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Chapter Title: Collateral, Rationing, and Government Intervention in Credit Markets

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2 Collateral, Rationing, and Government Intervention in Credit Markets

William G. Gale

2.1 Introduction

The federal government is the largest single lender in the country. As of the end of 1988, direct loans outstanding exceeded $222 billion, while outstanding loan guarantees were approximately $550 billion (Office of Management and Budget 1989). Federal credit assists borrowers across a wide variety of sectors, including housing, agriculture, small business, and education, in a bewildering array of over 100 programs.[1]

In order to analyze the effects of these policies, this paper focuses on two salient characteristics of virtually all credit programs. First, federal credit is usually intended for those who could not obtain private financing. For example, "a direct loan is best justified when the federal objective could not be met with financing from private sources" (Office of Management and Budget, 1988, F-15). Other programs, such as Small Business Administration loan guarantees, *require* applicants to prove that they could not obtain private financing.

Second, federal credit is provided on easier terms than comparable private credit. These terms can include reduced interest or collateral, longer maturities, grace periods, and so on. These provisions are estimated to reduce the discounted value of borrower payments by amounts that vary widely across programs, but typically range between 10% and 25% (Office of Management and Budget 1989).

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This paper analyzes the effects of policies with these characteristics in a model where rationing arises endogenously.[2] The underlying model is described in Section 2.2 and is closely related to Rothschild and Stiglitz (1976) and Besanko and Thakor (1987). Investors are divided into two groups, high risk and low risk, and have a choice between investing in a safe project or borrowing to invest in a risky project. For each group, lenders specify a probability of issuing a loan, an interest rate, and a collateral requirement.

Any given amount of collateral is assumed to be worth less to lenders than to borrowers. This feature implies that the use of collateral will generate an efficiency loss. In addition, all projects have expected gross returns greater than their social opportunity cost. Therefore, any amount of rationing represents an additional efficiency loss. The full information equilibrium arises when borrower type is known ex ante, implies no rationing and no collateral, and is thus efficient.

Section 2.3 analyzes situations where each borrower's type is private information. Now, lenders must collectively offer sets of contracts that induce borrowers to self-select into the appropriate contract. In equilibrium, high risks choose a contract with a relatively high interest rate and a zero collateral requirement. Low risks signal their type by choosing to pay high collateral in exchange for a lower interest rate. As long as low risks have sufficient wealth to post as collateral, the equilibrium involves no rationing. Nevertheless, because of the efficiency loss created by the use of collateral, there is a potential role for government.

The principal result of this section is to show that credit policies operate through their effects on the incentive-compatibility constraint, which limits the set of admissable contracts such that high-risk borrowers do not apply for the low-risk contract. For example, a guarantee to low-risk borrowers reduces their interest rate. Since the high-risk contract has not changed, the low-risk collateral requirement must *rise* in order to restore incentive compatibility. The increase in collateral means that guarantees to low risks reduce efficiency.

In contrast, a guarantee to high-risk borrowers makes the high-risk loan more attractive, thereby allowing lenders to reduce the collateral requirement on low-risk loans. Consequently, guarantees to high-risks raise efficiency. Equal guarantees to both groups have similar negative effects on the collateral requirement and positive effects on efficiency.

The major results of the paper are presented in Section 2.4, where it is assumed that borrower wealth is too low to support the collateral requirement outlined in Section 2.3. Because the high-risk contract involves no collateral, it does not change. However, since low-risks can only post a small amount of collateral, the low-risk contract must somehow be made less desirable in order to restore incentive compatibility. The only option is to reduce the probability of granting a low-risk loan; that is, to introduce rationing of low-risk borrowers.[3]

With the existence of rationing, it is now possible to analyze credit policies with the two salient features described above. Suppose the government agrees to offer subsidized credit (either direct or guaranteed loans) to some proportion of the (low-risk) borrowers who are turned down by the private market. The key point is that in the absence of any further changes, these subsidies make the low-risk contract more attractive to high-risk borrowers. Therefore, some other aspect of the low-risk contract must become less desirable in order to restore incentive compatibility. Since the collateral requirement cannot rise, the only alternative is for the overall (public and private) probability of obtaining a loan to fall. That is, increased subsidies to the rationed borrowers *raise* the extent of rationing. Private lending is crowded out on a more than one-to-one basis. It should be emphasized that this is an equilibrium response and is due to the existence of the incentive-compatibility constraint.

Although the subsidies increase the extent of rationing, they raise the ex ante expected utility of low-risk borrowers. This occurs because the benefits of the added cheap government loans outweigh the costs of the increased probability of being rationed. Thus, subsidies to low risks make the representative low-risk borrower better off ex ante but actually reduce the utility of some low-risk borrowers ex post. Since they increase the extent of rationing, the subsidies to low-risk borrowers reduce overall efficiency.

In contrast, subsidies to high-risk borrowers loosen the incentive-compatibility constraint. As a consequence, the extent of rationing of low-risk borrowers falls and efficiency rises.

Section 2.5 offers a short conclusion. The Appendix provides derivations of the various equilibria and proofs of the propositions.

2.2 The Basic Model

2.2.1 Description

The model describes a competitive credit market with many investors, but even more lenders. All agents are assumed to be risk neutral, thus eliminating any insurance role for federal credit, and there is no aggregate risk.

Investors can invest their initial endowment in a safe project that yields a gross return of Z.[4] Alternatively, each investor can borrow \$1 and invest that and the initial endowment in a risky project. Investors fall into two categories, which differ according to the probability of having a risky project succeed, π_i, and the gross return to that project, if it is successful, R_i. I assume $\pi_1 > \pi_2$, so that type 1's are low-risk borrowers. Projects that do not succeed yield a gross return of zero. The expected gross return to all projects are equal: $p_i R_i = k$, $i = 1, 2$, where k is a constant. Investors have a certain end-of-period endowment, W. I assume the existence of a sufficient enforcement technology such that W is acceptable to lenders as collateral. The proportion of borrowers that are low risk is given by φ.

Lenders have an alternative safe investment that earns ρ. They offer loans characterized by an interest rate (r), a collateral requirement (c), and a probability of issuing the loan to any particular applicant (p). Following several authors,[5] I assume there is a cost to collateralization. Specifically, the lenders' valuation of \$1 in collateral is given by β, $0 \leq \beta < 1$. Therefore, $1 - \beta > 0$ represents the social cost of transferring the collateral or realizing its value. Competition among lenders generates the following zero-profit condition on loans to each group:[6]

$$\rho = \pi_i r_i + (1 - \pi_i)\beta c_i, \quad i = 1, 2. \tag{1}$$

Investors are assumed to be able to apply for only one loan. The expected utility of an investor in group i applying for a loan contract meant for group j is

$$U_{ij} = p_j[\pi_i(R_i - r_j) - (1 - \pi_i)c_j - Z], \quad i, j = 1, 2. \tag{2}$$

Lenders always know the value of φ. In the full information equilibrium described below, lenders also know each borrower's type. In the asymmetric information equilibria, information on borrower type is unavailable to banks on an ex ante basis.

The Nash equilibrium concept is used throughout this paper. A set of contracts is a Nash equilibrium if, holding the current set of contracts fixed, no contract in the set earns negative profits and there is no additional contract which, if offered, would make positive profits (Rothschild and Stiglitz 1976).

2.2.2 Full Information Equilibrium

Although subsequent analysis will focus on markets with asymmetric information, the full information equilibrium is presented first as a benchmark. Because the types are identifiable ex ante, lenders face two distinct loan markets. In each submarket, optimal contracts maximize expected borrower utility U_{ii}, given in (2) subject to (1).

EQUILIBRIUM I. The full information equilibrium is characterized by

$$p_i^{\mathrm{I}} = 1, \quad i = 1, 2, \quad \text{(no rationing)} \tag{3a}$$

$$c_i^{\mathrm{I}} = 0, \quad i = 1, 2, \quad \text{(no collateral)} \tag{3b}$$

and

$$r_i^{\mathrm{I}} = \frac{\rho}{\pi_i}, \quad i = 1, 2. \tag{3c}$$

With full information, all borrowers receive loans. In addition, since borrowers are indifferent between committing to a dollar of expected interest payments and a dollar of expected collateral, while lenders prefer the former, equilibrium involves complete elimination of collateral. Formally, from (1), the slope of isoprofit curves is

$$\frac{dr_i}{dc_i} = \frac{-(1 - \pi_i)\beta}{\pi_i},$$

while the marginal rate of substitution for borrowers of type i is

$$\frac{dr_i}{dc_i} = \frac{-(1 - \pi_i)}{\pi_i}.$$

Indifference curves (U_1^I and U_2) and zero-profit curves (I_1 and I_2) for each group are shown in figure 2.1 below. For each group, $\beta < 1$ implies that the isoprofit curve is flatter than the indifference curve. Curves for high-risk borrowers are steeper than those for low-risk borrowers. The full information equilibrium is given by contracts α_1^F and α_2, along with $p_i^I = 1$.

Substituting (3a)–(3c) into (2) yields

(4) $$U_{ii} = \pi_i R_i - \rho - Z, \quad i = 1, 2.$$

Since $\rho + Z$ is the social opportunity cost of investment, (4) shows that investments are made ($U_{ii} > 0$) if and only if the expected total return exceeds the expected social cost. Therefore, equilibrium is efficient.

2.3 Asymmetric Information and Unconstrained Collateral

2.3.1 Private Equilibrium

When individual investors' types are private information, lenders must design sets of loan contracts that generate self-selection of each borrower type into the appropriate contract. Thus, lenders operate subject to (1) and a pair of incentive-compatibility constraints:

(5a) $$U_{11} \geq U_{12},$$

and

(5b) $$U_{22} \geq U_{21},$$

where U_{ij} is defined in (2). It can be directly verified that the full information equilibrium is not incentive compatible because both types would prefer the low-risk contract.

Instead, with asymmetric information, collateral is used as a sorting device.[7] High-risk borrowers have a stronger preference not to post collateral because they have a larger probability of having to pay it.

Whether collateral can induce complete separation depends crucially on W, the level of borrower wealth. This section examines equilibria and government policy when borrower wealth is sufficiently large to allow complete separation. Section 2.4 examines markets characterized by insufficient wealth.

EQUILIBRIUM II. When borrower type is private information, and borrower wealth is sufficiently large, equilibrium is characterized by:[8]

(6a) $$p_1^{II} = 1, \qquad p_2^{II} = 1,$$

(6b) $$c_1^{II} = \frac{(\pi_1 - \pi_2)\rho}{\pi_1(1 - \pi_2) - \pi_2(1 - \pi_1)\beta}, \qquad c_2^{II} = 0,$$

(6c) $$r_1^{II} = \frac{\rho}{\pi_1} - \frac{1 - \pi_1}{\pi_1}\beta c_1, \qquad r_2^{II} = \frac{\rho}{\pi_2}.$$

With imperfect information, high-risk borrowers obtain the same loan contract, and therefore the same utility, as in the full information equilibrium. Low-risk borrowers are not rationed, but their loan terms have changed. Specifically, low-risk borrowers indicate their type by posting collateral. In return, they pay a lower interest rate than in the full information equilibrium.

Substituting (6a)–(6c) into (2) for type 1's yields

(7) $$U_{11}^{II} = \pi_1 R_1 - \rho - Z - (1 - \pi_1)(1 - \beta)c_1^{II}.$$

Comparing (4) and (7), low-risk borrowers are worse off relative to the full information equilibrium by $(1 - \pi_1)(1 - \beta)c_1^{II}$. The magnitude of the welfare loss increases with c_1.

The equilibrium with asymmetric information and unconstrained collateral is shown in figure 2.1 as (α_1, α_2). High risks obtain α_2, as before. However, any contract offered to low risks must be incentive compatible with α_2. Of all such contracts, α_1 is the most desirable contract for type 1's that also earns nonnegative profits when extended to type 1's. The reduction in low-risk borrowers' utility to U_1^{II} from U_1^I is shown by the shift from α_1^F to α_1. Note that (5b) is binding in this equilibrium.

2.3.2 Government Credit

Although there is no rationing in the above model, there is still a role for government policy due to the efficiency losses created by the use of collateral. Because all investors receive loans in the private equilibrium, it seems natural to focus on loan guarantees (rather than direct loans) in this context.[9]

Loan guarantees ensure the lender of receiving an amount γ_i, where $0 \leq \gamma_i \leq \rho$. The government can set $\gamma_1 = \gamma_2 \equiv \gamma$, or choose the γ_i separately. In return for the guarantee, the lender passes on any collateral collected to the government. The net cost to the government of a defaulted loan is $\gamma_i - \beta c_i^{III}$, where c_i^{III} is the collateral requirement in the presence of the guarantee and is discussed further below. The government is subject to the same information constraints that private lenders face.

With the guarantees in place, expected borrower utility is still given by (2), but the zero-profit condition for lenders is now given by

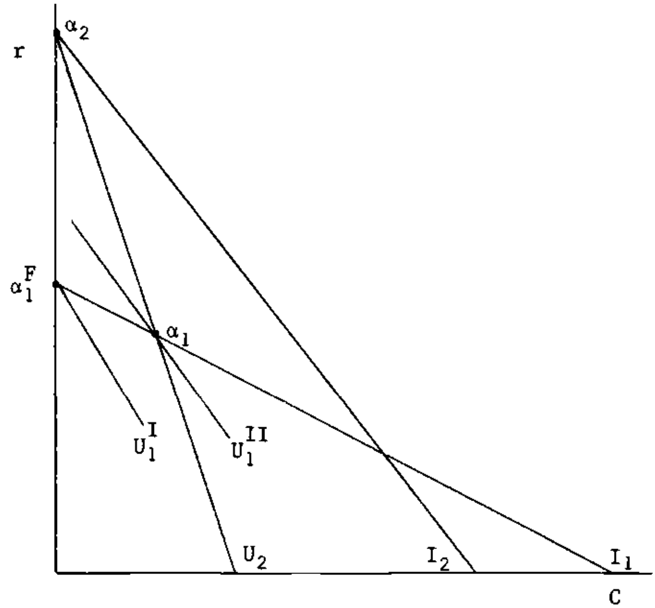

Fig. 2.1 Equilibrium with unconstrained collateral

(8) $$\rho = \pi_i r_i + (1 - \pi_i)\gamma_i, \quad i = 1, 2.$$

EQUILIBRIUM III. When borrower type is private information and $W > C_1^{II}$, equilibrium with loan guarantees is characterized by

(9a) $$p_1^{III} = 1, \qquad p_2^{III} = 1,$$

(9b) $$c_1^{III} = \frac{(\pi_1 - \pi_2)\rho}{\pi_1(1 - \pi_2)} - \gamma_2 + \frac{\pi_2(1 - \pi_1)}{\pi_1(1 - \pi_2)}\gamma_1, \quad c_2^{III} = 0,$$

(9c) $$r_1^{III} = \frac{\rho}{\pi_1} - \frac{1 - \pi_1}{\pi_1}\gamma_1, \qquad r_2^{III} = \frac{\rho}{\pi_2} - \frac{1 - \pi_2}{\pi_2}\gamma_2.$$

In the preceding private Equilibrium II, banks received βc_1^{II} and 0 in collateral on loans to type 1 and 2 borrowers, respectively. It is easy to verify that if $\gamma_1 = \beta c_1^{II}$ and $\gamma_2 = 0$, (9a)–(9c) reduce to the private equilibrium (6a)–(6c). Only higher guarantee rates have real effects.

Using (9b), increases in γ_1 cause c_1^{III} to rise in equilibrium. This result is contrary to standard intuition, which would suggest that as γ_1 rises, the necessary collateral should fall. However, as γ_1 rises, r_1 falls and the low-risk contract becomes more desirable to high-risk borrowers. Since (5b) binds, c_1 must rise to eliminate the possibility of having high risks masquerade as low risks.

This situation is depicted in figure 2.2. Increases in γ_1 shift the zero-profit line for low-risk lending from I_1 to I_1'. Equilibrium contracts, which are con-

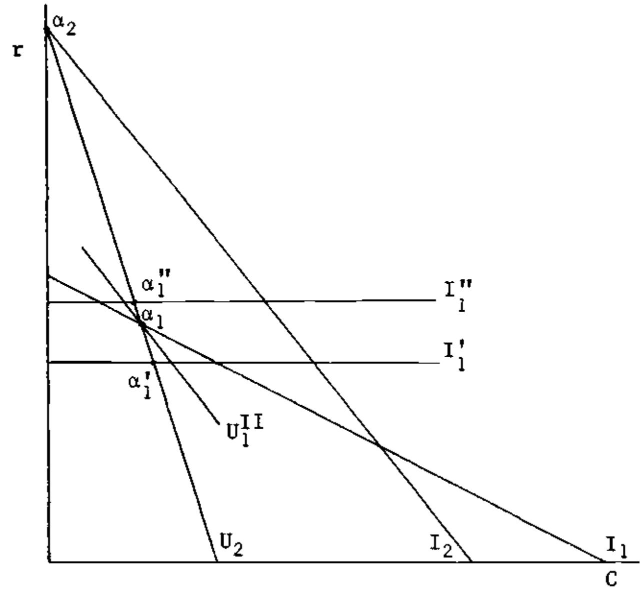

Fig. 2.2 Guarantees to low-risk borrowers

strained by (5a) and (5b), shift from (α_1, α_2) to (α_1', α_2). The collateral required at α_1' is greater than that required at α_1. Therefore, the existence of imperfect information reverses the usual intuition concerning the effect of γ_1 on c_1.

Any guarantee to low risks that reduced the collateral requirement would make them worse off and thus would be rejected in favor of α_1. This is illustrated by a guarantee that shifts the zero-profit curve to I_1'' from I_1' and the low-risk contract to α_1''. It is easy to show that all such guarantees correspond to $\gamma_1 < \beta c_1^{II}$.

Similar arguments show that c_1^{III} falls with increases in γ_2. As shown in figure 2.3, a rise in γ_2 shifts the zero-profit line for lending to high-risk borrowers from I_2 to I'_2, which raises low-risk utility to U_2' from U_2. The equilibrium thus shifts from (α_1, α_2) to (α_1^G, α_2^G). At the latter points, *both* groups are better off and the collateral requirement has fallen.

Since the use of collateral creates efficiency losses, these results will have important welfare implications. substituting (9a)–(9c) into (2) yields expected utilities:

$$(10) \qquad U_{11}^{III} = \pi_1 R_1 - \rho - Z - \frac{(1 - \pi_1)(\pi_1 - \pi_2)}{\pi_1(1 - \pi_2)}(\rho - \gamma_1) + (1 - \pi_1)\gamma_2,$$

and

$$(11) \qquad U_{22}^{III} = \pi_2 R_2 - \rho - Z + (1 - \pi_2)\gamma_2.$$

Increases in γ_1 raise U_{11}, even though they also raise c_1. For any $\gamma_1 > \beta c_1^{II}$, low-risk borrowers are better off than in private equilibrium. Increases in γ_2 raise both U_{11} and U_{22}. Thus, both types of borrowers are better off with guarantees.

Welfare calculations are based on total expected borrowers' utility minus expected government costs of funding the guarantees. Define overall welfare as

$$V = \varphi U_{11} + (1 - \varphi)U_{22} - \varphi(1 - \pi_1)(\gamma_1 - \beta c_1) - (1 - \varphi)(1 - \pi_2)(\gamma_2 - \beta c_2). \tag{12}$$

The first two terms represent utility of each borrower type, weighted by their population proportion; the last two terms represent net expected government costs of providing guarantees to low risks and high risks, respectively.[10]

PROPOSITION 1. When borrower type is private information, and $W > c_1^{II}$, the welfare effects of loan guarantees are as follows:

$$\left.\frac{\partial V}{\partial \gamma_1}\right|_{\gamma_2} \begin{array}{ll} = 0 & \text{if } \gamma_1 < \beta c_1^{II}, \\ < 0 & \text{if } \gamma_1 \geq \beta c_1^{II}, \end{array} \tag{13}$$

$$\left.\frac{\partial V}{\partial \gamma_2}\right|_{\gamma_1} > 0, \tag{14}$$

$$\left.\frac{\partial V}{\partial \gamma}\right|_{\gamma = \gamma_1 = \gamma_2} > 0. \tag{15}$$

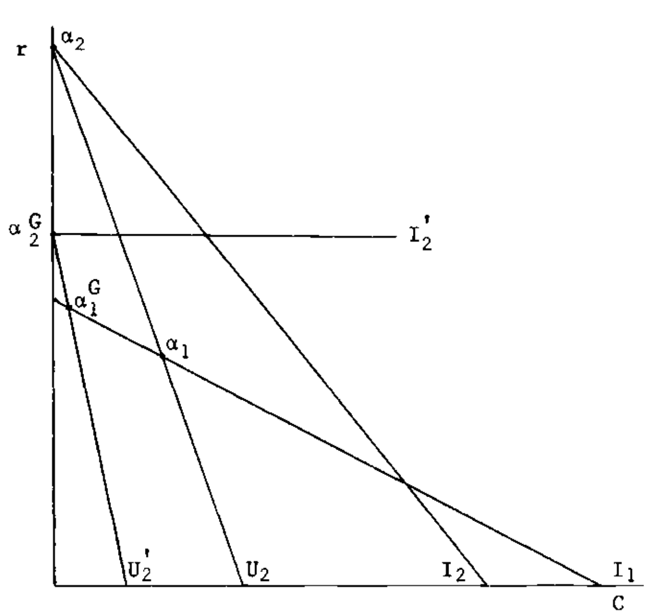

Fig. 2.3 Guarantees to high-risk borrowers

The main result from proposition 1 is that the effects of government intervention depend on how the incentive-compatibility constraint, and in particular the collateral requirement, is affected.

Equation (13) states that increases in γ_1, holding γ_2 constant, reduce welfare. From (10), the guarantee raises U_{11} by $[(1 - \pi_1)(\pi_1 - \pi_2)]/[\pi_1(1 - \pi_2)]$. However, from (12), the marginal cost, per low-risk borrower, of raising γ_1 is $(1 - \pi_1)$. It is easy to show that the marginal costs exceed the marginal benefits. This occurs because the rise in γ_1 raises c_1, which creates an efficiency loss.

The second result states that subsidizing the high-risk group is welfare improving. Using (11) and (12), the government's costs equal the benefits to high-risk borrowers. Since low-risk borrowers are also made better off, there is a welfare gain. The rise in γ_2 increases the attractiveness of the high-risk contract, so that low-risk borrowers can be offered more attractive terms. The fall in c_1 allows for an efficiency gain.

Equation (15) states that raising the guarantee rate on all loans is welfare improving. From (9b), setting $\gamma_1 = \gamma_2 = \gamma$ yields $c_1^{III}/\partial\gamma < 0$, so that collateral falls as the overall guarantee rate rises. Therefore, the net effect of universal guarantees is to weaken the incentive-compatibility constraint and raise welfare.

2.4 Asymmetric Information and Constrained Collateral

2.4.1 Private Equilibrium

The effectiveness of collateral as a sorting device is crucially dependent on the existence of sufficient end-of-period borrower wealth. For example, if $W = 0$, collateral cannot be used as a sorting device. More generally, suppose $W < c_1^{II}$, given in (6b); then the low-risk contract offered in Equilibrium II cannot be fulfilled by borrowers. Lenders, knowing this, will not offer the contract. Moreover, because (5b) is binding in equilibrium, if lenders simply reduced c_1 to W, the contracts offered would not be incentive compatible; both groups would prefer the contract meant for low-risk borrowers.

Lenders can resolve this problem by making the low-risk contract less attractive to high-risk borrowers.[11] Raising r_1 would discourage low-risk borrowers more than high-risk borrowers, since the latter have a smaller probability of actually having to pay the higher rate. The only alternative is to reduce p_1. This adjustment will discourage high-risks more and restore incentive compatibility. Therefore, when borrower wealth is insufficient to permit collateral alone to act as a sorting device, rationing of low-risk borrowers ($p_1 < 1$) is required to restore equilibrium. The intuition presented above is summarized in

Equilibrium IV. When borrower type is characterized by private information and $W < c_1^{II}$, the private equilibrium is characterized by[12]

$$\text{(16)} \qquad p_1^{IV} = \frac{U_{22}^{IV}}{\pi_2(R_2 - r_1) - (1 - \pi_2)W - Z} < 1, \quad p^{IV} = 1,$$

$$\text{(16b)} \qquad c_1^{IV} = W, \qquad c_2^{IV} = 0,$$

$$\text{(16c)} \qquad r_1^{IV} = \frac{\rho}{\pi_1} - \frac{1 - \pi_1}{\pi_1}\beta W, \qquad r_2^{IV} = \frac{\rho}{\pi_2}.$$

High-risk borrowers receive the same contract and utility as in Equilibrium II, when $W > c_1^{II}$. In contrast, because they cannot post c_1^{II} in collateral, low-risk borrowers are rationed. Their expected utility is given by

$$\text{(17)} \qquad U_{11}^{IV} = p_1^{IV}[\pi_1 R_1 - \rho - Z - (1 - \pi_1)(1 - \beta)W].$$

2.4.2 Government Credit

In the presence of rationing, the natural government policy to analyze is characterized by p_G, the probability of obtaining a government-guaranteed loan given (1) the borrower cannot obtain private credit and (2) γ_1, defined as before. Although the analysis will focus on subsidized loan guarantees, analogous results can be shown to hold for subsidized direct loans as well.

The interest rate charged on guaranteed loans gives lenders zero profits and is given by

$$\text{(18)} \qquad r_G = \frac{\rho}{\pi_1} - \frac{1 - \pi_1}{\pi_1}\gamma_1.$$

It is assumed that government charges W as collateral.[13]

With this policy, low-risk utility is given by[14]

$$\text{(19)} \qquad U_{11} = p_1 X_{11} + (1 - p_1)p_G X_{1G}.$$

The first term on the right-hand side of (19) represents the probability of obtaining a private loan, p_G, multiplied by X_{11}, the expected payoff to low-risk borrowers of obtaining a private loan meant for low-risk borrowers. The second term represents the probability of obtaining a government-guaranteed loan, $(1 - p_1)p_G$, multiplied by X_{1G}, the expected payoff to low-risk borrowers from that loan. If $\gamma_1 = \beta w$, then $X_{11} = X_{1G}$ and there is no gain to obtaining a government rather than a private loan. If $\gamma_1 > \beta W$, $X_{11} > X_{1G}$.

The incentive compatibility constraint (5b) is now given by

$$\text{(20)} \qquad U_{22} \geq p_1 X_{21} + (1 - p_1)p_G X_{2G},$$

where X_{21} is the expected payoff to high risks of taking a private loan meant for low risks, and X_{2G} is the expected payoff to high risks of taking a government loan meant for low-risks. (These are given in more detail in the proofs of Propositions 2 and 3 in the Appendix.)

Equilibrium V. When borrower type is private information, and $W < c_1^{II}$, the equilibrium with loan guarantees is characterized by

$$(21a)\qquad p_1^V = \frac{U_{22} - p_G X_{2G}}{X_{21} - p_G X_{2G}} < 1, \quad p_2^V = 1,$$

$$(21b)\qquad c_1^V = W, \qquad c_2^V = 0,$$

$$(21c)\qquad r_1^V = \frac{\rho}{\pi_1} - \frac{1 - \pi_1}{\pi_1} \beta W, \quad r_2^V = \frac{\rho}{\pi_2}.$$

These private loans are, of course, supplemented by government-guaranteed loans to some low-risk borrowers described by (p_G, r_G, W). High-risk borrowers obtain the same contract and utility as in Equilibria II and IV. Low-risk utility is given by (19) with appropriate substitutions for X_{11} and X_{1G}.

Define the probability of low risks obtaining any loan as

$$(22)\qquad p^* = p_1 + (1 - p_1)p_G.$$

Then a proportion $1 - p^*$ of low risks will be rationed.

PROPOSITION 2. When the initial allocation is given by Equilibrium V, increases in p_G or γ_1 increase the extent of rationing (reduce p^*).

Proposition 2 establishes that government subsidies to borrowers who cannot obtain private financing *increases* the number of borrowers who cannot obtain any financing, public or private. That is, private credit is crowded out on a more than one-to-one basis. Although this result may appear surprising, it is based on the equilibrium response of lenders to the shift in the incentive-compatibility constraint.

For example, in the equilibrium with $W > c_1^{II}$, an increase in γ_1 raised c_1 in order to restore incentive compatibility. Now, however, collateral is constrained to equal W, which is assumed to be less than c_1^{II}. As a consequence, increases in γ_1 (which reduce r_1) require that p_1 fall. From (22), a reduction in p_1, holding p_G constant, reduces p^*. Therefore, raising the guarantee rate to low-risk borrowers increases the extent of rationing.

Similarly, consider a small increase in p_G, holding γ_1 constant. If p_1 were held constant, all borrowers would have an increased chance of obtaining a cheap government loan, which would induce high risks to masquerade as low risks. If p_1 fell such that p^* were unchanged from its previous level, high risks would still prefer masquerading as low risks to taking the high-risk contract. To show this, rewrite (20) as

$$(23)\qquad U_{22} \geq p^* X_{21} + (1 - p_1)p_G(X_{2G} - X_{21}).$$

Recall that this constraint is binding in Equilibrium V. If p_G rises and p^* is held constant, (23) is violated. Therefore, in equilibrium, incentive compatibility requires p_1 to fall enough to make p^* fall in response to the rise in p_G. That is, increases in p_G raise the extent of rationing.

Although they increase the likelihood of any given low-risk borrower being rationed, guarantees raise the ex post utility of those who do obtain government credit; this is caused by the reduced interest rate r_G on guaranteed loans.

The effects of these guarantees on the ex ante expected utility of low-risk borrowers and on welfare is given in

PROPOSITION 3. When the initial allocation is given by Equilibrium V, the welfare effects of government intervention are

$$\frac{\partial U_{11}^{V}}{\partial \gamma_1} > 0, \tag{24a}$$

$$\frac{\partial U_{11}^{V}}{\partial p_G} > 0, \tag{24b}$$

$$\frac{\partial V}{\partial \gamma_1} < 0, \tag{25a}$$

$$\frac{\partial V}{\partial p_G} < 0. \tag{25b}$$

Equations (24a) and (24b) show that guaranteed loans raise the ex ante expected utility of the targeted group. This occurs even though the overall probability of obtaining a loan falls. Therefore, the targeted group prefers to have the policy, even though fewer lower-risk borrowers obtain credit when the subsidy is in place.

Equations (25a) and (25b) show that, as in Proposition 1, guarantees to low-risk borrowers reduce overall welfare. This occurs because the increase in rationing represents an overall efficiency loss. Following the approach taken in Section 2.3 it is straightforward to show that guarantees to the high-risk group reduce the extent of rationing. Intuitively, these guarantees make the high-risk contract (U_{22}) more attractive and, through (21a), raise γ_1. Because of the reduction in rationing, these guarantees raise efficiency.

2.5 Conclusion

This paper analyzes the effects of credit subsidies in markets characterized by adverse selection. The principal result is that the effects of credit subsidies depend on how eligibility is determined. Programs that subsidize the unrationed contract will reduce the extent of rationing and raise overall efficiency. In contrast, programs that target borrowers who cannot obtain private financing raise the extent of rationing and reduce efficiency. The distinction is important precisely because most government credit is designed to provide funds to those who do not receive private loans. In the model presented here, such policies raise the extent of rationing and create inherent tradeoffs among members of the same target group: fewer of them obtain any type of credit, but those that receive government loans are better off.

Two concluding comments should be made. First, the effects described above represent equilibrium responses. In particular, they take into account the need to deter high risks from pretending to be low risks. As a consequence of this incentive-compatibility constraint, whenever the government eases some of the terms on low-risk contracts, the others must be adversely affected in equilibrium.

Second, the paper has focused on a fairly standard adverse selection model, based on Rothschild and Stiglitz (1976). Numerous extensions of that and other models have shown that the nature of equilibrium can be affected by incentive effects (Stiglitz and Weiss 1986), project characteristics, the set of available financial instruments (de Meza and Webb 1987), alternative projects (Chan and Thakor 1987), information sharing (Yotsuzuka 1987), the shape of the production function (Milde and Riley 1988), and other characteristics. The effects of credit policies in such alternative models deserve further exploration.

Appendix

EQUILIBRIUM I. In each submarket, equilibrium contracts maximize expected borrower utility, U_{ii}, given in (2) subject to (1). Substitution of (1) into (2) for each U_{ii} yields

$$\max_{p_i c_i} L^{\mathrm{I}} = p_i[\pi_i R_i - \rho - Z - (1 - \pi_i)(1 - \beta_i)c_i].$$

Taking derivatives yields:

$$\frac{\partial L^{\mathrm{I}}}{\partial c_i} = -p_i(1 - \pi_i)(1 - \beta_i) < 0,$$

so that $c_i^{\mathrm{I}} = 0$, which implies through (1) that $r_i^{\mathrm{I}} = \rho/\pi_i$. In addition,

$$\left.\frac{\partial L^{\mathrm{I}}}{\partial p_i}\right|_{c_i^{\mathrm{I}} = 0} = \pi_i R_i - \rho - Z,$$

which is positive if type i investors are applying for loans, implying that $p_i^I = 1$.

EQUILIBRIUM II. The remaining equilibria follow a common pattern. Therefore, Equilibrium II is derived in some detail, while the derivations of Equilibria III–V are shorter.

With asymmetric information, equilibrium is obtained by maximizing the population-weighted average of borrowers' expected utility, given by

$$\varphi U_{11} + (1 - \varphi)U_{22}, \tag{A1}$$

subject to (1), (5a), and (5b). Following Besanko and Thakor (1987), the strategy employed is initially to ignore (5a) and assume $p_i = 1$. Later it will be shown that the optimal solution does satisfy (5a) and $p_i = 1$. Substituting (1) into (A1) and maximizing (A1) subject to (5b) yields the problem:

$$\max_{c_1, c_2} L^{II} = \varphi p_1[\pi_1 R_1 - \rho - Z - (1 - \pi_1)(1 - \beta)c_1]$$

(A2)
$$+ (1 - \varphi)p_2(\pi_2 R_2 - \rho - Z - (1 - \pi_2)(1 - \beta)c_2$$
$$+ \lambda\{p_2[\pi_2 R_2 - \rho - Z - (1 - \pi_2)(1 - \beta)c_2]$$
$$- p_1[\pi_2 R_2 - \frac{\pi_2}{\pi_1}\rho - \frac{\pi_1(1 - \pi_2) - \pi_2(1 - \pi_1)\beta}{\pi_1} c_1 - Z]\},$$

where λ is the LaGrange multiplier associated with (5b). Setting $p_i = 1$ and setting $\partial L^{II}/\partial c_1 = 0$ implies

(A3)
$$\lambda = \frac{\varphi(1 - \pi_1)(1 - \beta)\pi_1}{\pi_1(1 - \pi_2) - \pi_2(1 - \pi_1)\beta} > 0,$$

which implies that (5b) is binding. Using (A3), it is easy to show that $\partial L^{II}/\partial c_2 < 0$, which implies $c_2^{II} = 0$. Substituting for c_2^{II} in (5b) and solving (5b) for c_1^{II} yields the expression in (6b). Given c_i^{II}, r_i^{II} can be found using (1).

It remains to be shown that (5a) is satisfied at the solution presented above and that $p_i = 1$ is optimal. To show that (5a) is satisfied, note that $U_{11} \geq U_{12}$ if

(A4)
$$\pi_1(R_1 - r_1^{II}) - (1 - \pi_1)c_1^{II} \geq \pi_1(R_1 - r_2^{II}), \quad \text{or}$$
$$\pi_1(r_2^{II} - r_1^{II}) \geq (1 - \pi_1)c_1^{II}.$$

Using (1) and (6b) and some algebra, it can be shown that (A4) holds for the values given in proposition 2 above. By examining figure 2.1, $U_{11} > U_{12}$ can also be seen.

It is also straightforward to show that $\partial L^{II}/\partial p_2 > 0$, which implies $p_2^{II} = 1$. Finally,

(A5)
$$\frac{\partial L^{II}}{\partial p_1} = \varphi[\pi_1 R_1 - \rho - Z - (1 - \pi_1)(1 - \beta)c_1^{II}]$$
$$- \lambda\left\{\pi_2 R_2 - \frac{\pi_2}{\pi_1}\rho - \frac{\pi_1(1 - \pi_2) - \pi_2(1 - \pi_1)\beta}{\pi_1}c_1^{II} - Z\right\}.$$

This can be shown to be positive provided that $k - r_1 - Z > 0$. This condition captures the idea that high risks have a higher aversion to posting collateral because they have a larger probability of having to pay it. That is, the same condition implies, from (2), that dc/dp rises with π.

EQUILIBRIUM III. Guarantees shift the zero-profit condition to (8) from (1). Otherwise, the maximization follows as in Equilibrium II. That is, substitute (8) into (2) for (A1) and maximize subject to (5a), ignoring (5b) and setting $p_i = 1$ for now. The problem is to

$$\max_{c_1, c_2} L^{\mathrm{III}} = \varphi p_1[\pi_1 R_1 - \rho - Z + (1 - \pi_1)(\gamma_1 - c_1)]$$
$$+ (1 - \varphi)p_2[\pi_2 R_2 - \rho - Z + (1 - \pi_2)(\gamma_2 - c_2)]$$
$$+ \lambda\{p_2[\pi_2 R_2 - \rho - Z + (1 - \pi_2)(\gamma_2 - c_2)]$$
$$- p_1\left(\pi_2 R_2 - \frac{\pi_2 \rho}{\pi_1} + \frac{\pi_2(1 - \pi_1)}{\pi_1}\gamma_1 - (1 - \pi_2)c_1 - Z\right)\}.$$

Setting $\partial L^{\mathrm{III}}/\partial c_1 = 0$ implies that $\lambda > 0$, which implies that (5b) binds. Given $\lambda > 0$, it is easy to show that $c_1^{\mathrm{III}} = 0$. Solving (5b) for c_1^{III} yields the expression in (9b). Showing that (5a) is satisfied and that $p_i^{\mathrm{III}} = 1$ follow in the same way as Equilibrium II.

Proposition 1. Expected borrower utilities are given in (10) and (11). The government cost of providing guarantee γ_i is $(1 - \pi_i)(\gamma_i - \beta c_i^{\mathrm{III}})$ for each borrower in group i. The effects of raising γ_1 only or γ_2 only are described in the text. The effects of raising γ are as follows.

If $\gamma < \beta c_1^{\mathrm{II}}$, the only effect of raising γ occurs through raising γ_2, so the increase in γ is welfare improving. When $\gamma \geq \beta c_1^{\mathrm{II}}$, further increases introduce opposing welfare effects. However, the net effect is always welfare improving. Note that, for each high-risk borrower, the benefits of raising γ equal $(1 - \pi_2)\delta\gamma_2$, which equals the cost of providing guarantees for high-risk borrowers. The welfare effects thus depend on comparing $\partial U_{11}^{\mathrm{III}}/\partial\gamma$ and the cost of providing guarantees to low risks, $(1 - \pi_1)$. From (10),

$$\partial U_{11}^{\mathrm{III}}/\partial\gamma = \frac{(1 - \pi_1)(\pi_1 - \pi_2)}{\pi_1(1 - \pi_2)} + (1 - \pi_1) > (1 - \pi_1).$$

Therefore, increases in γ are welfare-improving.

Equilibrium IV. The problem is now to maximize (A2) subject to (1), (5b), and a wealth constraint, $W < c_1^{\mathrm{II}}$. As before, taking derivatives with respect to c_2 and p_2 yields $p_2^{\mathrm{IV}} = 1$ and $c_2^{\mathrm{IV}} = 0$, and the latter result implies $r_2^{\mathrm{IV}} = \rho/\pi_2$. The wealth constraint implies that $c_1^{\mathrm{IV}} = W$. The zero-profits condition determines r_1^{IV}. Only p_1 remains to be determined. Optimizing with respect to p_1 implies that $\lambda > 0$ so that (5b) is binding. Solving (5b) for p_1 yields the expression in (16a).

Equilibrium V. The problem is now to maximize (A2) subject to (1), (5a), $W < c_1^{\mathrm{II}}$, and (20), and where U_{11} is given by (19). Values for c_2^{V}, p_2^{V}, r_2^{V}, c_1^{V}, and r_1^{V} as derived as in Equilibrium IV. The equilibrium p_1 is determined by solving (20) for p_1.

Proposition 2. Note that $X_{21} = \pi_2 R_2 - \pi_2\rho/\pi_1 - \{\pi_1(1 - \pi_2) - \pi_2(1 - \pi_1)\beta/\pi_1\}W - Z$, and $X_{2G} = X_{21} + \{\pi_2(1 - \pi_1)/\pi_1\}(\gamma_1 - \beta W)$, where these terms are derived by substituting for r using (1) and (18). Thus X_{11} and X_{1G} are derived analogously. From (22)

$$\frac{\partial p^*}{\partial\gamma} = \frac{\partial p_1}{\partial\gamma}(1 - p_G) = \frac{p_G\pi_2(1 - \pi_1)(U_{22} - X_{21})}{\pi_1(X_{21} - p_G x_{2G})^2}(1 - p_G) < 0,$$

because from (21a), $U_{22} < X_{21}$. Similarly

$$\frac{\partial p^*}{\partial p_G} = \frac{\partial p_1}{\partial p_G}(1 - p_G) + (1 - p_1) = \frac{(U_{22} - X_{21})(X_{2G} - X_{21}}{(X_{21} - p_G x_{2G})^2} < 0,$$

using (21a) and some algebra.

PROPOSITION 3. Taking derivatives of (19) yields

$$\text{(A6)} \qquad \frac{\partial U_{11}^V}{\partial \gamma_1} = \frac{\partial p_1}{\partial \gamma_1}(X_{11} - p_G X_{1G}) + (1 - p_1)p_G \frac{\partial X_{1G}}{\partial \gamma_1}$$

$$= p_G(1 - \pi_1)(1 - p_1)\left\{1 - \frac{\pi_2(x_{11} - p_G X_{2G}}{\pi_1(X_{21} - p_G X_{2G}}\right\},$$

using (21a). The expression in brackets can be shown to be positive, so that (A6) is positive. In addition,

$$\frac{\partial U_{11}^V}{\partial p_G} = \frac{\partial p_1}{\partial p_G}(X_{11} - p_G x_{1G}) + (1 + p_1)X_{1G}$$

$$= \frac{(X_{21} - U_{22})\{X_{21}X_{1G} - X_{2G}X_{11}\}}{(X_{21} - p_G X_{2G})^2} > 0,$$

because the term in brackets can be shown to be positive.

To show the welfare effects of changing γ_1, note that the expresion in (A6) is less than $(1 - \pi_1)$, the marginal costs of raising γ_1 per low-risk borrower. Thus, increases in γ_1 reduce efficiency. Similar results hold for p_G.

Notes

1. For discussions of the features and overall effects of federal credit, see Bosworth, Carron, and Rhyne (1987), Gale (1988a), or Office of Management and Budget (1989).
2. Previous research on federal credit in markets with imperfect information includes Mankiw (1986) and Gale (1988b), who study models with a continuum of borrower types, and Gale (1987) and Smith and Stutzer (1989) who examine models with two types of borrowers. The current paper is based on Gale (1987, app. C). Some of the results are closely related to independent work by Smith and Stutzer (1989).
3. As described in Section 2.4 below, raising the interest rate on the low-risk loan cannot restore incentive compatibility, because low risks are more averse to accepting a higher interest rate than high risks are.
4. The initial endowment could also represent a unit of labor supply, in which case Z would be interpretable as the value of leisure forgone by investing.
5. See Barro (1976), Besanko and Thakor (1987), Bester (1985), and Williamson (1987).
6. In order to focus on the role of collateral and rationing as sorting devices, the paper focuses on separating equilibria. See nn. 8 and 13 below for a discussion of potential pooling equilibria.

7. Besanko and Thakor (1987), Bester (1985), Chan and Thakor (1987), Stiglitz and Weiss (1981, 1986) and Wette (1983) analyze the selection effects induced by collateral.

8. The existence of a separating equilibrium can be ensured by assuming that there is a sufficiently large proportion of high-risk borrowers or that the difference between $\pi_1 - \pi_2$ is large enough. See Rothschild and Stiglitz (1976) or Besanko and Thakor (1987). There are no pooling equilibria under the assumptions in this section.

9. Gale (1987) also examines government policies in which low-risk loans are taxed and high-risk loans are subsidized. These policies operate through the same channels as guarantees and will be ignored here.

10. Therefore, the welfare criterion is total surplus, rather than a Pareto measure. In addition, (12) assumes that taxes are raised in a lump-sum manner.

11. It is impossible to make the high-risk contract more attractive to high-risk borrowers without losing money on high-risk contracts. Because of the Nash assumption, cross-subsidization of contracts will not occur in this model, although it could in other contexts. See Stiglitz and Weiss (1989).

12. Besanko and Thakor (1987) show that, under these circumstances, (5a) requires that $W \geq Z\rho\,(\pi_1 - \pi_2)/\{\pi_1(\pi_2 R - \rho) - Z\pi_2[(1 - \pi_1) + \beta\pi_1]\}$. Loosely speaking, this requires that Z is small relative to W. In order to rule out a pooling equilibrium at $P_i = 1$, $c_i = W$, it if necessary to assume that $\beta < [\pi^*(1 - \pi_1)]/[\pi_1(1 - \pi^*)]$, where $\pi^* = \varphi\pi_1 + (1 - \varphi)\pi_2$. It may be thought that the allocation in Equilibrium V could be broken by an offer that raises p_1 and r_1, holding c_1 at W. Such an offer would earn positive profits if it attracted only low risks. However, from (2), $dr/dp = (k - c - Z)/p\pi - (r - c)/p$. Since this expression is decreasing in π, any offer that raised p_1 and r_1 relative to Equilibrium V would attract both types and thus would not be offered.

13. Reducing the collateral requirement on government loans has the same qualitative effects as raising p_G or γ_1.

14. In order to avoid the prospect of borrowers turning down private loans to accept public ones, I assume $X_{11} > P_G X_{1G}$.

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