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# A COMPARISON OF ROBUST AND VARYING PARAMETER ESTIMATES OF A MACRO-ECONOMETRIC MODEL

BY THOMAS F. COOLEY*

*Four estimators of econometric models are compared for predictive accuracy. Two estimators assume that the parameters of the equations are subject to variation over time. The first of these, the adaptive regression technique* (ADR), *assumes that the intercept varies over time, while the other, a varying-parameter regression technique* (VPR), *assumes that all parameters may be subject to variation. The other two estimators are ordinary least squares* (OLS) *and a robust estimator that gives less weight to large residuals. The vehicle for these experiments is the econometric model developed by Ray Fair.*

*The main conclusion is that varying parameter techniques appear promising for the estimation of econometric models. They are clearly superior in the present context for short term forecasts. Of the two varying parameter techniques considered,* ADR *is superior over longer prediction intervals.*

## 1. INTRODUCTION

Two recent studies of the performance of alternative estimation techniques indicate that considerable gains in forecasting accuracy may be achieved by using more advanced and difficult techniques. These studies by Fair [7, 8] apply a variety of estimation techniques to the stochastic equations of the model described in Fair [6]. The purpose of this paper is to extend this comparison of estimators by examining the performance of two varying parameter estimation techniques in the context of the same model. The two estimation techniques are compared in terms of the accuracy of ex ante predictions, with OLS and the most successful robust estimator obtained in Fair [8]. Some within-sample prediction results are also examined.

It is a well known fact that many of the macro-econometric models which are used for forecasting are incapable of producing accurate forecasts without the regular and extensive use of constant adjustments.[1] Fair [6] has argued that part of the need for constant adjustments in many models appears to be due to serial correlation in the error terms. Thus, the formal treatment of the serial correlation problem is one of the features of the estimation techniques he considers. The assumption of serial correlation in the error terms does not, however, completely resolve the apparent dichotomy between standard estimation theory and common forecasting practice. An examination of the constant adjustments often reveals what appear to be permanent structural shifts in the equations. One of the estimators considered in this study, the Adaptive Regression technique, resolves this dichotomy by assuming that the constant is subject to both permanent and transitory changes over the sample period.

The other estimator considered in this study is a logical extension of the first. Once one admits to the possibility of permanent structural shifts in the intercept it is reasonable to look into the structural stability of the relationship as a

[1] See for example Evans et al. [5].

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whole. In recent years there has been increasing recognition of the fact that the aggregative relationships we deal with in econometrics represent such complex interactions of behavioral and technical phenomena that it is not feasible to assume that relationships are stable over long periods of time. This feature of econometric relationships has been well explored in [3], [4] and [9], and Fair [6] in his original development of the model considered herein acknowledges that the objective is to develop a reasonably stable forecasting model rather than a "structural model". The problem of estimating relationships with time varying parameters has been approached imaginatively by several authors. Work by Rosenberg [10, 11] and Sarris [12] has greatly increased the feasibility of estimating relationships with time varying parameter structures. This study considers only the estimator developed in Cooley and Prescott [3, 4] because of its computational ease given the limited sample size and because it is a natural extension of the adaptive regression model.

It is worth noting at this juncture that this study is not intended to be a formal comparison of estimation techniques. Indeed, the proper way to compare the performance of varying parameter techniques is in the context of well designed Monte Carlo experiments in which we can compare their performance on models where the true structure is varying, the true structure is fixed and the structure is simply mispecified.[2] The only criterion of comparison considered here is the predictive accuracy of the estimators in the context of a model which has many unique features. Nevertheless, the Fair model provides a convenient vehicle for the comparison of estimation techniques because it has been used extensively for this purpose in other studies. Whether or not the results obtained in this study are likely to hold elsewhere is an open question but they at least indicate that varying parameter estimation methods are worthy of further investigation.

## 2. The Fair Model

The equations of the Fair model are presented in Table 1. The model is described completely in [6] and will not be elaborated upon here. There are few differences between the original Fair model and the version used in this study. These differences are discussed briefly in [8] and enumerated at the end of Table 1. The version of the model used in this study was kept identical to that reported in [8] to maintain the comparability of results. There are, however, some features of the model specification which should be commented on at this point.

Dummy variables D644, D651, D704 and D711 have been added to the CD, V and M equations and dummy variables D704 and D711 were added to the IP equation. The purpose of these variables is to account for the effect of two major auto strikes. The question that arises is whether these variables should be included when varying parameter estimation methods are applied. In this study it was decided to retain them because the comparison being made is a modest one and to the extent that these represent discrete disruptions and not part of the continuous pattern of variation it is reasonable to treat them as such.

[2] This work is currently being carried out at the NBER Computer Research Center.

The sample period used for estimation and prediction was 1960-II through 1973-I, the same as that used in Fair [8]. The choice of this sample period reflects the fact that this model is designed to be a forecasting rather than a long term structural model. This shorter sample period at least insures that the relationships are likely to be more stable than they would if data extending farther into the past were used. This is not really at variance with common practice in macroeconometric modelling which rarely employs data from before the early to midfifties even though such data are generally available. It is at variance with the statistical theory which underlies econometric method, however, in that it neglects sample information which could improve our knowledge of the parameters in these models. The fact that it is not feasible to use the information because of structural change simply highlights the fact that either the models need to be more carefully formulated or estimation techniques which assume structural change should be used or, preferably both.

**TABLE 1**
**THE EQUATIONS OF THE MODEL**

| | |
|---|---|
| *Stochastic Equations* | |
| (3.3) | $CD_t = \beta_{11} + \beta_{12}GNP_t + \beta_{13}MOOD_{t-1} + \beta_{14}MOOD_{t-2} + \beta_{15}D644_t + \beta_{16}D651_t + \beta_{17}D704_t + \beta_{18}D711_t$ |
| (3.7) | $CN_t = \beta_{21}GNP_t + \beta_{22}CN_{t-1} + \beta_{23}MOOD_{t-2}$ |
| (3.11) | $CS_t = \beta_{31}GNP_t + \beta_{32}CS_{t-1} + \beta_{33}MOOD_{t-2}$ |
| (4.4) | $IP_t = \beta_{41} + \beta_{42}GNP_t + \beta_{43}PE2_t + \beta_{44}D704_t + \beta_{45}D711_t$ |
| (5.5) | $IH_t = \beta_{51} + \beta_{52}GNP_t + \beta_{53}HSQ_t + \beta_{54}HSQ_{t-1} + \beta_{55}HSQ_{t-2}$ |
| (6.15) | $V_t - V_{t-1} = \beta_{61} + \beta_{62}(CD_{t-1} + CN_{t-1}) + \beta_{63}V_{t-1} + \beta_{64}(CD_{t-1} + CN_{t-1} - CD_t - CN_t) + \beta_{65}D644_t + \beta_{66}D651_t + \beta_{67}D704_t + \beta_{68}D711_t$ |
| (10.7) | $PD_t - PD_{t-1} = \beta_{71} + \beta_{72}\frac{1}{20}\sum_{i=1}^{20} GAP2_{t-i+1}$ |
| (9.8) | $\log M_t - \log M_{t-1} = \beta_{81} + \beta_{82}t + \beta_{83}(\log M_{t-1} - \log M_{t-1}H_{t-1}) + \beta_{84}(\log Y_{t-1} - \log Y_{t-2}) + \beta_{85}(\log Y_t - \log T_{t-1}) + \beta_{86}D644_t + \beta_{87}D651_t + \beta_{88}D704_t + \beta_{89}D711_t$ |
| (9.10) | $D_t = \beta_{91} + \beta_{92}t + \beta_{93}M_t$ |
| (9.11) | $\frac{LF_{1t}}{P_{1t}} = \beta_{10.1} + \beta_{10.2}t$ |
| (9.12) | $\frac{LF_{2t}}{P_{2t}} = \beta_{11.1} + \beta_{11.2}t + \beta_{11.3}\frac{M_t + MA_t + MCG_t + AF_t}{P_{1t} + P_{2t}}$ |
| *Identity Equations* | |
| Income Identity | $GNP_t = CD_t + CN_t + CS_t + IP_t + IH_t + V_t - V_{t-1} + EX_t - IMP_t + G_t$ |
| (10.5) | $GAP2_t = GNPR_t^* - GNPR_{t-1} - (GNP_t - GNP_{t-1})$ |
| (10.8) | $GNPR_t = 100\frac{GNP_t - GC_t}{PD_t} + YG_t$ |
| (10.9) | $Y_t = GNPR_t - YA_t - YG_t$ |

(9.2) $$M_tH_t = \frac{1}{\alpha_t}Y_t$$

(9.9) $$E_t = M_t + MA_t + MCG_t - D_t$$

(9.14) $$UR_t = 1 - \frac{E_t}{LF_{1t} + LF_{2t} - AF_t}$$

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*Definition of Symbols*

| | |
|---|---|
| $CD_t$ | = Consumption expenditures for durable goods, SAAR |
| $CN_t$ | = Consumption expenditures for nondurable goods, SAAR |
| $CS_t$ | = Consumption expenditures for services, SAAR |
| †$EX_t$ | = Exports of goods and services, SAAR |
| †$G_t$ | = Government expenditures plus farm residential fixed investment, SAAR |
| $GNP_t$ | = Gross National Product, SAAR |
| †$HSQ_t$ | = Quarterly nonfarm housing starts, seasonally adjusted at quarterly rates in thousands of units |
| $IH_t$ | = Nonfarm residential fixed investment, SAAR |
| †$IMP_t$ | = Imports of goods and services, SAAR |
| $IP_t$ | = Nonresidential fixed investment, SAAR |
| †$MOOD_t$ | = Michigan Survey Research Center index of consumer sentiment in units of 100 |
| †$PE2_t$ | = Two-quarter-ahead expectation of plant and equipment investment, SAAR |
| $V_t - V_{t-1}$ | = Change in total business inventories, SAAR |
| †$AF_t$ | = Level of the armed forces in thousands |
| $D_t$ | = Difference between the establishment employment data and household survey employment data, seasonally adjusted in thousands of workers |
| $E_t$ | = Total civilian employment, seasonally adjusted in thousands of workers |
| †$GG_t$ | = Government output, SAAR |
| $GNPR_t$ | = Gross National Product, seasonally adjusted at annual rates in billions of 1958 dollars |
| †$GNPR_t^*$ | = Potential GNP, seasonally adjusted at annual rates in billions of 1958 dollars |
| $LF_{1t}$ | = Level of the primary labor force (males 25–54), seasonally adjusted in thousands |
| $LF_{2t}$ | = Level of the secondary labor force (all others over 16), seasonally adjusted in thousands |
| $M_t$ | = Private nonfarm employment, seasonally adjusted in thousands of workers |
| †$MA_t$ | = Agricultural employment, seasonally adjusted in thousands of workers |
| †$MCG_t$ | = Civilian government employment, seasonally adjusted in thousands of workers |
| $M_tH_t$ | = Man-hour requirements in the private nonfarm sector, seasonally adjusted in thousands of man-hours per week |
| †$P_{1t}$ | = Noninstitutional population of males 25–54 in thousands |
| †$P_{2t}$ | = Noninstitutional population of all others over 16 in thousands |
| $PD_t$ | = Private output deflator, seasonally adjusted in units of 100 |
| $UR_t$ | = Civilian unemployment rate, seasonally adjusted |
| $Y_t$ | = Private nonfarm output, seasonally adjusted at annual rates in billions of 1958 dollars |
| †$YA_t$ | = Agricultural output, seasonally adjusted at annual rates in billions of 1958 dollars |
| †$YG_t$ | = Government output, seasonally adjusted at annual rates in billions of 1958 dollars |
| †$D644_t$ | = Dummy variable: 1 in 1964 IV, 0 otherwise |
| †$D651_t$ | = Dummy variable: 1 in 1965 I, 0 otherwise |
| †$D704_t$ | = Dummy variable: 1 in 1970 IV, 0 otherwise |
| †$D711_t$ | = Dummy variable: 1 in 1971 I, 0 otherwise |

*Differences between present model and model in Fair [4], Table 11-4*

1. Housing starts ($HSQ_t$) exogenous.
2. Imports ($IMP_t$) exogenous.
3. Price equation (10.7) linear and length of lag is 20 rather than 8.
4. In equation (9.12), $M_t + MA_t + MCG_t$ replaces $E_t$.
5. Strike dummy variables added to equations (3.3), (4.4), (6.5) and (9.8).

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Notes: † Exogenous variable.
SAAR = Seasonally adjusted at annual rates in billions of current dollars.

### 3. Estimation Methods

The estimation methods chosen for comparison in this study are ordinary least squares (OLS) and the most promising of the robust estimators investigated in [8]. This robust estimator is an approximate least-absolute-residual (LAR) estimator. If we write the typical structural equation of the model as

$$F_i(Y_t, X_t, \beta_i) = u_{it} \qquad \begin{array}{l} i = 1, \dots G \\ t = 1, \dots T \end{array} \tag{1}$$

where $Y_t$ is a row vector of endogenous variables, $X_t$ is a row vector of exogenous variables, $\beta_i$ is a vector of parameters and $u_{it}$ is an error term, the LAR estimates are obtained by minimizing

$$Q = \sum_{t=1}^{T} |u_{it}| \tag{2}$$

with respect to the unknown parameters. Typically, this is solved by linear programming, but, because the Fair model assumes serial correlation, $u_{it}$ is a non-linear function of the unknown parameters. Consequently, LAR is approximated by a weighted least squares (WLS) estimator in which the minimand is redefined as

$$Q = \sum_{t=1}^{T} \frac{(u_{it})^2}{|u_{it}|} \tag{3}$$

and is minimized iteratively.

The adaptive regression estimators (ADR) are discussed thoroughly in [1, 2] and the varying parameter estimators (VPR) are developed in [3, 4]. Briefly, these estimators assume that the $\beta_i$ of equation (1) can be represented by the following process[3]

$$\begin{aligned} \beta_{it} &= \beta_{it}^{p} + v_{it} \\ \beta_{it}^{p} &= \beta_{i,t-1}^{p} + \omega_{it} \end{aligned} \tag{4}$$

where $\beta_{it}^{p}$ represents the permanent component of the parameter process. The errors $v_{it}$ and $\omega_{it}$ are independent random variables with mean zero and covariance matrices

$$\begin{aligned} \mathrm{Cov}(v) &= (1 - \gamma)\sigma^2 \Sigma_v \\ \mathrm{Cov}(\omega) &= \gamma\sigma^2 \Sigma_\omega. \end{aligned} \tag{5}$$

If $\gamma$ is significantly different from zero the implication is that the parameters are subject to permanent change. Specification of the elements of $\Sigma_v$ and $\Sigma_\omega$ represent our prior beliefs about the parameters which are changing. In the ADR technique the covariances reduce to scalars and the appropriate elements of $\Sigma_v$ and $\Sigma_\omega(\sigma_v^{11}, \sigma_\omega^{11})$ are unity which makes estimation more efficient. The VPR estimates require specific prior assumptions about $\Sigma_v$ and $\Sigma_\omega$. In this study alternative plausible assumptions were tried and the final set used were chosen on the basis of the computed Bayesian posterior odds.

[3] The $u_{it}$ of equation (1) is then omitted.

Computation of both ADR and VPR estimates requires that the parameter process be normalized on some specific realization. For the purpose of generating the ex-ante predictions in this study the process was normalized on the value of the parameters one period beyond the sample.

## 4. Results

### 4.1 *Coefficient Estimates*

Four sets of coefficient estimates were generated for the model by both the ADR and VPR techniques. These are available from the author upon request.[1] The ADR technique was not applied to either the CN or CS equations since these did not have intercepts in the original version of the model. Equations were estimated with intercepts but these appeared to be less plausible than the original equations. The only relations which did not have any significant intercept variation were the PD and LF1 equations. Neither of these had any significant slope variation either. Estimation of the CN and CS equations by the VPR technique did not reveal any significant slope variation. All of the remaining equations had significant slope and intercept variation although the extent to which they vary is different for different equations. Of those subject to variation the most stable equation is the employment equation (M) while the least stable is the inventory equation (V). The investment equations (IP and IH) and the labor force equations (LF1 and LF2) were also subject to substantial variation.

### 4.2 *Within Sample Results*

Because the varying parameter estimation technique assumes that the parameters are subject to permanent changes over time, within sample comparisons of these estimators with others is rather difficult. It is possible, once we have estimated $\gamma$ for each equation, to trace out implied parameter values historically but this is time consuming and expensive. Consequently, within sample comparisons were made only for the ADR estimates which were traced out over the entire sample period and compared with the results for WLS-I and OLS over that period. Table 2 presents the results of this comparison.

TABLE 2

Within Sample Errors. 52 Observations

| | RMSE | | | MAE | | |
|---|---|---|---|---|---|---|
| Variable | OLS | WLS | ADR | OLS | WLS | ADR |
| GNP | 14.00 | 9.63 | 8.84 | 11.72 | 7.73 | 7.00 |
| PD | 2.99 | 2.16 | 2.08 | 2.57 | 1.97 | 1.89 |
| GNPR | 20.39 | 15.03 | 12.06 | 17.32 | 13.24 | 11.08 |
| M | 1618.0 | 1106.0 | 1195.0 | 1423.0 | 943.0 | 1030.0 |
| D | 804.0 | 586.0 | 609.0 | 733.0 | 523.0 | 551.0 |
| LF2 | 357.0 | 365.0 | 293.0 | 271.0 | 287.0 | 216.0 |

[4] The four sets of OLS and WLS estimates were supplied by Ray Fair.

It should be noted that the predictions are dynamic in the sense that lagged endogenous variables assume their predicted values. For the ADR predictions, the constant term is different in every period. As the results in Table 2 reveal ADR is the best at predicting GNP (in current dollars) followed by WLS and OLS in terms of both the root mean squared error (RMSE) and the mean absolute error (MAE).[1] For the output deflator PD, the ranking is just the same even though the PD equation displayed no significant variation in the intercept. This is explained by the fact that PD depends in large part on the accuracy with which GNP is predicted over the sample period and ADR and WLS are better at that. The variable GNPR (GNP in constant dollars) simply depends on GNP, PD and exogenous variables representing the government sector so it is natural that its ranking is the same as the first two.

For the employment variables M and D, OLS is again the worst, while WLS is slightly better than ADR, but not remarkably so. For the labor force variable LF2, ADR is clearly the best followed by OLS and WLS.

### 4.3 *Outside Sample Results*

The main focus of this study is on the ex-ante prediction properties of the ADR and VPR estimates. To examine these properties the model was estimated by OLS, WLS, ADR and VPR over three different sample periods. The first of these extends through 1968-IV and predictions are made for the 1969-I–1973-I period. The second sample period extends through 1970-II with predictions from 1970-III–1973-I while the final sample extends through 1971-IV with predictions over the period 1972-I–1973-I. It is of interest to know how each of the estimators being compared performs over different prediction intervals so the errors are examined for 1 period, 4 period, 8 period and longer predictions.

Table 3 presents the simple static 1 period prediction errors for each of the three sample periods and each of the four estimation methods. For the estimates through 1968-IV VPR has the smallest one period prediction error for GNP and four of the six components of GNP. ADR ranks a very close second followed by OLS and WLS. All estimators perform equally well for PD and hence the same ranking holds for the prediction of real GNP (GNPR). Both ADR and VPR do significantly worse at predicting employment (M) and significantly better at predicting the unemployment rate (UR) with the other results being mixed. The results based on the estimates through 1970-II are quite similar with some exceptions. Although ADR and VPR are better at predicting GNP and no worse at predicting PD, OLS does better at predicting GNPR because the errors are offsetting (errors reported in Table 3 are absolute values). The other notable change is that ADR and VPR are here dramatically more successful at predicting the recursive employment and labor force variables. The estimates through 1971-IV again show ADR and VPR to be more successful than either OLS or WLS in general, but the differences are much less pronounced than in the previous sample periods.

[5] The variables chosen for analysis here are the same as those presented in [8] and are the most important variables in the model. GNP is determined simultaneously while the other five are determined recursively.

TABLE 3

ONE PERIOD PREDICTION ERRORS

| Variable | Estimates Through 1968-IV | | | | Estimates Through 1970-IV | | | | Estimates Through 1971-IV | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | OLS | WLS | ADR | VPR | OLS | WLS | ADR | VPR | OLS | WLS | ADR | VPR |
| GNP | 3.36 | 4.41 | 1.52 | 1.19 | 2.16 | 3.67 | 0.94 | 0.72 | 3.95 | 4.24 | 3.84 | 3.71 |
| CD | 1.88 | 1.43 | 1.20 | 1.14 | 0.23 | 0.45 | 0.15 | 0.13 | 2.22 | 2.49 | 1.07 | 1.75 |
| CN | 0.46 | 1.30 | 1.07 | 1.05 | 1.86 | 2.25 | 2.40 | 2.41 | 0.17 | 0.32 | 0.30 | 0.30 |
| CS | 0.13 | 0.07 | 0.13 | 0.43 | 0.49 | 0.10 | 0.01 | 0.02 | 0.29 | 0.43 | 0.41 | 0.41 |
| IP | 1.38 | 1.53 | 0.75 | 1.09 | 0.68 | 0.87 | 0.38 | 0.24 | 1.97 | 1.94 | 2.38 | 1.85 |
| IH | 0.76 | 0.91 | 0.53 | 0.39 | 0.45 | 0.60 | 0.56 | 0.55 | 0.88 | 0.58 | 0.87 | 0.89 |
| V | 4.10 | 3.81 | 1.35 | 0.81 | 3.15 | 3.90 | 2.26 | 2.23 | 1.87 | 1.81 | 1.51 | 1.79 |
| PD | 0.04 | 0.05 | 0.05 | 0.05 | 0.23 | 0.24 | 0.24 | 0.24 | 0.30 | 0.28 | 0.28 | 0.28 |
| GNPR | 2.98 | 3.88 | 1.53 | 1.26 | 0.47 | 1.57 | 0.49 | 0.66 | 1.35 | 1.62 | 1.33 | 1.24 |
| M | 281.56 | 255.81 | 402.75 | 402.81 | 246.75 | 437.88 | 131.63 | 16.25 | 130.69 | 66.31 | 65.23 | 64.19 |
| D | 44.75 | 20.32 | 48.11 | 12.51 | 242.13 | 314.27 | 80.10 | 75.54 | 123.80 | 109.04 | 91.01 | 28.80 |
| LF1 | 8.52 | 13.83 | 19.15 | 12.92 | 96.19 | 84.49 | 58.70 | 58.70 | 122.82 | 135.69 | 118.07 | 90.75 |
| LF2 | 276.56 | 273.60 | 229.15 | 223.90 | 61.87 | 77.45 | 22.51 | 22.17 | 54.01 | 35.16 | 15.63 | 49.85 |
| UR | 0.0041 | 0.0043 | 0.0023 | 0.0017 | 0.0033 | 0.0033 | 0.0033 | 0.0021 | 0.0037 | 0.0039 | 0.0030 | 0.0015 |

TABLE 4

| | RMSE | | | | MAE | | | |
|---|---|---|---|---|---|---|---|---|
| | OLS | WLS | ADR | VPR | OLS | WLS | ADR | VPR |
| **Estimates Through 1968-IV. 4 Period Prediction** | | | | | | | | |
| GNP | 2.64 | 4.85 | 1.84 | 1.48 | 2.15 | 4.60 | 1.59 | 1.21 |
| CD | 1.24 | 1.20 | 1.59 | 1.66 | 1.04 | 1.10 | 1.37 | 1.38 |
| CN | 0.75 | 1.88 | 1.37 | 1.37 | 0.68 | 1.63 | 1.20 | 1.20 |
| CS | 1.91 | 1.79 | 1.98 | 3.00 | 1.56 | 1.45 | 1.62 | 2.52 |
| IP | 0.96 | 1.07 | 1.02 | 0.89 | 0.89 | 1.00 | 0.75 | 0.76 |
| IH | 1.42 | 1.55 | 0.98 | 0.65 | 1.38 | 1.51 | 0.95 | 0.63 |
| V | 12.70 | 12.62 | 4.25 | 4.05 | 11.21 | 11.09 | 3.43 | 3.08 |
| PD | 0.53 | 0.54 | 0.56 | 0.56 | 0.45 | 0.46 | 0.48 | 0.48 |
| GNPR | 4.53 | 6.59 | 3.83 | 3.71 | 4.13 | 6.18 | 3.12 | 3.09 |
| M | 563.86 | 491.46 | 740.60 | 738.99 | 539.22 | 471.48 | 714.5 | 713.52 |
| D | 156.08 | 177.07 | 181.71 | 209.14 | 137.86 | 145.08 | 155.05 | 162·39 |
| LF1 | 104.58 | 89.65 | 75.53 | 90.57 | 91.48 | 79.53 | 68.15 | 80.15 |
| LF2 | 910.18 | 881.01 | 732.68 | 732.87 | 820.72 | 795.95 | 667.46 | 666.45 |
| UR | 0.0053 | 0.0055 | 0.0017 | 0.0013 | 0.0052 | 0.0055 | 0.0016 | 0.0012 |
| **Estimates Through 1968-IV. 8 Period Prediction** | | | | | | | | |
| GNP | 4.82 | 6.20 | 4.13 | 6.92 | 3.73 | 5.21 | 2.91 | 4.80 |
| CD | 1.39 | 1.26 | 1.85 | 1.79 | 1.19 | 1.10 | 1.56 | 1.47 |
| CN | 6.70 | 5.01 | 5.22 | 5.92 | 4.68 | 3.77 | 3.83 | 4.26 |
| CS | 1.79 | 1.76 | 1.81 | 3.55 | 1.48 | 1.48 | 1.48 | 3.26 |
| IP | 3.04 | 3.37 | 2.53 | 2.60 | 2.54 | 2.81 | 2.08 | 2.16 |
| IH | 1.37 | 1.47 | 0.98 | 0.76 | 1.28 | 1.38 | 0.87 | 0.66 |
| V | 30.01 | 28.73 | 15.13 | 12.30 | 25.68 | 24.74 | 12.11 | 9.99 |
| PD | 0.74 | 0.74 | 0.80 | 0.82 | 0.66 | 0.67 | 0.72 | 0.73 |
| GNPR | 5.90 | 7.83 | 5.15 | 5.28 | 4.56 | 6.78 | 4.92 | 2.27 |
| M | 469.41 | 420.74 | 607.47 | 630.52 | 415.56 | 387.33 | 536.13 | 587.12 |
| D | 281.95 | 252.55 | 376.04 | 402.11 | 248.20 | 226.56 | 323.07 | 343.09 |
| LF1 | 128.19 | 111.17 | 94.45 | 112.22 | 118.32 | 102.74 | 87.32 | 103.64 |
| LF2 | 1535.88 | 1503.46 | 1005.9 | 1026.79 | 1383.75 | 1352.11 | 941.76 | 957.81 |
| UR | 0.0128 | 0.0131 | 0.0048 | 0.0037 | 0.0110 | 0.0113 | 0.0039 | 0.0030 |

TABLE 4 (continued)

Estimates Through 1968-IV. Prediction Through 1973-I

| | RMSE | | | | MAE | | | |
|---|---|---|---|---|---|---|---|---|
| | OLS | WLS | ADR | VPR | OLS | WLS | ADR | VPR |
| GNP | 13.93 | 9.97 | 9.17 | 20.47 | 11.06 | 8.30 | 6.93 | 16.41 |
| CD | 4.68 | 3.96 | 2.48 | 3.26 | 3.75 | 3.17 | 2.00 | 2.67 |
| CN | 11.35 | 8.30 | 8.34 | 10.74 | 9.64 | 7.13 | 7.14 | 9.12 |
| CS | 2.08 | 2.30 | 2.22 | 7.45 | 1.75 | 1.95 | 1.83 | 6.64 |
| IP | 2.87 | 3.36 | 2.45 | 2.46 | 2.42 | 2.82 | 2.03 | 2.03 |
| IH | 5.92 | 6.15 | 5.16 | 5.40 | 4.45 | 4.65 | 3.74 | 3.84 |
| V | 58.13 | 57.11 | 36.52 | 31.69 | 50.35 | 49.17 | 29.69 | 24.64 |
| PD | 0.87 | 0.82 | 0.91 | 1.11 | 0.74 | 0.69 | 0.77 | 1.00 |
| GNPR | 8.33 | 7.46 | 6.19 | 10.61 | 6.70 | 5.81 | 4.66 | 8.93 |
| M | 437.75 | 474.80 | 924.68 | 1355.62 | 374.33 | 435.64 | 794.70 | 1149.89 |
| D | 490.99 | 373.12 | 462.78 | 397.93 | 425.91 | 318.96 | 421.18 | 361.34 |
| LF1 | 219.65 | 239.68 | 220.69 | 240.80 | 229.22 | 209.33 | 190.04 | 210.44 |
| LF2 | 2285.65 | 2233.14 | 1223.73 | 1329.92 | 2118.48 | 2070.21 | 1171.51 | 1263.01 |
| UR | 0.0163 | 0.0164 | 0.0069 | 0.0088 | 0.0149 | 0.0150 | 0.0056 | 0.0065 |

Table 4 presents the results of estimating the model through 1968-IV and simulating through 1973-I. The VPR estimates do best at predicting both current and real GNP as well as three of the six GNP components over four periods. The ADR estimates do nearly as well, while OLS is generally superior to WLS at predicting GNP and its components. The predictions of the recursive labor force and employment variables are again somewhat mixed although ADR and/or VPR are generally superior for three out of the five and inferior for the other two. WLS seems to dominate OLS for these variables. These rankings of estimators generally remain the same for the eight period predictions although VPR does the worst at predicting current dollar GNP and the differences among the estimators are less pronounced. Over the longer prediction interval of 17 quarters the ranking of the estimators changes somewhat with respect to GNP and its components. The estimates generated by ADR are clearly superior to WLS, OLS and VPR in that order. The change of the VPR estimates appears to be due to the large errors in predicting CN and CS because it is clearly superior to WLS and OLS at predicting the other four GNP components. The rankings of the estimators with respect to the recursive variables remains the same over this period.

Table 5 presents the results of estimating the model through 1970-II and predicting through 1973-I. Here the pattern is changed somewhat. The varying parameter techniques are again better at forecasting real and current GNP as well as three of the six GNP components over four periods. These techniques also yield better forecasts for all of the five labor force and employment variables. When the prediction interval is extended to eight periods the superiority of ADR and VPR over WLS disappears where real and money GNP are concerned although they still do best at predicting three of the GNP components and all of the labor force and employment variables. When the prediction interval is extended to 1973-I (11 quarters) the ranking changes again with WLS being superior followed by ADR, OLS and VPR in that order where GNP and its components are concerned. For the remaining variables ADR and VPR do the best with the exception of M for which OLS dominates.

Finally, Table 6 presents the results of estimation through 1971-IV and prediction through 1973-I. Here again ADR dominates the other estimation techniques for all but a few of the variables. The VPR estimates are slightly better than OLS and significantly better than WLS. It is worth noting that all of the estimation techniques do noticeably worse over this latter period, mainly underpredicting the large increases in money GNP and its components.

## 5. Conclusions

From the results presented in the previous section we can draw some cautious conclusions. First, it seems that varying parameter techniques yield, in the present context, more accurate short term forecasts than either competitor. This is true of both the static one period predictions and the dynamic four period predictions. In general the ADR techique performed as well or better than the VPR technique which assumes all of the slope coefficients are varying, especially over longer prediction intervals. While the varying parameter estimation techniques also

## TABLE 5

| | Estimates Through 1970-II. 4 Period Prediction | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | RMSE | | | | MAE | | | |
| Variable | OLS | WLS | ADR | VPR | OLS | WLS | ADR | VPR |
| CNP | 4.91 | 2.60 | 1.86 | 1.79 | 4.03 | 2.24 | 1.73 | 1.32 |
| CD | 2.47 | 1.79 | 1.88 | 1.93 | 2.00 | 1.38 | 1.77 | 1.71 |
| CN | 3.81 | 4.27 | 3.75 | 3.72 | 3.65 | 4.12 | 3.57 | 3.54 |
| CS | 2.42 | 1.55 | 1.91 | 1.87 | 1.80 | 1.23 | 1.42 | 1.39 |
| IP | 2.25 | 2.19 | 1.12 | 1.18 | 2.01 | 1.89 | 0.84 | 1.01 |
| IH | 1.23 | 0.77 | 0.82 | 0.82 | 0.94 | 0.55 | 0.59 | 0.58 |
| V | 8.59 | 10.59 | 4.67 | 4.73 | 7.93 | 9.62 | 4.43 | 4.42 |
| PD | 0.44 | 0.41 | 0.44 | 0.44 | 0.42 | 0.39 | 0.41 | 0.41 |
| GNPR | 1.69 | 1.20 | 1.01 | 1.58 | 1.51 | 1.11 | 1.00 | 1.16 |
| M | 464.64 | 873.49 | 168.09 | 424.18 | 443.47 | 822.66 | 157.38 | 359.67 |
| D | 354.09 | 480.13 | 146.54 | 166.48 | 339.12 | 464.18 | 140.90 | 155.88 |
| LF1 | 153.36 | 139.83 | 104.15 | 104.15 | 145.11 | 131.07 | 94.47 | 94.47 |
| LF2 | 162.49 | 351.74 | 171.95 | 148.58 | 143.35 | 247.62 | 129.78 | 121.75 |
| UR | 0.0040 | 0.0037 | 0.0022 | 0.0024 | 0.0039 | 0.0036 | 0.0020 | 0.0020 |
| | Estimates Through 1970-II. 8 Period Prediction | | | | | | | |
| GNP | 4.57 | 2.12 | 2.79 | 2.70 | 3.95 | 1.75 | 2.03 | 2.08 |
| CD | 3.32 | 2.64 | 3.41 | 3.30 | 2.87 | 2.18 | 2.86 | 2.76 |
| CN | 3.24 | 3.62 | 3.40 | 3.32 | 3.04 | 3.42 | 3.27 | 3.18 |
| CS | 3.12 | 1.78 | 2.56 | 2.47 | 2.67 | 1.58 | 2.19 | 2.12 |
| IP | 2.40 | 2.45 | 1.37 | 1.34 | 2.06 | 2.09 | 1.08 | 1.11 |
| IH | 3.39 | 2.56 | 1.68 | 1.66 | 2.63 | 1.91 | 1.00 | 1.99 |
| V | 21.25 | 25.26 | 13.07 | 14.94 | 17.32 | 20.73 | 10.40 | 11.54 |
| PD | 0.42 | 0.44 | 0.42 | 0.42 | 0.40 | 0.41 | 0.39 | 0.39 |
| GNPR | 3.10 | 1.67 | 2.46 | 2.65 | 2.47 | 1.55 | 2.41 | 2.49 |
| M | 500.83 | 1172.15 | 395.28 | 745.37 | 479.75 | 1104.09 | 317.33 | 634.80 |
| D | 575.77 | 818.50 | 155.71 | 145.36 | 526.82 | 743.76 | 151.13 | 135.18 |
| LF1 | 200.47 | 186.75 | 148.2 | 148.2 | 184.70 | 170.09 | 129.35 | 129.35 |
| LF2 | 197.79 | 432.24 | 157.14 | 182.04 | 167.13 | 362.29 | 128.36 | 151.14 |
| UR | 0.0036 | 0.0035 | 0.0010 | 0.0012 | 0.0032 | 0.0031 | 0.0009 | 0.0009 |

TABLE 5 (continued)

| | Estimates Through 1970-II. 4 Period Prediction | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | RMSE | | | | MAE | | | |
| Variable | OLS | WLS | ADR | VPR | OLS | WLS | ADR | VPR |
| | Estimates Through 1970-II. Prediction Through 1973-I | | | | | | | |
| GNP | 14.94 | 10.63 | 13.31 | 15.98 | 10.62 | 6.65 | 9.34 | 12.40 |
| CD | 4.79 | 3.89 | 4.34 | 4.60 | 4.04 | 3.14 | 4.05 | 3.84 |
| CN | 6.24 | 6.19 | 6.62 | 7.31 | 5.10 | 5.23 | 5.58 | 6.33 |
| CS | 3.43 | 1.70 | 2.99 | 2.74 | 3.13 | 1.57 | 2.11 | 2.49 |
| IP | 2.29 | 2.21 | 2.51 | 2.44 | 1.88 | 1.87 | 1.90 | 1.89 |
| IH | 5.94 | 4.78 | 4.93 | 4.89 | 4.80 | 3.74 | 3.87 | 3.84 |
| V | 37.91 | 44.23 | 23.73 | 29.73 | 31.34 | 36.78 | 19.40 | 23.63 |
| PD | 0.61 | 0.71 | 0.57 | 0.58 | 0.52 | 0.61 | 0.49 | 0.50 |
| GNPR | 12.46 | 10.08 | 11.57 | 13.03 | 8.71 | 6.73 | 7.07 | 9.81 |
| M | 418.63 | 1291.42 | 773.54 | 1100.55 | 369.90 | 1237.61 | 622.55 | 960.8 |
| D | 647.89 | 1028.83 | 152.75 | 140.58 | 607.80 | 951.89 | 142.93 | 128.98 |
| LF1 | 240.33 | 225.44 | 183.44 | 183.44 | 224.64 | 209.16 | 165.58 | 165.58 |
| LF2 | 200.94 | 619.96 | 138.94. | 180.00 | 174.22 | 534.51 | 113.59 | 151.50 |
| UR | 0.0067 | 0.0070 | 0.0011 | 0.0013 | 0.0057 | 0.0057 | 0.0091 | 0.00108 |

TABLE 6

Estimates Through 1971-IV. 5 Period Prediction

| Variable | RMSE | | | | MAE | | | |
|---|---|---|---|---|---|---|---|---|
| | OLS | WLS | ADR | VPR | OLS | WLS | ADR | VPR |
| GNP | 18.73 | 20.97 | 16.72 | 18.00 | 15.93 | 18.03 | 14.18 | 15.34 |
| CD | 4.65 | 4.93 | 2.94 | 3.75 | 3.84 | 4.24 | 1.82 | 2.96 |
| CN | 6.69 | 7.14 | 6.76 | 6.88 | 5.23 | 5.67 | 5.36 | 5.45 |
| CS | 0.23 | 0.89 | 0.51 | 0.62 | 0.21 | 0.79 | 0.46 | 0.56 |
| IP | 3.57 | 3.87 | 3.97 | 3.46 | 3.17 | 3.45 | 3.61 | 3.08 |
| IH | 3.55 | 3.18 | 2.95 | 3.12 | 2.96 | 2.56 | 2.43 | 2.58 |
| V | 2.05 | 3.98 | 1.93 | 2.41 | 1.80 | 3.25 | 1.66 | 2.11 |
| PD | 0.40 | 0.41 | 0.41 | 0.41 | 0.38 | 0.39 | 0.39 | 0.39 |
| GNPR | 13.62 | 15.41 | 12.46 | 13.37 | 11.78 | 13.42 | 10.79 | 11.58 |
| M | 471.35 | 311.26 | 132.91 | 151.24 | 414.05 | 258.96 | 116.99 | 134.88 |
| D | 189.03 | 183.60 | 177.22 | 133.97 | 170.01 | 164.74 | 160.37 | 95.34 |
| LF1 | 138.90 | 163.73 | 89.76 | 77.08 | 130.93 | 155.65 | 81.38 | 66.78 |
| LF2 | 120.62 | 374.12 | 216.60 | 148.59 | 101.31 | 312.07 | 170.92 | 119.12 |
| UR | 0.0096 | 0.0108 | 0.0058 | 0.0045 | 0.0089 | 0.0099 | 0.0053 | 0.0039 |

performed well over longer prediction intervals their relative performance seemed to decline with the length of the prediction interval. The superiority of the ADR and VPR estimates appears to hold up better over longer intervals for the recursive equations than it does for the simultaneous equations.

These conclusions must be interpreted with caution since it is clear that they are drawn from a limited experiment and that further experimentation, particularly Monte Carlo experimentation, is needed before any real conclusions can be drawn. There can be no assertion that what is being captured here is variation in the "true" parameters. We may well be capturing variation that is due to specification or aggregation error. The relative performance of the varying parameter estimators might also well be improved by using the longer sample period to gain precision in the estimation of the parameter process. The application of a Kalman filtering and smoothing approach would also enable us to further differentiate parameter processes which might improve the estimation of the slope coefficients. Further work is also warranted in the consideration of simultaneous versions of adaptive regressions.[6] The results of this study do indicate, however, that varying parameter estimation techniques appear promising enough for the estimation of econometric models to warrant further investigation.

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