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## The Nontradable Goods' Real Exchange Rate Puzzle

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### I. Introduction

Real exchange rates are among the most volatile aggregate prices. The volatility of real exchange rates in a cross-section of countries is typically a multiple of the volatility of output, and the deviations are highly persistent. As demonstrated by Engel (1999) and Betts and Kehoe (2001, 2006, 2008), the decomposition of real exchange rate movements in the data suggests that the bulk of these movements is accounted for by the international deviations of the relative price of tradable goods—the so-called tradable component of the real exchange rate decomposition. This finding is grossly at odds with the predictions of the traditional theories of real exchange rate determination featuring one homogeneous tradable good for which the law of one price holds and distinct nontradable goods. Unlike in the data, in these theories, the real exchange rate is fully accounted for by the deviations of the relative price of nontradable goods across countries.

However disturbing these empirical results are from the perspective of the traditional theories, they are not readily inconsistent with the predictions of the standard international macro models featuring differentiated tradable goods by the country of origin—like the Backus, Kehoe, and Kydland (1995) model or the extension of this model due to Stockman and Tesar (1995). In this class of models, despite the fact that the law of one price is preserved at all times for individual tradable commodities, the tradable component of the real exchange rate can be volatile due to relative price movements of differentiated foreign and domestic varieties of tradable goods.

Motivated by this state of affairs, our paper asks to what extent standard international macro theories can be consistent with the data. More precisely, we ask whether a carefully parameterized standard theory,

extended to incorporate an explicit nontradable sector, á la Stockman and Tesar (1995), can quantitatively account for the properties of the decomposed real exchange rates and, in particular, for the small contribution of the nontradable component to the overall real exchange rate movements (as measured by its volatility relative to the overall index). Since the model that we subject to this test has been the backbone of a large strand of the literature, we view this exercise as an important stepping stone in guiding further research on the topic.

Our findings suggest that while the parameterized standard model can generate a volatile and persistent tradable component of the real exchange rate, the model still pervasively implies a too-important role for the nontradable component relative to the data. More specifically, the problem seems to lie in the nontradable component exhibiting a too strong negative correlation with the tradable component, resulting in a low volatility of the overall index (the product of the two components).<sup>1</sup> Our analysis shows that the key factor generating this failure is the response of the model to the shock in the tradable sector, which in the data turns out to be the key driver of the overall productivity. In response to this shock, the two components of the decomposition move in the opposite direction.

In terms of the literature, our work is related to the papers on the decomposition of the real exchange rates following Engel (1999). In this line of research, Betts and Kehoe (2001, 2006, 2008) extend Engel's original results by looking at a broad cross-section of country pairs, consider a broader set of decompositions, and propose a model with endogenous tradability of goods to account for the real exchange rate dynamics between the United States and Mexico. Burstein, Eichenbaum, and Rebelo (2006) study Engel's decomposition using import and export prices and conclude that, with such indices used as trade prices, the verdict is more favorable for the traditional trade theory of real exchange rate determination. Mendoza (2000) studies the bilateral real exchange rate between the United States and Mexico across different nominal exchange rate regimes and finds significant differences across regimes.<sup>2</sup> Relative to these papers, our contribution is to document how the standard international macro models fare in light of these findings. Specifically, we ask whether these properties can be accounted for by the simple theories, and, if not, what exactly precludes the model from reproducing the data.

In terms of measurement, to maintain consistency between productivity and output prices, we focus here solely on the value-added deflators as measures of tradable and nontradable output. Relative to the results

presented both in Engel (1999) and Betts and Kehoe (2006, 2008), our approach of using value-added deflators makes the results on the relative contribution of nontradable goods in the data higher—but still small in comparison to the model.

## II. Empirical Regularities

This section documents the key empirical regularities of the time-series of the bilateral real exchange rates. Specifically, we decompose the overall bilateral real exchange rate into a tradable part and a nontradable part and assess their relative contribution. Our finding is that the nontradable part we extract has, at best, a moderate contribution to the dynamics of the overall bilateral real exchange rates. This finding is broadly consistent with the related literature—even though our measurement methodology is slightly different.

We first sketch our approach to decomposing the movements of the real exchange rates and then describe the specifics of our data constructs.

We construct the overall price level of a country,  $P$ , to be a composite of tradable and nontradable components with tradable weight  $\zeta$ , according to

$$P = (P^N)^\zeta (P^T)^{1-\zeta}.$$

We next define the real exchange rate based on price index  $P$  as

$$rer = \frac{eP^*}{P}, \quad (1)$$

where  $e$  is the nominal exchange rate, and decompose it using the assumed by us internal structure of the composite price index  $P$  as follows:

$$rer = \frac{eP^*}{P} = \underbrace{\frac{eP^{T*}}{P^T}}_{rer^T} \times \underbrace{\left(\frac{P^{N*}}{P^N}\right)^\zeta}_{rer^N}. \quad (2)$$

In this decomposition, the first term captures international deviations in the relative price of local tradable output of each country—which we label “the tradable goods’ real exchange rate,” and the second term captures international deviations in the relative price of nontradable output to local tradable output—which we label the “nontradable goods’ real exchange rate.”

We now turn to the specifics of our data constructs. Our primitive series are the tradable and nontradable price measures,  $P^T$  and  $P^N$ , and the tradable weight in the price basket,  $\zeta$ . Our goal here is to come as close as possible to the interpretation given to each term above, while still being able to obtain enough data to cover a wide cross section of countries.<sup>3</sup>

To construct the key objects present in the decomposition (eq. [2]), we use annual data from 1970 to 2005 for 21 countries, giving us a total of 210 country pairs.<sup>4</sup> Our raw data include the official nominal exchange rates ( $e_{ij}$ ) from the World Bank Development Indicators database and two price measures—the manufacturing value-added deflator ( $P_i^T$ ) and total services value-added deflator<sup>5</sup> ( $P_i^N$ ) from the OECD STAN database. The weight  $\zeta$  is assumed common across countries and equal to the median share of tradable sector in the total output across countries (0.78).<sup>6</sup>

For each of the 210 country pairs in our sample, we construct the following objects:

- a value-added composite output deflator in country  $i$  constructed from the manufacturing and services value-added deflators (STAN series 1537, 5099) according to

$$P_i = \left(P_i^N\right)^\zeta \left(P_i^T\right)^{1-\zeta}, \quad (3)$$

where  $\zeta$  is assumed common and set equal to the sample median value of 0.78.

- the deflator-based bilateral real exchange rate between countries  $i$  and  $j$

$$rer = \frac{e_{ij}P_j}{P_i}, \quad (4)$$

- the tradable goods bilateral real exchange

$$rer^T = \frac{e_{ij}P_j^T}{P_i^T}, \quad (5)$$

- and the nontradable goods bilateral real exchange rate

$$rer^N = \left(\frac{P_j^N}{P_j^T}\right)^\zeta \bigg/ \left(\frac{P_i^N}{P_i^T}\right)^\zeta. \quad (6)$$

Together, these objects decompose the deflator-based bilateral real exchange rate (eq. [4]) into tradable (eq. [5]) and nontradable goods' real exchange rate (eq. [6]) as follows:

$$rer = rer_{ij}^T \times rer_{ij}^N. \quad (7)$$

We now proceed to studying the properties of the above objects in the data.

### A. Findings

This section focuses on equation (7) and the relative contribution of the tradable goods' and nontradable goods' bilateral real exchange rates,  $rer^N$  and  $rer^T$ , to the overall real exchange rate  $rer$ . In what follows, we report results for prices for the whole sample and then briefly discuss results from analyses focused on specific subsamples. All series have been logged and Hodrick-Prescott (HP) filtered with an annual smoothing parameter 100.

To summarize the properties of the decomposition in the data, we compute the median values and the 10th and 90th percentiles to give a feel of how spread out the observations are. Our main conclusions are

1.  $rer^N$  and  $rer^T$  are only moderately negatively correlated in the data ( $-0.26$ ), and  $rer^N$  carries slightly above a third of the volatility of  $rer^T$ .
2.  $rer^T$  and  $rer$  have very similar volatility, so that  $rer^N$  carries slightly above a third of the volatility of the overall index.

Table 1 presents summary descriptive statistics of our constructed series. As we can see from the first panel, the high volatility of the bilateral real exchange rate is driven predominantly by the tradable real exchange rate, with the nontradable real exchange rate contributing little. The real exchange rate for nontradable goods  $rer^N$  carries only 38.2% of the volatility of the real exchange rate as measured by the ratio  $\sigma(rer^N)/\sigma(rer)$ . The second panel of table 1 reveals the source of the small contribution of  $rer^N$  to the overall index  $rer$ . First,  $rer^T$  is much more volatile than  $rer^N$ —in fact, it is more volatile than  $rer$ . Second,  $rer^T$  and  $rer^N$  are weakly negatively correlated, with the median correlation of  $-0.26$  and a very wide range of numbers across pairs. This means that the two components do not systematically offset each other in the data, and, hence, both of their volatilities contribute to the volatility of the overall index.

**Table 1**  
Decomposition of *rer* in the Data

	Median	10th Percentile	90th Percentile
Comparison of <i>rer</i> and $rer^N$ :			
$\sigma(rer)$ (%)	6.55	2.90	11.40
$\sigma(rer^N)/\sigma(rer)$ (%)	38.20	18.70	80.10
$\rho(rer^N, rer)$	.09	-.38	.47
Comparison of $rer^T$ and $rer^N$ :			
$\sigma(rer^T)$ (%)	7.00	3.50	11.30
$\sigma(rer^N)/\sigma(rer^T)$ (%)	38.10	18.50	65.00
$\rho(rer^T, rer^N)$	-.26	-.72	.16

Note: All reported statistics are based on logged and HP-filtered series with a smoothing parameter 100.

### B. Robustness

The results discussed in this section are not unique to our sample of countries or years. Several authors have conducted similar exercises and reached similar conclusions, starting with Engel (1999) and extended in a series of papers by Betts and Kehoe (2001, 2006, 2008).

Using our data, we conducted a detailed analysis of the decomposition in several subsamples, with essentially the same conclusions. In one of our exercises, motivated by the fact that European countries look different in terms of trade statistics and policies, we have analyzed the decomposition in subsamples of non-European country pairs and of European pairs. For non-European country pairs, we found that the relative volatility of the nontradable real exchange rate is lower, bringing the relative volatility  $\sigma(rer^N)/\sigma(rer)$  down to 25%. This is not the case in the European case, where it stays at about 50%.<sup>7</sup> We have also studied how these statistics change depending on the bilateral trade intensity. We did not find any significant patterns.

### III. The Model

In this section, we formally set up the standard model of international business cycles under complete markets. To highlight the links between prices and quantities, we focus on a decentralized equilibrium. The model is closely related to the setup in Stockman and Tesar (1995), additionally including a distribution sector as in Burstein, Neves, and Rebelo (2003) to capture the fine details of how a nontradable component enters the final consumption. We also extend the model to a three-country setup to have a

more natural mapping between the productivity series available from the data and the process needed for the quantitative model.<sup>8</sup>

### A. *Physical Environment*

The world economy is composed of three countries: two small countries, home ( $H$ ) and foreign ( $F$ ), and one large country, the rest of the world ( $G$ ). Time is discrete and horizon infinite ( $t = 0, 1, 2 \dots$ ).

Each country produces a local nontradable good and a country-specific tradable good. The tradable good produced in the home country is labeled  $H$  (home good), the tradable good produced by the foreign country is labeled  $F$  (foreign good), and the tradable good produced by the rest of the world is labeled  $G$  (global good). The nontradable good in each country is labeled  $N$ .

There are three sectors in the economy: a production sector for tradable goods, a production sector for nontradable goods, and a distribution sector. Producers in the tradable and nontradable sectors employ labor supplied by households and produce a country-specific tradable good or the nontradable good, depending on the sector. Agents in the distribution sector, the distributors, buy tradable goods from each country's producers, aggregate them into a composite tradable consumption good, and pay a nontradable distribution cost to deliver the tradable consumption to households. Households consume tradables and nontradables, supply labor, and trade a complete set of state contingent assets. All markets are perfectly competitive.

### B. *Notation*

Variables in the model are subscripted and superscripted. Our convention is that the subscript denotes the country to which a variable pertains or in which a given activity actually takes place (e.g., consumption or investment). Depending on the context, the superscript denotes the country/sector of origin or the type of good involved. For example, the price of the home good in the foreign country is denoted as  $P_F^H$ .

All variables in the model are history dependent, where the history of shocks up to and including period  $t$  is denoted by  $s^t = (s_0, s_1, \dots, s_t)$ . The seed value  $s_0$  as well as the time-invariant product probability measure  $\mu(\cdot)$  over the space of all possible histories  $S^t$  are assumed given.

### C. Output and Productivity

In each country,  $i \in \{H, F, W\}$ , local producers have access to a linear production function that uses labor as the only input and is subject to the country- and sector-specific productivity shock  $z_i^j$

$$y_i^j(l) = z_i^j l, \quad j = T, N.$$

The only source of uncertainty in the economy is stochastic productivity  $z_i^j$ , which is assumed to follow a joint AR(1) process,

$$\mathbf{z}(s^t) = \mathbf{A}\mathbf{z}(s^{t-1}) + \varepsilon, \quad (8)$$

$$\mathbf{z} = [z_H^T \quad z_H^N \quad z_F^T \quad z_F^N \quad z_G^T \quad z_G^N],$$

where  $\varepsilon$ —identified with the primitive event  $s_t$ —is assumed to be an independently and identically distributed random variable with zero mean and a finite variance-covariance matrix  $\Sigma$ .

In the presentation of the setup, we exploit the assumption of constant returns to scale and summarize all production constraints by the marginal production costs, given by

$$v_i^T(s^t) = \frac{w_i(s^t)}{z_i^T(s^t)}, \quad (9)$$

$$v_i^N(s^t) = \frac{w_i(s^t)}{z_i^N(s^t)}, \quad (10)$$

for the tradable sector,  $T$ , and the nontradable and distribution sectors,  $N$ .

### D. Household's Problem

In each country  $i$ , there is a measure  $n_i$  of households, each endowed with one unit of labor. The population size is assumed equal between home and foreign country and larger in the rest of the world ( $n_H = n_F < n_G$ ).

Households supply labor inelastically, purchase tradable and nontradable goods in the local markets, and trade a complete set of financial assets in an integrated world asset market. Their objective is to

maximize the expected discounted stream of flow utility from the composite consumption  $c_i$ ,

$$\sum_{t=0}^{\infty} \beta^t \int_{S^t} u[c_i(s^t)] d\mu(s^t),$$

where  $c_i$  is determined by consumption of tradable and nontradable goods through a CES aggregator  $c(\cdot)$ ,

$$c_i = c(c_i^T, c_i^N). \tag{11}$$

In their choice, households are constrained by a sequence of budget constraints given by

$$P_i^T c_i^T + P_i^N c_i^N + \int Q(s_{t+1}, s^t) b_i(s_{t+1}, s^t) d\mu(s_{t+1}) = b_i(s^t) + w_i(s^t) n_i + \Pi_i(s^t).$$

Household's expenditures are comprised of expenditures on tradable and nontradable consumption and purchases of a set of one-period-forward state contingent bonds  $b_i(s_{t+1}, s^t)$ , priced by the kernel  $Q_i(s_{t+1}, s^t)$ . Household's income is derived from the payoff of previously purchased bonds  $b_i(s^t)$ , labor income  $w_i(s^t) n_i$ , and dividends paid out by home producers. To avoid Ponzi schemes, bond holdings of the household are assumed to be bounded from below.

The *numéraire* in each country is assumed to be the  $s^t$ -composite consumption  $c_i$ . By interest rate parity condition, we can recover the evolution of the relative price of the composite consumption in country  $j$  in the units of country  $i$ ,

$$x_i^j(s^t) = x_i^j(0) \frac{u'[c_j(s^t)]}{u'[c_i(s^t)]}. \tag{12}$$

The above condition states that the households perfectly share consumption risk in the sense of equalizing the cross-country marginal rate of substitution of consumption with the relative price of consumption baskets (as measured by  $x$ ). The constant  $x_i^j(0)$  in the above expression guarantees that in terms of the expected present discounted value, no net flows of wealth between countries are observed in equilibrium.

### E. Producers and Distributors

Both producers and distributors operate in a perfectly competitive market. Producers of tradable goods sell their goods to distributors, who

aggregate them, incur the distribution cost, and resell tradable consumption to households. Nontradable goods have no explicit distribution cost.

Producers sell their respective goods in a perfectly competitive market and face a marginal cost of production  $v_i^j(s^t)$ . Profit in state  $s^t$  of a producer of good  $k$  in country  $i$  is

$$\pi_i(s^t) = y_i^k(p_i^k - v_i^k),$$

and the production constraint is

$$y_i^k \leq z_i^k L_i^k.$$

The zero profit conditions imply that producer prices are equal to the marginal cost of production, that is,

$$p_i^k = v_i^k, \quad k = N, H, F, G.$$

Distributors purchase tradable goods from producers in each country and aggregate them into a composite tradable consumption good. They then resell tradable consumption in the local perfectly competitive market. The distribution cost, denoted  $\xi$ , is paid in the local nontradable good. Given prices, distributors choose  $(c_i^T, q_i^H, q_i^F, q_i^G)$  to maximize profit

$$\pi_i^D(s^t) = (P_i^T - \xi P_i^N) c_i^T - p_i^H q_i^H - p_i^F q_i^F - p_i^G q_i^G, \quad (13)$$

subject to

$$c_i^T \leq \left[ \sum_{j=H,F,G} \omega_i^j (q_i^j)^{(\gamma-1)/\gamma} \right]^{(\gamma-1)/\gamma},$$

where  $\sum_j \omega_i^j = 1$ , and  $\gamma > 0$  is the elasticity of substitution.<sup>9</sup>

#### F. Market Clearing and Feasibility

Equilibrium requires several market clearing and feasibility conditions to be satisfied. Consumption of tradables by all countries has to be equal to production

$$y_i^T = z_i^T L_i^T, \quad (14)$$

$$\sum_{i=h,f,g} q_i^j = y_j^T. \quad (15)$$

Consumption of nontradables in each country and expenditures on distribution must be equal to the production of nontradables, and labor markets must clear

$$c_i^N + \xi c_i^T = z_i^N L_i^N,$$

$$L_i^N + L_i^T = n_i.$$

The definition of equilibrium is straightforward and will be omitted.

#### IV. Dynamics of Prices and Quantities in the Model

This section studies the response of key prices to sectoral productivity shocks. In what follows, we study the forces behind  $rer^T$  and  $rer^N$  movements in the model and then link our analysis back to the decomposition of the real exchange rate and to the failure of the model to account for the modest contribution of the nontradable goods' real exchange rate to the overall real exchange rate. We will work out the mechanics of prices in a bilateral pair, which immediately maps to the general case. We do not include formal plots of the impulse responses, as the analysis below is rather straightforward.

##### A. *Dynamic Response of $rer^T$ and $rer^N$ to Tradable Sector Productivity Shock*

To understand the response of key prices to a tradable shock, it is instructive to focus on a simplified depiction of the market for the home good at home and abroad illustrated in figure 1. Note that due to home bias, the initial quantity sold (point *A*) is smaller in the foreign market than in the home market, and, thus, given the constant elasticity nature of the demand in the model, the foreign market demand line is also plotted as steeper than the home one.<sup>10</sup> The key driving force of the prices in the model is the fall in production cost in the home tradable sector after a positive productivity shock faced by the home producers and their subsequent attempt to expand supply both at home and abroad. As indicated in the figure, this behavior results in an asymmetric fall of the price of the home good in the home market versus the foreign market and a simultaneous real exchange rate depreciation.

The mechanics behind these responses are as follows. When the home producers attempt to expand quantity sold in both markets, they face a steeper demand abroad than at home—due to home bias. Thus, due to

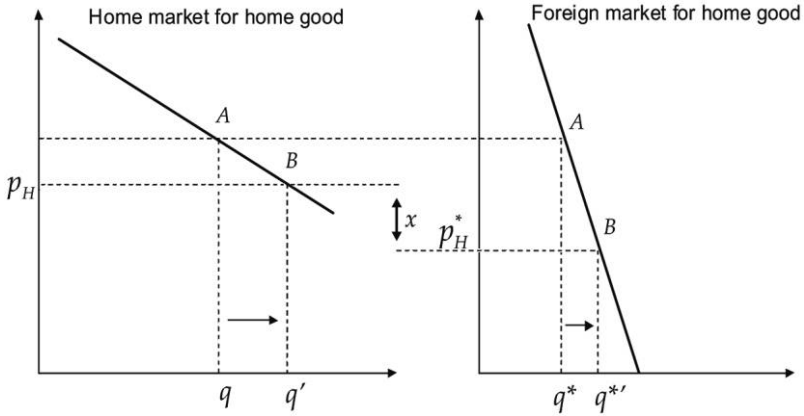


Fig. 1. Response of prices to positive shock to the home country tradable sector

arbitrage considerations, more quantity is directed to the home market than to the foreign market, which leads to an increase in the overall consumption at home relative to the consumption abroad and, by the risk-sharing condition shown in equation (12), to a simultaneous depreciation of the consumer price index (CPI) real exchange rate  $x$ . At the same time—since not much is happening to the foreign price of the foreign good  $p_F^*$ —the price of the foreign good at home  $xp_F^*$  goes up with the CPI real exchange rate  $x$ . Consequently, given the approximate formula for bilateral  $rer^T$ ,<sup>11</sup>

$$rer^T = \frac{xp_F^*}{p_H}, \tag{16}$$

an even more forceful depreciation of this object (relative to  $x$ ) is observed due to a simultaneous fall of  $p_H$ , in the model.

Under perfect labor mobility,  $rer^N$  can actually be traced back to the primitive productivities as follows:<sup>12</sup>

$$rer^N = \left( \frac{p^{N*}}{p_F^*} \right)^\zeta / \left( \frac{p^N}{p_H} \right)^\zeta = \left( \frac{z_T^*}{z_N^*} \right)^\zeta / \left( \frac{z_T}{z_N} \right)^\zeta.$$

Thus, following the shock it  $rer^N$  falls (appreciates).

*B. Dynamic Response of  $rer^T$  and  $rer^N$  to Nontradable Sector Productivity Shock*

Clearly, by the above formula, the increase in the productivity of the nontradable sector results in the increase of  $rer^N$ . However, due to an

increase in the home consumption  $c$ , by the risk-sharing condition shown in equation (12), the real exchange rate  $x$  also depreciates. This, in turn, results in a simultaneous reallocation of labor from the nontradable sector to the now more profitable home tradable sector (foreign price level increased), leading to a depreciation of  $rer^T$  through an analogous mechanism to the one described in the discussion of the response to the tradable sector shock. This implies that after a nontradable shock  $rer^N$  and  $rer^T$  positively comove—unlike in response to the tradable sector productivity shock.

The positive comovement between  $rer^N$  and  $rer^T$  after the nontradable shock makes this shock play a potentially important role in driving the real exchange rate  $rer$  movements as it brings the model closer to the weak correlation in the data. This is especially true compared to the tradable sector shock, for which  $rer^N$  and  $rer^T$  have moved in offsetting directions. However, because in the quantitative model the nontradable shock plays only a minor role due to its low volatility, the response to tradable shock plays a bigger role in generating the fluctuations of  $rer$ , implying an overall counterfactual performance of the model in terms of importance of  $rer^N$ .

## V. Quantitative Analysis

### A. Parameterization

This section describes the choice of functional forms and parameters. We will discuss two different parameterizations, labeled Benchmark and High Elasticity. In the Benchmark calibration, we perform a search over the set of admissible parameters for the risk aversion  $\sigma$  and the elasticity of substitution  $\gamma$ , to come as close as possible to reproducing the volatility of  $rer^T$  and the volatility of  $rer^N$  relative to  $rer$ . We describe the admissible set for these parameters below, together with our remaining parameter choices. As it turns out, lower values of the elasticity of substitution  $\gamma$  bring the model closer to data, hence our choice in the Benchmark calibration will be at the lower end of the admissible interval, significantly below one. Since such a low value for the elasticity of substitution is unconventional for the international business cycles literature, which uses values for  $\gamma$  closer to two, we provide the results from the second parameterization, labeled High Elasticity, in which we set the elasticity of substitution equal to the upper bound of our admissible region and recalibrate the rest of the parameters.

## 1. Functional Forms

We assume the utility function to be constant relative risk aversion with the intertemporal elasticity parameter  $\sigma$ :

$$u(c_i) = \frac{c_i^{1-\sigma}}{1-\sigma}.$$

The aggregator between tradable  $c_i^T$  and nontradable consumption  $c_i^N$  is assumed to be Cobb-Douglas, with the share on nontradables parameterized by  $\zeta$ :

$$c(c_i^T, c_i^N) = (c_i^N)^\zeta (c_i^T)^{1-\zeta}.$$

The built-in assumption of a unit elasticity between the two consumption components in the above aggregator is on the high side of the elasticity numbers used in the literature. For example, Stockman and Tesar (1995) use 0.44, and Corsetti, Dedola, and Leduc (2008) use 0.75. A lower value of the elasticity parameter acts similarly as an increase in  $\sigma$  but does not have a big effect quantitatively. The remaining functional forms are stated in the setup of the model.

## 2. Parameter Values

The values of all the parameters are listed in table 2. For both parameterizations, we use our own estimates of the productivity process, estimated to account for productivity fluctuations in a three-country, two-sector system. These estimates to our knowledge are new to the literature. Below, we provide a detailed description of how we have chosen the values for parameters and which moments from the data were used as calibration targets. In the actual parameterization exercise, most of the parameters have to be determined jointly, so our identification of a target with parameter serves as a guide to which moment a given parameter affects the most.

Population  $n_i$  of the relative rest of the world (country  $G$ ) has been set so that country  $G$  is 20 times bigger than  $H$  or  $F$ . The value of the intertemporal discount  $\beta$  is 0.96, and in the stationary equilibrium it implies a real risk-free interest rate of 4%. Factoring in an expected world growth of about 2% to 3%, it implies a real interest rate of about 6% to 7%.

The share of consumption of nontradable goods in the final consumption  $\zeta$  and the distribution cost  $\xi$  have been selected to account for the median 78% share of nontradable sectors in total output of countries from

**Table 2**  
Parameter Values

Description	Symbol	Parameterization	
		Benchmark	High Elasticity <sup>a</sup>
Common parameters:			
Discount factor	$\beta$	.96	
Risk aversion	$\sigma$	2.6	3.0
Share of $N$ consumption	$\zeta$	.417	.419
Distribution cost	$\xi$	3.0	3.5
Elasticity between $T$ goods	$\gamma$	.59	1.62
Country-specific parameters:			
Home country:			
Weight on home good	$\omega_H^H$	.10	.20
Weight on foreign good	$\omega_H^F$	.0053	.071
Weight on global good	$\omega_H^G$	.8947	.729
Population size	$n_H$	5.0	
Relative rest of the world:			
Weight on home good	$\omega_G^G$	.988	.8392
Weight on foreign good	$\omega_G^H$	.006	.0804
Weight on global good	$\omega_G^F$	.006	.0804
Population size	$n_G$	100.	
Productivity process (the same for both parameterizations):			
Spillover matrix	$A =$	$\begin{bmatrix} .956 & -.033 & 0 & 0 & -.098 & -.215 \\ .00 & .96 & 0 & 0 & .039 & -.078 \\ 0 & 0 & .956 & -.033 & -.098 & -.215 \\ 0 & 0 & .00 & .96 & .039 & -.078 \\ 0 & 0 & 0 & 0 & .8 & -.3 \\ 0 & 0 & 0 & 0 & .036 & .819 \end{bmatrix}$	
Variance-covariance matrix	$\Sigma =$	$\begin{bmatrix} .17 & .039 & .032 & .019 & .037 & .012 \\ .039 & .033 & .012 & .005 & .013 & .005 \\ .032 & .012 & .17 & .039 & .037 & .023 \\ .019 & .005 & .039 & .033 & .012 & .006 \\ .037 & .012 & .037 & .012 & .06 & .015 \\ .012 & .005 & .012 & .005 & .015 & .008 \end{bmatrix} \times 10^{-2}$	

<sup>a</sup>Values reported only when different from the Benchmark calibration.

our sample in the year 2000 and the estimate of a 50% share of nontradable inputs in the price of final goods on the consumer level as estimated by Burstein et al. (2003).<sup>13</sup>

The weights  $\omega_i^j$  on each tradable good have been chosen to account for the median bilateral trade intensity (imports from selected partner country to total imports) and the median trade openness (imports/gross domestic product [GDP]) for the set of countries excluding European countries. We exclude all European country pairs (also European with non-European pairs) because the model does not make any distinction

between gross and net trade flows and is ill-suited to match the trade openness of countries in which import figures are inflated by cross-border production sharing.<sup>14</sup> The targeted trade openness is 17.5%, and the targeted bilateral trade intensity is 3.45%. To obtain the target for trade openness of the relative rest of the world, we have calculated the median imports of the world from the pairs of countries from our sample (excluding European pairs) evaluated relative to the world GDP (less the GDP of the selected pair of countries), which gave us a target of 0.8%.<sup>15</sup>

### 3. Estimation of the Productivity Process

The productivity process is one of the most crucial elements determining the performance of the model, hence a proper quantitative exercise required its careful estimation. Parameters governing the forcing process,  $A$  and  $\Sigma$ , were obtained by fitting an AR(1) process to the panel of annual sectoral productivity constructed by us. To maximize the number of countries in our constructs, we use series starting in 1977. The productivity series have been calculated from output and employment series available from the STAN database. As sectoral measure of output, we have used value-added volumes (STAN code VALK 1537 and 5099). These two groupings account for about 85% to 90% of total output in a median economy from our sample—we left out agriculture, mining, and construction.<sup>16</sup>

To construct a labor productivity series from output and employment series, we have divided sectoral output by total hours worked (HRSN), and, when not available, we have used instead total employment series (EMPN).<sup>17</sup> For each pair of countries in the sample, we have obtained the relative rest of the world productivity time series by aggregating all the remaining countries together. To build this aggregate, we have first normalized individual productivity series so that the number in the year 2000 corresponds to the share of each sector in total output, and then we multiplied each country series by the corresponding purchasing-power-parity-based GDP in year 2000 to weight them properly (obtained from Penn World Tables). Finally, to render the resulting productivity series stationary, we subtracted exponential trend growth for each country pair equal to the median growth rate of all countries in the sample (by sector).<sup>18</sup> Using ordinary least squares, we have obtained a spillover matrix  $A$  and a variance-covariance matrix  $\Sigma$  for the order of variables given by  $z_H^T, z_H^N, z_F^T, z_F^N, z_G^T, z_G^N$  (reported in table 3).

**Table 3**  
Quantities: Data versus Models

Statistic	Data	Parameterization	
		Benchmark	High Elasticity
Standard deviations (%):			
GDP	1.76	1.93	1.94
GDP <sup>T</sup>	3.39	3.26	5.90
GDP <sup>N</sup>	1.49	1.85	1.39
L <sup>T</sup> /L	1.90	1.38	2.98
L <sup>N</sup> /L	.55	.36	.77
z <sub>h</sub> <sup>T</sup>	3.39	3.92	3.92
z <sub>h</sub> <sup>N</sup>	1.50	1.71	1.71
z <sub>h</sub> <sup>T</sup> /z <sub>h</sub> <sup>N</sup>	2.88	3.31	3.31
z <sub>rw</sub> <sup>T</sup>	2.69	2.39	2.39
z <sub>rw</sub> <sup>N</sup>	1.08	.89	.89
Correlations:			
GDP <sup>T</sup> , GDP <sup>N</sup>	.62	.83	.50
L <sup>T</sup> /L, L <sup>N</sup> /L	-.97	-1.00	-.99
z <sub>h</sub> <sup>T</sup> , z <sub>h</sub> <sup>N</sup>	.63	.54	.54

Note: The note in table 1 applies.

#### 4. Ranges for $\gamma$ and $\sigma$

The range for the elasticity of substitution  $\gamma$  has been selected based on the so-called short-run elasticity of trade flows—a measure of how trade flows between countries responding to relative price changes seen in the time-series. Instead of relying on microlevel estimates of such elasticity typically used in the literature, we use our own methodology based on the aggregate data, developed in Drozd and Nosal (2008). The advantage of our methodology is a more natural mapping between the aggregate model and the data and the avoidance of the use of correlation—which in simple regressions of this sort may create a bias due to lagged adjustment of quantities to prices (*J*-curve).

The details of our approach closely follow Drozd and Nosal (2008), with the only difference being our use of annual and not quarterly data. In particular, we define a measure of the short-run elasticity of trade, called the volatility ratio (*VR*), as

$$VR \equiv \sigma \left( \log \frac{f_t}{d_t} \right) / \sigma \left( \log \frac{p_{d,t}}{p_{f,t}} \right). \quad (17)$$

We use annual data for manufacturing the value-added volume index to measure  $d$ , annual data on imports in constant prices to measure  $f$ , and their respective deflators to measure their corresponding relative price.<sup>19</sup>

The range of estimates of  $VR$  that we obtain in our sample is between 0.59 and 1.62. We will take these extreme values as the bounds of our search interval for the elasticity of substitution  $\gamma$ , and for the Benchmark parameterization, we choose its value to come as close as possible to matching the relative volatility of  $rer^N$  to a  $rer$  of 38.2%.<sup>20</sup> For the High Elasticity parameterization, we choose the value of  $\gamma$  of 1.62, which is in the ballpark of values for elasticity more commonly used in business cycle literature.

The acceptable range for the risk-aversion parameter  $\sigma$  that we use in our search algorithm is defined by an interval whose center is the most common value used in business cycle literature, 2. We will consider values between 1 and 3, and choose  $\sigma$  so that the model comes as close as possible to matching the annual volatility of the tradable goods' real exchange rate  $rer^T$  (7% in the data).

## B. Quantitative Results

This section presents our main quantitative results. First, we briefly present results for quantities to demonstrate that the model's fit does not exhibit serious anomalies in this respect, and then we proceed with the presentation of the results for prices. The results for prices will be evaluated from the perspective of two stylized facts established in the data section:

1.  $rer^N$  and  $rer^T$  are only moderately negatively correlated in the data ( $-0.26$ ), and  $rer^N$  carries slightly above a third of the volatility of  $rer^T$ .
2.  $rer^T$  and  $rer$  have very similar volatility, so that  $rer^N$  carries slightly above a third of the volatility of the overall index.

### 1. Quantities

Before we proceed to the discussion of the results for prices, we need to take a brief look at what the models predict for quantities. Given its simplified supply-side structure, we would like to make sure that the model does not imply (i) excessive reallocations of labor over the business cycle or (ii) excess volatility of output and productivity. We would also like to investigate if the volatility of relative sectoral productivity  $z_H^T/z_H^N$  is consistent with the data, since it determines the level of the volatility of  $rer^N$ . The results, presented in table 3, confirm that the estimated productivity process does come very close to the median in the data and that the Benchmark parameterization comes very close

to reproducing the data for relocation of the sectoral labor and behavior of sectoral GDP.<sup>21</sup> The volatility of  $z_H^T/z_H^N$  is overpredicted by the model by about 20%, which contributes somewhat to the high volatility of the nontradable goods' real exchange rate. However, as we will see below, the difference is not as big as the failure of the model, and this overprediction turns out not to be the main culprit for the model's failure to reproduce the volatility of  $rer^N$  relative to  $rer$ . The High Elasticity parameterization implies too much relocation of labor across sectors and thus overshoots the volatility of the tradable GDP. This finding confirms that our choice of the elasticity of substitution between tradable and nontradable goods is on the high side.<sup>22</sup> We now proceed to the discussion of our main results for prices.

## 2. Prices

The results for prices are reported in table 4, which follows the structure of the tables in the data section. The first panel of table 4 shows the main moments of the decomposition of the real exchange rate. As we can see, both parameterizations of the model underpredict the volatility of the real exchange rate and overpredict the contribution of the nontradable goods' real exchange rate, as measured by the relative volatility  $\sigma(rer^N)/\sigma(rer)$ . For the Benchmark parameterization, this number is overpredicted by almost 60%, while for the High Elasticity parameterization, by over 140%. The second panel of table 4 gives us hints as to the source of this failure. It shows the relation between the tradable goods' real exchange rate and the nontradable goods' real exchange rate. For the Benchmark parameterization, we were able to reproduce

**Table 4**  
Prices: Data versus Models

Statistic	Data	Parameterization	
		Benchmark	High Elasticity
Comparison of $rer$ and $rer^N$ :			
$\sigma(rer)$ (%)	6.6	5.28	3.52
$\sigma(rer^N)/\sigma(rer)$ (%)	38.2	62	93
$\rho(rer^N, rer)$	.09	-.31	-.01
Comparison of $rer^T$ and $rer^N$ :			
$\sigma(rer^T)$ (%)	7.0	7.0	4.86
$\sigma(rer^N)/\sigma(rer^T)$ (%)	38.1	47	67
$\rho(rer^T, rer^N)$	-.26	-.70	-.69

Note: All series refer to bilateral statistics between home and foreign country. The note in table 1 applies.

the volatility of  $rer^T$  exactly, but the volatility of  $rer^N$  is about 20% higher than in the data. However, what precludes the model from matching the data is the predicted correlation between  $rer^T$  and  $rer^N$  (third row). The model predicts a value of roughly  $-0.7$  under both parameterizations, compared to the data value of  $-0.26$ . Such strong negative correlation brings down the volatility of  $rer$ , as implied by the following decomposition:

$$\sigma_{rer} = \sqrt{(\sigma_{rer^T} + \sigma_{rer^N})^2 + 2[\rho(rer^T, rer^N) - 1]\sigma_{rer^T}\sigma_{rer^N}}, \quad (18)$$

and, therefore, contributes to the failure to reproduce the relative volatility of  $rer^N$  and  $rer$ . In the High Elasticity parameterization, the upper-bound value of  $\sigma$  is still not able to deliver a high enough volatility of  $rer^T$ , and, thus, the model underpredicts the volatility of both  $rer^T$  to  $rer$ .<sup>23</sup>

## VI. Conclusions

In this paper, we study whether standard models, when extended to include nontradable sectors in a disciplined manner, can account for the decomposition of the real exchange rate into tradable and nontradable components. We find that while the parameterized standard model can generate a volatile and persistent tradable component of the real exchange rate, it implies a nontradable component that is too volatile. Moreover, comparing to the data, the nontradable component exhibits a too strong negative correlation with the tradable component, resulting in an insufficient volatility of the overall index. Because this is a pervasive feature of the theory across all parameterizations, we conclude that this property of the data should be thought of as a puzzle with respect to the standard models. Our analysis shows that the key factor generating this puzzle is the response of the model to the shock in the tradable sector, which in the data turns out to be the key driver of the overall productivity. In response to this shock, component real exchange rates move in the opposite directions, resulting in a low volatility of their product. The nontradable shocks, while pushing the model in the right direction, carry too little volatility to bring the model back on track.

What can possibly account for this puzzle? Our conjecture, based on the fact that mechanically more cushion is needed to isolate domestic prices from volatile international prices, is that theories that feature some form of the deviations from the law of one price may be more successful in accounting for the facts. Such resolution of the puzzle would be consistent with the anecdotal evidence suggesting stability

of the relative prices of home to foreign goods in the data. The list of promising theories would then include the models of pricing-to-market (e.g., work by Drozd and Nosal [2008] or Atkeson and Burstein [2008]) or sticky price models featuring local currency pricing. Future research will show to what extent a plausibly parameterized extended theory can match the actual data.

## Endnotes

1. The real exchange rate construct studied in this paper is based on value-added output deflators. For CPI-based real exchange rates, the mismatch of theory and data is even more pronounced.

2. Mendoza (2000) finds the managed exchange rate regime more favorable to the traditional theory prediction of the high contribution of nontradable components of the real exchange rate.

3. It is important to note that our definition of nontradable real exchange rate is slightly different than the one used by Engel (1999) or Betts and Kehoe (2008). Engel (1999) uses the relative nontradable CPI to tradable CPI in each country, while Betts and Kehoe (2008) use the overall CPI relative to the producer price index in each country. Both the tradable CPI and the overall CPI include imported goods, while our method includes only locally produced and sold goods.

4. This is the widest date range. For some pairs, data are limited to fewer years.

5. The advantage of using value-added deflators is their wide availability in the Organization for Economic Development (OECD) STAN database. The major concern—which is not going to be important for our results—is that they may not necessarily represent the actual market prices of tradable and nontradable goods at any level of aggregation. For example, when a firm producing a tradable good outsources some of the activities to a firm from a nontradable sector (e.g., business service sector), this activity may artificially inflate the volatility of the value-added deflator due to potential demand-side links between them. Yet, the value of the outsourced nontradable service will be included in the final good prices of tradables but not in the final good prices of nontradables (these nontradables are intermediate goods). Betts and Kehoe (2006) discuss this issue in detail and, by comparing value-added deflators to gross output deflators, find that the value-added decomposition results in a significantly higher variance of the nontradable goods' real exchange rate. Since our intention here is to find the upper bound of the contribution of nontradable prices to real exchange rate fluctuations, we are comfortable with this property.

6. We found modest variation of this weight and also experimented with weights varying by country. It did not change any of the results.

7. We conjecture that these differences are attributable to the fact that European countries pegged their nominal currencies. The results are reminiscent of Mendoza (2000) and Mussa (1986). Since European countries trade more with each other, it is not clear that theory would be more successful when restricted to this subset.

8. This structure allows better control shock spillovers between countries and the rest of the world. In addition, the three-country setup also disconnects bilateral trade intensity from trade openness—which in a two-country setup is the same thing. Thus, such an environment better disciplines possible endogenous demand spillovers across countries depending on how open they are vs. how much they trade with each other.

9. Note that nontradable goods are used separately as consumption and to distribute the tradable goods.

10. The demand lines in the model are constant elasticity demand lines as implied by the CES aggregator. However, the crucial thing for our argument here is the local difference in slopes due to home bias, which we highlight by plotting linear demand lines.

11. In the model, we take great care to measure data analogously to the way it is measured in the model. Therefore, this is only an approximate formula, and in our quantitative

exercise  $rer^T$  is defined using deflator prices of sectoral output. These prices, however, turn out almost identical to the actual prices.

12. We should note that perfect labor mobility is actually the most favorable assumption to find a modest contribution of the nontradable component of real exchange rate decomposition. Any friction precluding relocation of labor makes the relative price of nontradable goods only more important.

13. The data come from the STAN database. To obtain this number, we evaluated the ratio of value added in total services to total value added in all sectors.

14. With the share of nontradable goods unchanged, our model is not capable of matching any numbers in excess of 22% for trade openness. Trade openness in the full sample is 28%, and bilateral trade intensity is 1.25%. Our conservative approach to matching the trade numbers only reinforces our results, as more trade in this model deteriorates its performance.

15. Data source: International Financial Statistics Database, IMF, update 2005.

16. Note that this measurement of sectoral productivity exactly aligns with the way we measure prices of the corresponding sectoral output.

17. We have not included capital in the analysis. However, capital stock rarely affects the results in this kind of analysis.

18. This implies that on a corresponding balance growth path of our model, all agents effectively expect to see the same growth rate, equal to the growth rate in the rest of the world. We have experimented with several other detrending methods, and the numbers do change quantitatively, but qualitatively all results stand.

19. We have corrected the nominal price of imports so that it excludes highly volatile fuels—a feature of the data that is not modeled in our theory. Using data pulled from the World Bank Development Indicators on the local currency value of total imports, total imports of merchandise products, and the share of imports of merchandise products excluding fuels, we have constructed the time-series of local currency value of imports less fuels. Data range varies by country, but most series cover the years 1980–2006.

20. The table of all the estimated coefficients is available from the authors upon request. The generally low values of the elasticity obtained by us is consistent with other micro-level studies on import prices and quantities (e.g., Blonigen and Wilson 1999). We have also verified the numbers for the United States using the U.S. Bureau of Labor Statistics series of import prices excluding fuels. We have obtained the value 1.04, which is very close to our volatility ratio for the United States of 1.11.

21. In the measurement of employment in the model, it is important to note that labor is in fixed supply and therefore, for consistency, we will compare it with the share of each sector in total employment in the case of the data.

22. As discussed in Sec. IV, lowering this elasticity will hurt the price statistics even more.

23. As a side comment, we should stress here that the particular channel of generating real exchange rate movements in our complete markets economy is not really essential for the results. In fact, we obtain exactly the same results under the assumption of financial autarky. Under financial autarky, the tight link between real exchange rate and consumption is severed, and all our results still stand.

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