_i^t)_h + (\rho_i^t)_h,$$

and

$$r = \frac{(\pi_i^t)_h - (\pi_i^{t-1})_h}{(\pi_i^{t-1})_h} + \frac{(\rho_i^t)_h}{(\pi_i^{t-1})_h}.$$

(b) A value standard commodity (in a specific region) has to be selected whose current price remains unchanged from $t-1$ to t:

$$(\pi_{i'}^{t-1})_h = (\pi_{i'}^t)_{h'},$$

where the value-standard commodity is commodity i' in region h'.

As an immediate consequence the rate of interest is revealed as the current rent on the value-standard commodity provided that finite non-zero stocks of the latter are being held in the optional solution.

6. National Income, Product and Wealth in Current Prices

If we transcribe the national income/product identity into current prices we get (remembering that in period $\underline{t}$ current and shadow prices are equal):

$$\pi^t.(H^t-H^{t-1})+\pi^t.C^t=\rho^t.H^{t-1}+\omega^t.Q^t,$$ where $\underline{\omega}$ is a current wage.

The defining relationship for the rate of interest from Sec. 5 can be restated in terms of accounting values that sum to zero:

$$-P^{t-1}.H^{t-1}+P^t.H^{t-1}+R^t.H^{t-1} \quad =0$$

$$\therefore \quad -\pi^{t-1}.H^{t-1}.(1+r)+\pi^t.H^{t-1}+\rho^t.H^{t-1}=0$$

Expressing $\rho^t.H^{t-1}$ from this relationship and substituting we obtain:

$$\pi^t.(H^t-H^{t-1})+\pi^t.C^t=((1+r).\pi^{t-1}-\pi^t).H^{t-1}+\omega^t.Q^t$$

$$=\pi^t.H^{t-1}.r-(\pi^t-\pi^{t-1}).H^{t-1}+\omega^t.Q^t,$$

where $\pi^t.H^{t-1}$ is the value of all stocks available for production in period $\underline{t}$, and $\pi^t.H^{t-1}.r$ is the total interest imputed to the former; while $(\pi^t-\pi^{t-1}).H^t$ is the net value increase on all stocks between period $\underline{t-1}$ and $\underline{t}$. Thus in terms of the interest rate, stock rents are replaced by the difference between interest income and stock appreciation.

We can redefine both national income and investment (and thus national product) by adding the stock appreciation to both sides of the identity:

$$\pi^t.(H^t-H^{t-1})+\pi^t.C^t+(\pi^t-\pi^{t-1}).H^{t-1}=\pi^t.H^{t-1}.r+\omega^t.Q^t$$

$$(\pi^t.H^t-\pi^{t-1}.H^{t-1})+\pi^t.C^t \qquad =\pi^t.H^{t-1}.r+\omega^t.Q^t.$$

National income thus redefined is the sum of wage and interest incomes, while investment in national product is redefined as the difference of stock values at time $\underline{t}$ valued at the prices of time $\underline{t}$, and stock values at time $\underline{t-1}$; i.e., the difference of aggregate stock values in current prices.

A rearrangement of the last equation yields the following relation for the evolution of national wealth (national aggregate stock value):

$$\pi^t . H^t = \pi^{t-1} . H^{t-1} . (1+r) + \omega^t . Q^t - \pi^t . C^t,$$

i.e., aggregate national wealth in current prices at time t equals the same concept at time t-1 increased by the interest accrued, plus the difference of aggregate wages and aggregate consumption.

Bibliographical Note

After this paper was finished I learned of the existence of a study addressed to the same over-all problems of locational-regional goal setting in economic planning and had an opportunity to exchange ideas and to inspect a preliminary version of the paper resulting from the latter study (T. A. Reiner, "Sub-National and National Planning," to be published in *Regional Science Association, Papers,* 1966). While the principal problems identified in my paper correspond closely to those in Reiner's, the analytical approaches taken are quite different. Reiner's paper contains a detailed literature survey and many practical examples of particular problems. The two papers are complementary rather than overlapping.

The basic reference on methods of regional and locational economics is Isard *et al.* (1960). A more recent survey of regional economics will be found in Meyer (1963). A number of articles on regional economic planning will be found in Isard and Cumberland (1961). The *Papers* of the Regional Science Association and the *Journal of Regional Science* contain much valuable material closely related to the topic of this paper.

The basic reference on mathematical programing in relation to economic problems is Dorfman, Samuelson, and Solow (1958). Standard references on linear programing include Dantzig (1963), Gass (1958), Hadley (1961), and Simonnard (1962); on nonlinear programing see Kuhn and Tucker (1951) and Wolfe (1963).

The first discussion of a mathematical programing approach to locational problems apart from simple transport problems was by Samuelson (1952). Such models have been developed further by Beckman and Marschak (1955), Vietorisz (1956), Moses (1957), Lefeber (1958, 1959), Isard (1958), Stevens (1958, 1959). A survey of interindustry models with a regional dimension, including both input-output and linear programing, is given in Chenery and Clark (1959, Chap. 12). The effects of economies of scale are explored in Vietorisz (1956, 1964) and Vietorisz and Manne (1963); the externalities arising from the fact that locations cannot be shared between several production processes are explored by Koopmans and Beckman (1957).

Applications of locational linear programing models to the study of particular industries will be found in Fox (1953, 1955, 1963), Henderson (1955, 1956, 1957, 1958), Snodgrass and French (1958), Koch and Snodgrass (1959), Heady and Egbert (1963), and Marschak (1963).

The handling of transport in the models given in Tables 1 and 2 in the text of the present paper is simple but highly sketchy; alternative ways of handling transport and a discussion of the merits and disadvantages of these alternatives appear in Stevens (1958), Lefeber (1958, 1959) and the discussion by Henderson (1958).

On interindustry growth models, see, for example, Dorfman, Samuelson, and Solow (1958), Solow (1963), and Koopmans (1964). The latter reference contains a detailed survey of the history of such models and in particular of the celebrated "turnpike theorem." A discussion of interregional growth models without interindustry detail will be found in Rahman (1963; see also the Comment by Dorfman, 1963), and Ward (1963). A regional interindustry growth model for a single region appears in Moore (1955).

REFERENCES

Arrow, K. J., *Social Choice and Individual Values*, New York, 1951.

Beckman, M., and T. Marschak, "An Activity Analysis Approach to Location Theory," *Kyklos*, 1955.

Chenery, H. B., "Development Policies and Programs," *United Nations Economic Bulletin for Latin America*, March 1958, pp. 51–77.

———, "Patterns of Industrial Growth," *American Economic Review*, September 1960, pp. 624–54 (mimeo. appendix on data and methodology is available from Research Center, for Economic Growth, Department of Economics, Stanford University).

Chenery, H. B., and K. A. Kretschmer, "Resource Allocation for Economic Development," *Econometrica*, October 1956, pp. 365–99.

Chenery, H. B., and P. G. Clark, *Interindustry Economics*, New York, 1959.

Dantzig, G. B., *Linear Programming and Extensions*, Princeton, N.J., 1963.

Dorfman, R., "Regional Allocation of Investment—Comment," *Quarterly Journal of Economics*, February 1963, pp. 162–65.

Dorfman, R., P. A. Samuelson, and R. Solow, *Linear Programming and Economic Analysis*, New York, 1958.

Fox, K. A., "A Spatial Equilibrium Model of the Livestock-Feed Economy in the United States," *Econometrica*, October 1953, pp. 547–66.

———, "Spatial Price Equilibrium and Process Analysis in the Food and Agricultural Sector," *Studies in Process Analysis*, eds. A. S. Manne and H. M. Markowitz, Cowles Foundation Monograph 18, New York, 1963, pp. 215–33.

Fox, K. A., and R. C. Taeuber, "Spatial Equilibrium Models of the Livestock-Feed Economy," *American Economic Review*, 1955, pp. 584–608.

Gass, S. I., *Linear Programming: Methods and Applications*, New York, 1958.

Hadley, G., *Linear Programming*, Reading, Mass., 1961.

Heady, E. O., and A. C. Egbert, "Spatial Programming Models to Specify Surplus Grain-producing Areas," *Studies in Process Analysis*, eds. A. S. Manne and H. M. Markowitz, Cowles Foundation Monograph 18, New York, 1963, pp. 161–214.

Henderson, J. M., "A Short-run Model for the Coal Industry," *Review of Economics and Statistics*, November 1955, pp. 336–46.

———, "Efficiency and Pricing in the Coal Industry," *Review of Economics and Statistics*, February 1956, pp. 50–60.

Henderson, J. M., "The Utilization of Agricultural Land—A Regional Approach," *Regional Science Association, Papers and Proceedings*, 1957, pp. 99–114.

———, *The Efficiency of the Coal Industry—An Application of Linear Programming*, Cambridge, Mass., 1958.

———, "Discussion—Interregional Equilibrium and Linear Programming," *Regional Science Association, Papers and Proceedings*, 1958, pp. 87–89.

Isard, W., "Interregional Linear Programming—An Elementary Presentation and a General Model," *Journal of Regional Science*, Summer 1958, pp. 1–59.

Isard, W., *et al., Methods of Regional Analysis: An Introduction to Regional Science*, Cambridge, Mass., and New York, 1960.

Isard, W., and J. H. Cumberland (ed.), *Regional Economic Planning—Techniques of Analysis*, Organization for European Economic Cooperation, Paris, 1961.

Koch, A. R., and M. M. Snodgrass, "Linear Programming Applied to Location of and Product Flow Determination in the Tomato Processing Industry," *Regional Science Association, Papers and Proceedings*, 1959.

Koopmans, T. C., "Economic Growth at a Maximal Rate," *Quarterly Journal of Economics*, August 1964, pp. 355–94.

Koopmans, T. C., and M. J. Beckman, "Assignment Problems and the Location of Economic Activities," *Econometrica*, January 1957, pp. 53–76.

Kuhn, H. W., and A. W. Tucker, "Nonlinear Programming," *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*, ed. J. Neyman, Berkeley, Cal., 1951, pp. 481–92.

Lefeber, L., "General Equilibrium Analysis of Production, Transportation, and the Choice of Industrial Location," *Regional Science Association, Papers and Proceedings*, 1958, pp. 77–86.

———, *Allocation in Space*, Amsterdam, Holland, 1959.

Marschak, T. A., "A Spatial Model of U.S. Petroleum Refining," *Studies in Process Analysis*, eds. A. S. Manne and H. M. Markowitz, Cowles Foundation Monograph 18, New York, 1963, pp. 75–135.

Meyer, J. R., "Regional Economics—A Survey," *American Economic Review*, March 1963, pp. 19–54.

Moore, F. T., "Regional Economic Reaction Paths," *American Economic Review*, May 1955, pp. 133–48.

Moses, L. N., *An Input-Output, Linear Programming Approach to Interregional Analysis*, Report, Harvard Economic Research Project, 1956–57, Cambridge, Mass., 1957.

Rahman, M. A., "Regional Allocation of Investment," *Quarterly Journal of Economics*, February 1963, pp. 26–39.

Reiner, T. A., "Sub-national and National Planning—Decision Criteria," *Regional Science Association, Papers and Proceedings*, 1966.

Samuelson, P. A., "Spatial Price Equilibrium and Linear Programming," *American Economic Review,* June 1952, pp. 283–303.

Simonnard, M., *Linear Programming,* Paris, 1962.

Snodgrass, M. M., and C. E. French, *Linear Programming Approach to Interregional Competition in Dairying,* Lafayette, Ind., 1958.

Solow, R. M., *Capital Theory and the Rate of Return,* Amsterdam, Holland, 1963.

Stevens, B. H., "An Interregional Linear Programming Model," *Journal of Regional Science,* Summer 1958, pp. 60–98.

———, "Interregional Linear Programming," unpublished Ph.D. thesis, Massachusetts Institute of Technology, 1959.

Tinbergen, J., *Economic Policy—Principles and Designs,* Amsterdam, Holland, 1956.

Tucker, A. W., "Combinatorial Theory Underlying Linear Programs," *Recent Advances in Mathematical Programming,* eds. R. L. Graves and and P. Wolfe, New York, 1963, pp. 1–16.

Vietorisz, T., "Regional Programming Models and the Case Study of a Refinery-Petrochemical-Synthetic Fiber Industrial Complex for Puerto Rico," unpublished Ph.D. thesis, Massachusetts Institute of Technology, 1956.

———, "Industrial Development Planning Models with Economies of Scale and Indivisibilities," *Regional Science Association, Papers and Proceedings,* 1964.

Vietorisz, T., and A. S. Manne, "Chemical Processes, Plant Location, and Economies of Scale," *Studies in Process Analysis,* eds. A. S. Manne and H. M. Markowitz, Cowles Foundation Monograph 18, New York, 1963, pp. 136–58.

Ward, B., *Problems of Greek Regional Development,* Athens, Greece, 1963.

Wolfe, P., "Methods of Nonlinear Programming," *Recent Advances in Mathematical Programming,* ed. R. L. Graves and P. Wolfe, New York, 1963, pp. 67–86.

COMMENT

Alan S. Manne, *Stanford University*

Plausible-looking linear allocation models display a disconcerting tendency to arrive at optimal solutions of an all-or-none character. For example, with the assumptions underlying Samuelson's substitution theorem, it will be optimal to produce all of a commodity via a single technique of production and none by any other technique. Other examples of all-or-none optimal solutions occur in the warehousing model of Charnes and Cooper and the minimum-time-to-balanced-growth model of Stoleru. Similarly, the Rahman-Dorfman multiperiod interregional growth model implies that in each time period all of the funds available for reinvestment should be absorbed by a single region, and none by any others.

To a policymaker concerned with equity between regions, this form of optimal solution will undoubtedly be unacceptable, and Dorfman himself has commented: "The resultant plan is a little outrageous, though suggestive, and should not be taken literally." Vietorisz has not taken the Rahman-Dorfman results literally. Most of his paper is devoted to discussing the qualifications that might cause a more realistic multiperiod interregional growth model *not* to have these unacceptable extreme solutions. Some of the qualifications suggested by Vietorisz are rather straightforward, e.g., that there might be upper bounds on the levels of individual activities so that there could be positive levels of investment in several activities (i.e., regions) within a single time period.

To this reader, perhaps the most appealing modification to the Rahman-Dorfman model is Vietorisz's hypothesis that there will be incentive effects operating to bring about a positive correlation between the current rate of savings in a region and the past rate of investment in that region. If this type of incentive effect is really operative, then the optimal policy need no longer be of an all-or-none character. Typically, it will be optimal to invest simultaneously in more than one region, and indeed there will be a strong presumption in favor of a policy of self-financing for each region. Self-financing constraints are anathema to those who are impressed with the workings of perfectly competitive capital markets—as well as to would-be global optimizers. It will take more than casual introspection to establish whether Vietorisz's savings incentive hypothesis is an empirically tenable one.

References

Charnes, A., and W. W. Cooper, "Generalizations of the Warehousing Model," *Operational Research Quarterly*, December 1955.

Dorfman, R., "Regional Allocation of Investment: Comment," *Quarterly Journal of Economics*, February 1963.

Rahman, M. A., "Regional Allocation of Investment," *Quarterly Journal of Economics*, February 1963.

Samuelson, P. A., "Abstract of a Theorem Concerning Substitutability in Open Leontief Models," *Activity Analysis of Production and Allocation*, ed. T. C. Koopmans, New York, 1951.

Stoleru, L. G., "An Optimal Policy for Economic Growth," *Econometrica*, April 1965.