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Chapter Title: Functions and Determinants of Stocks in Business Enterprises

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2. *Functions and Determinants of Stocks in Business Enterprises*[1]

Two attributes of stocks are critical to their management: first, stocks serve a wide variety of purposes in a business enterprise; second, they are costly to carry. These two facts mean that business firms must carry stocks, that they do not wish to carry more than they need, and that what is needed depends on the many functions that stocks serve and the influences that play upon them. What are these functions and influences? How, in view of the factors that determine the size of stocks, are they likely to vary in the course of economic fluctuations? In endeavoring to answer these questions, it will be useful to include stocks on order along with those on hand.

METHOD AND DIRECTION OF THE ANALYSIS

The Vantage Point of the Firm

The chapter examines the stock-carrying problem within business firms. Ideally it would combine economic analysis, management expertise, and descriptions by businessmen of how they operate and why, but the last approach has unfortunately gone by default in this study. As compensation in small measure, one end product of the book is a list of particular questions that need to be explored with business executives. The second source of information, management expertise, may require some justification.

There is an ample literature on inventories; it constitutes one of the major fields explored in management science or operations research. The literature is normative; it says what *should be* done. Does this bear an instructive relationship to *what is* done, the matter of interest here? I think the answer is yes. For one thing, the analysis that underlies the normative prescriptions helps to specify the relevant factors at work and thus helps the student discover what business does. For another thing, many of the prescriptions seem to have their counterpart in seat-of-the-pants judgments of businessmen. Finally, in a group of businesses, what "should be" and what "is" can differ in degree but not in basic kind. Even if actual rules have only a rough resemblance to normative rules, action is likely to resemble them more closely. Firms practicing successful procedures are imitated by their rivals, whereas firms persistently practicing unsuccessful procedures tend to disappear as Darwin has prescribed.[2]

[1] Norman Agin (MATHEMATICA and Columbia University, Department of Industrial Management and Engineering) has collaborated in the preparation of this chapter both in specific ways and by general aid and counsel throughout.

[2] Armen Alchian has gone much farther and argued that trial and error and survival of the fittest can go a long ways toward duplicating classic optimizing procedure without assuming classic rationality (which, because of uncertainty, is not even roughly realistic). "Uncertainty, Evolution and Economic Theory," *Journal of Political Economy,* June 1950, pp. 211–221.

Ownership of Materials

The first insight yielded by viewing inventories from within a business firm concerns the boundaries of the stock-carrying and procurement problem. For many purposes these boundaries appear to encompass materials on order along with those on hand—what I call materials "ownership." There are several reasons for arriving at this judgment.

Management literature speaks in these terms, often implicitly. Decision rules concern when and how much to order, and are based on correcting the difference between actual and desired stock. Obviously, if stock refers only to stock on hand, the implicit assumption is that the time required for the delivery of an order is constant and short—more rigorously, zero (since lead time affects the proper size of stocks and the forecasting error). I know of no exception in the literature to the fact that decision rules governing inventories stipulate what to order; of necessity, therefore, it is inventories on hand and on order which are thereby governed.

Sometimes authors make this attention explicit. To quote at random: "I_i = minimum stock of i at time of analysis including stock ordered." [3] "If the level system stock (stock at hand plus outstanding order) is dangerously low . . ." [4] "The foregoing analysis of optimal periodic order placement has shown that the on-hand plus on-order purchased materials inventory is brought into line each period with the prevailing usage forecast." [5]

In business firms also, reference to stocks both on hand and on order is common. It is sometimes referred to as the "position." The term "ownership" is one I encountered in discussing materials buying problems with executives in shoe and leather manufacturing concerns. Department store retailers call the same thing "in sight." [6]

The reasons why stock and procurement planning must comprehend goods on order as well as on hand will become clearer as the functions of stocks are discussed. But because the concept is unfamiliar, it may be useful to analyze it for a moment.

Production operations ordinarily involve acts that must be performed in sequence. The purpose of stocks is to provide, at each station where work is to be performed, an adequate supply of materials to facilitate efficient performance of the work at that station (under the constraint that the total system also operate efficiently, as defined).

Finished stocks supply the point where goods are ready for shipment to customers and sometimes some intermediate storage spots. Stocks in process service a number of operations that convert goods from less to

[3] Richard M. Cyert and James G. March, *A Behavioral Theory of the Firm,* Englewood Cliffs, N.J., 1963, p. 136.

[4] Yoichiro Fukeda, "Optimal Policies for the Inventory Problem with Negotiable Leadtime," *Management Science,* July 1964, pp. 690–708.

[5] Charles C. Holt and Franco Modigliani, "Firm Cost Structures and the Dynamic Responses of Inventories, Production, Work Force, and Orders to Sales Fluctuations," Joint Economic Committee, *Inventory Fluctuations and Economic Stabilization,* Washington, 1961, Part II, p. 44.

As the title suggests, the authors interest themselves in how the problems that businessmen must solve influence the size of their stocks. This and other publications by the same authors are replete with analysis and information about many of the same questions that I touch on, often in a far more impressionistic fashion, in this chapter and elsewhere in this volume. See particularly Modigliani and Franz E. Hohn, "Production Planning over Time and the Nature of the Expectations Horizon," *Econometrica,* 1955, pp. 46–66; Modigliani and Kalman J. Cohen, "The Role of Anticipations and Planning in Economic Behavior and Their Use in Economic Analysis and Forecasting," Bureau of Economic and Business Research, *University of Illinois Bulletin,* January 1961.

[6] See, for example, testimony of Vincent J. Graham, General Merchandise Controller, Sears, Roebuck and Company, in Hearings Before the Subcommittee on Economic Stabilization, Automation, and Energy Resources of the Joint Economic Committee, July 9, 1962, *Inventory Fluctuation and Stabilization:* "I am talking about the overall combination, of the inventory and the on-order, which we call in sight" (p. 29). Also "Most retailers . . . operate on a turnover basis, maintaining a flow of goods in sight related to a predetermined number of weeks or months of anticipated future sales" (p. 5).

more finished states. "Raw" materials stocks service not only the first production station but also earlier preparatory stations where cleaning, sorting, and ticketing operations may have taken place. Finally, the first appearance of goods in the establishment has been made possible by outstanding orders which have achieved their delivery date. Materials currently required in stock must have been ordered earlier by whatever time is required for delivery to take place; they must be on order during this interval in the same sense in which current finishing operations imply the previous presence of stocks of materials undergoing cleaning or sorting. Thus the essence of the work that is done by stocks is that of providing for a sequence of things to occur, each of which takes time. From the point of view of the orderly sequence whereby goods become available to the operations that a particular enterprise performs, stocks on hand and on order can be very much of a piece.

Of course, the parallelism is far from complete. Work can be performed at the discretion of the owner of stocks on hand, whereas the seller as well as the buyer make determinations about stock on order. The movement of stocks through a business has physical counterparts associated with income flows and other matters that do not apply to stock on order. The transformation effected by an order is executive and informational rather than physical.

A second reason for thinking in terms of ownership has already been mentioned—the most usual action that must be taken in order to increase or decrease stocks is to order more or fewer materials, and thus change the level of stocks both on hand and on order. Of course, if lead times were invariant, the distinction would be empty. Since they are not, the purchaser decides both what to order and, within constraints, whether to hold it on hand or on order.

In making the second decision, there are advantages and disadvantages which mean that, in effect, stock on hand has a competitive relationship to stock on order and vice versa. Stock held on order has the advantage that it does not imply carrying and financing costs. Under some circumstances purchasers may not have to live up to the commitment to accept delivery. There may be further advantages in the form of better prices and selections if suppliers are given ample time to make deliveries. On the other hand, stock on order implies uncertainty about just when materials will actually arrive and the extent to which they will meet specifications. There are other ways in which stocks on hand and on order are only partial substitutes for one another and therefore have a competitive relationship expressed by a differential price that buyers choose to pay.

Stocks on hand and on order also have a complementary relationship to one another. The complementary aspect follows from the fact that orders precede receipts. An intention to increase stocks of materials, or for that matter to increase the flow of goods toward its final state, of necessity involves some temporary increase in outstanding orders unless delivery is immediate. In this sense, outstanding orders constitute a vestibule through which materials must pass before they enter the door of factory or store. If the entrance flow is to be increased, either to add to stocks or to feed the flow of production, this "vestibule effect" will precede the increase. If, on the other hand, the intention focuses on the increase in outstanding orders, there is a "reverse vestibule effect" and the orders will eventually be delivered; when this occurs, stocks will increase as a deferred reaction to changes in outstandings, other things the same. Of course, other things do not need to be the same; the flow into production, or to customers, may have increased in line with the increase in orders for material. In this event no increase in stock need occur.

I conclude that it is essential to study stocks on order and those on hand together, as well as individually.

A "System" of Alternatives

The second major insight that the firm's-eye view affords is the wide system of alternatives of which stocks constitute a part. Carrying stocks provides one way of meeting a large variety of business problems each of which may also be met in a number of other ways.[7] Thus an alternative to solving the problem, say, of meeting variable demand promptly by means of stocks is that of doing so by flexible production schedules, and each of the two methods are pursued to the point where, other things the same, their marginal costs are equal. Similarly, an alternative to using funds for financing inventories is to use them for financing new plant capacity, and their potential earnings in one use constitute their opportunity costs for employing them in the other. To pursue this point of view it is necessary to set the cost of carrying stock against the contribution of stock to efficient business operation.

Plan for the Chapter

These generalizations point the way to how fluctuations in stocks may be probed. We need to outline the major functions that stocks serve and note by what other means the same functions can be carried out. This, along with information on costs, indicates how the size of stocks is determined and how it is affected by changes in the level of sales and in other relevant factors. Next we consider how the cost of carrying stock, and of the alternative ways of meeting the management objectives that stocks serve, may tend to change during business cycles.

The purpose of this last phase is not to arrive at conclusions or even firm hypotheses about cyclical behavior. Information is still too fragile for that. Rather, the analysis is intended to provide the background for understanding the evidence yielded by the examination of time series in the body of the book. The time series, it should be added, deal only with materials stock on hand and on order. But to understand the functions that these stocks serve, it is necessary to use a broader perspective for the purpose of this chapter and to cover all stocks, whether finished, in process, or purchased materials on hand and on order.

It may be useful to have the main conclusions in mind at the outset. The major functions that stocks serve are:

1. Bridging the time required for processes (economic transformations) to be performed.
2. Efficient production or purchasing lots.
3. Insurance against losing sales because of individually unpredictable fluctuations in demand or other matters.
4. Smoothing operations by provision for more or less foreseeable fluctuations.
5. Grasping the potential advantage (or avoiding the disadvantage) of actual or expected changes in conditions in markets in which purchases or sales are made.
6. Providing elective freedom from the tyranny of planning for uncertain events.

The first is, as far as I can judge, a unique function of stocks. All the other functions, in effect, "buy efficiency" by substituting the lesser cost of carrying stock for a greater cost

[7] The appropriate context in which to consider particular business problems—the system—has received growing attention in recent years. The usual conclusion seems to be that the system should be broadened. Arthur D. Hall presents this general approach in *A Methodology for Systems Engineering*, Princeton, N.J., 1962. An interesting article which makes the point for inventories is by Herbert Simon and Charles Holt, "The Control of Inventories and Production Rates—A Survey," *Journal of the Operations Research Society of America*, August 1954, pp. 289 ff. The authors discuss research relating to ordering decisions, production rate decisions, and scheduling decisions. "Since ordering and production decisions both involve this weighing of costs against gains from the holding of inventories, we should expect that fundamental research directed at either one should have applicability to the other." This, the authors say, has not been so in the past, but "Currently these two separate lines of investigation are converging rapidly" (p. 298).

of coping with a particular management problem in some other way.

In each case, these functions may be served by stocks at all levels of processing—from finished goods to purchased materials. They are often also served by purchased materials on order (as well as those on hand), and consequently these outstanding purchase orders must be considered along with stocks physically in the possession of the enterprise.

If we think of each of the six functions as covered by stocks serving just that function and no other, patterns of appropriate variation may be ascertained. This is, of course, an abstraction since the same physical stocks serve several functions. In any event, analysis leads to the conclusion that the efficient servicing of sales does not require stocks that vary in proportion to sales but that vary substantially less, other things the same. In other words, the stock-sales ratio could well have a pattern inverse to that of sales if nothing else changed. Yet, of course, other things do change. The analysis suggests that costs may often shift in favor of the stock-carrying alternative when business is good.

THE COST OF STOCKS

To achieve the purposes that stocks serve, the cost of carrying them must be borne. What then are these costs and how are they likely to change with business conditions?

Components of Cost

Appropriate to a decision to increase or decrease stocks are the costs that actually do change as a result of the decision—the marginal or incremental costs. Whether this solid economic rule is more honored in the breach than in practice is not clear.[8]

Physical care of physical goods includes the cost of storage, handling and guardian functions, insurance against risk of fire, theft, or other hazards. An allowance for physical depreciation may also be required. Though these costs may be substantial in some businesses, it seems likely that they usually add up to materially less than that of economic depreciation and finance. They apply to stock on hand at all stages; they do not apply to those on order.

[8] Practice may differ in different contexts. For example, unused storage space may typically not be considered a charge against stocks, as it should not be; whereas unused and unusable financial leeway may be considered such a charge, at least in the management rules.

Economic depreciation or obsolescence results from the need to immobilize resources in, and make a commitment to buy, specific inventory items. As a result there may be losses due to markdowns or other costs of having the wrong goods on hand. Though the time covered by the commitment is far shorter than for most capital investment, it nevertheless can involve a serious risk. The less is known about what sales will be (either with respect to volume or kind), and the more differentiated are the materials required for particular finished articles, the higher is the risk. It increases also with the length of time over which resources must be committed.

The risk of obsolescence starts when a commitment to purchase is made. Accordingly it applies to stock on order as well as on hand. In the former case, however, the commitment may sometimes have a modicum of elasticity which is lost once delivery takes place.

Financing costs, the cost of funds invested in stocks is calculated as a percentage, per unit of time, of the value of inventory goods. The figure is determined in one of several ways. It may be defined as an actual interest expense when stocks are in fact financed by borrowed funds. This cost, though small relative to other ways of determining financing cost, may be an important burden in com-

panies that are short of funds and operate with high materials costs and low value added. But even when no actual borrowing takes place, a common situation, the "opportunity costs" of funds invested in inventories are ordinarily charged against them.

This rate may be the average rate of return on invested capital for the company. It may also be considerably higher; ". . . a rate of return or imputed interest rate between ten and thirty per cent is not unreasonable." [9] Concerning one large company I was specifically told that its average return was 14 per cent and the inventory financing charge 30 per cent. The higher end of the range no doubt includes an allowance for risks of the kind discussed in the previous paragraph. But high financing rates may also serve a partly strategic managerial function in a large decentralized company. A high cost of inventories keeps management on its toes to discover other ways of achieving the objectives that inventories serve—production schedules are examined to increase flexibility; suppliers are pushed to make frequent, swift, and reliable deliveries; sales pressure aims at filling in seasonal lows.

In any event, it seems clear that the bookkeeping charge for carrying inventories has a very substantial judgmental and even strategy-linked element. If so, the formal rules, and what they yield by way of cost figures, may be only part of what upper management takes into account when it reviews the inventory position of departments and of the company as a whole. Implicit if not explicit judgment may also be made concerning the current applicability of the rules themselves.

Financing costs apply in the first instance to stock on hand only. However, if on-order positions are extended, their ultimate delivery may cause stock on hand to be larger than it otherwise would. If so, this inverse vestibule effect will imply a delayed financing cost for stock on order also. Per unit of finished goods, financing costs are of course higher as the finished state is approached.

[9] John F. Magee, *Production Planning and Inventory Control,* New York, 1958, p. 40.

Cyclical Changes in Inventory Costs

The previous discussion suggests that changes in inventory costs may be of two sorts: changes recognized in explicit costing rules and changes that concern extra-rule evaluations that determine changes in the conclusions drawn from the same formal cost figures. As far as I know, there are no studies of cyclical patterns in the first type of cost, much less the second. Accordingly, only the most tentative statements are possible. However, I would like to venture the thought that, contrary to the most obvious suppositions, costs are on balance more likely to fall than to rise in prosperity.

Bank interest rates, it is true, may rise, but the difference of one, two, or even three percentage points a year is so small a portion of total stock costs in most businesses as to be of most questionable general significance. This is not to minimize the importance of these costs in some business for which funds are short and stock financing an important part of value added. Moreover, availability of credit, which is likely to parallel its costs, can seriously tend to discourage inventory investment in such businesses when credit is tight. But for many businesses, cyclical variations in the cost and availability of bank credit may be a minor influence in stock-carrying decisions except under most exceptional circumstances. In any event, painstaking empirical search has failed to establish the expected association.[10]

On the other hand, lower financing costs during prosperity are suggested by several considerations. First, the average period for which goods are held in stock is likely to fall be-

[10] See Paul F. McGouldrick, "The Impact of Credit Cost and Availability on Inventory Investment," *Inventory Fluctuations and Economic Stabilization,* Part II. The author reviews the literature on the subject, and the burden of his conclusion may be summarized in his words, "On the whole, results were disappointing" (p. 105).

cause stocks resulting from overestimating future sales are lower than during at least the early months of recession. And virtually all carrying costs are a function of the time that goods are held in stock. Second, risk of economic obsolescence is reduced by the shift from a buyers' to a sellers' market, which typically accompanies periods of strong demand; the company's customers are not as choosy.

Third, actual opportunity costs of capital are probably reduced. For one thing, the risk charge that is included in the financing rate is less for the reasons just mentioned. Further, substantial profits during peak periods may provide funds which companies are willing to invest in liquid assets but presently fear to commit to permanent capital improvements. If funds were really compartmentalized in this way, firms that customarily borrow to finance stocks might be able to substitute more internal funds when profits were high; firms that customarily finance stocks from internal funds could dip into some portion of prosperity profits for which the alternative form of investment would be that of other liquid assets, such as financial instruments. If so, the opportunity cost would be far lower than if the alternative was investment in fixed plant. The argument implies that though the explicit rules for stock-carrying costs may remain the same, actual behavior may change. The grapevine communicates a permissive attitude from the front office. As will be seen later, the time series look as if this could be the case. In any event, for present purposes it is sufficient to conjecture that stock-holding costs will typically not have clear positive cyclical conformity.

PROCESS-TIME STOCKS

Against the cost of carrying stock is set the benefit that stocks provide. The first major type of benefit to be considered is that of supporting the time required to effect economic transformations.

The transformations may be physical—cloth and findings are transformed into a suit; they may be locational—the cloth is moved from Raleigh to Rochester; they may be executive—an order for cloth is received by the mill, recorded, scheduled, and eventually shipped. For an individual company, total processing time is meaningfully defined as the sum of all three types. It applies to stocks at all stages of processing and to materials on order.

Stocks that support some properly determined minimum time required for processing to take place are *necessities* of business existence, whatever their cost. As the next section indicates, the minimum is defined under normally prevailing efficient eco-engineering conditions. What determines this minimum size?

Determinants: Link to Sales

The amount of stocks that are required for physical transformation depends on the length of the process. If output is continuous, then some goods in each condition of processing will reside in stock at any given moment. The longer the time required for the process to be completed, the larger, relative to the output per day or week, stocks must be.

The time required for a process to take place is in one sense an engineering problem. But goods seldom pass through a production process at the maximum speed that engineering techniques would allow. Economic considerations demand that assembly belts move at rates that do not cause excessive spoilage, that goods wait until enough units queue up so that the process can be performed in economical lots, that some waiting occurs between sequential processes, and so on. All of these economic considerations must be superimposed on engineering efficiency, and

therefore process-time stocks are enlarged by some admixture of "efficiency" stocks serving the many other functions that stocks perform. The point is underscored by the fact that minimum production time is also subject to some control, at a price; men and machines can be required to move more or less swiftly or production lineups can be altered. But for present purposes, the conceptual annoyances that this ambiguity involves can be kept to a minimum by thinking of process-time stocks, somewhat loosely, as those required to support the time required to move goods through the process under *normally prevailing* efficient eco-engineering conditions. These conditions are, by definition, thought of as not changing during business fluctuations, though of course they can undergo marked trends.

Process-time stocks form a significant portion of stocks of goods in process in any factory. But this is by no means the only place where they exist. Sorting, marking, packaging, carting, and storing operations, all of which take time, may be performed on goods after production has been completed and they have entered "finished" stocks. Materials stocks of "purchased goods" are also often subjected to several processes that require time, as likewise are distributors' "stock in trade."

Process time is also present for materials outstanding. The purchaser, in planning for a flow of materials designed to feed production, must consider not only the time required for the transportation of goods from the maker's shipping sheds to his own receiving docks, but also for the time required for the maker to get around to shipping it to him. The whole period is the "replenishment time" or "lead time"—the interval between placement of the order by the purchaser and replenishment of his stocks of purchased materials by the receipt of the goods.

It is particularly important in connection with this aspect of process time, which is of course supported by materials stocks outstanding, to think of the "normally prevailing efficient eco-engineering conditions" as being constant during business fluctuations. Actually conditions, needless to say, are not, since delivery periods and other aspects of market conditions tend to vary during business cycles. Perhaps the best way to identify the process-time part of outstanding orders is in terms of the amount required under "hand-to-mouth" conditions. The phrase is not meant literally; it applies under a "buyers' market"—a time, that is, when markets are unexcited.

If processing time is uniform, the size of process-time stocks (I_p) is a function of the volume of goods (D) which is required to emerge from the stockpile per calendar interval, and the time, in terms of that interval, required for processing (T).[11] Thus, $I_p = DT$. This means that if there is no change in processing time, desired process-time stocks will increase or decrease in *direct proportion to demand.* This is, of course, the model of derived demand in its generic sense, if one assumes that total stocks serve the process-time function. Actually, of course, all stocks are not caught up in the march of goods through a plant at its normal pace, and the constant ratio applies only to those that are.[12]

Other Influences

Processing time, in terms of the calendar interval in which demand is recorded, can change in two ways. First, the calendar time required for the process can change as a result of changing technology—it can take two days of processing, whereas previously it took

[11] If statistics are in dollars rather than physical units, and value is added evenly and continuously, then the value of a unit of stock will be one-half of the unit value of demand D minus the value of the equivalent unit value of "raw" materials.

[12] Another way to put it is in terms of a constant incremental ratio equal to the average ratio of process-time stocks to sales. If sales are 1,000 units, total stocks are 3,000, and process-time stocks 2,000, then as sales increase to 1,100 process-time stocks rise to 2,200.

four. Second, the relation between calendar and processing time can change; the factory can operate longer, as in a change from a one-shift to a two-shift operation. Work that took one shift two days now takes two shifts one day. Accordingly, to represent a change in process time, a correction factor is required. The ratio represents the change in the proportion of the 24-hour day or 168-hour week that processing is taking place on the same line sequence. It is the ratio of processing time to calendar time at time zero, divided by the ratio of processing time to calendar time at time 1. Thus, $I_p = \alpha DT$.[13]

The ratio α can change characteristically with business conditions. For example, if additional demand is accompanied by a second shift, α is approximately halved.[14] Thus an approximate doubling of demand can be accommodated without much change in processing stocks. Overtime work has an analogous impact, though to a far less extent. If, on the other hand, additional output is accommodated by adding parallel production lines, process time is unchanged and this means that stocks will increase proportionately to output.

Cyclical Patterns

These considerations suggest that stocks identified as process-time stocks would have an underlying tendency to maintain a generally proportional relation to sales, after allowing for the appropriate lags associated with the change in the level of output.[15] But overtime work and multiple-shift operations (when in line sequences rather than parallel sequences) will introduce a tendency for stocks to rise less than sales during prosperous times. This could, I imagine, be quite important in some industries. Some countervailing tendency may arise if overcrowding of facilities slows up operations.

Process-time stocks on order share the tendency to change in proportion to sales, and there is no reason to expect a change in the relation of process to calendar time, α. Therefore the relation is strictly proportional, other things the same.

However, replenishment periods themselves often change. As business expands and markets tighten, suppliers tend to quote delivery dates that extend replenishment periods. Purchasers very typically accept these dates and extend their materials outstanding correspondingly, rather than pay the premium prices (including poorer selections, quality, etc.) that quick deliveries would imply. Indeed they often anticipate these market stringencies for a number of reasons, thereby further extending outstandings. These resulting stocks on order are a type of "efficiency stock" covering market prospects and are discussed below. This does not, however, exempt them from their process-time characteristics of $I_p = DT$. Thus T increases; but if D does also, stocks are a product of the two.

[13] α has a value of 1 under unchanged conditions. An alternative way of formulating the concept would be to assign the value of 1 to "normal" conditions.

[14] I ignore the lower productivity of second shifts. Also, insurance stocks should be increased since the cost of stockouts in terms of lost time increases with multiple shift operation. I assume as previously that the second shift takes over where the first one ended and continues to move the same inventory pools along.

[15] The lag is present in process-time stocks as defined, though other functions of stocks also contribute. Assuming that there is no foreknowledge of change in demand, the lag in process-time stocks relative to shipments, both measured at book value, increases with the length of process-time and the ratio of value added to the value of product. If output is increasing, the higher unit value of more nearly completed goods will cover a smaller proportion of stocks (measured in equivalent physical units of finished goods) than the lower unit values of more nearly raw goods, which reflect the increase in demand more promptly. Therefore the ratio of stocks to output measured in dollars will be lower than when the level of output has remained unchanged during the processing time. Conversely, when output is declining, the ratio of stocks to output is higher.

EFFICIENCY STOCKS: THE RANGE OF ALTERNATIVES

The fundamental notion in the designation "efficiency stocks" is that of *opportunity costs.* These stocks "buy" managerial efficiency at a cost that is equal to that of the next best way of achieving an analogous marginal benefit.[16]

There are a wide variety of managerial problems that are moderated by an increase in stocks, on the one hand, or by some other management device, on the other hand, and it is to these situations that efficiency stocks apply. Correct solutions inevitably involve some combination of the two sorts of devices —the point, theoretically, where incremental (marginal) costs are equal. The range of the problems and of the devices by which they may be dealt with are indicated by Exhibit 1. For manufacturing, section 1 applies primarily to finished stock, section 2 to in-process stock, and section 3 to materials on hand and on order (see column 4). For distribution, sections 1 and 3 encompass most alternatives.

Glancing down the first column, it may be seen that one recurrent type of problem involves variability in demand; it appears at the selling stage (line 1.2), at the producing stage (line 2.1), or at the purchasing stage (line 3.1). There are many ways of dealing with this variability. For example, at the selling stage, price concessions (column 2, line 1.2.1) may encourage off-season sales, but this involves some lost revenue (relative to the full price) even if the incidence of price reduction can be narrowly contained; besides, return to normal prices may encounter customer resistance (column 3, line 1.2.1). Similarly, the extra expense of additional selling effort, including directed advertising campaigns, might help to smooth sales (lines 1.2.2 and 1.2.3).

An alternative to any of these ways of reducing sales fluctuation is simply to carry an inventory of finished goods which provides a reservoir large enough to service the existent pattern of sales. Such stocks are of two sorts according to the variability against which they protect: the first are "insurance" or "buffer" stocks, which provide for random ups and downs in weekly or monthly sales; the second we call "fluctuation" stocks, which provide for variability due to more or less predictable seasonal or other patterns of demand. Insurance stocks would afford faster delivery for perhaps a wider selection of items with a lower acceptable chance of stock-outs. Fluctuation stocks might cover higher seasonal peaks as alternative to the methods in column 2 applied to smoothing the monthly pattern of demand. Both sorts of stocks may provide alternatives to price concessions or sales pressure in dealing with variability of demand at the selling stage.

Another alternative is that of doing nothing (line 1.2.7) and simply tolerating the cost of disappointing or turning away customers. Somewhat analogous situations reappear at the earlier stages of the sequence of operations that a company performs. At the purchasing stage the stock function of insuring against variable demand is served both by stocks on hand and on order. (Column 4b indicates when this is or is not the case.)

Stocks also permit production or buying to take place in efficiently sized batches. Job lots that are too small (line 2.2) involve high labor costs and machine down-time (column 3). Larger job lots imply larger "lot-size" stocks (about half the size of the lot) and resultant inventory costs (column 4). Orders too have optimal sizes. The high unit purchasing costs that result when orders are too small

[16] The notion of stocks that "buy efficiency" was used and emphasized in Magee's very useful *Production Planning and Inventory Control.* I have borrowed from his analysis in a number of other ways as well.

EXHIBIT 1

Business Problems Which Can Be Dealt With by Methods That Include an Increase in Stock

Problem	Method Other Than Increasing Stock		Stock Method[a]
	Method	Type of Cost Involved	
(1)	(2)	(3)	(4)
1. *Selling*			*Finished Goods Stocks*
1.1 Sales promotion	1.1.1 Price adjustments	Possible lost revenue	Ins., Fl.
	1.1.2 Selling pressure	Sales costs	Ins., Fl.
	1.1.3 Advertising	Advertising expense	Ins., Fl.
1.2 Variable sales (expected or random)	*Reduce variability:*		
	1.2.1 Price concessions	Lost revenue Difficulty in reversing policy	Ins., Fl.
	1.2.2 Selling pressure	Selling; costs	Ins., Fl.
	1.2.3 Advertising	Advertising expense	Ins., Fl.
	Predict variability:		
	1.2.4 Research	Research cost	Ins., Fl.
	Shift variability:		
	1.2.5 Buy vs. make	Higher cost	Ins., Fl.
	1.2.6 Counteract via requiring advance orders	Lost sales or selling expense minus the advantage of preknowledge of sales	Ins., Fl.
	1.2.7 Accept the cost of stock-outs	Lost sales	Ins., Fl.
1.3 Achieving an optimal selling price if prices are expected to rise	1.3.1 Forgo opportunity to wait	Lost revenue	MP
2. *Producing*			*In-Process Stocks*
2.1 Variable demand (expected or random) for completed product	*Reduce variability:*		
	2.1.1 Reducing sales variability as above	Costs as above	Ins., Fl.
	2.1.2 Support finished inventories	Finished inventory costs as above (col. 4)	Ins., Fl. Un.
	Meet variability:		
	2.1.3 Add facilities	Capital expense with risky payoff	Ins., Fl.
	2.1.4 Buy rather than make	High cost	Ins., Fl.
	2.1.5 Achieve more flexible production	Labor cost Idle capacity cost Too small job lots as below	Ins., Fl.
	2.1.6 Accept the cost	Lost sales	Ins., Fl.

(continued)

EXHIBIT 1 (concluded)

Problem	Method Other Than Increasing Stock		Stock Method[a]
	Method	Type of Cost Involved	
(1)	(2)	(3)	(4)
2.2 Job lots that are too small	2.2.1 Accept the cost	Labor cost Machine down-time	LS
3. *Purchasing*			*Materials Ownership*[b]
3.1 Variable demand	*Reduce variability:*		
	3.1.1 Reduce variability of production or sales as above	Costs as above	Ins., Fl., H, O
	3.1.2 Support finished and in-process inventories	Cost of inventories as above (col. 4)	Ins., Fl., H Un.
	Meet variability:		
	3.1.3 Reduce order-lot size	Costs as below	Ins., Fl., H, O
	3.1.4 Demand fast deliveries	High purchase price	Ins., Fl., H
	Accept the cost:		
	3.1.5 Accept	Production delays, etc.	Ins., Fl., H, O
3.2 Variable receipts: risk of tardy or unavailable goods of desired specification	3.2.1 Pay premium prices for desired goods	High purchase price	MP, Ins.
	3.2.2 Accept the cost of late, short, or incorrect deliveries	High-cost manufacture	MP, Ins., H, O
	3.2.3 Support inventories of "raw" materials	Cost of inventories (col. 4)	Un, H, O
3.3 Order lots that are too small	3.3.1 Accept	High-cost purchasing	MP, H, O
3.4 Achieving an optimal purchase price	3.4.1 Forgo the advantage	High-cost purchasing	MP, H, O
3.5 Supplier-announced increase in replenishment period	3.5.1 Pay premium price elsewhere	High-cost purchasing	PT, H, O

Note: Most of the entries are probabilistic; that is, they involve the *chance* of costs rather than sure costs.

[a]Type of stock involved: process time (PT), lot size (LS), insurance (Ins.), fluctuation (Fl.), market prospect (MP), unplanned (Un.), on hand (H), on order (O).

[b]The "Stock Method" can utilize material stocks either on hand or on order, except in the case of 3.1.2 and 3.1.4.

must be balanced against higher stockholding costs for larger order sizes (line 3.3).

Other management problems in connection with which stocks can "buy efficiency" include variability in receipts of goods of desired specification (line 3.2) and the need to sell at what is judged to be the highest possible price (line 1.3) and buy at the lowest (line 3.4).

In each case, the problem arises out of actual or anticipated changes in the condition of the markets in which finished goods are sold or materials are purchased. One way to cope with the problem is to carry larger stocks of goods on hand and on order. Increases in these "market-prospect" stocks involve costs which are alternative to those of other devices —paying premium prices for swift deliveries (line 3.2.1), accepting the higher manufacturing cost of, say, substandard materials (line 3.2.2), or (because of failure to anticipate a rise in prices) buying at future hypothetically higher prices rather than at present low ones (line 3.4).

Further study of Exhibit 1, which is intended to be amply illustrative rather than entirely comprehensive, may be rewarding, but its basic message has been brought out: stocks are one of a number of management devices for solving a number of management problems. Efficient solutions involve the proper combination of the stock and other alternatives. They involve, also, a proper combination of the other alternatives which are often partial substitutes for one another. How much of each to use depends on comparison of incremental costs at the margin of decision. We need to examine the character of these appropriate comparisons.

The subject is adorned by an elaborate and highly technical literature, which constitutes a large part of the work in operations research and management science. Most of it applies to stocks of finished goods and those that serve primarily what I have called the lot-size, insurance, and variability functions. It specifies the optimal size of stocks serving designated purposes. At the growing edge of study, interrelations are taken into account, though often mathematical solutions are indeterminate.[17] *Changes* in the cost functions receive very little attention.

The basic results of these studies are learned at business schools. Restricted aspects of them are embodied in nomographs and simplified tables for the use of stock clerks.[18] How often they are actually used in their full quantitative form by business is difficult to say. However, some sort of qualitative consideration of the relevant factors can hardly be avoided.[19] But what-

[17] For relatively nontechnical summaries, see J. F. Magee, "Guides to Inventory Policy," *Harvard Business Review*, January–February 1956, March–April 1956, and May–June 1956; D. W. Miller, and M. K. Starr, *Executive Decisions and Operations Research*, Englewood Cliffs, N.J., 1960, Chapter 10.

For a swift idea of the range of problems dealt with, see Robert Dorfman, "Operations Research," *American Economic Review*, September 1960, pp. 589–598; J. Laderman, S. B. Littauer, and Lionel Weiss, "The Inventory Problem," *Journal of the American Statistical Association*, December 1953, pp. 717–732.

Good comprehensive treatments are available in many books, including M. K. Starr and D. W. Miller, *Inventory Control: Theory and Practice*, Englewood Cliffs, N.J., 1962; G. Hadley and T. M. Whitin, *Analysis of Inventory Systems*, Englewood Cliffs, N.J., 1963; T. M. Whitin, *The Theory of Inventory Management*, Princeton, N.J., 1963.

More advanced treatment, including work with dynamic models and efforts to explore interreactions among inventories at various stages or serving various purposes, is contained in K. J. Arrow, S. Karlin, and H. Scarf, *Studies in the Mathematical Theory of Inventory and Production*, Stanford, Calif., 1958.

[18] See, for example, Joseph Buchan and Ernest Koenigsberg, *Scientific Inventory Management*, Englewood Cliffs, N.J. The preparation and use of these devices are described on pp. 263–277 and many applications given. See, for example, pp. 72–73, 152–153.

[19] John F. Magee, research director of the Operations Research Group of Arthur D. Little, Inc., refers to the question in "Guides to Inventory Policy, Functions and Lot Size," *Harvard Business Review*, January–February 1956. He discusses the matter in connection with optimum lot-size formulas: "Even though formulas for selecting the optimum lot size are presented in many industrial engineering texts, few companies make any attempt to arrive at an explicit quantitative balance of inventory and change-over or set-up costs." He speaks of the difficulty of calculating these costs in many companies and particularly their marginal cost: "Oftentimes companies therefore attempt to strike only a quantitative balance of these costs to arrive at something like an optimum

ever the extent of their direct use, or use in a common-sense approximate way, their particular value here is primarily analytic. They help to indicate, as I mentioned at the beginning of this chapter, what factors are relevant to inventory control and approximately in what way. Thereby they should help to suggest how inventories may be expected to behave during business fluctuations.

The essential point is that the correct solutions for the management problems involve almost universally some *combination* of supporting the cost of stocks and of recourse to one or more other devices. Solutions should proceed according to traditional economic principles of equimarginal advantage, whereby the combined costs are minimized. They must be analyzed separately for each of the five sorts of efficiency stocks: lot-size, insurance, fluctuation, market-prospect, and error stocks.

THREE TYPES WITH DAMPED ASSOCIATION TO SALES

Lot-Size Stocks

THE NEED SERVED AND RELATION TO LOT SIZE

These stocks permit bunching of processing or ordering in efficient lots as suggested in lines 2.2 and 3.3 of Exhibit 1. Bunching for processing applies most particularly to goods in process, but it can also apply to such preparatory processes as are performed on purchased materials; finished-goods stocks may also present a bunching problem particularly in connection with appropriate lot sizes for shipping. For example, the cost of machine processing is ordinarily increased by a changeover in the work performed by a machine. Costs include the labor of setting up the new operation, and often labor inefficiency in getting into the swing of its production (shakedown cost); the fact that equipment is idle while the change is under way gives rise to a further opportunity cost. All these costs occur less often when lots are large than when they are small; on a per-unit-of-output basis, they are, in other words, inversely associated with the size of lots. Inventory costs, on the other hand, are directly associated with lot size. Stocks that serve the lot-size function range between a maximum of the lot sizes and a minimum of zero; assuming a uniform rate of withdrawal, they average one-half the lot size.

Orders for materials also need to be assembled in economical bunches: order quantities. Otherwise costs of processing the order, receiving it, and particularly the cost of transportation mount; quantity discounts may be foregone. For stock on hand, associated stocks will again tend to equal one-half the size of the order. But for stock on order the withdrawal rate is necessarily discontinuous; the whole lot is delivered at the same time or at a stipulated discrete time. As a result, outstanding orders associated with economical order quantities are larger relative to the lot size than are stocks on hand.[20]

DETERMINANTS

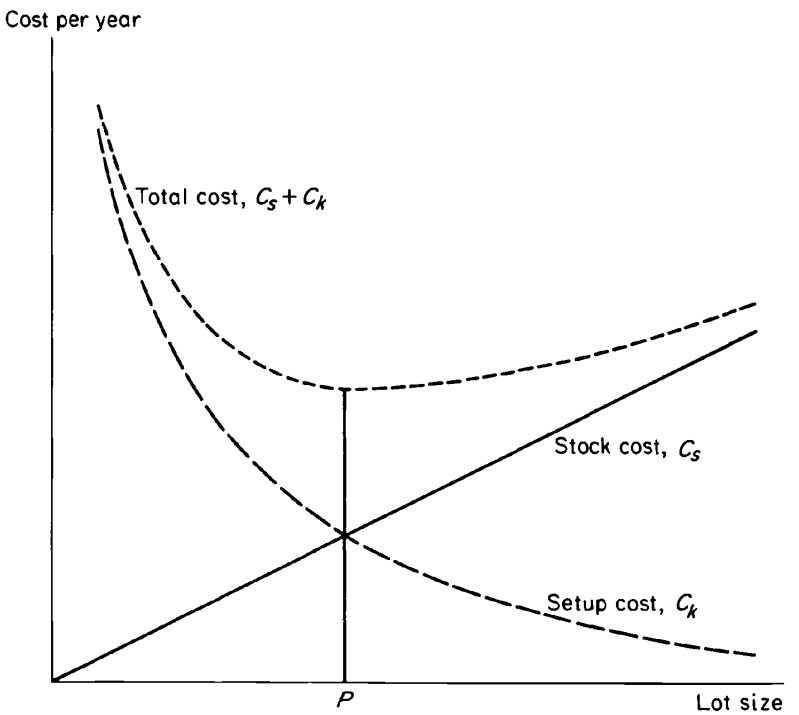
The choice of the appropriate lot or order size (which, divided by about two, is also the choice of the average inventory size) selects the point at which the *sum* of the cost of carrying stock and of supporting the setup costs, as described above, is at a minimum. Where the point is located depends on the pattern of substitution of stock costs for setup costs. Figure 1 depicts such a pattern. It is

of minimum-cost re-order quantity" (p. 9). I take it that the distinction is between an accurate and a very approximate optimization. However, the article was written ten years ago and there has been a large increase in "calculating" over the period.

[20] The size of stocks, I, is a function of lead time (T weeks), lot size (L units) and utilization rate (D units). Without specifying the particulars, note that when $L = TD$, $I = L$. When $L < TD$, $I > L$; when $L > TD$, $I < L$.

FIGURE 1

Optimum Lot Size



characteristic that as stocks increase, they "buy" marginally a declining reduction in setup costs. The optimum size of lots is located where the slopes of the two curves are equal and opposite, which is also the low point on the total-cost curve.

The curves compare the cost, over some period of time (say, a year) for each possible lot size, of the carrying charge for stock on the one hand and the setup costs on the other hand. Stock cost per year in dollars C_s increases more or less proportionately to the size of stocks (assuming the unit value of the stocks is constant) and the stock-cost curve, $C_s Z/2$, is a straight line. The setup cost C_k is a constant; cost per year is this constant multiplied by the number of setups that need to occur—a function of the year's demand D and lot size Z. Thus the setup cost per year varies with lot size according to the formula $C_k D/Z$. The sum of the two $C_k D/Z + C_s Z/2$, represents total cost. To obtain the point of minimal total cost—the point where marginal costs of the two alternatives are equal—we find the derivative of total cost, and set it equal to zero.[21] Thus,

$$Z = \sqrt{2C_k D/C_s}$$

The formulations of the previous paragraph apply also to order-lot-size with the costs of placing, shipping, receiving, and accounting

[21] I have used (with a minor change) the notation given in David W. Miller and Martin K. Starr, *Executive Decisions and Operations Research,* Englewood Cliffs, N.J., pp. 249–250. Chapter 10 has an excellent brief discussion of certain aspects of the inventory problem; see pp. 244–280.

The assumptions as to the nature of the two types of costs in the square-root formula imply that the point p in the diagram is at the intersection of the two curves. However, the basic argument holds under a range of assumptions which do not necessarily imply that lowest total costs occur at the lot size at which the two cost curves intersect.

for an order substituted for setup costs. If orders are delivered immediately, order-lot-sizes will affect only the size of stocks on hand. However if lead times are present, the size of stocks on order will also be influenced, as indicated in note 20, above, and the textual discussion.

The construction carries two important messages: First, optimal lot size or its associated stocks on hand or on order vary proportionately to the *square root of sales.* The square-root relationship derives from the character of the lot-size term. Setup or order cost, a constant which is multiplied by the number of orders per year, approaches infinity as the lot size approaches zero. Second, optimal lot size increases (or decreases) if setup or order costs increases (or decreases) in proportion to the square root of the lot costs.

PATTERNS OF CHANGE

These conclusions would presumably apply to changes in sales and relative costs associated with business cycles. Consider what they may be. Sales or shipments tend to conform to cyclical fluctuations, so the rules just mentioned would cause a damped positive reaction of stock. The relative cost structure is also likely to shift in response to changed level of sales and collateral conditions. Setup costs tend to rise when plants are busy. When a plant is operating on a tight schedule, actual cost of labor for resetting and other purposes is likely to be higher than when many workers have extra time for which they have to be paid in any event. Similarly, when factories are well utilized, down time for machines has genuine opportunity cost, which is not the case when machines are idle. In terms of Figure 1, this means that the setup cost (C_k) curve shifts upward. If so, the optimum lot size will be larger than previously, even if there is no shift in the cost of carrying stock. However, as I indicated earlier, I believe that the realities (though perhaps not the bookkeeping) of stock costs may actually decline. If this were the case, the C_s curve would drop and the optimal point would indicate a still larger lot size.

Insurance or Buffer Stocks

The uncertain pattern of sales, or procurement, and even production schedules, calls for stocks that insure against cost of failing to meet demand. Demand in the sense relevant to stock problems occurs at each station where work is performed. It consists of the desired flow of materials into that operation. Thus the schedule for production starts is the demand relevant to stocks of purchased materials after preliminary operations have been completed. Demand at each station is subject to random variability, but the greatest variability, and an important source of variability at other stations, is at the point of final demand.

The risk of failure to meet demand is a function of the variability of the actual demand per relevant time interval—a day, or week, or month. The cost of protection against stockouts increases rapidly as 100 per cent insurance is attempted. Accordingly, policy objectives, explicitly or implicitly, designate a tolerable incidence of stockouts. Stocks, then, must be adequate to meet, at all times, a demand of a size which is exceeded no oftener than this acceptable frequency, say, 2 per cent of the time. Insurance stocks per se cover the variation from the mean level.

But in order to insure this adequacy, the frequency with which stocks are replenished must also be considered. The problem is perhaps most critical at the purchased-materials stage, where the replenishment period can be relatively long and there is no leeway supplied by anterior stocks.

DETERMINANTS

The critical determinants, then, of the size of insurance stocks of, say, "raw" materials on hand and on order are (1) variability of demand, (2) replenishment period, (3)

tolerable percentage of stockouts, and, in some cases, (4) the time between orders.

Assume experience shows that demand averages 1,000 units a week and ranges between 900 and 1,100. It is larger than 1,080 units only one per cent of the time. Then if mean demand plus 80 units were always on hand, customers would be disappointed on the average only one time in one hundred and this would be the insurance stock required. If the level of sales, mean demand, increases, the range of variation is also likely to increase to something more than the 80 units, but typically the increase is less than proportional. When actual distributions are not known on the basis of the company's history, it may be approximated by a Poisson distribution for which the standard deviation is the square root of the mean. Reference to standard probability tables indicates that $D + 2.326\sqrt{D}$ approximates the stocks that would insure against the same level of stockouts as above. According to this formula, the 1 per cent level of stockouts would be covered by stocks of 1,073 units (rather than the 1,080 units that the previous example assumed that experience had shown); larger levels of demand imply an increase to cover the new mean level plus an insurance factor that increases according to the square root principal.

To effectuate this degree of insurance, it is of course necessary to consider how fast goods can be procured, that is, the replenishment period, T. It is the mean demand plus likely variation over this entire period which must be *on hand and on order* at the beginning of the period; thus required ownership is $DT + 2.326\sqrt{DT}$.[22]

A number of implications of the formula are of interest. First, insurance or buffer stocks consist of stock on order as well as on hand; second, the impact on stock of the variability of sales tends to be muted by the square-root relationship or probably something like it; third, insurance stocks must also keep a constant incremental relationship to sales (the DT term); fourth, the size of the increment depends on the length of the replenishment period; fifth, if the replenishment period varies, the size of buffer stocks on hand or on order will vary proportionately.

[22] The formula applies to an ordering system in common use, the "two-bin system." If the "order-cycle system" is used, the time between orders, N, must be covered and the formula becomes $D(T + N) + 2.326\sqrt{D(T + N)}$. This tends to increase stock on hand somewhat, though stock on order is the same under either system. For a discussion of the two systems see, Whitin, *The Theory of Inventory Management,* pp. 44, 48.

PATTERNS OF CHANGE

As to the cyclical pattern of these variables, that of sales is obvious enough, but how about the variability of sales? Other things the same, it seems reasonable to assume, as has the previous discussion, that random components do not increase proportionately. But they may indeed even decrease if the unforetold fluctuation of required shipments to customers is reduced by increasing lead time (and the resulting increase in backlog of orders) for the articles the company makes (Exhibit 1, line 1.2.6). If so, the size of insurance stocks could be reduced.

There seems every reason to suppose that an increase in lead time for sales orders will be a favored choice of the several ways by which sales can be tailored to available capacity. It is more easily reversed than a rise in price.. Nor will it typically alienate customers if many competitors are doing likewise. At the same time, it produces several benign results affecting stocks. It is likely to reduce the cost of carrying stock by markedly diminishing economic obsolescence, since requirements are known farther ahead. It reduces the need for buffer stocks, since the variance of the shipment schedule is reduced. The last two results have an opposing influence on the size of stocks: the first decreases it and the second tends to increase it because of lower cost. The net result is unknown.

The same logic just applied to sales orders (and unfilled order backlogs) applies to purchase orders (and outstanding-materials or-

ders) since these are the sales orders of some other company, the supplier. And the previous discussion showed that the size of total ownership of materials responds in a somewhat more than proportionate fashion to the relative change in the lead time for materials. How much of the impact will fall on stocks on hand rather than on order is a question that will be discussed later.

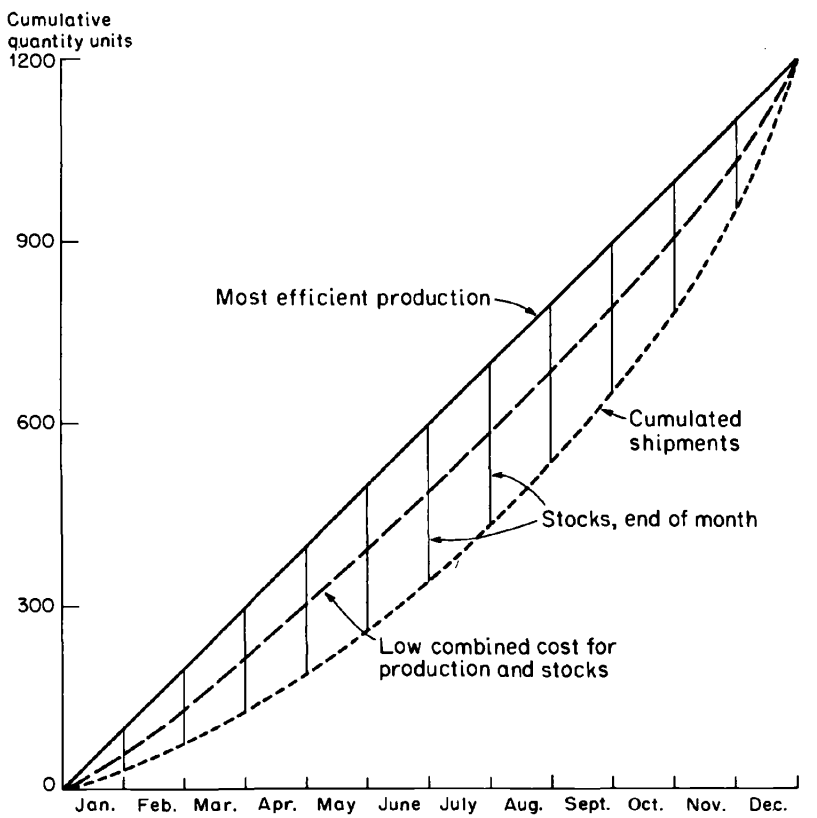
Fluctuation Stocks

The pattern of monthly sales often varies in ways that can be at least roughly foretold. Seasonal patterns, for example, may be reasonably well understood. The expectation of a strike may demand some advance preparation. Far more hazardous and infrequent prophecies about cyclical conditions and trends are, willynilly, incorporated in decisions concerning the size of plants and other things which carry implications about accommodating change via inventories or via overtime work.

Assume, for example, that sales of a given enterprise have a strong seasonal peak toward the end of the year. The cumulative pattern of shipments is shown in Figure 2 by the dotted line. For the year, shipments are 1,200 units and the monthly amounts are indicated by the slope of the line month by month. What path should production follow? If production costs were the only consideration, they would be minimized by the smooth flow implicit in equal monthly lots of 100 units—the path shown by the diagonal solid line. But if this course were followed, inventories would pile up during the first part of the year and then gradually reduce to zero by the end of

FIGURE 2

Cumulative Forecast Shipment, and Two Production Schedules



the year. The size of inventories month by month is shown by the vertical lines on the chart.[23] The carrying costs of these stocks constitutes the opportunity cost of this most efficient production schedule. However, the combined costs of inventory and production are almost inevitably lower at some path intermediate between the two shown, perhaps that of the dashed line. This is especially true in view of the uncertainty that would in practice surround the sales estimate.[24]

A similar type of construction applies to purchased-materials stocks that provide a way of coping with expected seasonal variability in production requirements, or in the sales of retail stores. Materials on order perform an identical function for less immediate requirements. They do so, moreover, without corresponding financing and storage costs.

Just how fluctuation stocks vary with the level of sales is not subject to general formulation. However, the cost of the inventory alternative would tend to increase linearly with the size of stocks as for lot-size stocks depicted in Figure 1. The alternate or opposing cost would tend to be convex to the origin, since ordinarily at least a substantial component of costs would be of the relatively fixed variety. This implies that efficiency of substitution of stocks for the alternative would increase as stocks increase, and increase also as sales increase. Thus, though the square-root relations of stocks to sales would not apply, at least stocks should increase substantially less than proportionately to sales.

Changes in the relative cost of holding stocks and of supporting uneven production schedules would presumably affect fluctuation stocks in much the same way that it affects lot-size stocks. If so, these stocks would tend to increase, other things the same, during expansion. On the other hand, the expansion phenomenon of larger backlogs of sales orders can flatten the required-shipments curve in Figure 2, thereby decreasing optimal stock.

MARKET-PROSPECT STOCKS

Market prospects concern the purchaser's estimate of conditions in markets in which materials are bought. The estimate may be based on sure knowledge; for example, the seller may simply have stipulated that he now will promise delivery in four weeks, whereas formerly the period was two weeks. It may, at the other end of the confidence spectrum, be based on highly uncertain guesses about how delivery periods or prices are likely to change. Delivery periods are a "process time" that must be bridged by stocks. If the length of the period changes, so does the amount of stock required to cover the delivery process. However, as stated earlier, it is useful for analytic purposes to distinguish between the time associated with customary efficient eco-engineering conditions, loosely called hand-to-mouth conditions, and departures from that time associated with particular conditions in the materials markets. Stocks associated with the former are process-time stocks, and those which accommodate the departures are market-prospect stocks.[25]

Note, however, that if market prospects cause replenishment periods to lengthen, then the ownership covering the longer period has the same association with expected sales as

[23] John Magee uses virtually the diagram here presented in "Guides to Inventory Policy, Anticipating Future Needs," *Harvard Business Review,* May–June 1956, p. 32.

[24] For a discussion of the impact of various patterns of sales, see Morton Klein, "On Production Smoothing," *Management Science,* April 1961.

[25] The reader may think that the distinction could more usefully be made on the basis of delivery periods as stipulated by the seller and actions primarily initiated by the buyer. But as will be seen as the evidence is examined, there is so intimate and complex an association between the supply and demand blades of the shears that a distinction along these lines is not useful.

does any process-time stock—it will fluctuate in direct proportion to sales volume. However, market-prospect ownership may reflect changes other than in lead times—in expected prices, or selections, for example; and for these the constant relation to sales does not necessarily apply.

Action based on market prospects is sometimes branded as "speculation." This is a misnomer. Most business life is necessarily speculative, and it serves no useful purpose to single out changing market prospects (including that of expected change in materials prices) as particularly so, and consequently a "bad thing." The purchasing agent should aim to insure that required materials of the desired quality are on hand at the desired place and time, and at the lowest possible price. This objective has its counterpart in the size of stocks of materials on hand and on order. Changes which have occurred, or which he anticipates, in the speed and reliability of deliveries, in their quality, in the range of selections, or in the prices he will pay must influence his behavior. Proper regard for these matters is as much a part of efficient business operation as is attention to future sales, or indeed to any unsure event which will perforce leave its impact on profits. Therefore, though there may be some difference in emphasis, it seems correct to include market-prospect stocks under the general heading of stocks that "buy efficiency."

Determinants

Two sorts of determinants of ownership linked to actual or expected conditions in the materials markets need to be covered. The first group concerns the judgment about what those conditions are, the second, how the judgments affect ownership.

Three types of conditions likely to engage attention are those bearing (1) on prices, (2) on delivery conditions, and (3) on quality, variously defined. These matters are considered more carefully in Chapter 10, but a preview is needed here.

One reason for an increase in ownership is the expectation that materials prices will rise (Exhibit 1, line 3.4). If the expectation were certain, it would be appropriate to increase ownership until the carrying cost of the last increment of stock over a specified period of time equaled the expected increase in price during the period. Thus, under certainty, the *level* of ownership is associated with the expected rate of *change* of prices. If a price expectation is unsure, the expected increase in prices required to instigate a given level of ownership must be greater than if the expectation is certain.[26]

Two other sorts of expectation that may motivate an increase in market-prospect stocks is the belief that the replenishment time for goods of desired quality will lengthen or quality, including selections, will deteriorate (Exhibit 1, line 3.2). These expectations confront management with four alternatives: materials on hand can be increased; orders for the more distant deliveries can be placed; premium prices can be paid for rapid delivery if and when the expectation proves to be correct; adverse quality and delivery schedules, should they materialize, can be compensated for at some other level of the enterprise. These matters involve costs which need to be balanced against the cost of increasing the amount of stocks on hand and on order.

But a judgment is also required (probably most typically an implicit judgment) about how the cost of extending ownership may itself have changed. Risk is a chief part of the cost of outstandings—risk of buying the wrong thing or paying the wrong price. But, as the following section explains, it is likely that risk costs will decrease at just the time when markets are expected to tighten.

Location and Size

The list of factors that motivate market-prospect stocks implies that they may be found

[26] This subject is discussed at some length in Chapter 13.

anywhere in a business. Even finished stocks may be involved under appropriate circumstances—circumstances that make it important to buy materials advantageously and preferable to hold them in finished rather than raw state. But the emphasis falls on purchased-materials ownership, and especially on materials on order. Thus, when markets are expected to tighten, materials stocks on order are likely to increase. But stocks on hand can have a competitive relation to stocks on order in that the additional buying may carry short delivery dates. In any event stocks on hand are likely to have the complementary association of the inverse vestibule effect: they will tend to increase as the larger orders are delivered. The matter is somewhat complicated and deserves a moment's thought.

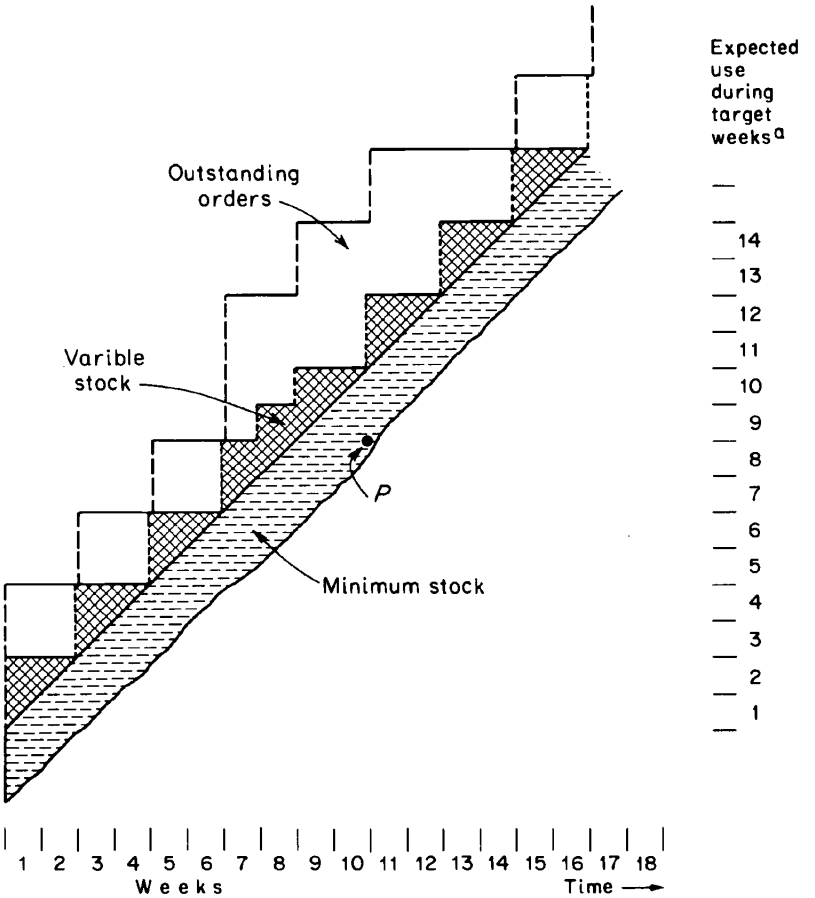
Consider first a situation in which the only change in market conditions is that delivery periods are expected to lengthen; sales are expected to be unchanged and are precisely foretold. Under this admittedly highly unlikely circumstance (delivery periods for materials are probably never expected to lengthen without some expected increase in sales of finished goods), the change in ownership could be almost entirely confined to outstandings. Figure 3 describes materials ownership at a sequence of one-week periods (horizontal axis). Requirements are reviewed and orders placed every two weeks at the beginning of the week. Orders are all delivered exactly at the end of the expected term—first at the end of two weeks, and later (starting with the order of week 7) at the end of four weeks. Stocks on hand are the shaded portion of the diagram and are of two sorts. First, there is some minimum loosely defined amount which is required at all times; it covers insurance stock, error stock, and everything else; it is shown as the unsegmented band at the bottom of the columns. Second, there is a variable stock intended to cover two weeks' expected sales.

Beginning at the left, picture the purchasing agent who starts with his required two weeks' stock on hand plus the minimum, and contemplates what he will order. He should cover expected sales for weeks 3 and 4, since the orders will not be delivered until week 3. The forecast sales for those weeks are shown on the right vertical axis. He places the orders shown by the dashed line at the extreme left. Now time ticks on. In the first week sales use up one week's supply. During the second week, sales continue at the same rate and draw down variable stocks to zero at the end of week 2. At the beginning of the third week, the orders placed at the start of week 1 are delivered, as shown by the dotted vertical line. At the same moment the purchasing agent once again reviews the scene and places orders to cover forecast sales for weeks 5 and 6 (dashed line). The process repeats until the beginning of period 6. Variable stock on hand has averaged one-half of sales over the order period. Stocks on order are uniformly two weeks' supply.

But now the expected delivery period doubles; orders placed at the beginning of week 7 with the regular supplier will not be delivered for four weeks—the beginning of week 11. If the purchasing agent simply placed orders covering forecast sales for the period, by the beginning of week 11 sales would have cut into the minimum safety band for stock on hand to point *P*. Accordingly he places orders covering weeks 11 and 12 with his regular supplier; finds a manufacturer or jobber who will deliver one week's supply of his expected needs in one week and another in two weeks. For this he pays through the nose, but no matter. At the end of two weeks, the beginning of week 9, at his usual order period, he now contemplates expected sales for weeks 13 and 14, and orders these requirements. As a matter of fact, his orders over this period should be somewhat larger than I have described since the longer replenishment period implies larger potential sales variance and therefore larger insurance stock. I simply indicate their impact by a slightly larger (responding to a square-root principle) mini-

FIGURE 3

Materials Stocks on Hand and on Order During Sequential Weeks as Delivery Periods Change



[a] Weeks are numbered successively from the present.

mum-stock. Over this period of the longer replenishment period, outstandings nearly double, in line with the doubled period. But stocks on hand increase only during the period of transition in weeks 7 and 8 (except for the variance buffer).

At the beginning of period 13, the scene shifts. The union contract has been signed, or whatever, and the purchasing agent expects deliveries to return to their customary two-week period. Accordingly he places no new orders but waits for the previous ones of two weeks ago to arrive. He starts ordering in the customary fashion (adjusted to reduce the variance buffer) at the beginning of week 15.

Figure 3 is concerned with market prospects confined to sure and correct estimates of a change in the delivery period, sales unchanged and correctly forecast. If sales had also been expected to increase when delivery periods did, the vertical distance for expected sales for weeks 9 and thereafter would have lengthened, and outstandings, starting at week 7,

would have been larger by four times the weekly increase plus some allowance for the increase in the buffer. Beginning in week 9, the slant of the bottom of the columns, indicating weekly use, would become steeper. But again, so long as sales are correctly forecast, stock on hand can still cover one week's sales on the average (except during the transition). Change in ownership is confined almost entirely to stock on order. Even when sales are expected to decline, an increase in stock on hand could be forestalled by appropriate changes in new orders *if the decline had been correctly forecast.*

The example has dealt with market prospects focused entirely on delivery periods. But if some of the advance buying had been intended to take advantage of selections which were expected to deteriorate, though delivery periods did not lengthen, the additional purchases would move into stocks on hand and increase their size over the longer period covered. If forestalling an expected increase in prices had been the target, the buyer would no doubt prefer to delay deliveries until the material was needed, thereby reproducing as nearly as possible the patterns described in the diagram. But the seller might be quite unwilling to cooperate. The steel industry, for example, has for some years refused to fix prices on customers' orders until their date of delivery.

Finally, uncertainty influences what happens. For one thing, sales cannot be exactly foretold, as we have been assuming. As a result, if forecasts were too optimistic, variable stock on hand will be larger than desired when orders are delivered and contrariwise if they were too pessimistic. However, in the latter case orders for "at once" delivery can often correct the discrepancy. For another thing, the fact that delivery periods, and anything else, cannot be forecast with certainty means that a planned flow of outstandings intended to meet contingencies needs to be reinforced by provisions in terms of stock on hand.

Cyclical Fluctuation

For each of the various sorts of stocks, some influences affecting their size are often external to the sales history of the particular company—changing labor cost and labor availability, financing costs, for example. But in the case of market-prospect stocks these external influences tend as we have seen, to be powerful in their impact. Therefore it is important to know the circumstances that cause them to vary significantly.

Do lengthening delivery periods ordinarily accompany rising demand? Or must plant capacity be fully utilized first? Do rising materials prices usually accompany large order backlogs? Do these things occur at quite different times in different industries or do they tend to cross industry lines and be broadly present? Unfortunately there is little empirical work on the basis of which to answer these and many other relevant questions. The time series to which we turn presently move toward some very tentative answers.

But a few guesses can be tendered on the basis of the logic of the procurement problem in business firms and a few investigations.[27]

[27] Tangential evidence on absence of parallelism between a firm's buying, production, and, sometimes, selling is scattered through the literature on individual industries. It also appears in market reviews in current periodicals, of which the *Commercial & Financial Chronicle,* starting well before the turn of the century, and, more recently, *Business Week* are notable examples. It is seen on financial pages of newspapers.

There have been a few studies of individual industries in which these problems have appeared to play an important part. Cf. Thomas M. Stanback, Jr., "Short Run Instability in the Cotton Broad Woven Goods Industry, 1946–51," unpublished Ph.D. dissertation, Duke University, 1954; Bert G. Hickman, "Cyclical Fluctuations in the Cotton Textile Industry," unpublished Ph.D. dissertation, University of California at Berkeley, 1951; Ruth P. Mack, *Consumption and Business Fluctuations: A Case Study of the Shoe, Leather, Hide Sequence,* New York, National Bureau of Economic Research, 1956. Explicit analysis of buying waves appears in *ibid.,* Chapters 9 and 16; also in Ruth P. Mack and Victor Zarnowitz, "Cause and Consequence of Changes in Retailers' Buying," *American Economic Review,* March 1958, pp. 19–22; Mack, "Business Expectations and the Buying of Materials,"

I might add that the evidence developed in this book has turned out to be entirely in line with these suppositions.

The willingness to extend the number of weeks for which materials are purchased is fostered by (1) increasing and reasonably strong demand, (2) increasing backlogs of orders for the product the company sells, (3) conditions in markets in which materials are bought that suggest rising prices and a reduction in the speed and certainty with which materials of desired quality can be bought and delivered. All three developments are interrelated.

Rising demand for the product an industry sells has a multiple relation to prospects in the markets in which the industry buys. Favorable market prospects are encouraged by rising demand, since materials markets are not likely to tighten when demand is slack and demand for materials is derived from demand for the products in which the materials will be incorporated.

If strong and rising demand for the product a company sells causes increasing backlogs of unfilled orders, a further link to materials markets is forged. With orders on the books, the purchasing agent can elect *when* to buy materials. He can buy them at the time the sales order is written or he can wait more nearly until the time when materials are needed to meet production schedules. During this entire "period of option," the deliberate timing of buying is relatively free of two important elements in the cost of advance buying—that of buying the wrong thing and of paying the wrong price. Indeed, *failure* to buy materials when sales orders are written could imply the risk of paying the wrong price, if the price at which materials were eventually purchased was higher than the materials cost embodied in the selling price of finished goods.

But though favorable demand for final products is necessary to optimistic expectations about the materials markets, it is not sufficient to it. These markets themselves are a tracery of signs and footprints that the purchasing agent reads daily. Telephone wires, salesmen, competitors, trade journals, selling floors, affairs in industries buying the same materials—all contribute to the sensitive reporting that helps to form the judgment about whether this is a good time to buy short or long of requirements and to what extent.

One consideration may be that of paying the right price. Another, and usually more important, is the need to have physical goods where they are wanted when they are wanted, and in the assortments and quality wanted, in view of the prevailing or expected delivery conditions.[9] But typically the two considerations dictate doing the same thing—buying ahead or refraining from doing so—at the same time.

These considerations on which market expectations are based in an individual firm are at any given time likely to change in similar ways for most firms in an industry. The judgments are actually addressed, for the most part, to conditions in sales and materials markets as a whole, rather than simply to the affairs of the customers or suppliers of a particular firm. Consequently, an entire industry is likely to move toward an increase in market-prospect stocks more or less at the same time.

The analysis has indicated that when materials markets are expected to tighten, stocks on order increase. If extension is dictated by actual or expected lengthening in delivery periods, stocks on order will increase proportionately. If extension is caused by expectation of rising prices, the quantitative dimensions are obscure. If optimistic market prospects occur when sales are rising, and they are not likely to occur at other times, sales and delivery periods interact in their impact on stocks on order. Stocks on hand increase because of the need to increase buffers; they may increase as an alternative response to optimistic expectations; they are likely to in-

in Mary Jean Bowman (ed.), *Expectations, Uncertainty, and Business Behavior,* New York, 1958, pp. 106 ff.

crease as the larger outstanding orders are delivered and move through the plant, the inverse vestibule effect. These are the direct effects; there are many indirect ones, among which are those associated with errors in guesses about market prospects and about sales over the longer period for which commitments have been made.

UNPLANNED STOCKS

Imagine a firm whose executives were thoroughly versed in management lore. They knew all about the appropriate size of stocks serving each of the five purposes that I or anyone else have described. They understood their interrelations and the cost structure of their own business. Theoretically, then, stocks could be set and kept at optimal levels.

But stocks would not in fact be at these theoretical levels. Instead "unplanned stocks" would exist. They would exist, first, because it was not worthwhile to plan for the last little contingency. They would exist, second, because most plans involve only partially foreknown future events, and cannot, therefore, except by luck, be precisely executed, a failure to which some ineptitude is also likely to contribute.

However, the theoretical levels themselves do not apply in most firms because ignorance obscures knowledge necessary to plan the proper size of stocks and to enforce such plans as are formulated. Ignorance, then, is a third reason for unplanned stock.

Passive and Unintended Stock

We can group unplanned stocks that result from ignorance and from the limits to advantageous planning, and think of them as "passive." [28]

Passive stocks reflect the decision not to decide. They result from the election that further control of stocks, given the degree of control that actually could be achieved, is not worth the executive time and other costs that would be involved. Thus they are always a matter of degree. Stocks are never passive in the sense that there are no limits to their size, but only in the sense that the limits are much broader than usual and only roughly determined. Passive stocks, then, constitute a residual category after process stocks and the other efficiency stocks have been provided for. They are included in Exhibit 1, primarily in lines 2.1.2, 3.1.2 and 3.2.3—coping with variability in demand at one level by carrying stocks at a later level, or with variability in supply by carrying stocks at the same level.

Note that the difference between unplanned and insurance or fluctuation stocks is somewhat subtle, and there are doubtless several ways of drawing the line between them. My own thinking focuses on the impact of uncertainty and the cost of information. If all relevant factors were known and if their impact could be evaluated at reasonable cost, then it would be worthwhile to plan the size of any stock and there would by definition be no passive stock. But for many purposes it seems clear that relevant factors cannot be assessed with sufficient accuracy to justify the *executive time* that would be involved in planning, and in enforcing the plans to the degree necessary for the planning to pay off.

[28] Cf. Ruth P. Mack, "Characteristics of Inventory Investment: The Aggregate and Its Parts," *Problems of Capital Formation*, Studies in Income and Wealth 19, Princeton University Press for National Bureau of Economic Research, 1957, p. 480: "Passive inventory investment or disinvestment takes place in part because plans about the proper size of stocks are hardly ever precise figures; instead, they are ranges, and variation within the range or band is a matter of indifference. Passive stock change also occurs when business objectives that focus on other matters than the appropriate size of stocks nevertheless affect their size." Some of this last group I would now exclude because attention to opportunity costs means that this behavior may represent a close weighing of relative costs of the two means of promoting efficiency.

Of course, what is feasible differs from time to time and firm to firm. Several decades ago stocks of finished goods were more commonly passive than they are today. Both management expertise and computers have made it feasible to plan their size within far narrower limits than formerly. The cost of planning can be reduced by rules of thumb, and if they are poor rules it is a question whether they define planned or passive stock. As we shall see later, the constant average stock-sales ratio may sometimes fall in this category. My point is simply that in matching empirical evidence with the presumptions of management theory, or the firm's-eye view of the procurement problem, it is realistic, I believe, to have in mind, in addition to the various explicit functions that stocks serve, and for which explicit planning is called for, the further category of stocks which in view of the uncertainties of the real world are the tail unwittingly wagged by the dog. It does not pay to try to plan them except in most approximate terms.

The second type of unplanned change in stock, unintended stock, results from the failure to *enforce* plans. Stock change of this sort constitutes divergence from plans. The divergence is recognized by the firm as errors which need to be reversed. It is unintended in the sense that it is not tolerated for long. Unplanned stock change of this sort has long been acknowledged in economic analysis. The distributed lag mentioned in the previous chapter is one way of coping with it at an econometric level.

Positive or negative unintended stock results in part from management lags. Discipline may be inadequate to enforce instructions; information may be poor or tardy. Unintended stocks may also result from the fact that the correct size of stocks is typically a function of conditions at a later period than the time when action that partly determines their size is taken. If the relevant conditions are incorrectly anticipated, some part of the actual change in stocks will be unintended.

The relevant conditions may be the level of sales. If materials are bought on the basis of an incorrect forecast of sales, stocks will not conform to plans and the divergence must be corrected. We shall see that the definition of error, and consequently the efforts to reverse it, appear to differ in connection with stocks of department stores on the one hand and materials stocks of durable-goods manufacturing on the other. The relevant conditions may also be changes in opportunity costs of the sort that we have discussed. Those concerned with market conditions are particularly subject to forecast error, and consequently unintended change in ownership based on erroneous expectations can be an important type of error stocks which subsequent action endeavors to reverse.

Location and Patterns of Change

Unplanned stocks of either the passive or unintended sort can of course occur at any point in the productive process. When they are of the unintended variety, efforts to reverse them can take the form of changes in selling prices or changes in purchasing of materials. Since the latter method is a very usual one, and since it can take effect quite swiftly, it seems reasonable to suppose that errors in materials stocks on hand will imply a compensating change in stock on order, outstandings, and vice versa, and the sum of the two, ownership, will be less subject to unintended change than either component. The time series that we shall be examining support this supposition.

Patterns of change in passive stock are hard to anticipate, but in general it seems likely that they will be unresponsive to the things that prescribe changes in stocks and thus will tend to have generally inverse cyclical patterns. But there may also be a tendency for this inverse behavior to be less than it otherwise would be because of a shift in attitudes. When business is good there is a general feeling both on the part of management and customers that service rather than economy is

called for. At such times, there also is a natural tendency for stock problems to take care of themselves. This may hide for a while the fact that unplanned stocks are in effect gaining relative to the rest. The general effect would be to dampen the possible countercyclical behavior of these stocks.

For unintended changes, errors associated with incorrect forecasts of sales depend on how sales forecasts are made and how errors are adjusted; and these things we do not know. Many surveys have shown that changes in sales tend to be underestimated. If so, when sales are rising, error stocks may tend to be negative. At the onset of recession, on the other hand, the level of sales is likely to be overestimated, since not to do so would imply that prized accomplishment, prophesying the turn. Yet here again adjustment is not likely to be overswift, and in the meantime stocks are larger than optimum; that is, error stocks are positive. The effect might well be exaggerated by the fact that replenishment periods tend to be long around business peaks and consequently the period for which sales must be forecast long.

There is no need to rehearse how failure to anticipate delivery periods, materials prices, or any other events, such as strikes or weather, can cause stocks to be larger or smaller than they would be had the event been correctly foreseen. But it is important, in view of the great emphasis in the literature on sales forecasts, to remember that these things too must be forecast if inventory holdings are to be optimal.

SUMMARY AND CONCLUSIONS

The first chapter pointed to unsolved problems in aggregative analysis of inventory investment. Efforts to solve them must be based on what businessmen actually try to do, and that is what the present chapter has attempted to discover. It will be useful to summarize this discussion, keeping the purpose of aggregative analysis in mind. What has been learned about how stocks are likely to change in the course of fluctuation in economic affairs? Particularly, what has been learned about how to study these changes and their causes?

The first set of conclusions concern how the information conveyed by orders can be integrated with the stock-carrying problem. Unfilled orders for a particular firm are of two sorts: the orders for materials which the supplier has not yet filled are a secondary materials stock; the unfilled orders for the product that the company makes affect the risk and other costs of carrying stocks or other assets. The new orders placed for materials afford a net picture of the current procurement situation as the company views it; new orders received for the product the company makes are a major source of information on which future plans are founded. That new orders for materials are the orders for the product that another company makes is no reason to smudge the differences in the informational content and the action implications of the same piece of paper viewed from the two points of view. This point is further developed in the next chapter.

Here it is prelude to a specific conclusion: it is necessary to examine materials stocks on order along with those on hand. Like stocks anywhere in a productive process, stocks on order provide availability at the appropriate time to a particular processing station. Further, none of the functions that stocks serve are unaffected by a change in the time required for orders to be delivered. Therefore, the correct size of stocks on hand must be determined with a view to how fast new orders arrive. At the same time, the length of the replenishment period is itself one of the choices that the purchaser must make in terms of trade-offs between stocks on order and

stocks on hand, or between other ways of handling procurement problems.

This line of thought explains why it may be useful to confine empirical analysis of time series to stocks of materials rather than to all stocks. The focus forgoes the advantage of a comprehensive analysis of stocks on hand. But it enjoys the advantage of studying the relation between materials on hand and those on order, without confusing the picture by the different price levels of "raw," in process, and finished goods.

The effort to understand the forces shaping the cyclical patterns of stocks has been built up in three parts: (1) by picturing the functions that stocks perform, (2) by spotting the influences within a business and external to it to which stocks need to react in order to do their job, (3) by speculating as to how these factors may vary during business fluctuations.

The functions of stocks are, on the one hand, to support the time required for an operation to be performed and, on the other hand, to buy efficiency by substituting a lesser cost of carrying stocks for a larger cost of accomplishing the same management purpose in other ways. Both of these functions and each of the subfunctions of buying efficiency are sensitive to the volume of sales, but their degree of sensitivity varies.

How stocks move relative to sales is a basic determinant of their cyclical behavior; therefore, it will be useful here to list the functions in the order of their sensitivity to sales. I start with the most sensitive. The analysis is phrased in terms of stocks serving each function. In actuality, of course, the same physical inventories can serve several functions.

1. *Process-time stocks* support the time required for economic processes to take place when they are executed with usual efficiency in an eco-engineering sense. The processes include the physical transformations involved in preparation of materials, processing, and preparing to sell; they also include the time normally required between the writing of an order for materials and their delivery to purchasers. Stocks serving this function tend to maintain a constant proportional relationship to the volume of sales unless the use of overtime or multiple-shift operations changes the relation of process time to calendar time. Since increased plant utilization is likely to take these forms during prosperity, process-time stocks on hand, other things the same, are likely to change somewhat less than in proportion to sales during business expansions and contractions. Process-time stocks on order would presumably tend to maintain a strictly proportional relation, other things the same.

2. *Market-prospect stocks* reflect the opinion that market conditions may depart from the hand-to-mouth condition. Insofar as delivery periods lengthen, the additional stock on order will cover the expected volume of sales over the additional period for which provisions are made; thus the impact of changing delivery periods and sales have a multiplicative relation. Insofar as changing market conditions affect judgments about prices, selections, or the like, the association with sales of either stocks on hand or on order is indeterminate.

3. *Fluctuation stock* has a pattern not subject to general formulation; it is likely to be somewhere between that of insurance and process-time stocks.

4. *Insurance stock* per se has something like a square-root association to sales. But since mean demand over the replenishment period must also be on hand or on order at the start of the period, a constant incremental relation is also present which would need to be covered by stock of some sort.

5. *Lot-size* (including order-size) *stock* has an approximately square-root association.

6. *Unplanned stock* provides a break in the link to sales except within a broad band. This may tend to give the passive aspect of unplanned stock an inverse association with sales proper, other things the same. For the unintended portion, resulting from the inevitable failures of control instruments in the first instance, the inverse pattern could feature the

rate of change in sales if forecasts tended to be conservative.

Differences in the functional character of the sales link imply differences in the impact of changing sales on stocks at the several stages of processing. Likewise, shifts in particular opportunity costs have more influence on stocks at one place in a business than at another. The numerical functional listings just set down will serve to illustrate these varying impacts. Stocks of finished goods are likely to be weak in functional classes 1 and 2, and strong in 3, 6, and perhaps 4. In-process stocks are likely to consist of all functional classes, though perhaps usually less of 2 and 6. Materials stocks will also be of diverse sorts, with perhaps the exception of 6, but with special emphasis on 2; materials stocks on order will be particularly dominated by 1 and 2. The emphasis on the higher-numbered functional classifications for finished stocks is in accord with statistical findings that they characteristically show a damped positive or inverse association to sales. It suggests that cyclical behavior of the stocks at the several stages will respond to different influences and that separate analysis of each sort is bound to be useful.

A chief implication of the analysis is that the link to sales, other things the same, does not prescribe a constant ratio to sales of stock on hand or of both stocks on hand and on order. The stock-sales ratio should, purely on the basis of efficient servicing of sales, have a pattern inverse to that of sales.

Just how marked the inverse pattern would be depends on matters such as the size of passive stocks, the length of replenishment periods, sales variance, and the like. Of all the stock functions that have been described, the only one requiring as much as a proportional association to sales is process-time stocks, and these only if there were no increase in operations in line sequence. All the rest, unplanned stock particularly, would tend to dampen changes in the level of stock relative to that of sales, other things the same.

A second finding of the analysis is that many other objective considerations can influence the size of stocks and how they change. They include changes in the costs of carrying stocks and of all of the alternative methods of achieving the efficiencies which stocks can provide. Thus as business improves, the unit costs of carrying stock may decline because of lower financing costs and lessened risk of obsolescence when sales and perhaps backlogs of orders are rising. On the other hand, the cost of competing ways of coping with management functions that stocks serve may rise in prosperity. For example, flexible production schedules encounter higher hire-and-fire and other labor costs, small-lot buying requires premium prices when suppliers are busy, and fast replenishment periods can only be achieved at premium prices or by sacrificing selections and quality. Such changes would tend to accentuate the rise in stocks during prosperity. Thus the stock alternative becomes more desirable because of both lower costs and higher productivity.

But the changes that occur during business fluctuations are not simply the function of objective situations and how they impinge on business decisions concerning buying and carrying stocks. Situations must be observed and appreciated, and this is a subjective process that may have patterns of its own. Even objective facts must filter through this learning process before they affect decisions. But we have seen that expectations as well as facts affect ownership decisions, and these are particularly sensitive to how information is generated, collected, and appreciated. Certainly there is some sort of group process involved in how expectations are formulated and acted upon. Whatever it is, changes in ownership, especially changes associated with market prospects, must bear its stamp.

The examination, then, of the functions that stocks serve has opened up a wealth of possibilities. The picture is much in need of the verification, modification, and enrichment which empirical study of the management of

stocks in business firms could supply. But instead, such empirical studies as are undertaken in this book make use of aggregative time series alone. They are confined to stocks of materials. However, they utilize information on stocks both on hand and on order, thereby making it possible to observe elements affecting stocks which otherwise would remain invisible or hopelessly obscured.

Study of the time series should help to sharpen the picture of fluctuation and its possible causes. We have seen that materials stocks on order and on hand can be influenced by a very considerable number of occurrences normally associated with rising and ebbing tides in business or industry affairs. The changes involve expectations and the information on which they are based as well as actual events of many sorts. One cannot say now how significantly these influences will impinge on the size of stocks, but the analysis has served to indicate that if stocks behave in one fashion it is likely to imply one sort of relative weighting of a number of causal factors, whereas if they behave in another fashion a different weighting system is implied.

This is not as meager a result as it may seem. We shall see that the aggregate time series for stocks and ownership do appear to change far more than the sales-service link alone implies. This over-responsiveness does seem to bear a strong resemblance to changes in market conditions and other influences to which this chapter has pointed. New and unfilled orders received and placed tell distinctive yet interrelated stories.

In addition, more subtle implications of our study of business problems fall into place in the interpretation of the behavior of aggregates. Aspects of that behavior are not entirely explicable in terms of the objective facts of the business environment and how they change. First, they seem to reflect a process that involves expectations and how they are formulated, acted upon, and how these actions generate new information and further expectations. And certainly this learning and reacting process has kept winking at us from behind every fact that has been reported. Second, there is an implication of the material here presented which it has not been possible to display without further complicating a too complicated story. It is this: where the functions that inventories serve are so numerous and so intimately involved in the *particulars* of business problems, sensitivity to any single influence will vary greatly among different inventory goods, different firms, and, of course, different industries. If so, aggregate response to change in any single influence will have to creep over a frequency distribution of firms that reflects these varying sensitivities —a time-consuming process. This notion is one of the cornerstones in my final explanation.