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A Study of Liquidity Premiums on Federal and Municipal Government Securities

Phillip Cagan

A rate premium on long- relative to short-term securities of comparable quality may be due to transaction costs, expectations of a rise in rates, or the greater "liquidity" of short-term securities. Recent studies have helped to clarify the nature and magnitude of these influences. An important study by Kessel[1] shows that U.S. long-term bond yields typically exceed bill rates by more than the difference in transaction costs plus any differential attributable to expectations. The premium apparently reflects the greater liquidity of short securities, that is, their greater marketability at relatively stable prices, a characteristic of financial instruments which declines as term to maturity increases. Liquidity provides a nonpecuniary return that substitutes in part for interest payments and accounts for the higher interest rate on longer-term securities. The premium on the higher yielding security measures the marginal advantage of holding the more liquid security.

NOTE: Conversations with Stanley Diller on earlier drafts of this paper have helped greatly to clarify the argument. I also received useful comments from Jack Guttentag and Geoffrey H. Moore of the National Bureau and from Herschel Grossman, Reuben Kessel, and Burton G. Malkiel.

I wish to thank also Josephine Trubek, who collected the data and supervised the initial, exploratory computations, and Jae Won Lee, who helped in the final stages of the research.

[1] Reuben Kessel, *The Cyclical Behavior of the Term Structure of Interest Rates,* NBER Occasional Paper 91, New York, 1965.

Such a premium is analytically distinct from yield differentials due to expectations of changes in rates and to risk of borrower default. This study examines the changes in liquidity premiums over long and short periods and tests two theories of their behavior. To abstract from default risk, the data pertain to U.S. securities and municipal bonds of uniform quality. Allowing for expectations is more difficult, and the analysis explores various ways of removing their effects.

Average Liquidity Premiums Over Business Cycles

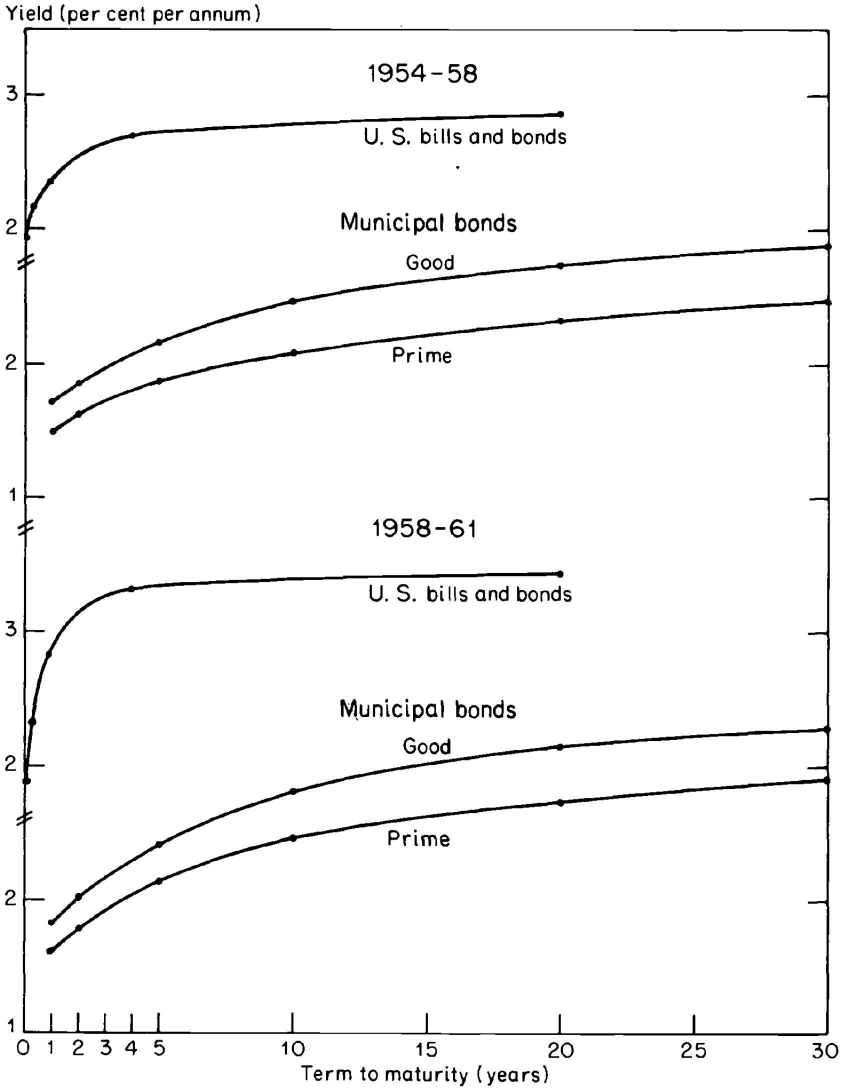
A curve showing yields by term to maturity incorporates the cyclical influence of expectations. Investors will ordinarily expect short-term rates to rise during business expansions and to fall during business contractions.[2] An expected rise in rates would make long rates higher than short rates in anticipation of greater capital losses on the longer maturities, thus contributing to an upward sloping yield curve, and conversely for an expected fall in rates. These varying expectations over a cycle will tend to cancel out in an average curve for each full cycle. Given that expansions are usually much longer than contractions, it is not clear that an unweighted average of monthly data is best. It may be better to give each stage rather than each month of the business cycle an equal weight, in order to approximate a yield curve for which expectations forecast no cyclical change in rates. If investors expect the cyclical movement of rates to proceed more rapidly in business contractions than it does in expansions, such stage averages overweight contraction periods, when rates are generally expected to fall, and so make the estimated slope of the yield curve, if anything, too flat. An unweighted monthly average, in contrast, probably makes the slope too steep. Neither the weighted nor the unweighted cycle average, however, eliminate any expected secular trend in rates; the upward trend in rates during the 1950's, if expected, could have added an upward tilt to the curve throughout that decade. Trend is discussed subsequently.

Chart 4-1 presents yield curves which, to eliminate most of the cyclical effects of expectations, are averages of National Bureau reference stages. There are nine stages in reference cycles. The initial

[2] See my study of "Changes in the Cyclical Behavior of Interest Rates," *Review of Economics and Statistics,* August 1966, reprinted as NBER Occasional Paper 100. There have been changes in the timing and amplitude of interest rates, but their conformity to reference cycles has always been high.

and terminal trough stages are averages of data for the three months surrounding the two trough months, and the peak stage for the three months surrounding the peak month. The period of expansion is divided equally into three stages, as is the contraction period. These nine

CHART 4-1. Yield Curves of U.S. and Municipal Securities, Reference Cycle Averages



stage averages are then averaged with equal weight except for the initial and terminal trough stages, each of which receives one-half weight. The curves in the chart are for U.S. and municipal securities in the 1954–58 and 1958–61 cycles. The municipal series is composed of new issue yields in two quality groups.[3] Most of the series underlying the chart begin after World War II and do not allow comparisons with earlier decades.

The curves exhibit a strong upward slope, somewhat greater for the later cycle than for the earlier one. For U.S. securities, the upward slope tapers off rapidly in the maturity segment of one week to about two or three years. The slope for municipals starts out smaller, judging by the one- to two-year maturities, but declines more gradually. The slope is somewhat greater for good than for prime municipals.[4] The stage averages, as noted, probably even understate the degree of upward slope. The characteristic upward slope of government yields is well known and has been widely commented on; the results presented here quantify this general impression and extend it to municipal securities.

How much of the upward slope reflects transaction costs or expected trends, and how much liquidity premiums? Brokerage costs for the purchase and sale of Treasury bills (the spread between dealer bid and ask prices) vary, but the range seldom exceeds 4 to 20 cents per $1,000 security on maturities of three months or less, while the comparable cost for long-term U.S. bonds is about $2.50.[5] The implication for the yield differential depends upon the holding period between purchase and sale. (A new issue acquired from the Treasury and held to maturity involves no brokerage costs.) In purchasing a bond from brokers and selling back after three months, the brokerage cost is

[3] For U.S. securities, market yields on one-week bills (kindly supplied by Jacob Michaelsen from data he obtained from the first Boston Corporation for his "The Term Structure of Interest Rates and Holding-Period Yields on Government Securities," *Journal of Finance*, Sept. 1965, pp. 444–463), three- and nine- to twelve-month bills and three- to five-year bonds (*Federal Reserve Bulletin*), twenty-year bonds (Morgan Guarantee Company). For municipal securities, new issues (Salomon Brothers and Hutzler, *An Analytical Record of Representative Municipal Yield Scales by Quality and Maturity 1950–June 1965*, n.d.).

[4] Possibly because, in addition to relative differences in marketability, the risk of default increases more with maturity on the lower than on the higher grade bonds. See R. E. Johnson, "Term Structures of Corporate Bond Yields as a Function of Risk of Default," *Journal of Finance*, May 1967, pp. 313–345.

[5] Reuben Kessel, "Market Segmentation in the Treasury Bill Market," May 9, 1967, dittoed; and Allan H. Meltzer and Gert von der Linde, *A Study of the Dealer Market for Federal Government Securities*, Joint Economic Committee, 1960, pp. 111–112.

100 basis points (that is, 1 per cent per annum), a sizable amount when compared with the zero cost for bills held for a full term. Between three-month bills and twenty-year U.S. bonds, the premium (Chart 4-1) was 69 points in 1954–58 and 113 points in 1958–61. We cannot take the 100-point figure, however, as the relevant difference in transaction costs. Bonds are typically held much longer than three months, and the difference in transaction costs between bonds of any two maturities is small. If transaction costs accounted for the upward slopes, the differential between one- and thirty-year municipal yields should be quite small; in fact, it exceeds the differential between bills and twenty-year U.S. bonds. Transaction costs appear far from sufficient, therefore, to account for the actual slope of these yield curves, even though we cannot assign an exact figure to such costs without knowing the average length of holding periods.

What about trend? Three-month bill rates rose 128 basis points from their average monthly level during the 1949–54 cycle to that during the 1958–61 cycle, or (dividing by the interval between the reference peaks taken as the cycle midpoints) rose 17.1 basis points per year. Suppose that investors had anticipated the actual trend and assumed it would continue for several decades. The market would have required that the yield curve rise by 8.5 points per year of term to maturity,[6] and so could have accounted for more than the entire upward slope in

[6] A long rate can be expressed as a geometric average of the current short rate and the expected (forward) short rates for each subsequent period to maturity (ignoring coupon payments and liquidity premiums). If these expected short rates rise by a trend factor,

$$(1 + R_n)^n = (1 + R_1)(1 + r_2 + T)(1 + r_3 + 2T)(1 + r_4 + 3T) \ldots (1 + r_n + [n - 1]T),$$

where R_n and R_1 are the current yields to maturity on maturities of n and 1 periods, r_i the 1 period rate in the ith period ahead expected now *exclusive of trend,* and T the expected increase in rates per period.

An arithmetic approximation, ignoring the compounding of interest, gives

$$R_n = \frac{R_1 + \sum_{2}^{n} r_i + \sum_{1}^{n} (i - 1)T}{n}.$$

If investors expect no changes, aside from trend, all r_i equal R_1, and the expression reduces to

$$R_n = R_1 + \frac{n - 1}{2} T.$$

The term structure then rises (approximately) linearly with a slope of $T/2$. From ten- to twenty-year bonds, it would rise $(\frac{19}{2} - \frac{9}{2})\ 17.1 = 85.5$ basis points. (Incorporating the higher-order terms ignored in the approximation produces a curve in which the slope is not constant but increases with maturity.)

Chart 4-1 beyond a certain point. On the 1958–61 yield curves, a slope of 8.5 points per year occurs at a maturity of only about three years for U.S. securities and only about seven years for municipals.

Judging by the long end of the curve, the actual trend was considerably underestimated. If investors expected a trend to continue for two decades, we can measure the expected trend by the average differential between ten- and twenty-year prime municipals, assuming a negligible liquidity premium between them. The figure is about 25 basis points per decade. Adjusting the short maturities for that estimate of expected trend still leaves a steep slope at the very short end of the curve. (The slope, as noted, tapers off more rapidly for U.S. securities than it does for municipals.) Whatever adjustments for expected trend should be made, it cannot explain a yield curve with a tapering slope, unless investors expect the upward movement to taper off.[7]

The sharp rise at the lower end of the yield curves is more plausibly explained by liquidity premiums. Such premiums are consistent with the traditional theory of liquidity preference, which posits a widespread demand in the economy for the stable market value of liquid assets. For example, federal and municipal bonds, as they age to within several years of maturity, become especially attractive to commercial banks and other financial intermediaries as secondary reserves, and short-term Treasury bills and certificates appeal to corporate treasurers as investment outlets for funds needed on short notice. To be sure, some institutions with long-term obligations, such as life insurance companies and pension funds, desire a predictable income more than stable capital value. A desire to hedge against the risk of changes in interest rates by matching the maturity of acquired assets with the maturity of given liabilities makes long-term securities the preferred investment for these institutions. The evidence of yield curves, however, implies that the demand for liquidity is large relative to the total market supply of short-term securities.

The demand for liquid assets relative to total financial assets is not infinitely elastic; it declines with respect to the price paid for liquidity, that is, the lower pecuniary yield on liquid assets. Given

[7] A trend expected to terminate in a certain number of years could produce such a curve, but the same expectation would produce a flatter curve in the next cycle, not a steeper one as we observe, unless the expected terminal date of the trend was progressively moved forward in time. By such *ad hoc* assumptions, of course, any yield curve can be "explained."

a downward sloping demand curve, the price paid for liquidity declines with increases in the supply of such assets. If borrowers of funds were indifferent to the maturity of the securities they issued, liquidity premiums could not exist. Short-term securities would then be issued to take advantage of the lower yield until they saturated the market's preference for short securities and eliminated the differential yield relative to long debt. The existence of liquidity premiums therefore implies that the supply is not infinitely elastic at the same yield as prevails on long-term bonds. Most borrowers prefer to issue long-term debt; this is certainly true of state and municipal governments and most business corporations. Their desire to hedge leads to long-term financing of long-lived physical assets. The federal government also limits the issue of short securities according to the goals of debt management. One implication of a less than infinitely elastic demand for liquidity and supply of short liquid assets is that an exogenous shift in the supply curve of one class of short securities affects the liquidity premiums on all. As a case in point, commercial banks have in recent years taken advantage of increases allowed in the deposit rate ceiling to issue marketable certificates of deposit in large volume. These are good substitutes for Treasury bills and add to the total supply of liquid assets. It is plausible that their expanded issue during the early 1960's made the yield curve flatter than it would otherwise have been.[8]

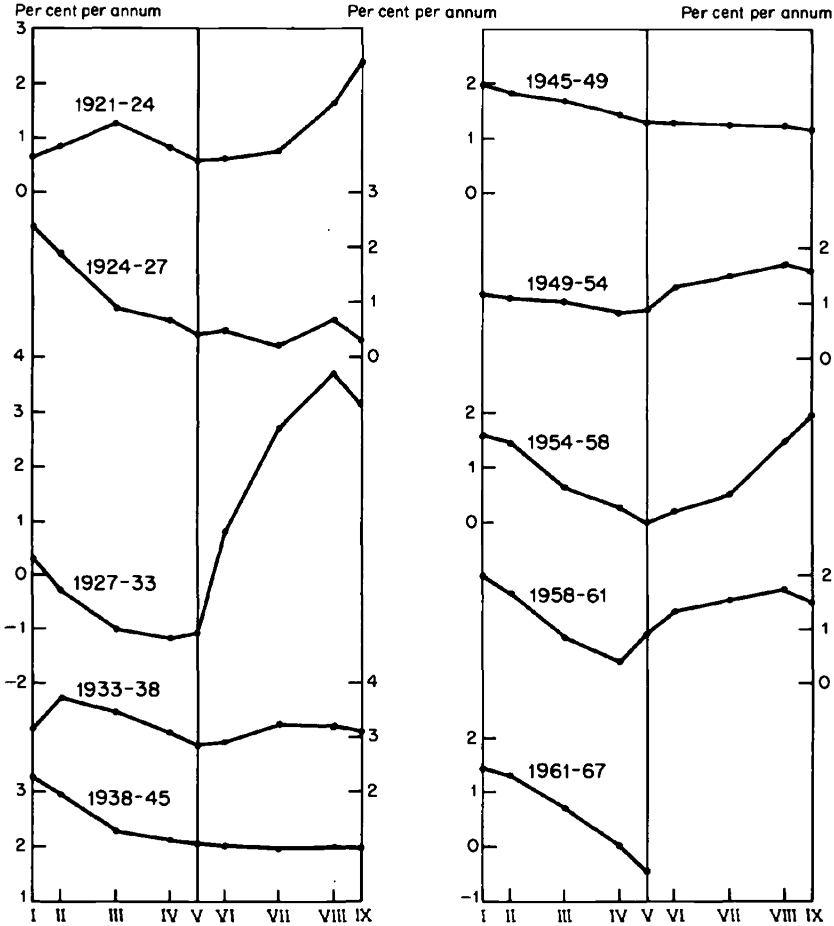
Accepting the evidence of Chart 4-1 that liquidity premiums exist, we may ask whether the premiums vary over time and if so why, the two questions examined in the following sections.

Fluctuations in Liquidity Premiums

EVIDENCE OF FLUCTUATIONS. Chart 4-2 shows reference cycle patterns of the differential yield between long- and short-term U.S. securities. This series has no adjustment for expectations, which must therefore be taken into account. Cyclical fluctuations in the differential display an inverse conformity to business activity, though often during contractions there was no increase, only a slackening in the rate of decline. The trough generally came at stage V, though sometimes, as

[8] F. Modigliani and R. Sutch ("Innovations in Interest Rate Policy," *American Economic Review*, May 1966, pp. 195–196) argue that CDs enhance the ability of commercial banks to arbitrage the yield differential between long and short rates, a similar point to that made here.

CHART 4-2. Reference Cycle Patterns of Yield Differential Between U.S. Bonds and Certificates or Bills, 1921–67



SOURCE: *Federal Reserve Bulletin.* Long-term U.S. bond series; and three- to six-month Treasury certificates through 1930, spliced to three-month bills thereafter.

in the 1958–61 cycle, at stage IV. The pattern for 1961–67 shows a large decline in the differential during that period (these stages were computed on the basis of a hypothetical business peak in December 1967). The individual bond and bill rates composing the differential have positive conformity to business cycles and similar timing, but the short rate has the larger amplitude, which accounts for the inverse conformity of the differential. The inverse conformity may be attributed, at least in part, to anticipations of cyclical fluctuations in the level of rates. During the later stages of business expansion, interest rates rise to heights which investors feel cannot last; hence long rates do not rise as sharply as do short rates in the expansion phase, and the differential gradually declines. During business contractions when rates fall, investors form the opposite expectation which causes the differential to rise. If investors anticipated cyclical turning points in rates perfectly, the differential would turn ahead of business, contrary to its actual timing. But the absence of leading turns in the differential yield does not mean that it is not affected by expectations. They may not be so precise. Vague expectations of cyclical movements, with a belief that turning points will occur but no clear idea of when, could produce the observed inverse pattern.

If we could adjust for expectations, therefore, the resulting differential might have any pattern. The behavior of the adjusted pattern is important, because the two leading explanations of the premium have opposite implications about the cyclical pattern. The next subsection discusses these explanations, after which additional evidence is examined.

TWO THEORIES OF FLUCTUATIONS IN LIQUIDITY PREMIUMS. The preference of most investors for short securities, giving rise to liquidity premiums, can be attributed to the relatively stable market prices of such securities. There is little disagreement that the existence of liquidity premiums reflects an aversion to the risks of capital losses due to changes in interest rates. The question of how the premiums fluctuate over time, however, is not settled. One theory views short securities as partial substitutes for money balances, performing to a degree the same functions, at least in large portfolios. These securities provide a nonpecuniary return, representing the value of the services they perform as substitutes for money holdings. On the margin this nonpecuniary return equals the liquidity premium. Another theory is that investors believe that interest rates tend to return to "normal" levels. Given an aversion to the risk of capital losses on long-term securities, these securities will carry a yield premium depending upon the relation of current yields to what is considered normal. This theory can be distinguished from the standard expectations hypothesis that expected holding-period yields are equal for all maturities.

These two theories of liquidity premiums, developed further below, are the leading interpretations in the literature.[9] While not incompatible with each other, they give opposite implications for the behavior of liquidity premiums, which allows us to test them against the data.

Short Securities As Substitutes for Money. Investors are presumed to equate the marginal returns from the various financial assets in their portfolios. If short securities are closer substitutes for money than are long securities—which seems plausible though there is little direct evidence on the question—the slope of the yield curve is affected by a change in its average level. Suppose the entire yield curve rises because of a shift in the demand or supply of loanable funds. The foregone interest income in holding money is higher, and the public attempts to exchange part of its money balances for securities. For a time this prevents yields from rising as much as they otherwise would. Moreover, because investors prefer short to long securities as substitutes for money, they favor shorts in the exchange, which prevents the yield on shorts from rising as much as that on longs. Hence the premium on longs over shorts increases (aside from any effects of expectations) as a result of a rise in the entire yield curve, and the converse is true for a decline.

On the margin investors will adjust their portfolios so that the marginal services of liquidity just compensate for differences in pecuniary yields. The pecuniary yield differential, apart from differences due to expected changes in interest rates, therefore measures the nonpecuniary services of the lower yielding security. This equality allows us to relate yield differentials to the marginal differences in liquidity. Suppose a security of term n, because of its services as a liquid asset, allows a person to reduce his average money holdings by ΔM_n. We may then define $S_n = \Delta M_n / P_n$, where P_n is the purchase price of the security. S_n measures the fractional amount by which the security substitutes for money. No longer wanting to hold as much money as before, investors can purchase nonliquid bonds or nonfinancial assets. If r_M is the nonpecuniary return to money balances, the total return

[9] On the money-substitute theory, see Kessel, *Cyclical Behavior*, p. 25, and Cagan, "A Partial Reconciliation Between Two Views of Debt Management," *Journal of Political Economy*, December 1966, pp. 624–628. On risk aversion and normal rates, see F. de Leeuw, "A Model of Financial Behavior" in Duesenberry *et al.*, *Brookings Quarterly Econometric Model of the United States Economy*, Chicago, 1965, Chap. 13; J. Van Horne, "Interest-Rate Risk and the Term Structure of Interest Rates," *Journal of Political Economy*, August 1965, 344–351; Modigliani and Sutch, *ibid.;* and B. G. Malkiel, *The Term Structure of Interest Rates*, Princeton, N.J., 1966, pp. 59–65. On normality in expectations, see S. Diller, "Extrapolations, Anticipations and the Term Structure of Interest Rates," forthcoming NBER study.

on the security of term n is

$$R_n + S_n r_M, \tag{1}$$

where R_n is the pecuniary return on the security and $S_n r_M$ the non-pecuniary return from its services as a money substitute. If, on the margin, an investor finds two securities the same except for differences in maturity (abstracting from expected changes in rates), the total returns will be equal. For example, indifference between thirteen-week bills and twenty-year bond implies

$$R_{13w} + S_{13w} r_M = R_{20y} + S_{20y} r_M. \tag{2}$$

This is an equilibrium condition attained through appropriate changes in market yields.

If money holdings are in equilibrium with respect to long-term bonds, we have, for example, $r_M = R_{20y} + S_{20y} r_M$.[10] There appears to be little error in assuming that S_{20y} is a relatively small fraction of unity; hence $r_m \simeq R_{20y}$. Then the measured liquidity premium, based on (2), is

$$R_{20y} - R_{13w} = S_{13w} R_{20y}. \tag{3}$$

If S_{13w}, which represents the marginal substitutability of thirteen-week bills for money, is constant, the differential yield between the long and short securities (abstracting from expected changes in rates) varies directly with the level of long-term rates.[11]

The values of S may not, of course, remain constant. Aside from shifts in preferences for liquidity, we may expect relative supplies to alter to some extent the marginal services of liquidity. When the relative supply of short securities increases, their marginal substitutability for money diminishes, signified by a decline in the values of S. An

[10] This treats any pecuniary return to money holdings as negligible.

[11] At the short end of the yield curve, indifference between one- and thirteen-week securities implies

$$R_{13w} + S_{13w} r_M = R_{1w} + S_{1w} r_M.$$

The measured liquidity premium is

$$R_{13w} - R_{1w} = r_M (S_{1w} - S_{13w}).$$

Setting $r_M = R_{20y}$ and using $R_{20y} = R_{13w}/(1 - S_{13w})$ from (3), we have

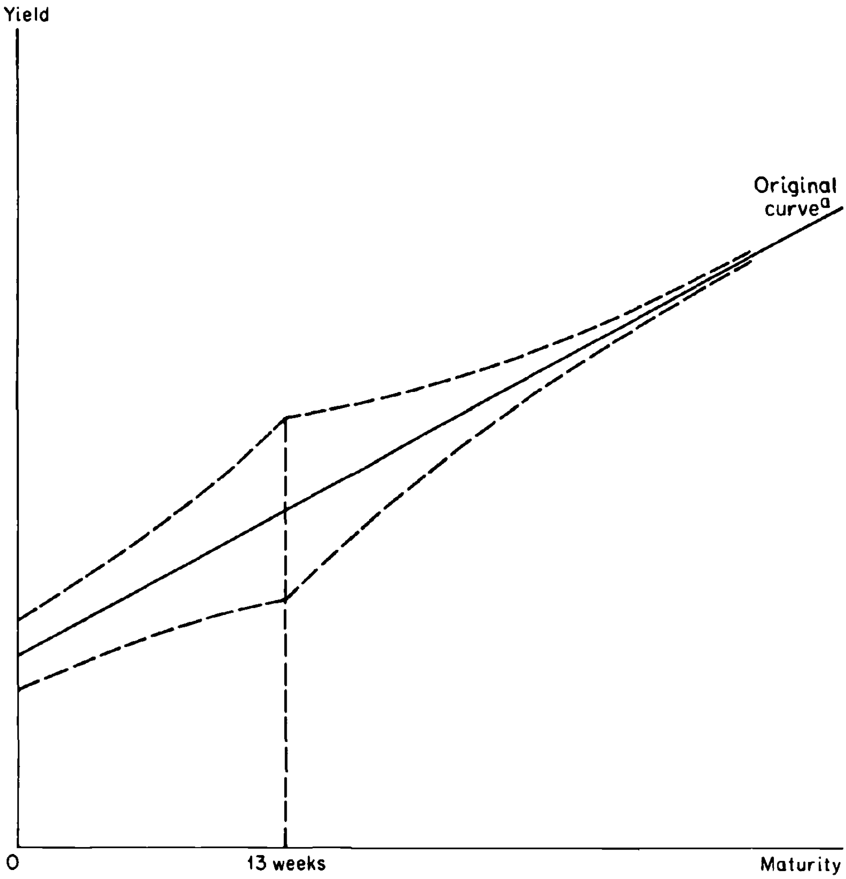
$$R_{13w} - R_{1w} = R_{13w}(S_{1w} - S_{13w})/(1 - S_{13w}).$$

Similarly, the liquidity premium between any two securities depends upon the level of the rates (here represented by the longer of the two) and their relative liquidity.

increased supply of any one substitute for money reduces the marginal substitutability for money of all of them, but not by the same proportion; a change in supply of one substitute affects its own marginal liquidity the most. Hence an increased supply of thirteen-week bills would reduce its liquidity premium relative to other securities. This will be manifested in a reduced yield differential relative to longer-term securities and an increased differential relative to shorter-term securities. The effect is to tilt the entire yield curve of U.S. securities toward the horizontal, and at the same time to produce a peak at the thirteen-week maturity. The change is analogous to pulling a loose string lying on top of the previously existing yield curve and fastened to it at the longest maturity, as in Figure 4-1. An increase in the relative supply of a given maturity swings and bends the curve, equivalent to pulling the string upward at that point. (This analogy ignores the possibility that an increase in supply of securities relative to the money stock may temporarily raise the yield on even the longest maturity, if it too is a partial substitute for money.) Given an upward sloping curve to begin with, the transformation makes the slope more horizontal and slightly peaked at the point of pull. For a decrease in supply, the effect is just the reverse. In the following weeks, the new supply approaches maturity, of course; if no further changes in the relative supplies occur, the point of pull on the string would slide down the curve toward a zero maturity and then disappear.

Different implications follow from the often stated view that the market demand for securities is divided into separate compartments. If the demands for different maturities are independent, an increased supply of thirteen-week bills would raise their yield relative to both longer and shorter securities equally, producing a hump just in that section of the curve. The effect of supply on liquidity premiums presented above, however, assumes that liquidity is a property shared by all maturities in varying degree. Consequently, a change in relative supply affects differential yields all along the curve; the slope of the entire curve changes.

The "Normal" Level of Interest Rates and Risk of Capital Loss. A belief that interest rates sooner or later gravitate toward a "normal" level provides an explanation of the fluctuations in liquidity premiums which contains different implications. (Some of the studies cited in footnote 9 claim to find evidence of such an attitude by investors in the behavior of interest rates.) Given a preference for short over long securities to avoid unanticipated capital losses, the belief in a return to normal levels can produce variations in liquidity premiums. Like the first theory, this effect relies on a preference for the stability of capital



[a] Original curve is drawn as a straight line for graphic simplicity only.

FIGURE 4-1. Hypothetical Effect on Yield Curve of Change in Relative Supply of Thirteen-Week Securities

value, but it disregards any difference between long and short securities as substitutes for money while at the same time implying variations in premiums opposite to the first theory. The second theory is implicit in much recent literature and may be formulated as follows.

Investors' expectations of the short-term rate can be viewed as a probability distribution of the possible rates, as illustrated in Figure 4-2. The illustration is shown there as symmetrical, but it need not be. The unweighted mean of the distribution is "the" expected short rate for a future date. Current yields on longer-term securities reflect the short rates expected in coming periods. If there is no liquidity premium,

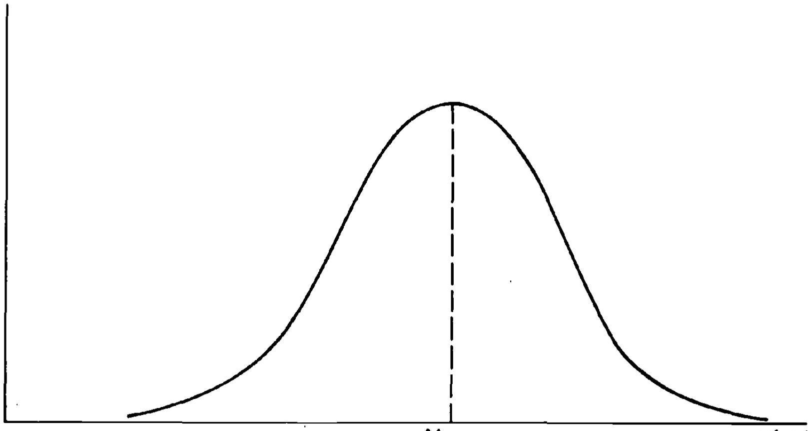


FIGURE 4-2. Investors' Probability Distribution of the Expected Short Rate

security prices will be adjusted to make the expected yields over any given period, composed of coupon payments and expected changes in price, equal on comparable securities of varying maturities. If the mean expected short rate for a later date turns out to be correct, the actual return on longer securities over the period will be as expected; the coupon payments plus price changes will provide the same return (aside from any liquidity premium) on all maturities. If the short rate turns out to be higher than the mean of the distribution, the yield to maturity on longer-term securities will then also be higher than expected, produced by an unanticipated decline in their market prices and resulting in a capital loss over the period. Conversely, if the short rate turns out to be lower than the mean, holders of longer-term securities will receive an unanticipated capital gain over the period.

A preference for stability of principal implies an asymmetrical view of price changes: A potential capital gain does not offset an equal potential loss. We can imagine that investors attach weights to the probabilities according to the importance of *avoiding* particular outcomes. Rates higher than the mean value will receive progressively greater weight, in reflection of their undesirability, since it is the higher than expected rates which produce unanticipated capital losses. The weighting pattern attaches a lower importance to potential rates below the mean expected value. The *weighted* mean value of the distribution will thus be pulled to the right, leading to a demand for higher yields on longer-term securities to partially compensate for the risk of capital loss. The amount of compensation will depend upon the probability of various rates and the capital loss each rate would produce for a security of given maturity, as well as the weighting pattern which quantifies the aversion to risk. (The relative supply of short securities would also be relevant, because the willingness to accept lower yields on short securities to avoid the risk of capital loss will diminish as the average maturity of an investor's total portfolio declines.) Since the amplitude of fluctuation in security prices is known to increase with term to maturity,[12] the risk of capital losses increases correspondingly. Insistence by investors on receiving compensation for such risk can account for liquidity premiums on long relative to short securities. Such a premium, increasing with term to maturity, modifies the "pure" expectations hypothesis (that investors adjust security prices to make expected returns equal regardless of maturity).

This explanation of liquidity premiums, though it says nothing about short securities as substitutes for money, shares with the first theory an assumption of risk aversion. But the two theories may have different implications about fluctuations in the premiums. If the shape of the probability distribution (as well as the weighting pattern) does not change over time, the compensation demanded by investors for incurring risk will remain the same. Premiums will be constant over time. If, on the other hand, the shape of the distribution varies systematically over business cycles, cyclical fluctuations in liquidity premiums will occur. One kind of variation, based on expectations of a return to normal levels, implies a certain skewness of the distribution.

Suppose that, as is often contended, investors have in mind a "normal" level of interest rates toward which they expect actual rates to gravitate in the long run; that level would indicate the likely direction of any large change in rates. The probability distribution of expected rates would then be skewed toward the normal level. When the mean expected rate is well above normal, investors would not rule out a large decline in the rate toward or beyond the normal level. The probability distribution is then skewed to the left, as in the top panel of Figure 4-3. Conversely, when the mean expectation is below normal, the distribution is skewed to the right, as in the bottom panel. The shaded areas of the distributions represent short rates above the mean expectation. If any one of those short rates turned out to be the actual future rate, unanticipated capital losses occur on longer-term securities. Actual rates below the mean produce unanticipated gains. If investors weight the probabilities to reflect an aversion to risk of capital losses, the weighted

[12] This is not a mathematical necessity but has generally been true.

mean expectation would, as noted above, lie to the right of the unweighted mean, giving rise to a liquidity premium (larger for securities of longer term) to partially compensate for the risk of capital loss. As the expected level of short-term rates fluctuates, therefore, liquidity premiums move inversely, being larger for a given maturity when expected rates are low relative to the normal level than when they are relatively high.[13]

It seems reasonable to suppose that the level of rates at any time considered to be normal reflects past experience, can be approximated by an average of past rates, and does not change greatly over the duration of a business cycle. In contrast, the rates expected for periods immediately ahead will ordinarily not differ greatly from current actual rates because it is difficult to foresee sizable short-run changes in rates. Consequently, the difference between the expected short rate and the normal level will vary similarly to actual short rates, and to actual long rates too, since cyclical fluctuations in long and short rates are highly correlated. For practical purposes, therefore, the foregoing theory implies an inverse dependence of liquidity premiums on the level of interest rates, the opposite of the relation implied by the money-substitute theory discussed previously.

Similarities and Differences Between the Two Theories. Let us summarize the two theories in terms of an investor's behavior. When interest rates are low, we might imagine him to demand relatively large money balances because the cost of holding money is low, in terms of the foregone interest earnings on financial assets. According to the first theory, the marginal value attached to the liquidity provided by short-term assets is then also low, and for that reason he is willing to purchase short securities only if their yield is not too far below the return on long securities. Liquidity premiums are then comparatively small.

[13] A recent study finds that expectations of near-term rates are skewed in the direction of recent changes in rates—that is, investors expect that any unusually large changes in rates are more likely to reinforce recent changes than to reverse them (see Edward J. Kane and Burton G. Malkiel, "The Term Structure of Interest Rates: An Analysis of a Survey of Interest-Rate Expectations," *Review of Economics and Statistics,* August 1967, pp. 343–355, esp. p. 349).

With risk aversion, such skewness of near-term expectations would be positively correlated with current or recent changes in rates. This relationship was not tested in the subsequent statistical analysis. As explained below, our estimates of the liquidity premium are *negatively* correlated with changes in rates chiefly because of errors in expectations. There is no way to test for a positive effect of changes in the rates while, at the same time, holding the errors in expectations constant.

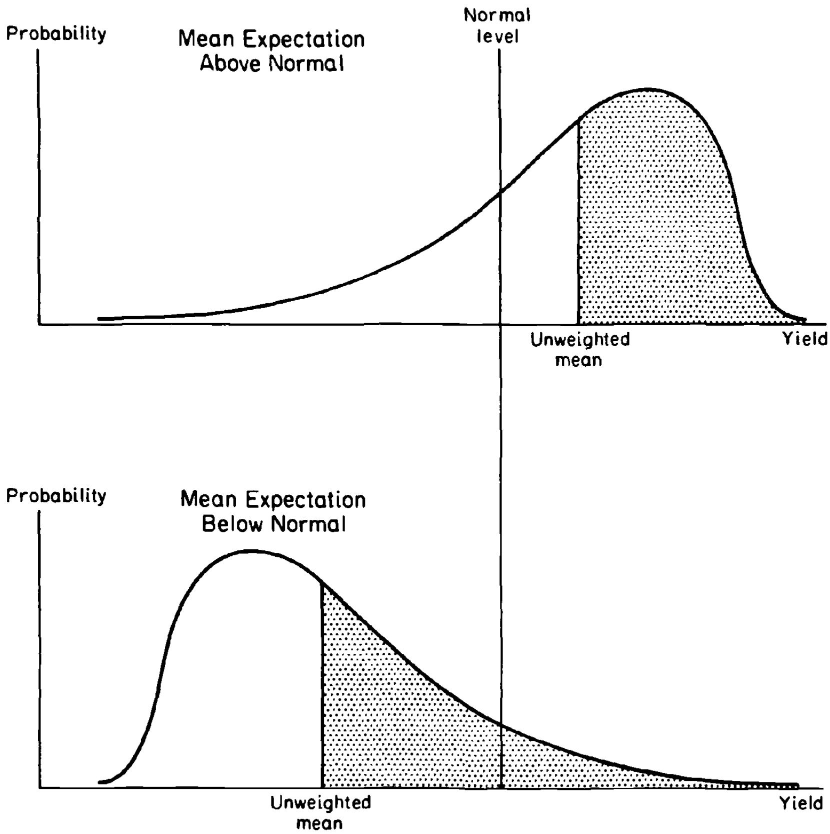


FIGURE 4-3. Investors' Probability Distribution of the Expected Short Rate in Relation to the Expected "Normal" Level

At the same time, his expectations may alert him to an impending rise in short-term rates toward normal levels, and he will acquire long-term securities only if they carry a higher current yield in reflection of a later rise in short rates. (Such expectational effects, though they do not produce a premium, are compatible with any theory of liquidity premiums.) Furthermore, according to the second theory he wants to avoid the risk of capital losses even at the expense of giving up what he considers an equal chance of capital gains. Due to the lingering recollection of the normal level of rates, he regards the currently low short rate as temporary, and his probability distribution of expected rates is skewed toward a return to the higher, normal level. Thus he regards an extra large capital loss on long securities as more probable than an extra large gain even though the expected dollar values of gains and losses are equal, and he avoids long securities unless they carry an extra premium yield. Liquidity premiums are then comparatively large. When interest rates are high, these considerations apply in reverse.

Both theories therefore help to explain the existence of liquidity premiums, but the fluctuations they imply in the premiums are opposite in direction—the first positively associated with the level of rates, the second associated inversely. They might both be valid descriptions of behavior for different groups of investors, or even perhaps as two influences on the same investors. Whether the fluctuations they produce in liquidity premiums cancel each other out or one of the two generally prevails can only be determined empirically.

Either of the two theories can explain the "humped" yield curve often observed in periods of tight credit. At such times the curve slopes upward at the lower end and downward at the longer end, forming a hump usually in the range of the two- to five-year maturities. When explained by expectations alone, such a curve implies that investors anticipate a rise in short-term rates for the next year or two, and thereafter a decline. Yet the short rates of such periods seldom remain high more than a few months, and it seems unlikely that investors would consistently misjudge actual developments. A more appealing explanation of the hump, as Kessel has emphasized,[14] is to combine liquidity premiums and expectations that rates will decline shortly. The premiums contribute to an upward slope in the yield curve, and the expectations to a downward slope. If the slope due to liquidity premiums tapers off sharply for maturities beyond a year or two, a combination of the two effects can produce the observed humped curve.

The occurrence of these curves in times of tight credit indicates that liquidity premiums impart a strong upward slope to the yield curve especially when the level of rates is high. This seems to suggest that the premiums vary directly with the level of rates, but such evidence is not inconsistent with an inverse variation, so long as the level of rates at such times is not so high that the risk of capital loss becomes negligible and the premium, due to that risk, falls to zero.

The inverse conformity to business cycles of the U.S. bond-bill rate differential (Chart 4-2) seems to support the second theory of liquidity premiums. The differential is lowest near business peaks when the level of rates tends to be highest, as that theory predicts.

[14] *Cyclical Behavior,* Chap. 4.

But that evidence is relevant only if expectations play a negligible part in the fluctuations, which seems most unlikely. Near business peaks, investors who are alert to business prospects will expect rates to decline sooner or later, and they will push long rates down close to current short rates. (That effect does not require any skewness in the probability distribution of expected rates or an aversion to capital losses.) Consequently, whether liquidity premiums vary directly or inversely with the level of rates, or change at all, is not implied by such cyclical movements in the long-short differential. Those movements can be said to suggest that investors expect unusually high or low short-term rates to return to normal levels.[15] But if we also want to distinguish between the two theories of liquidity premiums, it is necessary to isolate the effect of expectations.

Tests of the Two Theories

In his study, Kessel concluded that liquidity premiums fluctuate positively with the level of interest rates, based on data for the short end of the yield curve in the post-World War II period. He found that the premium on eight-week bill rates relative to four-week rates, adjusted for expectations,[16] had a positive correlation with the level of four-week rates from October 1949 to February 1961, and that the adjusted premium on six-month rates relative to three-month rates also exhibited a positive correlation with the level of three-month rates from January 1959 to February 1961.

To estimate liquidity premiums, Kessel compared the yields on two securities of different maturity for the same periods. This eliminates the effects of expectations, because as noted the market adjusts prices so that expected holding-period yields (aside from liquidity and risk premiums) on different securities will be the same. The yield differential then reflects influences other than anticipated changes in rates. The yield period he selected for comparison was the period to maturity of the longer security. With maturities of three months and one year, for example, the yield to maturity on the one-year security is compared with the total return from investing in the three-month security, then

[15] That is the explanation for those movements given by B. G. Malkiel, *The Term Structure,* Chap. 4. He does not distinguish between such expectations and fluctuations in the liquidity premium.

[16] *Cyclical Behavior,* p. 26. Kessel measured the premium by the difference between the forward rate and the corresponding future spot rate, which allows for expectations.

upon its maturity reinvesting the proceeds three more times in successive three-month issues, thus keeping the funds invested for a full year. Such comparisons are only feasible, however, for fairly short maturities. (Kessel used this procedure in the results just cited for maturities of eight weeks and six months.) For bonds, the procedure introduces statistical difficulties. It would involve, for example, comparing the yields on five-year bonds and three-month bills over a holding period of five years; that is, each observation of their differential yield would pertain to a five-year period. Whether or not overlapping periods are used, such long intervals render time-series analysis quite impractical, as well as involve large errors in investors' expectations because of the long lead time.

An alternative procedure, used here, is to calculate yields on all securities for the same holding period of some short duration, regardless of their individual maturities, which eliminates the necessity of comparisons for widely separate dates. The holding-period yields of long-term securities still involve large errors in expectations—the problem cannot be avoided. But these errors can be taken into account to some extent, so that estimates of premiums at the long end of the yield curve adjusted for expectations appear feasible for time-series analysis.

ESTIMATING LIQUIDITY PREMIUMS BY DIFFERENCES BETWEEN HOLDING-PERIOD YIELDS. Holding-period yields measure the rate of return on a security purchased at the beginning of a given period and sold at the end, which ordinarily will not coincide with the term to maturity. The data presented below are one-week holding-period yields on Treasury securities.[17] No special appropriateness can be claimed for the one-week period. Other periods, while worth experimenting with, would probably give similar results.

The one-week holding-period yield on a security of maturity n at

[17] Ideally we want to use the holding period for which the error variance of expectations is lowest, which seems most likely to be the actual period over which investors plan to hold. Some speculation in bills may occur on a daily or weekly basis, while bond investors may purchase to hold for months, quarters, or years. Actual holding periods for which investors form expectations and make comparisons among securities probably differ widely among investors and over time. Data on average holding periods are not available, moreover, which makes the one-week period somewhat arbitrary. Yet if expectations equalize yields for periods of one week or less, the equality will carry over to longer periods as well. Errors of forecast may or may not be larger for longer periods; it depends upon whether public anticipations are more accurate for short- or long-run changes in rates.

time t, $H_{n,t}$, expressed as an annual rate with continuous compounding, is

$$H_{n,t} = 52 \log_e \frac{(P_{n-1,t+1} + C_{n,t})}{P_{n,t}} \tag{4}$$

where $P_{n,t}$ is the price of a security at the beginning of week t with n weeks to maturity (disregarding buying and selling costs); and $C_{n,t}$ is the coupon payment, if any, on that security at the end of the week.[18] The expectations theory, altered to allow for liquidity premiums, assumes that

$$H_{n,t} = E_t - L_{n,t} + \xi_{n,t}, \tag{5}$$

where E_t is the expected total yield during week t, the same for all maturities; $L_{n,t}$ is the known nonpecuniary yield during week t due to liquidity services from a security with n weeks to maturity ($E - L$ is the expected pecuniary yield); and $\xi_{n,t}$ is the error of expected pecuniary yield for this security in week t. All yields are expressed as annual rates. For any two maturities n and m, $n > m$:

$$H_{n,t} - H_{m,t} = L_{m,t} - L_{n,t} + \xi_{n,t} - \xi_{m,t}, \tag{6}$$

where the error terms are assumed to have zero means. Hence, as an approximation, for sufficiently large T,[19]

$$\sum_{t=1}^{T} \frac{(H_{n,t} - H_{m,t})}{T} = \bar{L}_m - \bar{L}_n > 0. \tag{7}$$

This is the measure of liquidity premiums used here. T is thirteen weeks.

The error term ξ can be quite large at times. Although the difference between the errors, $\xi_{n,t} - \xi_{m,t}$, will be much smaller than either one separately owing to positive correlation, the difference may still be large at times, because unanticipated changes in rates usually have a

[18] The formula follows from the properties of exponential growth. If $\frac{P_T}{P_0}$, the growth in a price from 0 to T, equals e^{rT}, r is the constant exponential rate of growth. Hence

$$r = \frac{1}{T} \log_e \left(\frac{P_T}{P_0}\right).$$

[19] Since n and m do not change over time in (7), the average value of $H_{n,t}$ for a period of n weeks is *not* the same as $R_{n,t}$, the yield on holding the same security to maturity. Eq. (7) is based on selling each week a $n - 1$ week bill and replacing it with an n week bill. See the Appendix to this chapter for further discussion of measuring liquidity premiums.

greater affect on the prices of securities with longer maturities. The possibility of large errors means that fluctuations in measured liquidity premiums must be interpreted with some caution.

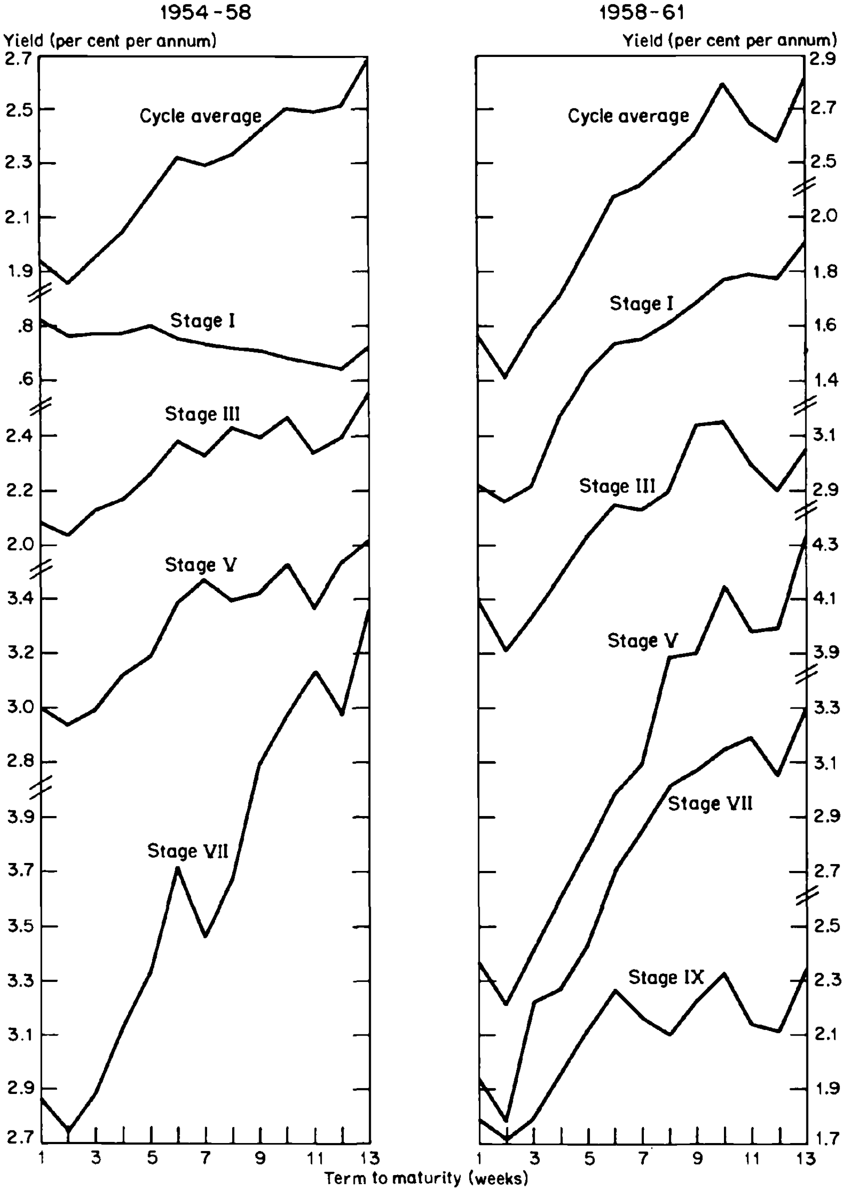
Chart 4-3 presents one-week holding period yields of one- to thirteen-week Treasury bills for various reference stages and their average for two full cycles,[20] showing in detail the steepest part of the yield curve—the short end. These curves are not smooth, no doubt mainly as a result of unanticipated changes in actual yields, but generally they rise up to the seven- to eight-week maturities. The data are averages of bid and ask prices and so do not allow for transaction costs in the form of dealer spreads. The spread is usually the smallest on newly issued thirteen-week bills and nearby maturities than on others, which may help explain why the curve levels off from the seven- to thirteen-week maturities. In any event, the over-all upward slope of these curves can be interpreted as evidence of liquidity premiums, since investors presumably adjust prices so that expected holding-period yields (apart from premiums) are equal on all maturities.

There is also evidence of cyclical fluctuations in the premiums; the peak stages (V) appear to have steeper slopes than do the trough stages (I and IX). This is brought out more clearly by Chart 4-4, which presents the quarterly differences between the thirteen- and one-week yields for the period 1951–65 (top panel), and some other related series discussed later. Reference contractions are shaded. (Differences between twelve- and two-week yields, not presented, exhibit similar movements.) The thirteen- and one-week yield differential fluctuated during this period generally in the same direction as business activity, though not consistently from quarter to quarter. The high point came considerably after the 1957 reference peak, for example, and the decline following the 1960 peak was slower and more prolonged than any previous one.

Much of the fluctuation in the differential appears to reflect errors in expectations, and to be negatively correlated with changes in the three-month bill rate (second panel). This suggests that the error terms in (6) are inversely associated with changes in interest rates, because

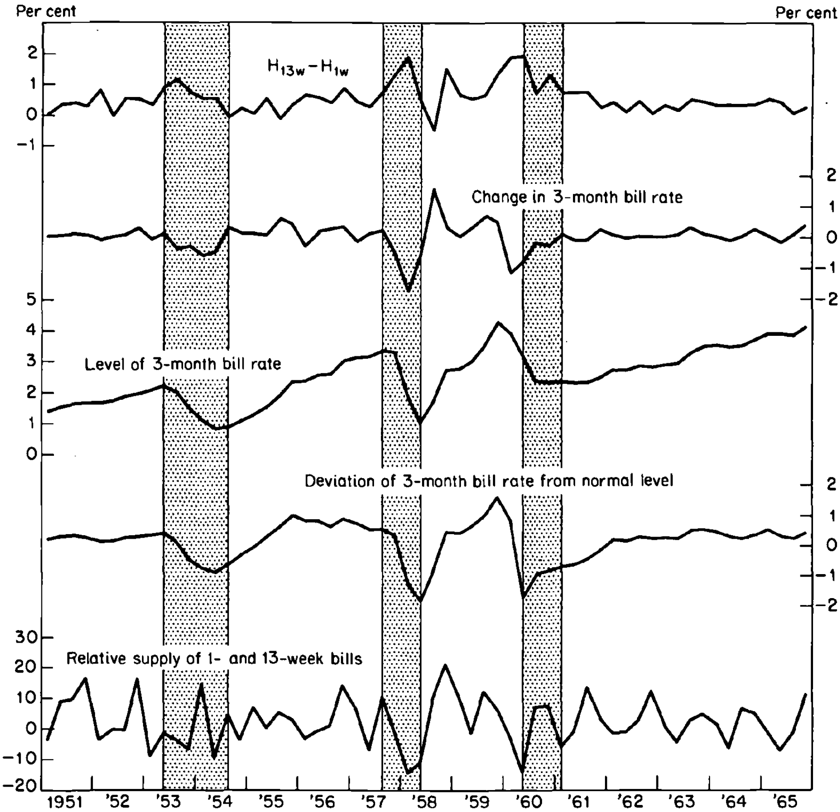
[20] Most of these data were kindly supplied to me by Jacob Michaelsen from his study. His compilation, from The First Boston Corporation, begins with 1951, ends with 1962, and is for Friday closing prices. The data were extended by the National Bureau back to 1948 and forward to 1965 and are for Tuesday prices from the quotation sheets kindly made available by the New York government securities department of Merrill Lynch. The difference in the day of the week should not make any difference for present purposes. The basic data are averages of bid and ask prices, except for the maturity value of one-week bills which was taken to be par.

CHART 4-3. Yield Curve of Treasury Bills, One-Week Holding-Period, Reference Stages and Cycle Averages



SOURCE: See footnote 20.

CHART 4-4. Estimated Liquidity Premium on Treasury Bills, Change, Level, and Deviation of Three-Month Rate, and Relative Supply, Quarterly, 1951–65



NOTE: Shaded areas represent reference cycle contractions.
SOURCE: See Table 4-1; data not seasonally adjusted.

changes in the level of rates, if not fully anticipated, produce greater capital gains and losses on long than on short maturities. This causes the difference between the error terms to duplicate their individual movements. Why should errors in expectations be negatively correlated with *changes* in rates? An obvious answer is "inertia" of expectations—a tendency of some investors to expect future rates to be the same as current ones. As a simple formulation of this interpretation, let us suppose that some constant fraction α of the market demand for securities is based on the expectation of no change in rates and the remainder is based on an expectation having a prediction error

ϵ with zero mean.[21] Then, assuming the Ls involve no error, the error terms in (6) are

$$\xi_{n,t} - \xi_{m,t} = \alpha[H_{n,t} - H_{n,t-1} - (H_{m,t} - H_{m,t-1})] + (1 - \alpha)(\epsilon_{n,t} - \epsilon_{m,t})$$
$$= \alpha[f(-\Delta R_t)] + (1 - \alpha)\, \epsilon_t. \tag{8}$$

The first term on the right is simply the difference between the changes in holding-period yields for the two maturities, which for $n > m$ is a negative function of the change in level of interest rates. The exact relationship depends upon the maturity of the securities, but we can treat it as approximately proportional to the negative of the change in the three-month bill rate. The second term is the prediction error, assumed independent of the level of rates. We may substitute (8) into (6) with a view to improving the estimate of liquidity premiums. The change in the bill rate will account for part of the errors in expectations and allow a better estimate of the effects of other variables on liquidity premiums.[22] If α is not constant, the estimates may be biased, though

[21] A more sophisticated hypothesis would be that expectations are formed from an autoregressive scheme of past rates plus an autonomous component with a mean error of zero.

[22] In *The Behavior of Interest Rates: A Progress Report,* NBER, New York, 1966, Appendix to Chapter 7, Joseph Conard questioned Kessel's empirical findings on the grounds that the liquidity premium is spuriously correlated with changes in the level of rates. Kessel's measure of the premium is the difference between the forward rate and the relevant future actual rate. Conard's point is that the forward rate, because of "inertia" in the formation of expectations, generally differs very little from the current actual rate; consequently, this measure of the liquidity premium approximates the current minus the future actual rate, which is the negative of the change in the rate. Then, if there is a tendency for the change in the rate over any period to be negatively correlated with the level of the rate at the beginning of the period, as is not unlikely because of cyclical fluctuations and as Conard presented evidence to confirm, one has identified a source of spurious correlation between the premium and the level of the rate to account for at least part of Kessel's results. In an unpublished work Kessel tested this interpretation by running regressions with the current spot rate in place of the forward rate in the estimate of the liquidity premium. On Conard's interpretation, these regressions should have higher correlation coefficients. Kessel finds higher coefficients using the forward rates, supporting the validity of his interpretation of the correlations.

The difference in holding-period yields used here as a measure of the liquidity premium is also negatively correlated with changes in the rate for a related reason: Unanticipated changes in the level of rates have an effect on the price of securities which increases with the length of maturity, and hence affect differences in holding-period yields. By holding the change in rates constant, we remove spurious correlation of the kind Conard attributed to Kessel's results and absorb part of the errors of expectations as well.

perhaps not seriously. For the tentative results derived below, such bias will be ignored.

MULTIPLE REGRESSIONS. The two theories of liquidity premiums discussed above imply certain testable relations among the variables. The relevant variables are depicted in Chart 4-4 for comparison with the differential holding-period yields between thirteen- and one-week bills. The cost of holding money is represented by the three-month bill rate—it should by the first theory relate positively to the differential. The relative supply is represented by the average difference between the amounts of one- and thirteen-week bills held by the public, expressed as a percentage of total bills held—it should by either theory relate negatively to the differential. The risk of capital losses is represented by the deviation of the three-month bill rate from normal levels estimated by a weighted average of the past nine quarters, the weights declining linearly[23]—it should by the second theory relate negatively to the differential. Multiple regressions of these variables, including the change in the three-month bill rate to represent errors of expectation, as in (8) are presented in Table 4-1.

The change in rates is highly correlated (negatively) with the difference in holding-period yields, as was evident in the chart. The level of bill rates is also significant, and has a positive regression coefficient consistent with the money-substitute theory. By this estimate, the premium increases 15 basis points for each 1 percentage point rise in the level of rates.[24]

The relative supply is not significant, however, and has a positive coefficient, inconsistent with both theories. Changes in this variable—the supply of one-week bills minus the supply of thirteen-week bills—should affect the premium inversely, since a relative rise in the supply of one-week bills can be expected to bring their yield closer to the thirteen-week rate, reducing the differential. To be sure, we may not have measured the relative supply properly. Other very short-term

[23] That is, normal level $= \sum_{i=1}^{9} \frac{10-i}{45} R_{t-i}$, as a rough approximation.

[24] Kessel (*Cyclical Behavior,* p. 26) found increases of 22 and 45 basis points in his two regressions, which, as noted, covered shorter periods and estimated liquidity premiums differently.

The regression results reflect the fact that thirteen-week yields fluctuate with greater amplitude than do one-week yields. The money-substitute theory is an acceptable explanation only if there is no inherent tendency of an institutional character for the cyclical amplitude of bill yields to increase with term to maturity. In particular, any cyclical variations in relative dealer spreads are ignored here.

TABLE 4-1. Multiple Regressions of Difference Between Holding-Period Yields for Thirteen- and One-Week Bills on Change, Level, and Deviation of Three-Month Rate, and Relative Supply, Quarterly, 1951–65

Equation Number	Regression Coefficient (and t value): Change in Rate	Level of Rate	Relative Supply	Deviation of Rate from "Normal"	Multiple Correlation Coefficient
1	−.71 (6.8)	.15 (2.9)			.69
2	−.81 (6.8)	.15 (2.9)	.01 (1.6)		.70
3	−.84 (6.8)	.12 (1.8)	.01 (1.5)	.09 (0.9)	.71

SOURCE: Holding-period yields, see footnote 20. Bill rate, *Federal Reserve Bulletin.* Relative supply based on Treasury bills held by the public, *Monthly Treasury Bulletin.* Data not seasonally adjusted.

NOTE: Dependent variable is $H_{13w} - H_{1w}$, as defined by eq. (4); quarterly average of weekly yields, per cent per annum. The constant terms are not shown. Independent variables are:

Change in rate: change over the quarter in three-month bill rate, per cent per annum.

Level of rate: three-month bill rate, quarterly average of monthly data, per cent per annum.

Relative supply: difference between one- and thirteen-week bills held by the public, quarterly average of weekly data, as percentage of midquarter total bills held [actually, the numerator was estimated by the quarterly change in total bills held divided by 13, $(TB_{13} - TB_0)/13$, which approximates for each quarter the average of weekly differences:

$$\sum_{t=1}^{13} \frac{(B_t^1 - B_t^{13})}{13} = \frac{\sum_{n=1}^{13} B_{13}^n - \sum_{n=1}^{13} B_1^n}{13} = \frac{TB_{13} - TB_1}{13},$$

where B^n is bills of maturity n outstanding in week t and TB_t is total bills outstanding in week t].

Deviation from normal: three-month bill rate minus weighted average of past rates for nine quarters, linearly declining weights (see footnote 22), per cent per annum. Quarterly data are averages of monthly rates.

Signs of t values have been dropped. At .05 level of significance, $t > 2.0$.

bills are close substitutes for the one-week bills, and thirteen-week bills are substitutable with longer-term bills. It is not clear how to treat substitutes in measuring the relative supply. In addition, appreciable bunching in the maturity distribution of bills is fairly infrequent, occurring temporarily only when the rate of new issues is suddenly changed; consequently, relative supplies may not have affected the premiums between the bill yields sufficiently to register in these regressions.

The deviations of the rate from normal levels have a low and insignificant coefficient. This variable and the level of the rate have similar movements apart from trends.[25] Each correlates positively with the premium, even when the other is held constant, though the rate level has the stronger association. The deviations should affect the premium negatively, since a fall in rates below what is considered the normal level increases the risk of capital loss and requires as compensation a yield which is higher on longer than on shorter maturities. On this evidence, changes in the premium at the short end of the yield curve do not support the second theory.

Examining the long end of the yield curve is more relevant than the short end to theories of liquidity premiums. The relevant series are shown in Chart 4-5. The top series is the difference between holding-period yields on two and a half-year bonds and thirteen-week bills. Corresponding differentials using bills and five-, seven and a half-, and ten-year bonds (not shown) have similar movements. These data, compiled by Michaelson (see footnote 20) through 1962, were not extended. For ease of comparison the change and level of the rate and its deviations from normal are the same series as in Chart 4-4, based on the three-month bill rate. It would perhaps be better (see footnote 11) to compare the premium on bonds relative to bills with the level of the bond instead of the bill rate, but any mismatching on that score does not appear serious, since movements in bond and bill rates are similar.

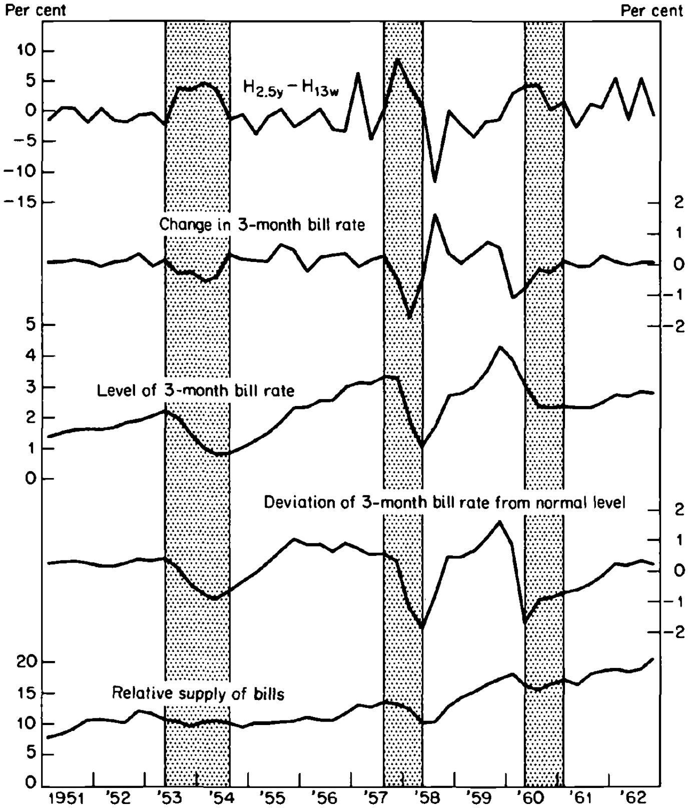
The relative supply variable is necessarily different here, however. According to the money-substitute theory of liquidity premiums, bills are used as a substitute for money balances, while bonds serve that purpose only to a very limited extent. The marginal substitutability of bills for money depends upon the supply of bills relative to money.

[25] The simple correlation between these two variables for the period covered in Table 4-1 is +.58. Lengthening the weighted average which estimates the normal level of the rate (footnote 23) would increase this correlation.

Multicollinearity between the change in rates and the deviations from normal may have turned a negative coefficient for the latter into a positive one. The correlation coefficient between the two is +.35. However, the simple correlation between the deviations and the differential is virtually zero.

Multicollinearity can also result from common cyclical fluctuations. We can hold common cyclical fluctuations in the variables constant by means of dummy variables. A separate dummy variable can be added to represent each of the stages of reference cycles in business activity. When that was done, the coefficient of the deviations variable became negative, but again it remained low and insignificant while the significance of the rate level was increased.

CHART 4-5. Estimated Liquidity Premium on Treasury Bonds, Change, Level, and Deviation of Three-Month Rate, and Relative Supply, Quarterly, 1951–62



NOTE: Shaded areas represent reference cycle contractions.
SOURCE: See Table 4-2; data not seasonally adjusted.

The chart therefore shows the ratio of bills (and CDs [26]) to the money stock outside commercial banks. It might be preferable to use instead the ratio pertaining to business and financial institutions, which hold most outstanding bills, but such data were not readily available.

[26] The statistical analysis implicitly assumes that the pecuniary yield on money is zero. With the rise of time deposit rates toward the end of the 1950's and expansion of certificates of deposit, this assumption was no longer valid. CDs were therefore treated as a substitute for bills, and were included in total bills outstanding and excluded from the money stock. Nevertheless, CDs were not quantitatively important until after 1962.

The ratio of bills to bonds outstanding is also relevant to the second theory, since a

The bond-bill differential in the chart naturally shows much greater volatility than the corresponding differential in Chart 4-4 (note that the scale is compressed here). Unanticipated changes in rates affect holding-period yields more for longer maturities. The bond-bill differential is subject to frequent and substantial errors of expectations, and some allowance for these errors seems necessary to improve the estimates of the premium. As before, we may use (the negative of) changes in the three-month rate as an index of the direction and magnitude of the errors.

Table 4-2 presents multiple regressions of the differential on the other variables in Chart 4-5. The bottom three regressions use longer-term bonds for the differential rate. The results, though generally in the same direction as in Table 4-1, are materially weaker. Changes in the rate are inversely related to the bond-bill differential, reflecting unanticipated capital losses on bonds when rates rise and unanticipated gains when rates fall. The other variables are all positively related, but none are significant. Since the relative supply and the deviations from normal are supposed to have negative coefficients but have positive ones here, we may tentatively conclude that these variables do not have the effects implied by the theory, or at least were not important in the period covered. The main qualification is that the supply variable may as before fail to include relevant substitutes for bills, and thus may be misspecified.

The level of the rate has a positive effect consistent with the money-substitute theory, and, though not significant as in Table 4-1, the coefficient has a reasonable magnitude. The coefficients for the level of the rate in Table 4-2 provide estimates of the marginal substitutability of bills for money balances. If we assume that ten-year bonds have negligible liquidity, then, as in eq. (2), $R_{13w} + S_{13w}R_{10y} = R_{10y}$. The regression may be written as

$$R_{10y} - R_{13w} = 1.2\,R_{13w}.$$

Hence

$$S_{13w}R_{10y} = 1.2\,R_{13w} = 1.2(R_{10y} - S_{13w}R_{10y}),$$

and

$$S_{13w} = \frac{1.2}{2.2} = .5;$$

rise in the ratio helps to satisfy the preference for bills and leads to a smaller differential. What total quantity of bonds should be used in the denominator, however, appears arbitrary. Yet, such a ratio, however defined, might behave similarly to the ratio of bills to money stock used here.

TABLE 4-2. Multiple Regressions of Difference Between Holding-Period Yields for Various Bonds and Thirteen-Week Bills on Change, Level, and Deviation of Three-Month Rate, and Relative Supply, Quarterly, 1951–62

Maturity of Bond in Dependent Variable (years)	Regression Coefficient (and *t* value)				Multiple Corre-lation Coeffi-cient
	Change in Rate	Level of Rate	Relative Supply	Deviation of Rate from "Normal"	
2.5	−4.7 (5.9)	.51 (1.1)			.66
	−4.6 (5.7)	.06 (0.1)	.16 (1.0)		.67
	−4.8 (5.7)			.38 (0.6)	.66
5	−9.6 (5.6)	1.0 (0.9)			.64
7.5	−12.2 (5.7)	1.3 (1.0)			.65
10	−11.8 (5.0)	1.2 (0.9)			.60

SOURCE: Same as for Table 4-1. Money stock, M. Friedman and A. Schwartz, *A Monetary History of the United States, 1867–1960,* Princeton for NBER, 1963, Table A1, extended and revised. Negotiable CDs of $100,000 or more, weekly reporting banks, from Federal Reserve Bank of St. Louis, *Review,* August 1965, Chart 1, with logarithmic interpolation.

NOTE: The dependent variable for the first regression is $H_{2.5y} - H_{13w}$, as defined by eq. (4), per cent per annum, and similarly for the others with longer bond maturities. Independent variables are the same as for Table 4-1, except for relative supply, which is total Treasury bills held by the public as a percentage of the money stock (adjusted for CDs—see footnote 26). The constant terms are not shown.

Signs of t values have been dropped. At .05 level of significance, $t > 2.0$.

that is, the liquidity premium for the given supply makes the bill yield on the average one-half the bond yield.[27]

The failure of the level of the rate to achieve statistical significance in Table 4-2 may be due to the volatility of the bond-bill differential and the failure of changes in the rate to account adequately for errors of expectations. Whether a more reliable method of adjusting for the errors would alter these results is not known. By all indications, the

[27] The liquidity premium measured from the slope of the yield curve for 1954–58 in Chart 4-1 is

$$1 - R_{13w}/R_{10y} = .22.$$

This is an alternative estimate of S_{13w}. Aside from the periods covered, the two estimates should differ only in that the method of holding expectations constant is not the same, and the measurement from the yield curve (based on a cycle average) assumes that the relation between the premium and the level of the rate is log linear and goes through the origin, while the regression assumes arithmetic linearity and does not require the constant term to be zero.

money-substitute theory outperforms the theory based on normal rates and risk aversion. The first is strongly supported by the regressions for the short end of the yield curve. Its importance to the premium between bonds and bills, though suggested by the results here, remains tentative.

Summary and Conclusions

The yields of U.S. and municipal securities, plotted by maturity, have an upward slope which appears *not* to reflect differences in transaction costs or expectations. The slope represents a lower pecuniary yield on the shorter-term securities apparently due to the greater stability of their market prices. This "liquidity" premium can be observed on U.S. securities back, at least, to 1920 when Treasury certificates were first traded.

Two theories were examined to explain short-run fluctuations in the premium. The first theory views the premium as reflecting the greater substitutability of short-term securities for money balances. When an expansion, say, in business conditions raises the level of interest rates, money becomes comparatively more expensive to hold, and it is exchanged for new securities and other assets until the marginal value of the services from the remaining money balances rises to equal the foregone return available on nonliquid long-term bonds or on capital goods. In exchanging money for other assets, the more liquid short-term securities are preferred. Short-term yields are therefore held down relative to long-term yields; although interest rates as a whole rise, the differential yields between short and long securities widen. Conversely, in a business contraction producing a decline in interest rates, the differential narrows. This increase in slope of the yield curve when the level rises, and decrease in slope when the level falls, occurs independently of any expectation of changes in rates.

The second theory views liquidity premiums as resulting from a general belief that interest rates gravitate toward the "normal" level expected to prevail over the long run and from an aversion to the risk of capital losses on long securities. If we take an average of past yields to reflect what appears to investors as normal at any time, we should find a higher yield on long relative to short securities when interest rates are relatively low, because long securities are then especially subject to capital losses; and conversely when rates are high. This implies an inverse relationship, opposite to the first theory. A test of these theories requires data on yield differentials from which the effects of expected changes in rates have somehow been eliminated.

Investors are presumed to bid bond prices up or down until, taking account of expected capital gains or losses, the expected returns on all options are equally attractive. On the average, therefore, returns from all investments are supposed to be equal over a given period except for errors in expectations and any liquidity premiums. If the errors tend to cancel out, the premiums can be estimated by the difference between the actual returns on two optional investments. The usual method has been to compare the yield to maturity of one security with the accumulated yield from successive reinvestments in a shorter-term security; for example, the yield to maturity on a six-month bill compared with the total return on two successive three-month bills. For bonds with maturities of several years or more, however, the method becomes impractical since the period covered by each comparison is as long as the maturity of the bond. Hence an estimate of the premium on two and a half-year bonds (say) gives one observation every two and one-half years; if we permit overlapping comparisons, the observations are then serially dependent and unfit for time-series analysis. To avoid such disadvantages, this study has used one-week holding-period yields, which compare the returns on buying two securities and selling both after one week regardless of their maturity. This provides one observation per week (though the statistical analysis used quarterly averages). The main drawback of this method is that unanticipated changes in interest rates produce large discrepancies between realized and expected holding-period yields on bonds. These errors in expectations are incorporated into the estimates of the liquidity premium and obscure the analysis. We attempted to account for these errors by adding changes in the market bill rate as an independent variable to the regressions. The volatility of holding-period differentials nevertheless remains troublesome, and it should be profitable in future research on liquidity premiums to explore other methods of dealing with the errors in expectations.

Although limited and necessarily tentative, the results give no support to a theory of liquidity premiums based on normal rates and favor instead the money-substitute theory. The deviations of interest rates from the normal level, representing the risk of capital loss, has the wrong sign (and is not significant) for regressions covering both the short and long ends of the yield curve. The level of interest rates does have a correct and significant positive correlation with the estimated premium on thirteen-week bills relative to one-week ones, as Kessel also found for slightly longer maturities. For the premium on bonds relative to bills, the regression coefficient of the rate level is again positive though not significant. It is unclear whether the lack of statistical significance here indicates no relation or simply our failure to measure it properly, perhaps because errors in expectations were not adequately eliminated. In any event, the size of its regression coefficient appears reasonable, suggesting that during the 1950's a $1000 Treasury bill substituted on the margin for about $500 of money balances.

Although the relative supply of money substitutes should also affect the liquidity premium, the supply variable is insignificant in the regressions. This is not, however, a conclusive test of the importance of the supply. Most changes in the distribution of Treasury bills seem too small to have measureable effects on premiums between the very short maturities. And supply effects on the bond-bill premium may not be adequately represented by changes in the ratio of Treasury bills to the money stock, as is used here. What to include in the total supply of substitutes cannot be settled theoretically but requires further empirical study. Developments during the early 1960's, when the sharp growth of certificates of deposit accompanied a decline in the bond-bill premium, suggest that supply effects may be important.

Supply effects on liquidity premiums have acquired a practical importance in recent years because of the Federal Reserve's "operation twist"—an effort during the early 1960's to raise short-term interest rates relative to long-term rates. The purpose was to alleviate the adverse U.S. balance of payments by attracting capital funds from abroad for domestic investment in short-term securities, while at the same time to reduce domestic unemployment by keeping long-term rates low enough to encourage investment expenditures. The twofold policy required a reduction in the differential between short- and long-term rates. Based on subsequent events, the policy appeared to reduce the differential, though only somewhat more than had occurred in the two previous business expansions (Chart 4-2). Whether the reduction reflected Federal Reserve market operations at all is subject to question. By the money-substitute theory, the policy required an increase in the supply of bills relative to the money stock or a decline in bond yields. But neither change appears to have been large enough to account for the decline that occurred in the differential. That is why one looks for alternative explanations such as the increased issue of CDs.

While we are not yet able to measure liquidity premiums accurately or to understand fully the causes of fluctuations, recent developments in the technique of measurement, reviewed and extended here, hold out the promise of answering such questions by quantitative analysis.

Appendix

A COMPARISON OF TWO ESTIMATES OF LIQUIDITY PREMIUMS: HOLDING-PERIOD YIELDS AND YIELDS TO MATURITY

A holding-period yield which happens to coincide with the period to maturity of a security equals the yield to maturity; otherwise the two are different. When the yields contain liquidity premiums, there are also conceptual differences in measurement of the premium. Although for most practical purposes these differences do not appear important, they should be made explicit. A simple example will help to clarify the two definitions of the premium.

Let us compare four- and eight-week bills, and suppose we calculate a holding-period yield of four weeks. That yield for four-week bills is simply their yield to maturity, $R_{4,t}$ (all yields expressed at an *annual* rate). The eight-week bill, with maturity value $P_{0,t+8}$, can be purchased at the price $P_{8,t} = P_{0,t+8}/(1 + R_{8,t})^{8/52}$, and sold in four weeks at the price then of four-week bills: $P_{4,t+4} = P_{0,t+8}/(1 + R_{4,t+4})^{4/52}$. Hence the first four-week holding-period yield on eight-week bills, at an annual rate, is

$$H_{8,t} = \left(\frac{P_{4,t+4}}{P_{8,t}}\right)^{13} - 1 = \left[\frac{(1 + R_{8,t})^{8/52}}{(1 + R_{4,t+4})^{4/52}}\right]^{13}. \tag{A1}$$

The estimate of $L_{4,t} - L_{8,t}$ by eq. (6) is therefore

$$H_{8,t} - H_{4,t} = \frac{(1 + R_{8,t})^2}{(1 + R_{4,t+4})} - 1 - R_{4,t}, \tag{A2}$$

which by an arithmetic approximation equals

$$2R_{8,t} - R_{4,t+4} - R_{4,t}. \tag{A3}$$

As originally defined by Hicks in *Value and Capital,* liquidity premiums attach to forward rates. An estimate of the premium is the forward rate minus the actual future rate. For eight- and four-week rates, we have

$${}_4F_{4,t} - R_{4,t+4} = L_{4,t+4} - L_{8,t}, \tag{A4}$$

where ${}_4F_{4,t}$ is the forward rate at time t on a four-week security beginning in four weeks, which covers eight weeks of time. The four-week forward rate, at an annual rate, is

$${}_4F_{4,t} = \left[\frac{(1 + R_{8,t})^{8/52}}{(1 + R_{4,t})^{4/52}}\right]^{13} - 1. \tag{A5}$$

The arithmetic approximation to (A5) is

$$2R_{8,t} - R_{4,t} - R_{4,t+4}, \tag{A6}$$

the same as for holding-period yields in (A3), though conceptually the premium estimated there is $L_{4,t} - L_{8,t}$, not $L_{4,t+4} - L_{8,t}$ as here. This equivalence of formulas, of course, holds only if the term of the predicted rate equals the length of the holding period. With a shorter holding period, there is no equivalent formula using forward rates and no equivalent estimate of liquidity premiums pertaining to the same time period.

In fact, since forward rates are defined in terms of yields to maturity, the liquidity premiums estimated from them are an average for a series of holding periods. Consider the premium attached to $R_{n,t}$ as implied by the slope of the yield curves in Chart 4-1. Its relation to $L_{n,t}$ used in eqs. (3)–(7) may be derived as follows.

Express $R_{n,t}$ as a geometric average of successive holding-period yields (footnote 6),

$$(1 + R_{n,t})^n = (1 + H_{n,t})(1 + H_{n-1,t+1}) \ldots (1 + H_{1,t+n}),$$

or by the arithmetic approximation,

$$R_{n,t} = \frac{\sum_{i=0}^{n-1} H_{n-i,t+1}}{n}.$$

By (5), this may be written, ignoring the error terms,

$$R_{n,t} = \frac{\sum_{i=0}^{n-1} E_{t+i} - \sum_{i=0}^{n-1} L_{n-i,t+i}}{n}.$$

For an average yield curve, let us assume that interest rates are expected to remain the same (that is, the Es for all periods are equal) and that L_n expected at t is the same for all time periods. Then, for any two successive maturities and all t,

$$R_n - R_{n-1} = -\frac{1}{n}\left(L_n - \frac{\sum_{1}^{n-1} L_i}{n-1}\right).$$

By holding-period yields, the corresponding premium is

$$H_{n,t} - H_{n-1,t} = L_{n-1,t} - L_{n,t}.$$

It seems unlikely that these differences in the measurement of liquidity premiums have much practical importance, however.