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5 Rational Expectations and the Foreign Exchange Market

Peter R. Hartley

In this paper I test the hypothesis that expectations of exchange rate movements are formed rationally. To do so, I need, in addition to the hypothesis of rational expectations, a theory of the determinants of exchange rate movements. I shall first consider a very simple monetary approach model of exchange rate determination (section 5.1). A serious defect of the model considered in this paper is that it ignores the possibility of a simultaneous determination of the exchange rate along with macroeconomic variables. However, it extends previous models in this genre by attempting to distinguish the effects of changes in expectations on exchange rates from the effects of changes in underlying determining variables apart from expectations. Furthermore, it does this in a context where the assumption of rationality of expectations can be tested.

In section 5.3 I shall present some results for the dollar/mark and dollar/pound exchange rates in the most recent floating rate period. In section 5.4 I examine a model similar to one studied by Frenkel (1981). However, I am able to test for rationality of expectations where Frenkel could not.

I have chosen to emphasize the test of rationality in this paper for two reasons. First, the test of rationality, unlike the tests of the restrictions implied by the simple monetary model, does not depend on the validity of the exogeneity assumptions. If we do find a rejection of the cross-equation restrictions implied by rationality, this is indeed a rejection

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either of the assumption that expectations are formed rationally or that the forward premium differs from the rationally expected depreciation or appreciation by no more than a constant term. Second, I have tested two alternative models of exchange rate determination and, while both lead to valid tests of rationality (given our assumption on the forward rate), they do not arise from a single, simple model.

5.1 Simple Monetary Approach Model

Proponents of the monetary approach to exchange rate determination view the exchange rate as the relative price of two monies. They therefore argue that variables affecting the supply of and demand for two monies will affect the rate of exchange between them. Quite a few studies have tested the monetary approach to exchange rate determination and some of the earlier ones are collected in Frenkel and Johnson (1978).

Since money is a durable asset, it has been argued that expectations about the values of variables affecting its future supply of demand ("exogenous" variables)[1] will be important determinants of current demand. Suppose expectations of future movements in exogenous variables are influenced by current movements in the same exogenous variables.[2] Movements in the exogenous variables would then affect money supply and demand directly, but they would also affect expectations and hence money demand. More significantly, it is most probable that anticipated and unanticipated movements in the exogenous variables will have quite different effects on exchange rates. Frenkel (1981) has suggested that short-run movements in exchange rates are dominated by the effect of unanticipated movements in the exogenous variables.

Many previous tests of simple monetary models have included lagged exogenous variables among the explanatory variables. Insofar as the justification for including these variables is that they are useful for proxying expectations, an important source of restrictions on the distributed lags has been ignored.

The present study focuses on explaining errors in forecasting exchange rate movements rather than the exchange rate movements themselves. This is one way (also used in Frenkel (1981)) to separate out the effects of anticipated and unanticipated movements in the exogenous variables. Only unanticipated movements in the exogenous variables should lead to unanticipated movements in the exchange rate. Rationality of expecta-

1. I shall use the term "exogenous" for these determining variables for ease of exposition. Some of them might, in fact, be simultaneously determined with money supply and/or demand. This shall be discussed further below.

2. This will be true, for example, if the evolution of these exogenous variables can be explained by a stable, low-order autoregression and if agents are aware of this fact and form expectations rationally.

tions implies a set of cross-equation restrictions on distributed lags. The conformity of these restrictions with the data provides a test of rationality which I shall implement in this paper.

The statistical theory I shall use derives from a paper by Abel and Mishkin (1979) and has been applied to a study of bond yields by Mishkin (1981).

Let $\hat{S}$ denote the $T \times 1$ vector of observations in the one-period percentage change in the exchange rate, and let Δ denote the $T \times 1$ vector of observations on the errors in the forecast of one-period exchange rate changes. Hence,

(1) $$\Delta_{t+1} \equiv \hat{S}_{t+1} - E(\hat{S}_{t+1} | \phi_t),$$

where ϕ_t is the information relevant to the pricing of foreign exchange at time $t+1$ available at time t.

Now suppose we have a theory which predicts

(2) $$\hat{S}_{t+1} = X_{t+1}\beta + u_{t+1},$$

where X is the $T \times k$ matrix of observations on variables which determine the exchange rate change $\hat{S}_{t+1}$ (X_{t+1} is observed by agents at time $t+1$ but not before), and β is the $k \times 1$ vector of coefficients. Suppose the variables X follow a stochastic process

(3) $$X = Z\gamma + v,$$

where Z denotes the $T \times 1$ matrix of observations on past information z_t (i.e., $z_t \varepsilon \phi_t, t - 1, 2, \ldots, T$) which is useful for predicting the elements of X, γ is the $1 \times k$ matrix of coefficients, and v is the $T \times k$ matrix of errors.

Now from (1) and (2),

(4) $$\begin{aligned}\Delta_{t+1} &= [X_{t+1} - E(X_{t+1} | \phi_t)]\beta + [u_{t+1} - E(u_{t+1} | \phi_t)] \\ &\equiv (X_{t+1} - X^e_{t+1})\beta + \varepsilon_{t+1},\end{aligned}$$

where I have defined X^e_{t+1} as the $T \times k$ matrix of the one-period-ahead optimal forecasts of X, and ε_{t+1} as the $T \times 1$ vector of errors with $E(\varepsilon_{t+1} | \phi_t) = 0$. Now if expectations are rational, then agents should use the process (3) in forming expectations in (4). In other words, we should find

(5) $$\Delta = (X - Z\gamma)\beta + \varepsilon.$$

To test for rationality of expectations, we estimate (3) and (5) jointly and test for the equality of the γ coefficients in the two sets of equations.

An alternative procedure to estimating (3) and (5) jointly would be to first estimate (3) and then use the residuals from that regression in (5). The joint estimation is preferred for several reasons:

(i) The two-step procedure does not test whether expectations are optimal linear forecasts, given the data on the right-hand side of (3).

(ii) The joint estimation will use information in both (3) and (5) to estimate β and γ and will deliver more efficient estimates of these parameters.

(iii) It is unlikely that the test statistics derived from the two-step procedure would be consistent since they do not take account of the variance-covariance structure in the regression from which the residuals are derived.

To proceed we need an observable proxy for $E(\hat{S}_{t+1}|\phi_t)$, a theory of exchange rate determination (2), and a forecasting equation (3) for the right-hand variables in (2).

If traders in the forward foreign exchange market were not risk averse and future prices were known with perfect foresight or were not correlated with the future level of the exchange rate,[3] we would expect to find

$$ {}_tF_{t+1} = E(S_{t+1}|\phi_t), $$

where ${}_tF_{t+1}$ is the one-period forward rate at time t.

To allow for the possibility of risk aversion, I shall assume

$$ {}_tF_{t+1} = E(S_{t+1}|\phi_t) - aS_t, $$

where a is the constant. Then

$$ (6) \qquad E(S_{t+1}|\phi_t) = \frac{{}_tF_{t+1} - S_t}{S_t} + a. $$

In appendix A a simple monetary model is used to derive a version of equation (2):

$$ (2') \qquad \hat{S}_{t+1} = \hat{M}_{t+1} - \hat{M}^*_{t+1} - \alpha_1 \hat{Y}_{t+1} + \alpha_2 \hat{Y}^*_{t+1} + \alpha_3 \Delta(i_{t+1} - i^*_{t+1}) + \xi_{t+1}, $$

where Y and Y^* are domestic and foreign real income, M and M^* are domestic and foreign money supplies, i and i^* are the domestic and foreign (nominal) interest rates, and ξ_{t+1} is a composite error term reflecting deviations from purchasing power parity, as well as random components in domestic and foreign money demand. ξ_{t+1} can be autocorrelated.

However, the model (2′) is not estimated since the interest parity condition implies that $i_{t+1} - i^*_{t+1}$ is related to the expected devaluation of the exchange rates. In appendix A, I substitute

$$ i_{t+1} - i^*_{t+1} = E_{t+1} \hat{S}_{t+2} - a $$

into (2′) and then "solve forward" to get an expression for $\hat{S}_{t+1}$ involving

3. See Frenkel and Razin (1980).

the expected values of all future money supply and income changes.[4] This expression can also be written[5]

$$(7)\quad \Delta_{t+1} \equiv \hat{S}_{t+1} - E_t\hat{S}_{t+1} = \sum_0^\infty \left(\frac{\alpha_3}{1+\alpha_3}\right)^i (E_{t+1}\hat{M}_{t+1+i} - E_t\hat{M}_{t+1+i})$$
$$- \sum_0^\infty \left(\frac{\alpha_3}{1+\alpha_3}\right)^i (E_{t+1}\hat{M}^*_{t+1+i} - E_t\hat{M}^*_{t+1+i})$$
$$- \alpha_1 \sum_0^\infty \left(\frac{\alpha_3}{1+\alpha_3}\right)^i (E_{t+1}\hat{Y}_{t+1+i} - E_t\hat{Y}_{t+1+i})$$
$$+ \alpha_2 \sum_0^\infty \left(\frac{\alpha_3}{1+\alpha_3}\right)^i (E_{t+1}\hat{Y}^*_{t+1+i} - E_t\hat{Y}^*_{t+1+i}) + \varepsilon_{t+1}.$$

Now, we can take the variables $\hat{M}_{t+1}$, $\hat{M}^*_{t+1}$, $\hat{Y}_{t+1}$, and $\hat{Y}^*_{t+1}$ as the exogenous variables X in (2). As forecasting equation (3) for these variables, I use bivariate autoregressions so that:

$$(3')\quad \begin{aligned} \hat{M}_{t+1} &= \gamma_1(L)\hat{M}_t + \gamma_2(L)\hat{Y}_t + v_{1t+1}; \\ \hat{Y}_{t+1} &= \gamma_3(L)\hat{M}_t + \gamma_4(L)\hat{Y}_t + v_{2t+1}; \\ \hat{M}^*_{t+1} &= \gamma_1^*(L)\hat{M}_t + \gamma_2^*(L)\hat{Y}_t^* + v_{3t+1}; \\ \hat{Y}^*_{t+1} &= \gamma_3^*(L)\hat{M}_t + \gamma_4^*(L)\hat{Y}_t^* + v_{4t+1}; \end{aligned}$$

If we use these forecasting equations in (7), it can then be shown that

$$(5')\quad \begin{aligned} \Delta_{t+1} = {} & (\hat{M}_{t+1} - E_t\hat{M}_{t+1})\beta_1 - (\hat{M}^*_{t+1} - E_t\hat{M}^*_{t+1})\beta_2 \\ & - (\hat{Y}_{t+1} - E_t\hat{Y}_{t+1})\beta_3 + (\hat{Y}^*_{t+1} - E_t\hat{Y}^*_{t+1})\beta_4 + \varepsilon_{t+1}, \end{aligned}$$

which is an equation of the form (5).[6] To test for rationality of expectations, we can estimate (3′) and (5′) jointly and require that the γ coefficients in equation (5′) equal those in (3′).

It is important to realize that the β coefficients in (5′) depend on the forecasting parameters γ in (3′), as well as elasticities in the underlying money demand functions. In particular, therefore, changes in policy that alter the γ parameters in (3′) will alter the β parameters in (5′). This is a feature of rational expectations models which has been emphasized by Lucas.[7] In the present context it might make us pessimistic about the chances of getting precise estimates of the β parameters in (5′). If the forecasting equations (3′) have varied over the sample period and if agents have been aware of these changes, then fitting a single time series over the whole period will produce at least two sources of imprecision in

4. This was also done in Mussa (1978) and Bilson (1978).

5. $\epsilon_{t+1} = A(\xi_{t+1} - E_t\xi_{t+1})$ for some constant A (which depends on the autoregressive process followed by ξ). See note 15.

6. See Appendix A.

7. See Lucas (1976).

the estimates of the β parameters. First, there will be errors in the right-hand side variables in (5′). Second, the "true" β parameters will have changed over the period.

The tests of rationality can be strengthened by estimating the system (3′) and (5′) jointly for two exchange rates. In this paper, I have jointly estimated a dollar/pound equation (5′) along with a dollar/mark equation (5′). If expectations are formed rationally, then the forecast of U.S. variables that agents use to predict the dollar/pound exchange rate should be the same forecast they use to predict the dollar/mark exchange rate. Note, however, that the ϵ's for these two (5′) equations may be correlated. The ϵ's represent sources of forecast errors apart from errors in forecasting money and income growths. It is quite likely that the same unaccounted source of error will affect both exchange rates each period. I report one set of estimates which do, and another which do not, allow for this correlation between the ϵ's.

I have been treating the unanticipated money and income shocks as exogenous with respect to the unanticipated exchange rate error Δ. If monetary policy is varied in response to current innovations in ϵ, then the v_{it}'s will be correlated with ϵ_t. This will bias the estimates of the β coefficients in (5′). In fact, it is shown in Abel and Mishkin (1979) that if $\sigma_{v\epsilon}$ is unknown the β parameters are not identified. Some set of k identifying restrictions on the k elements of $\sigma_{v\epsilon}$ in (3′) and (5′) is needed to identify the β parameters. The system (3′) and (5′) cannot be estimated using standard, full information, maximum likelihood techniques. The covariance matrix must be constrained if we are to get unique estimates for the β parameters. In addition, unless the covariance matrix is restricted, in a test of restrictions on the β parameters the degrees of freedom of the test statistic could be seriously overestimated. In all the estimates reported below, I restricted $\sigma_{v\epsilon} = 0$, and if this is invalid the estimated β coefficients will be biased.

5.2 Estimation of the Simple Monetary Model

I used M1 money stocks as reported in the IMF *International Financial Statistics*. Data on industrial production were obtained from the same source to serve as proxies for Y and Y^*. The exchange rate data were taken from the Harris Trust and Savings Bank *Weekly Bulletin*. The monthly observations were taken on the last Friday of each month.

We want a parsimonious set of forecasting equations (3′) to keep the number of estimated parameters to a minimum. We cannot get much guidance from theory on which lags should be included and which excluded from (3′). I regressed each exogenous variable on twelve of its own lagged values and twelve lagged values of the other exogenous variable from the same country. In all cases this produced white noise residuals. Insignificant variables were then dropped from the regressions.

At each point the residuals were checked to ensure they were still white noise. The forecasting equations arrived at in this way were not altered after the joint estimation had been completed.

Appendix B sets out the likelihood function for the joint model (3′) and (5′) for the two exchange rates and discusses the method used to maximize the likelihood function. If we denote the covariance matrix of the error terms in (3′) as Σ_ν, the covariance matrix of the error terms in (5′) as Σ_ϵ, and the covariance between ν and ϵ as $\Sigma_{\nu\epsilon}$, then we can distinguish two situations:

(a) Both Σ_ν and Σ_ϵ are diagonal so that the covariance matrix of the system (3′), (5′) is diagonal.

(b) Σ_ν and Σ_ϵ are unconstrained, although $\Sigma_{\nu\epsilon}$ is constrained to be zero.

In (a) the likelihood function can be maximized by iterative nonlinear least squares, whereas in (b) an explicit maximum likelihood algorithm is required. Assumption (b) is more general but alas more expensive to implement. The results using the model (a) are set out in table 5.1.

Most of the β coefficients are not significantly different from zero. However, as noted above, the β coefficients predicted by the simple monetary model will be functions not only of the parameters in the money demand function but also the parameters in the forecasting equations for money and income growth. The fact that most of the coefficients are not significantly different from zero cannot be taken as evidence against the simple monetary model. More explicit tests of the simple monetary model will be considered in section 5.3.

When the covariance structure is generalized to model (b) and the log likelihood function is explicitly maximized, to reduce the number of parameters to be estimated a more parsimonious parameterization for the forecasting equations is required. Hence the results in table 5.2 are not directly comparable with those of table 5.1.

As in table 5.1, few of the β coefficients in table 5.2 are significantly different from zero. Furthermore, the coefficients on errors in forecasting U.S. money growth changed sign in moving from table 5.1 to table 5.2. The most robust β coefficients appear to be those on German money and income forecasting errors. The cross-equation restrictions implied by rationality were not rejected in the models of either table 5.1 or table 5.2.

5.3 Testing the Simple Monetary Model

In appendix A, a result from Hansen and Sargent (1980) is used to express the β coefficients in (5′) in terms of the forecasting parameters γ in (3′) and the income elasticities and interest semielasticity of demand for money. As long as we restrict the covariance matrix $\Sigma_{\nu\epsilon}$ to be zero, the β coefficients will be identified, and the restrictions on those coefficients implied by the simple monetary model can be tested. At the same time, we can recover estimates of the income elasticities and interest semielas-

Table 5.1 Joint Estimation of (3′) and (5′) with Covariance Structure (a)[a]

Dependent Variable	Period	β_0	β_{USM}	β_{USI}	β_{UKM}	β_{UKI}	β_{GM}	β_{GI}	Test for Rationality[b]
Δ_{UKUS}	2/72–4/79	.00003	.348	.293	−.130	.130	—	—	
		(.0029)	(.373)	(.470)	(.247)	(.143)			
									$\chi^2_{49} = 54.54$
Δ_{GUS}	2/72–4/79	.00499	.058	−.101	—	—	−.699	−.867	
		(.0040)	(.446)	(.615)			(.383)	(.309)	

[a]Asymptotic standard errors are in brackets below the coefficients.
[b]Twice the difference in the maximized log likelihood functions.

Table 5.2 Joint Estimation of (3′) and (5′) with Covariance Structure (b)[a]

Dependent Variable	Period	Adj. for Hetero.	β_0	β_{USM}	β_{USI}	$\beta_{UKM,GM}$	$\beta_{UKI,GI}$	Corr.[b]	Test for Rationality[c,d]
Δ_{UKUS}	2/72–4/79	No	.0011	−.2321	.2258	.0008	.0563		—
			(.0037)	(.4820)	(.7458)	(.2865)	(.2250)	.597	
Δ_{GUS}	2/72–4/79	No	.0049	−.1590	.4157	−.2159	−.8561		—
			(.0051)	(.6017)	(.7138)	(.3382)	(.2635)		
Δ_{UKUS}	2/72–4/79	Yes	−.0174	−.0539	.2356	−.1142	.0328		
			(.0295)	(.4780)	(.6946)	(.2749)	(.1780)	.735	$\chi^2_{23} = 23.6504$
Δ_{GUS}	2/72–4/79	Yes	.1778	−.0481	.4912	−.1115	−.8615		PRV = 57.7%
			(1.7195)	(.6184)	(.7236)	(.3664)	(.2744)		

[a]Asymptotic standard errors are in brackets below the coefficients.
[b]Corr. = correlation between the errors in the Δ_{UKUS} and Δ_{GUS} equations.
[c]Twice the difference in the maximized likelihood functions.
[d]PRV = the probability that the chi-square variable with twenty-three degrees of freedom would produce a value as stated.

ticity of demand for money which can be compared with the values of these same parameters obtained from estimates of money demand functions.

I attempted to test for the conformity of the simple monetary model with the data, using either the real GNP or bank clearings divided by the wholesale price index as the income variable.[8] I also tried estimating the model, imposing the requirement that Σ_v and Σ_ϵ be diagonal or leaving them unconstrained. In all cases I had difficulty getting the algorithms to converge. The problem appeared to be that the likelihood function was maximized for values of the interest semielasticity of demand for money (α_3) which were very large, so that $\lambda = [\alpha_3/(1 + \alpha_3)]$ approached one. One example from these results is reproduced in table 5.3. Note that these parameter estimates are not maximum likelihood estimates as the algorithm was still diverging at these values.

The results must cast considerable doubt on the ability of the simple monetary model to adequately account for the data I examined. However, it should be emphasized again that some of the difficulty might be the result of changes over the period in the stochastic processes (3′) governing the evolution of the money and income variables.

As a further test of the simple monetary model, I used the term structure of the forward rate to test an alternative implication of the model. I show in appendix A that the error in forecasting the change in the exchange rate over three months, if exchange rates are determined in accordance with the simple monetary model in appendix A and expectations are formed rationally, can be written:[9]

$$
\begin{aligned}
(8)\quad & \frac{S_t - S_{t-3}}{S_{t-3}} - \frac{{}_{t-3}F_t - S_{t-3}}{S_{t-3}} = \beta_0 + \beta_1 v_{1t} + \beta_2 v_{3t} + \beta_3 v_{2t} + \beta_4 v_{4t} \\
& + \left[\beta_1 + \frac{(\beta_1 - 1)}{\alpha_3}\right] v_{1t-1} + \left[\beta_2 + \frac{(\beta_2 - 1)}{\alpha_3}\right] v_{3t-1} + \left[\beta_3 + \frac{(\beta_3 - 1)}{\alpha_3}\right] v_{2t-1} \\
& + \left[\beta_4 + \frac{(\beta_4 - 1)}{\alpha_3}\right] v_{4t-1} + \left[\beta_1 + \frac{(\beta_1 - 1)}{\alpha_3}\,\frac{(1 + 2\alpha_3)}{\alpha_3} - \frac{(\gamma_{11} - \alpha_1\gamma_{31}}{\alpha_3}\right] v_{1t-2} \\
& + \left[\beta_2 + \frac{(\beta_2 - 1)}{\alpha_3}\,\frac{(1 + 2\alpha_3)}{\alpha_3} - \frac{(\gamma_{11}^* - \alpha_2\gamma_{31}^*)}{\alpha_3}\right] v_{3t-2} \\
& + \left[\beta_3 + \frac{(\beta_3 - 1)}{\alpha_3}\,\frac{(1 + 2\alpha_3)}{\alpha_3} - \frac{(\gamma_{21} - \alpha_1\gamma_{41})}{\alpha_3}\right] v_{2t-2} \\
& + \left[\beta_4 + \frac{(\beta_4 - 1)}{\alpha_3}\,\frac{(1 + 2\alpha_3)}{\alpha_3} - \frac{(\gamma_{21}^* - \alpha_2\gamma_{41}^*)}{\alpha_3}\right] v_{4t-2} + \epsilon_t + \epsilon_{t-1} + \epsilon_{t-2},
\end{aligned}
$$

8. Bank clearings and the wholesale price indices were obtained from the *International Financial Statistics*. I used this variable to capture the transactions demand for money.

9. Note that the theory implies the error term in equation (8) will be a moving average. This fact has been ignored in the estimation and will lead to inefficient estimates of the parameters of the model.

Table 5.3 **Estimation of the Simple Monetary Model[a] (Σ diagonal)**

Period	Independent Variables	Income Elasticities U.S.	U.K.	G.	Interest Semielasticity
2/72–4/79	M1 growth	−.01482	−.0832	.6149	21,578.88
	Real clearings growth	(.1757)	(.0717)	(.3655)	(94,754,325.1)

[a]Asymptotic standard errors are in brackets below the coefficients.

where β_i and v_{it} have exactly the same definition as in equations (3′) and (5′), while γ_{i1} is the coefficient of L^1 in the polynomial $\gamma_i(L)$ in the lag operator in (3′). The parameters α_1 and α_2 are the income elasticities of demand for money in the United States and the foreign country (the United Kingdom or Germany in our case), while α_3 is the common interest semielasticity of the demand for money.

We can get a test of the simple monetary model by estimating (3′), (5′), and (8) jointly and then testing the restrictions on the coefficients in (8). The values of the unrestricted coefficients in (8) are of interest in themselves. If the so-called overshooting hypothesis is correct,[10] then we might expect to see the coefficients on unanticipated money growth change sign as the lag increases from one to three periods in equation (8). The results of jointly estimating the equations (3′), (5′), and (8) are given in table 5.4. No adjustments were made for heteroscedasticity. Although the constraints on the coefficients in (8) are not rejected, it is apparent that the β coefficients in the unconstrained version of (8) are estimated very imprecisely. The results in table 5.4 would give one very little confidence that the models (3′), (5′), and (8) are consistent with the data. As far as the overshooting hyothesis is concerned, most of the coefficients in the unconstrained version of (8) follow a pattern, as illustrated in figure 5.1. Exceptions to this pattern are the coefficients on U.S. and German income in the German three-month forecasting error equation. Both of these behave monotonically as a function of the lag, but only the coefficients on U.S. income forecasting errors change sign.

5.4 Interest Rate Model

Following Frenkel (1981), I estimated a model

(9)
$$i_{t+1} = \delta_1(L)i_t + \delta_2(L)\hat{M}_t + \delta_3(L)\hat{Y}_t + v_{1t};$$
$$i^*_{t+1} = \delta^*_1(L)i^*_t + \delta^*_2(L)\hat{M}^*_t + \delta^*_3(L)\hat{Y}^*_t + v_{2t};$$

and

10. See Dornbusch (1976), for example.

Table 5.4 **Joint Estimation of Exogenous Variable Forecasting Equation, One-Month Exchange Rate Forecasting Error Equation and Three-Month Exchange Rate Forecasting Error Equation**

Dependent Variable	Unconstrained Model:[a] One-Month Equations									
	β_O	β_{USM}	β_{USI}	$\beta_{UKM,GM}$	$\beta_{UKI,GI}$		Δ_{UKUS}	Δ_{GUS}	Δ^3_{UKUS}	Δ^3_{GUS}
Δ_{UKUS}	.0009 (.0068)	−.2227 (.6767)	.2444 (1.1752)	.0157 (.6112)	.1086 (.4823)	Δ_{UKUS}	1	.592	.627	.326
						Δ_{GUS}	—	1	.412	.567
Δ_{GUS}	.0049 (.0083)	−.1810 (.8901)	.4239 (1.1111)	−.3535 (.3989)	−.9089 (.4905)	Δ^3_{UKUS}	—	—	1	.608
						Δ^3_{GUS}	—	—	—	1

Unconstrained Model:[a] Three-Month Equations

	β_O	β_{USM}	β_{USI}	$\beta_{UKM,GM}$	$\beta_{UKI,GI}$	$\beta_{USM_{-1}}$	$\beta_{USI_{-1}}$	$\beta_{UKM,GM_{-1}}$	$\beta_{UKI,GI_{-1}}$	$\beta_{USM_{-2}}$	$\beta_{USI_{-2}}$	$\beta_{UKM,GM_{-2}}$	$\beta_{UKI,GI_{-2}}$
Δ^3_{UKUS}	.002	−.838	−.204	.372	.307	−.004	.366	.394	.063	−.624	.092	−.482	.054
	(.011)	(1.44)	(2.241)	(.797)	(.724)	(1.128)	(1.828)	(.680)	(.468)	(.979)	(1.471)	(.549)	(.481)
Δ^3_{GUS}	.016	−.737	.838	−1.215	−1.172	−.176	.204	−.435	−.881	−.445	−.549	−.937	−.764
	(.023)	(1.962)	(2.157)	(.881)	(.889)	(1.782)	(1.711)	(.820)	(.766)	(1.460)	(1.820)	(.801)	(.762)

Constrained Model

	β_0	β_1	β_2	β_3	β_4	α_{US}	α_{UK}	α_G	$\alpha_{INTEREST}$
UKUS	.0012	−.1870	.3487	.0083	.0700		−6.5789	—	
	(.0041)	(.3713)	(.6012)	(.2289)	(.2653)		(88.4177)		
						−9.8315			24.681
						(53.2765)			(81.7341)
GUS	.0041	−.2069	.2531	−.3385	−.8229		—	22.4224	
	(.0059)	(.5627)	(.6507)	(.3111)	(.2754)			(98.4515)	

NOTE: Test for validity of constraints χ^2_{22} = 18.6019. Probability level = 33.0%.

[a]Cross-equation constraints on the γ coefficients are retained.

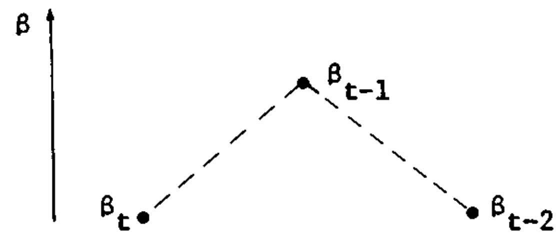

Fig. 5.1

$$(10) \qquad \Delta_{t+1} = \beta_0(i_{t+1} - E_t i_{t+1})\beta_1 + (i^*_{t+1} - E_t i^*_{t+1})\beta_2 + \epsilon_{t+1}.$$

Although this model has a less rigorous theoretical foundation than the simple monetary model studied in appendix A, Frenkel found it was capable of explaining part of the exchange rate forecasting error Δ_{t+1}. He suggested that interest rates and exchange rates might both be affected by the same news. Further, if both bonds and foreign exchange are traded in efficient financial markets, the time lag between the arrival of the news and its subsequent effect on prices will be similar in the two markets. This begs the question as to the exact nature of the news. It also suggests we should set up a simultaneous equation model where exchange rates and interest rates are both endogenous variables. Frenkel uses instrumental variables to cope with the simultaneity problem. I have ignored it. As above, I shall assume that ϵ and ν in (8) and (10) are uncorrelated. On the other hand, I can test for rationality by testing the validity of the cross-equation restrictions on the δ parameters in (9) and (10). Note that the validity of this test does not depend on the validity of the assumption that $E(\epsilon, \nu) \equiv \Sigma_{\epsilon\nu} = 0$.

In the results reported in table 5.5, I used the one-month Eurocurrency rates reported in the Harris Trust and Savings Bank *Weekly Bulletin* for interest rates. Again, I used the rate on the last Friday in each month. Date limitations prevented me estimating (9) and (10) over the full period of February 1972 to April 1979. I used maximum likelihood estimation as I did for the simple monetary model results reported in table 5.2. In contrast to the tests based on the simple monetary model, we find some weak evidence against rationality of expectations in table 5.5. Also, it is rather interesting to note that although the German equation produced more robust results for the simple monetary model, the U.K. equation gives more significant β coefficients when the simple interest rate model is estimated.

5.5 Conclusion

We have uncovered very little evidence unfavorable to the hypothesis that expectations are formed rationally in the foreign exchange market.

Table 5.5 Joint Estimation of Equations (9) and (10)

Dependent Variable	Period	Adj. for Hetero.	β_0	β_{USINT}	$\beta_{UKINT,GINT}$	Corr.	Test for Rationality
Δ_{UKUS}	10/72–4/79	No	.1938	.7217	−.5531		
			(.3875)	(.6250)	(.1877)		
						.543	—
Δ_{GUS}	10/72–4/79	No	.5554	1.2304	−.0802		
			(.7058)	(.8254)	(.5330)		
Δ_{UKUS}	10/72–4/79	Yes	−2.2500	.7175	−.5399		
			(9.9163)	(.4616)	(.1720)		
						.551	$\chi^2_{34} = 44.9922$
Δ_{GUS}	10/72–4/79	Yes	−.6508	1.0428	−.1905		PRV = 90.15%
			(7.7214)	(.7420)	(.5472)		

However, proponents of a simple monetary model of exchange rate determination can find little comfort in the results. There is some evidence that a single, simple model may not be satisfactory for explaining all currency movements. This might be related to the different way monetary policy is conducted in different countries. If the monetary authorities followed an interest rate rule, then unanticipated innovations in interest rates may provide more relevant information than unanticipated movements in the money stock. The opposite might be the case for a country which followed a money stock growth rate rule. If the authorities did not follow any stable rule over the period examined, then it would be very difficult to test any model of exchange rate determination. This is just another implication of the Lucas critique of econometric policy evaluation (Lucas 1976). Instability in policy over the estimation period could have been a major factor leading to the very imprecise estimates of the β parameters found in this paper.

Appendix A A Simple Monetary Model

The model discussed here is a rather simple variant of the models discussed in the monetary approach literature. This has been done since the model:

(i) delivers strong restrictions on the effect of lagged values in the exogenous variables on exchange rate forecast errors,[11] and

(ii) enables the derivation of testable restrictions on the effect of unanticipated changes in the exogenous variables on forecast errors.

We begin with an equation representing deviations from purchasing power parity:

$$\hat{S}_{t+1} = \hat{P}_{t+1} - \hat{P}^*_{t+1} + \xi_{t+1}, \tag{A1}$$

where the caret denotes percentage rates of change, and P and P^* are the domestic and foreign price levels.[12] Equilibrium in the domestic and foreign money markets requires

$$M/P = L(Y, i) \text{ and } M^*/P^* = L^*(Y^*, i^*), \tag{A2}$$

11. If, for example, partial adjustment parameters were appended to the present model, a far wider range of estimated lagged effects would be consistent with the model. However, I would consider that a weakness and not a strength of the extended model. If one is to postulate a model with lagged adjustments, it would be preferable to have a theory explaining the source of the lags so that one could get restrictions on the adjustment parameters more open to refutation.

12. Frenkel (1978) discusses the use of the relevant price index for use here. To the extent that purchasing power parity pertains to traded goods only, (A1) would also contain terms involving the relative price of traded to nontraded goods.

where Y and Y^* are domestic and foreign real income, and i and i^* are the domestic and foreign (nominal) interest rates.

Then from (A2)

$$P/P^* = [M/L(\cdot)][L^*(\cdot)/M^*].$$

Now take $L = k_1 Y^{\alpha_1} e^{-\alpha_3 i}$ and $L^* = k_2 Y^{*\alpha_2} e^{-\alpha_3 i^*}$,[13] then

$$\text{(A3)} \quad \hat{P} - \hat{P}^* = \hat{M} - \hat{M}^* - \alpha_1 \hat{Y}_{t+1} + \alpha_2 \hat{Y}_{t+1} + \alpha_3 \Delta(i_{t+1} - i^*_{t+1}),$$

and from (A1) and (A3)

$$\text{(A4)} \quad \hat{S}_{t+1} = \hat{M}_{t+1} - \hat{M}^*_{t+1} - \alpha_1 \hat{Y}_{t+1} + \alpha_2 \hat{Y}_{t+1} + \alpha_3 \Delta(i_{t+1} - i^*_{t+1}) + \xi_{t+1},$$

which is equation (2′) in the text.

Now we impose the interest parity condition. In the absence of transaction costs, arbitrage in assets ensures[14]

$$\text{(A5)} \qquad \frac{{}_{t+1}F_{t+2}}{S_{t+1}} = \frac{1 + i_{t+1}}{1 + i^*_{t+1}},$$

where ${}_{t+1}F_{t+2}$ is the forward rate at $t + 1$ for $t + 2$. Exanding $1/(1 + i^*)$ and ignoring squares and higher powers of interest rates, we get

$$\frac{{}_{t+1}F_{t+2}}{S_{t+1}} \simeq 1 + i_{t+1} - i^*_{t+1},$$

or

$$\text{(A6)} \qquad i_{t+1} - i^*_{t+1} \simeq \frac{{}_{t+1}F_{t+2} - S_{t+1}}{S_{t+1}} = E_{t+1}\hat{S}_{t+2} - a$$

from equation (6) in the text. Substitute (A6) into (A4):

$$\text{(A7)} \qquad \hat{S}_{t+1} = \hat{M}_{t+1} - \hat{M}^*_{t+1} - \alpha_1 \hat{Y}_{t+1} + \alpha_2 \hat{Y}^*_{t+1} + \alpha_3 E_{t+1}\hat{S}_{t+2} - \alpha_3 E_t \hat{S}_{t+1} + \xi_{t+1}.$$

Take expectations of (A7) at time t and use $E_t(E_{t+1}\hat{S}_{t+2}) = E_t\hat{S}_{t+2}$ to get

$$\text{(A8)} \qquad E_t\hat{S}_{t+1} = E_t\hat{M}_{t+1} - E_t\hat{M}^*_{t+1} - \alpha_1 E_t\hat{Y}_{t+1} + \alpha_2 E_t\hat{Y}^*_{t+1} + \alpha_3 E_t\hat{S}_{t+2} - \alpha_3 E_t\hat{S}_{t+1} + E_t\xi_{t+1}.$$

This can be written using the backshift operator as

$$\text{(A9)} \quad (1 + \alpha_3 - \alpha_3 B^{-1})E_t\hat{S}_{t+1} = E_t\hat{M}_{t+1} - E_t\hat{M}^*_{t+1} - \alpha_1 E\hat{Y}_{t+1} + \alpha_2 E_t\hat{Y}^*_{t+1} + E_t\xi_{t+1}.$$

But

$$1 + \alpha_3 - \alpha_3 B^{-1} = (1 + \alpha_3)\left(1 - \frac{\alpha_3}{1 + \alpha_3} B^{-1}\right) \text{ with } \left|\frac{\alpha_3}{1 + \alpha_3}\right| < 1.$$

13. The functional forms proposed here for the money demand functions are common in monetary economics and exchange rate literature.

14. Equation (A5) has been tested previously in quite a few studies and appears to hold up reasonably well (see, for example, Frenkel and Levich 1975, 1977; Levich 1978, 1979).

Hence,

$$\text{(A10)}\quad (1+\alpha_3)E_t\hat{S}_{t+1} = \sum_{i=0}^{\infty}\left(\frac{\alpha_3}{1+\alpha_3}\right)^i E_t\hat{M}_{t+1+i} - \sum_{i=0}^{\infty}\left(\frac{\alpha_3}{1+\alpha_3}\right)^i E_t\hat{M}^*_{t+1+i}$$
$$- \alpha_1\sum_{i=0}^{\infty}\left(\frac{\alpha_3}{1+\alpha_3}\right)^i E_t\hat{Y}_{t+1+i} + \alpha_2\sum_{i=0}^{\infty}\left(\frac{\alpha_3}{1+\alpha_3}\right)^i E_t\hat{Y}^*_{t+1+i}$$
$$+ \sum_{i=0}^{\infty}\left(\frac{\alpha_3}{1+\alpha_3}\right)^i E_t\xi_{t+1+i}.$$

We can lead (A10) one period to get $E_{t+1}\hat{S}_{t+2}$ and the substitute into (A7):

$$\text{(A11)}\quad \hat{S}_{t+1} = \hat{M}_{t+1} - \hat{M}^*_{t+1} - \alpha_1\hat{Y}_{t+1} + \alpha_2\hat{Y}^*_{t+1}$$
$$+ \frac{\alpha_3}{1+\alpha_3}\left[\sum_{i=0}^{\infty}\left(\frac{\alpha_3}{1+\alpha_3}\right)^i E_{t+1}\hat{M}_{t+2+i}\right.$$
$$- \sum_{i=0}^{\infty}\left(\frac{\alpha_3}{1+\alpha_3}\right)^i E_{t+1}\hat{M}^*_{t+2+i} - \alpha_1\sum_{i=0}^{\infty}\left(\frac{\alpha_3}{1+\alpha_3}\right)^i E_{t+1}\hat{Y}_{t+2+i}$$
$$\left.+ \alpha_2\sum_{i=0}^{\infty}\left(\frac{\alpha_3}{1+\alpha_3}\right)^i E_{t+1}\hat{Y}^*_{t+2+i} + \sum_{i=0}^{\infty}\left(\frac{\alpha_3}{1+\alpha_3}\right)^i E_{t+1}\xi_{t+2+i}\right]$$
$$- \frac{\alpha_3}{1+\alpha_3}\left[\sum_{i=0}^{\infty}\left(\frac{\alpha_3}{1+\alpha_3}\right)^i E_t\hat{M}_{t+1+i} - \sum_{i=0}^{\infty}\left(\frac{\alpha_3}{1+\alpha_3}\right)^i E_t\hat{M}^*_{t+1+i}\right.$$
$$- \alpha_1\sum_{i=0}^{\infty}\left(\frac{\alpha_3}{1+\alpha_3}\right)^i E_t\hat{Y}_{t+1+i} + \alpha_2\sum_{i=0}^{\infty}\left(\frac{\alpha_3}{1+\alpha_3}\right)^i E_t\hat{Y}^*_{t+1+i}$$
$$\left.+ \sum_{i=0}^{\infty}\left(\frac{\alpha_3}{1+\alpha_3}\right)^i E_t\xi_{t+1+i}\right] + \xi_{t+1}.$$

From (A11):[15]

$$\text{(A12)}\quad \hat{S}_{t+1} - E_t\hat{S}_{t+1} = \sum_{0}^{\infty}\left(\frac{\alpha_3}{1+\alpha_3}\right)^i (E_{t+1}\hat{M}_{t+1+i} - E_t\hat{M}_{t+1+i})$$
$$- \sum_{0}^{\infty}\left(\frac{\alpha_3}{1+\alpha_3}\right)^i (E_{t+1}\hat{M}^*_{t+1+i} - E_t\hat{M}^*_{t+1+i})$$
$$- \sum_{0}^{\infty}\left(\frac{\alpha_3}{1+\alpha_3}\right)^i (E_{t+1}\hat{Y}_{t+1+i} - E_t\hat{Y}^*_{t+1+i})$$
$$+ \alpha_2\sum_{0}^{\infty}\left(\frac{\alpha_3}{1+\alpha_3}\right)^i (E_{t+1}\hat{Y}^*_{t+1+i} - E_t\hat{Y}^*_{t+1+i}) + \epsilon_{t+1},$$

15. Note that if ξ is autocorrelated so $\xi_{t+1} = \alpha(L)\xi_t + \nu_{t+1}$, then

$$\epsilon_{t+1} = \sum_{0}^{\infty}\left(\frac{\alpha_3}{1+\alpha_3}\right)^i (E_{t+1}\xi_{t+1+i} - E_t\xi_{t+1+i}),$$

which is equation (7) in the text.

Now we want to derive equation (5′) in the text. We begin with the forecasting equations (3′). These can be written, using lag operator notation as:

$$\text{(A13)}\quad \begin{bmatrix} \gamma_{11}(L) & \gamma_{12}(L) \\ \gamma_{21}(L) & \gamma_{22}(L) \end{bmatrix} \begin{bmatrix} \hat{M}_{t+1} \\ \hat{Y}_{t+1} \end{bmatrix} = \begin{bmatrix} \nu_{1t+1} \\ \nu_{2t+1} \end{bmatrix},$$

where $\gamma_{ij}(L) \equiv \sum_{k=0}^{n_{ij}} \gamma_k^{ij} L^k, \quad i, j = 1, 2, \text{with } \gamma_0^{ij} = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases}.$

Put
$$\lambda \equiv \frac{\alpha_3}{1+\alpha_3} \text{ and } r = \max_{ij} \{n_{ij}\}$$

and
$$\gamma_k = \begin{bmatrix} \gamma_{11}^k & \gamma_{12}^k \\ \gamma_{21}^k & \gamma_{22}^k \end{bmatrix}, k = 1, \ldots, r.$$

Then, as shown in Hansen and Sargent (1980), we can write

$$\sum_{i=0}^{\infty} \lambda^i E_{t+1} \hat{M}_{t+1+i} = U_1 \gamma(\lambda)^{-1} \left[I + \sum_{j=1}^{r-1} \left(\sum_{k=j+1}^{r} \lambda^{k-j} \gamma_k \right) L^j \right] \begin{bmatrix} \hat{M}_{t+1} \\ \hat{Y}_{t+1} \end{bmatrix},$$

and

$$\text{(A14)}\quad \sum_{i=0}^{\infty} \lambda^i E_{t+1} \hat{Y}_{t+1+i} = U_2 \gamma(\lambda)^{-1} \left[I + \sum_{j=1}^{r-1} \left(\sum_{k=j+1}^{r} \lambda^{k-j} \gamma_k \right) L^j \right] \begin{bmatrix} \hat{M}_{t+1} \\ \hat{Y}_{t+1} \end{bmatrix},$$

where $U_1 = [1\ 0]$ and $U_2 = [0\ 1]$.

But
$$\sum_{j=1}^{r-1} L^j \begin{bmatrix} \hat{M}_{t+1} \\ \hat{Y}_{t+1} \end{bmatrix}$$

is known at time t. Therefore,

$$\text{(A15)}\quad \sum_{i=0}^{\infty} \lambda^i E_t \hat{M}_{t+1+i} = U_1 \gamma(\lambda)^{-1} \begin{bmatrix} E_t \hat{M}_{t+1} \\ E_t \hat{Y}_{t+1} \end{bmatrix} + U_1 \gamma(\lambda)^{-1} \sum_{j=1}^{r-1} \left(\sum_{k=j+1}^{r} \lambda^{k-j} \gamma_k \right) L^j \begin{bmatrix} \hat{M}_{t+1} \\ \hat{Y}_{t+1} \end{bmatrix},$$

with a similar expression for $\Sigma_{i=0}^{\infty} \lambda^i E_t \hat{Y}_{t+1+i}$.

---

and by applying the same manipulations to ξ as are done for $\hat{M}$ and $\hat{Y}$ in equations (A.14)–(A.15), then $\epsilon_{t+1} = A \cdot (\xi_{t+1} - E_t \xi_{t+1}) = A \cdot \nu_{t+1}$ for some constant A, which depends on the α coefficients in $\alpha(L)$. Without loss of generality we may assume ϵ is uncorrelated.

Substitute into (A12) to get

$$\text{(A16)}\quad \hat{S}_{t+1} - E_t\hat{S}_{t+1} = \frac{[\gamma_{22}(\lambda) + \alpha_1\gamma_{21}(\lambda)]}{\det\gamma(\lambda)}(\hat{M}_{t+1} - E_t\hat{M}_{t+1}) - \frac{[\gamma_{12}(\lambda) + \alpha_1\gamma_{11}^*(\lambda)]}{\det\gamma(\lambda)}(\hat{Y}_{t+1} - E_t\hat{Y}_{t+1}) - \frac{[\gamma_{22}^*(\lambda) + \alpha_2\gamma_{11}^*(\lambda)]}{\det\gamma^*(\lambda)}(\hat{M}_{t+1}^* - E_t\hat{M}_{t+1}^*) + \frac{[\gamma_{12}^*(\lambda) + \alpha_2\gamma_{11}^*(\lambda)]}{\det\gamma^*(\lambda)}(\hat{Y}_{t+1}^* - E_t\hat{Y}_{t+1}^*) + \epsilon_{t+1},$$

where $\qquad \det\gamma(\lambda) = \gamma_{11}(\lambda)\gamma_{22}(\lambda) - \gamma_{21}(\lambda)\gamma_{12}(\lambda),$

and $\qquad \det\gamma^*(\lambda) = \gamma_{11}^*(\lambda)\gamma_{22}^*(\lambda) - \gamma_{21}^*(\lambda)\gamma_{12}^*(\lambda).$

Equation (A16) corresponds to (5′) in the text. The simple monetary model can also be tested using the term structure of the forward rate. We could use expression (A14) above, but I decided to test an alternative expression which is also implied by the simple monetary model.

First, observe that

$$\text{(A17)}\qquad \frac{S_t - S_{t-3}}{S_{t-3}} \simeq \ln S_t - \ln S_{t-3} = \ln S_t - \ln S_{t-1} + \ln S_{t-1} - \ln S_{t-2} + \ln S_{t-2} - \ln S_{t-3} \simeq S_t + S_{t-1} + S_{t-2}.$$

So, $$\frac{S_t - S_{t-3}}{S_{t-3}} - \frac{{}_{t-3}F_t - S_{t-3}}{S_{t-3}} \simeq (\hat{S}_t - E_{t-3}\hat{S}_t) + (\hat{S}_{t-1} - E_{t-3}\hat{S}_{t-1}) + (\hat{S}_{t-2} - E_{t-3}\hat{S}_{t-2}).$$

Now use (A12) together with the forecasting equations in (3′) in the text:

$$\text{(A18)}\qquad \begin{aligned} \hat{M}_{t+1} &= \gamma_1(L)\hat{M}_t + \gamma_2(L)\hat{Y}_t + \nu_{1t+1};\\ \hat{Y}_{t+1} &= \gamma_3(L)\hat{M}_t + \gamma_4(L)\hat{Y}_t + \nu_{2t+1};\\ \hat{M}_{t+1}^* &= \gamma_1^*(L)\hat{M}_t^* + \gamma_2^*(L)\hat{Y}_t^* + \nu_{3t+1};\\ \hat{Y}_{t+1}^* &= \gamma_3^*(L)\hat{M}_t^* + \gamma_4^*(L)\hat{Y}_t^* + \nu_{4t+1}; \end{aligned}$$

where

$$\gamma_i(L) \equiv \sum_{j=1}^{n}\gamma_{ij}L^j, \qquad \gamma_i^*(L) \equiv \sum_{j=1}^{n}\gamma_{ij}^*L^j, \qquad i = 1,2,3,4.$$

Then $\hat{S}_t - E_{t-3}\hat{S}_t$, for example, will contain terms like

$$\hat{M}_t - E_{t-3}\hat{M}_t, \qquad E_t\hat{M}_{t-1} - E_{t+3}\hat{M}_{t+1}, \ldots,$$

and

$$E_{t-1}\hat{M}_t - E_{t-3}\hat{M}_t, \qquad E_{t-1}\hat{M}_{t+1} - E_{t-3}\hat{M}_{t+1}, \ldots$$

Now use (A15) to evaluate these. For example,

$$\begin{aligned}\hat{M}_t - E_{t-3}\hat{M}_t = (\gamma_{11}^2 + \gamma_{12} + \gamma_{21}\gamma_{31})v_{1t-2} \\ + [\gamma_{22} + (\gamma_{11} + \gamma_{41})\gamma_{21}]v_{2t-2} \\ + \gamma_{11}v_{1t-1} + \gamma_{21}v_{2t-1} + v_{1t},\end{aligned}$$

and

$$\begin{aligned}E_{t-1}\hat{M}_t - E_{t-3}\hat{M}_t = (\gamma_{11}^2 + \gamma_{12} + \gamma_{21}\gamma_{31})v_{1t-2} \\ + [\gamma_{22} + (\gamma_{11} + \gamma_{41})\gamma_{21}]v_{2t-2} \\ + \gamma_{11}v_{1t-1} + \gamma_{21}v_{2t-1}.\end{aligned}$$

Put

$$\begin{aligned}E_t\hat{M}_{t+i} - E_{t-1}\hat{M}_{t+i} \equiv \xi_{1i}v_{1t} + \zeta_{1i}v_{2t}, \\ E_t\hat{Y}_{t+i} - E_{t-1}\hat{Y}_{t+i} \equiv \xi_{2i}v_{1t} + \zeta_{2i}v_{2t},\end{aligned}$$

and

$$\beta_1 \equiv \sum_0^\infty \left(\frac{\alpha_3}{1+\alpha_3}\right)^i (\xi_{1i} - \alpha_1\xi_{2i}),$$

$$\beta_3 \equiv \sum_0^\infty \left(\frac{\alpha_3}{1+\alpha_3}\right)^i (\zeta_{1i} - \alpha_1\zeta_{2i}),$$

Define ξ^*, ζ^* and β_2, β_4 analogously for the foreign variables. Then it can be shown that

$$\begin{aligned}\hat{S}_t - E_{t-3}\hat{S}_t = \beta_1 v_{1t} + \beta_2 v_{3t} + \beta_3 v_{2t} + \beta_4 v_{4t} \\ + \frac{1}{\alpha_3}[(\beta_1 - 1)v_{1t-1} + (\beta_2 - 1)v_{3t-1} + (\beta_3 - 1)v_{2t-1} \\ + (\beta_4 - 1)v_{2t-1}] + \left(\frac{1+\alpha_3}{\alpha_3^2}\right)\Bigg\{\Bigg[\beta_1 - 1 \\ - \frac{\alpha_3}{1+\alpha_3}(\gamma_{11} - \alpha_1\gamma_{31})\Bigg]v_{1t-2} \\ + \Bigg[\beta_2 - 1 - \frac{\alpha_3}{1+\alpha_3}(\gamma_{11}^* - \alpha_2\gamma_{31}^*)\Bigg]v_{3t-2} \\ + \Bigg[\beta_3 - 1 - \frac{\alpha_3}{1+\alpha_3}(\gamma_{21} - \alpha_1\gamma_{41})\Bigg]v_{2t-2} \\ + \Bigg[\beta_4 - 1 - \frac{\alpha_3}{1+\alpha_3}(\gamma_{21}^* - \alpha_2\gamma_{41}^*)\Bigg]v_{4t-2}\Bigg\} + \epsilon_t.\end{aligned}$$

Similarly,

$$\begin{aligned}\hat{S}_{t-1} - E_{t-3}\hat{S}_{t-1} = \beta_1 v_{1t-1} + \beta_2 v_{3t-1} + \beta_3 v_{2t-1} \\ + \beta_4 v_{4t-1} + \frac{1}{\alpha_3}[(\beta_1 - 1)v_{1t-2} + (\beta_2 - 1)v_{3t-2} \\ + (\beta_3 - 1)v_{2t-2} + (\beta_4 - 1)v_{4t-2}] + \epsilon_{t-2},\end{aligned}$$

and

$$\hat{S}_{t-2} - E_{t-3}\hat{S}_{t-2} = \beta_1 v_{1t-2} + \beta_2 v_{3t-2} + \beta_3 v_{2t-2} + \beta_4 v_{4t-2} + \epsilon_{t-2}.$$

Substitute into (A17). The simple monetary model predicts that

$$\frac{S_t - S_{t-3}}{S_{t-3}} - \frac{{}_{t-3}F_t - S_{t-3}}{S_{t-3}}$$

will be a moving average of the one-period forecasting errors

$$v_{it-j}, \begin{cases} i = 1, 2, 3, 4. \\ j = 0, 1, 2. \end{cases}$$

Furthermore, if we jointly estimate

$$\frac{S_t - S_{t-1}}{S_{t-1}} - \frac{{}_{t-1}F_t - S_{t-1}}{S_{t-1}} = \beta_1 v_{1t} + \beta_2 v_{3t} + \beta_3 v_{2t} + \beta_4 v_{4t} + \epsilon_t,$$

and

$$\begin{aligned}\frac{S_t - S_{t-3}}{S_{t-3}} - \frac{{}_{t-3}F_t - S_{t-3}}{S_{t-3}} &= \beta_1 v_{1t} + \beta_2 v_{3t} + \beta_3 v_{2t} \\ &+ \beta_4 v_{4t} + \left(\beta_1 + \frac{\beta_1 - 1}{\alpha_3}\right) v_{1t-1} + \left(\beta_2 + \frac{\beta_2 - 1}{\alpha_3}\right) v_{3t-1} \\ &+ \left(\beta_3 + \frac{\beta_3 - 1}{\alpha_3}\right) v_{2t-1} + \left(\beta_4 + \frac{\beta_4 - 1}{\alpha_3}\right) v_{4t-1} \\ &+ \left[\beta_1 + \frac{\beta_1 - 1}{\alpha_3}\frac{1 + 2\alpha_3}{\alpha_3} - \frac{(\gamma_{11} - \alpha_1\gamma_{31})}{\alpha_3}\right] v_{1t-2} \\ &+ \left[\beta_2 + \frac{\beta_2 - 1}{\alpha_3}\frac{1 + 2\alpha_3}{\alpha_3} - \frac{(\gamma_{11}^* - \alpha_2\gamma_{31}^*)}{\alpha_3}\right] v_{3t-2} \\ &+ \left[\beta_3 + \frac{\beta_3 - 1}{\alpha_3}\frac{1 + 2\alpha_3}{\alpha_3} - \frac{(\gamma_{21} - \alpha_1\gamma_{41})}{\alpha_3}\right] v_{2t-2} \\ &+ \left[\beta_4 + \frac{\beta_4 - 1}{\alpha_3}\frac{1 + 2\alpha_3}{\alpha_3} - \frac{(\gamma_{21}^* - \alpha_2\gamma_{41}^*)}{\alpha_3}\right] v_{4t-2} \\ &+ \epsilon_t + \epsilon_{t-1} + \epsilon_{t-2},\end{aligned}$$

then the restrictions on the β coefficients implied by the simple monetary model can be tested. As above, estimates of the elasticity parameters in the money demand functions will also be obtained, and these can be compared with estimates of these same parameters which have been obtained by other methods.

Appendix B The Likelihood Function for the Model (3), (5)

To simplify the exposition, I will use the notation of equations (3) and (5) in the text rather than the more explicit but more cumbersome notation of (3′) and (5′).

We want the likelihood function for the simultaneous system

$$X = Z\gamma + v;$$
$$\Delta = (X - Z\gamma)\beta + \epsilon.$$

Let $(X_{1t}, \ldots, X_{kt}, \Delta_t)$ be the $1 \times (k+2)$ vector of observations on the endogenous variables at time t, and let $(Z_{1t}, \ldots, Z_{\ell t})$ be the $1 \times \ell$ vector of observations on the exogenous variables at time t.

Then the system can be written

$$(X_{1t}, \ldots, X_{kt}, \Delta_t)B + (Z_{1t}, \ldots, Z_{\ell t})\,\Gamma = (v_t, \epsilon_t),$$

where

$$B = \left(\begin{array}{c|c} \mathrm{I}_k & -\beta \\ \hline 0 & I_2 \end{array}\right),$$

$$\Gamma = \left(\begin{array}{c|c} \gamma & \gamma\beta \end{array}\right).$$

Since $\det B = 1$, we can write the log likelihood function[16]

$$L^* = -\frac{1}{2}n(k+1)\log 2\pi - \frac{n}{2}\log\det\Sigma - \frac{1}{2}\sum_{t=1}^{n}(v_t \epsilon_t)\Sigma^{-1}\begin{pmatrix} v_t' \\ \epsilon_t' \end{pmatrix}$$

where

$$\Sigma = \left(\begin{array}{c|c} \Sigma_v & 0 \\ \hline 0 & \Sigma_\epsilon \end{array}\right).$$

If Σ_v and Σ_ϵ are diagonal, the log likelihood can be maximized by dividing each of the equations by the estimated variance of the residual of that equation and then using nonlinear least squares to obtain the parameter estimates γ and β.

In the more general case, where Σ_v and Σ_ϵ are unrestricted, I used an algorithm specified in a paper by Berndt, Hall, and Hausman (1974).

In both cases, I corrected the residuals for heteroscedasticity using the time trend procedure outlined by Glejser and discussed in Johnston (1963). Initial parameter estimates were obtained and the absolute values

16. See, for example, Schmidt (1976), p. 216.

of the residuals were then regressed on a constant and a time trend to get two parameters β_{i0} and β_{i1} for each equation i. The data of equation i were then corrected by dividing by the square root of $(\beta_{i0} + \beta_{i1} \cdot \text{time})$. Parameter estimates using the corrected data were then obtained. While this procedure is satisfactory when Σ_v and Σ_ϵ are diagonal, it would have been preferable in the more general case to have included time trend terms in Σ_v and Σ_ϵ. However, this would have greatly increased the number of parameters to be estimated. Results for both the unadjusted and adjusted data are reported when Σ_v and Σ_ϵ are not constrained to be diagonal.

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Comment Debra Glassman

I will begin my discussion with two specific comments on the data and estimation in this paper and then follow with some general remarks about testing rational expectations in the framework used by Peter Hartley.

The exchange rate is one of the few economic variables that we can observe changing from day to day, or hour to hour, or even minute to minute. This is a mixed blessing. On the one hand, a wealth of exchange rate data is available. On the other hand, it is hard to test models explaining or forecasting exchange rate behavior because there are features of foreign exchange market activity that also change with the date or time of day. Consequently, it makes a difference exactly when you observe the market.

Peter Hartley tested his model with data on exchange rates taken on the last Friday of each month. There are two potential problems associated with this choice. In the foreign exchange market, each day of the week has its own character. On Monday, for example, there can be substantial catching up with the news of the weekend. Thursday is often called "technical factors day," in part because it is heavily influenced by preweekend activity in the Eurocurrency markets (there being little

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opportunity for transatlantic trading on Friday because of the time difference). On Friday, North American traders position themselves for the weekend in their home markets (for example, to take advantage of the extra day of float on contracts whose value date is usually one or two days after the contract date). In addition, regularly scheduled announcements, like those of U.S. money supply figures (formerly on Thursday, now on Friday), can have a substantial impact on foreign exchange markets. Clearly it is hard to say which day of the week exhibits typical market behavior. The days mentioned, including Friday, are probably not good choices. When foreign exchange data are collected on a weekly basis, it is usual to choose Wednesday (or, less frequently, Tuesday) rates to avoid the regularly occurring disruptive influences.

The end of the month is also a time when special factors alter foreign exchange market activity. There can be a substantial number of trades made at the end of the month by corporations and institutions for tax or balance sheet purposes (for example, in response to the accounting rules that require reporting of unrealized foreign exchange gains or losses). This activity is even more pronounced at the end of a quarter or a year. Such trades are not motivated by the kind of economic fundamentals and expectations that we usually include in our models of exchange rate determination. Therefore we may misinterpret the patterns caused by the calendar, time zones, taxes, and so forth as reflecting nonrational expectations when in fact they are the result of rational behavior. Estimating Mr. Hartley's model with Friday, end-of-month data may produce misleading results for these reasons.

The second issue I would like to address relates to the results reported in table 5.2. Mr. Hartley jointly estimated equations (3′) and (5′) (with covariance structure [b]) both with and without a correction for heteroscedasticity. He reports the test statistic for the test of rationality (the cross-equation restrictions) only for the estimation *with* the heteroscedasticity correction. The correction is described in appendix B: the disturbance variance in each equation is modeled as being proportional to a time trend; the weights are the square roots of the fitted values from a regression of the absolute values of the residuals on time. No motivation for this time trend model is given, and, in my opinion, it is unjustified. Why should we expect the disturbance variance to monotonically increase or decrease over the 1972–79 sample period? Other studies show that the variance of exchange rate forecast errors fluctuated substantially and nonmonotonically during this period.

In view of the comments I made about the changes in market activity by day of the week and calendar date, Mr. Hartley is on the right track in considering the possibility of heteroscedasticity disturbances in his model. For instance, given the tax and balance sheet considerations that motivate additional trades at the end of a quarter, it does not seem unreasonable to propose that the variance of forecast errors is greater for end-of-quarter data points than for others. Even if this calendar-induced problem is avoided by choosing, say, mid-month, Wednesday observations, the potential for heteroscedastic disturbances remains. In examining the 1972–79 period, we can identify subperiods when the foreign exchange markets were very active (e.g., early and late in the sample) and subperiods when they were relatively quiet. There is evidence that the variance of exchange rate forecast errors differs significantly across these subperiods. Therefore, I suggest continuing to investigate heteroscedasticity corrections based on the possibilities just outlined.

I would like to conclude by discussing the issue of testing rational expectations in more general terms. Mr. Hartley provides us with an example of an innovative class of models that tests the rationality of expectations in the form of the cross-equation restrictions implied by the combination of a model of exchange rate determination that incorporates expectations of the determining variables with a model of the process generating those variables. This approach, however, inevitably involves testing the joint hypothesis of the rationality of expectations and the model specification. Rejection of such a joint hypothesis can be caused *either* by nonrational expectations *or* by an incorrect specification of the model. I wonder whether this is the direction that we should be going in investigating the rationality of expectations.

Being very pessimistic, I could suggest that we may never succeed in correctly specifying models for the exchange rate and the variables that determine it. In that case, the tests described here cannot answer our questions about rationality. Taking a more optimistic view, I suggest that, by using this approach, our testing of rational expectations will be handicapped until we get the model right.

Allow me to propose an alternative to explicitly modeling the processes on which exchange rate expectations are based. There is an increasing number of sources of direct or indirect exchange rate forecasts, including other active auction markets (e.g., futures markets, options markets, stock markets) and surveys of consumer, business, and trader opinion. Data from these sources (or an indicator measure extracted from a group of such sources) can provide exchange rate forecasts whose rationality we can examine by testing whether the forecasts differ from the realized rates only by a white noise error.

To the extent that the forecasts are based on variables that have high correlations with exchange rate movements rather than on a structural model, this approach produces a predictive rather than an explanatory model of exchange rate behavior. In other words, it would not increase our understanding of the economic processes underlying exchange rate determination. However, this alternative approach does separate the problem of determining whether exchange rate expectations are rationally formed from that of determining what moves exchange rates.

With these concluding remarks, I do not intend to leave the impression that the approach taken by Mr. Hartley and others is invalid; instead I wish to suggest that it simply may be slightly ahead of its time.

Comment Maurice Obstfeld

The avowed goal of Peter Hartley's paper is to test the hypothesis that exchange rate expectations are rational. But, as the author acknowledges, his tests are, inescapably, *joint* tests of rationality *and* the assumption that the nominal interest rate differential (the forward premium) differs from the expected rate of exchange rate depreciation over the holding period only by a constant risk premium. Given rational expectations, the assumption of a constant, time invariant risk premium can be strongly rejected for at least one of the currencies in Hartley's sample, as the Hansen-Hodrick paper in this volume shows. It is therefore difficult to see how we can learn much more about the rational expectations hypothesis from Hartley's elaborate tests.

If the constant risk premium assumption is regarded as an empirical approximation, however, Hartley's estimates can tell us much about the ability of the monetary approach to the exchange rate to explain developments over the recent decade of floating. A number of authors (for example, Dornbusch 1980 and Frankel 1983) have noted the apparently poor performance of the monetary approach exchange rate equation in recent years. I say "apparently" because the estimates these authors discuss are typically ordinary least squares (OLS) estimates of equations which include at least one endogenous variable (the nominal interest differential) as a regressor.[1] As Hodrick (1979) and others have emphasized, OLS estimates of the monetary equation's parameters are inconsistent, and thus provide little insight into the usefulness of the monetary approach to the exchange rate. Hartley's maximum likelihood, rational expectations procedure is essentially the one proposed by Hodrick (1979) as a means of circumventing the simultaneity problem that arises in OLS

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1. It is also unlikely that the real income variables in monetary exchange rate equations are predetermined with respect to the structural disturbance.

estimation. Here, I wish to discuss an alternative estimation strategy and its costs and benefits relative to Hartley's approach. I retain Hartley's notation but concentrate, for simplicity, on the problem of estimating a single, bilateral exchange rate equation.[2]

The Model

Hartley's basic monetary equation is

$$\hat{S}_t = \hat{M}_t - \hat{M}_t^* - \alpha_1 \hat{Y}_t + \alpha_2 \hat{Y}_t^* + \alpha_3 \Delta (i_t - i_t^*) + \xi_t. \tag{1}$$

For simplicity, I assume that $E(\xi_t | \phi_{t-1}) = 0$.[3] The customary monetary approach assumption that money demand income elasticities (α_1 and α_2) are equal across countries is relaxed in (1), but the interest rate semielasticities in the two countries (α_3) are constrained to equality. The latter assumption is crucial for Hartley's procedure, for it allows him to substitute

$$i_t - i_t^* = E(\dot{\hat{S}}_{t+1} | \phi_t) \tag{2}$$

into (1) to obtain a first-order difference equation in the expected depreciation rate.[4] The assumption could be relaxed, and consistent estimates could be obtained, if instrumental variables correlated with i_t and i_t^* but uncorrelated with ξ_t were available. Of course, one can still use this type of limited information approach if specification (1) is adopted.

Under the assumption of rational expectations, more efficient estimation techniques are available if the econometrician has information about the process that agents use to forecast future exchange rate movements. The costs of this efficiency gain include a greater computational burden and a greater risk of specification error. Let $x_t = [\hat{M}_t \; \hat{Y}_t \; \hat{M}_t^* \; \hat{Y}_t^*]'$, and substitute (2) into (1), iterating forward, to obtain

$$\hat{S}_t = \delta \left[\sum_{i=0}^{\infty} \lambda^i E(x_{t+i} | \phi_t) - \lambda \sum_{i=0}^{\infty} \lambda^i E(x_{t+i} | \phi_{t-1}) \right] + \xi_t, \tag{3}$$

where $\delta \equiv [1 \; -1 \; -\alpha_1 \; \alpha_2]$ and $\lambda \equiv \alpha_3/(1 + \alpha_3)$. (Equation [3] follows from Hartley's [A11].) Assume that conditional expectations coincide with best linear predictors and that the vector x_t follows the autoregressive process

$$\gamma(L)x_t = v_t, \tag{4}$$

2. Hartley estimates two exchange rate equations jointly in an attempt to increase the power of his tests.

3. This assumption is stronger than what is necessary to implement the instrumental variable approach described below. The assumption is not made by Hartley, and is made here only to cut down on notation. The possible autocorrelation of ξ_t is an important empirical issue, however, and is handled quite differently by Hartley's approach and by the instrumental variable approach. I return to this issue at the end of my discussion on the instrumental variables estimation below.

4. It is assumed for simplicity that a, the constant risk premium, is zero.

where L is the lag operator, $v_t = [v_{1t}\ v_{2t}\ v_{3t}\ v_{4t}]'$ is a vector of innovations, $\gamma(L) = I - \gamma_1 L - \gamma_2 L^2 - \ldots - \gamma_r L^r$, and each γ_j is a 4×4 matrix of coefficients. Using (4) and the prediction formulas of Hansen and Sargent (1980), one can write

$$(5) \quad \sum_{i=0}^{\infty} \lambda^i E(x_{t+i}|\phi_t) = \{[\gamma(L)^{-1} - L^{-1}\lambda\gamma(\lambda)^{-1}]/(1-\lambda L^{-1})\} v_t,$$

and[5]

$$(6) \quad \sum_{i=0}^{\infty} \lambda^i E(x_{t+i}|\phi_{t-1}) = \{L^{-1}[\gamma(L)^{-1} - \gamma(\lambda)^{-1}]/(1-\lambda L^{-1})\} v_{t-1}.$$

Equations (4), (5), and (6) imply that (3) can be written:

$$(7) \quad \hat{S}_t = \delta\{[(1-\lambda)I - \lambda(1-L^{-1})\gamma(\lambda)^{-1}\gamma(L)]/(1-\lambda L^{-1})\} x_t + \xi_t.$$

Equations (4) and (7) may be estimated jointly, subject to the cross-equation restrictions implied by the rational expectations hypothesis. Estimates of α_1, α_2, α_3, and the parameters in (4) are thereby obtained.

There exist several full information estimation strategies. Each relies on different assumptions about the stochastic process $[x_t'\ \hat{S}_t]'$. In evaluating the strategies, one encounters a trade-off between asymptotic efficiency on the one hand and robustness and tractability on the other. The trade-off becomes apparent in comparing Hartley's maximum likelihood (ML) approach to an alternative, instrumental variables (IV) approach.

Maximum Likelihood Estimation

For the purpose of estimation, Hartley writes the exchange rate equation in innovation form. Because

$$E(\hat{S}_t|\phi_{t-1}) = (1+\alpha_3)^{-1}\delta \sum_{i=0}^{\infty} \lambda^i E(x_{t+i}|\phi_{t-1})$$

[Hartley's equation (A10) when $E(\xi_t|\phi_{t-1}) = 0$], the unanticipated change in the logarithm of the exchange rate is, using (3),

$$(8) \quad \hat{S}_t - E(\hat{S}_t|\phi_{t-1}) = \delta\left\{\sum_{i=0}^{\infty} \lambda^i [E(x_{t+i}|\phi_t) - E(x_{t+i}|\phi_{t-1})]\right\} + \xi_t.$$

According to (8), the depreciation innovation is a function only of the

5. To derive equation (6), note that by (5),

$$\sum_{i=0}^{\infty} \lambda^i E(x_{t+i-1}|\phi_{t-1}) = \{[\gamma(L)^{-1} - L^{-1}\lambda\gamma(\lambda)^{-1}]/(1-\lambda L^{-1})\} v_{t-1}.$$

Therefore,

$$\sum_{i=0}^{\infty} \lambda^i E(x_{t+i}|\phi_{t-1}) = \lambda^{-1}\{[\gamma(L)^{-1} - L^{-1}\lambda\gamma(\lambda)^{-1}]/(1-\lambda L^{-1})\} v_{t-1} - \lambda^{-1}\gamma(L)^{-1} v_{t-1}$$
$$= \{L^{-1}[\gamma(L)^{-1} - \gamma(\lambda)^{-1}]/(1-\lambda L^{-1})\} v_{t-1}.$$

unpredictable shock ξ_t and news concerning current and future levels of the forcing variables. Hartley substitutes the lagged forward premium $i_{t-1} - i^*_{t-1}$ for $E(\hat{S}_t|\phi_{t-1})$ and uses (4), (5), and (6) to write (8) in the form

$$\hat{S}_t - i_{t-1} + i^*_{t-1} = \delta\gamma(\lambda)^{-1}v_t + \xi_t. \tag{9}$$

Equation (9) shows, in compact form, how agents process the new information v_t to revise previous forecasts of current and future levels of the forcing variables. The equation can also be expressed as

$$\hat{S}_t - i_{t-1} + i^*_{t-1} = \delta\gamma(\lambda)^{-1}(x_t - \sum_{i=1}^{r} \gamma_i x_{t-i}) + \xi_t. \tag{10}$$

If β denotes the row vector $\delta\gamma(\lambda)^{-1}$, (10) becomes

$$\hat{S}_t - i_{t-1} + i^*_{t-1} = \beta(x_t - \sum_{i=1}^{r} \gamma_i x_{t-i}) + \xi_t, \tag{11}$$

which is Hartley's equation (5′) when $E(\xi_t|\phi_{t-1}) = 0$. In section 5.2 of his paper, Hartley estimates (11) jointly with the equation

$$x_t = \sum_{i=1}^{r} \gamma^*_{t-i} + v_t \tag{12}$$

and tests the restrictions $\gamma_i = \gamma^*_i (i = 1, \ldots, r)$ implied by the rational expectations hypothesis. The coefficient vector β, which measures the effects of unexpected movements in forcing variables on the depreciation forecast error, is left unconstrained in these tests even though β can be written as $\delta\gamma(\lambda)^{-1}$ under rationality. One can conclude from tables 5.1 and 5.2 only that Hartley's test of rationality is quite weak, for it provides little evidence against a joint hypothesis that has been rejected in other studies.

More interesting, to my mind, than the test of rationality is the task undertaken in section 5.3. That task is the joint estimation of (4) and (11) subject to all cross-equation constraints, including the constraints $\beta = \delta\gamma(\lambda)^{-1}$. Hartley employs a maximum likelihood procedure for this purpose; but the ML algorithm fails to converge even after Hartley imposes a number of exclusion restrictions on (4) in an attempt to economize on free parameters.[6] This disappointing outcome makes evident the computational diffficulty of the ML approach to estimating rational expectations systems.

The consistency of ML estimates in the present framework requires some strong exogeneity assumptions. In particular, it must be assumed

6. Hartley assumes that domestic forcing variables do not Granger-cause foreign forcing variables, and vice versa. This set of assumptions is implausible and could have been tested. There is no statistical justification for the procedure Hartley uses to determine which lags of the forcing variables are to be included in agents' forecasting equations. It would have been preferable to economize on free parameters through formal tests of lag length. such as those described by Geweke and Meese (1981).

that the innovation in the exchange rate equation, ξ_t, is uncorrelated with v_t, the vector of innovations in the forcing variables. The assumption ensures that the regressors in (9) are uncorrelated with that equation's structural disturbance. However, if money growth responds systematically to contemporaneous exchange rate movements, perhaps because central banks "lean against the wind," ξ_t will in fact be correlated with the innovations in money growth. There is substantial evidence that monetary growth in several countries is influenced by exchange market developments, so Hartley's identifying assumption that $E(v_t \xi_t) = 0$ may be inappropriate.

One might question also the distributional assumption underlying the specification of the likelihood functions. The assumption of normally distributed disturbances is particularly hard to swallow in the interest rate model of section 5.4, where nominal interest rates are the dependent variables in the forecasting equations.

Instrumental Variables Estimation

The author's ML procedure delivers consistent and asymptotically efficient parameter estimates if (and only if) certain stringent assumptions are made. In view of the implausibility of those assumptions, however, an estimator consistent under a broader set of assumptions is desirable, even if that estimator is inefficient relative to ML in some cases. Instrumental variables estimators of the type described by Cumby, Huizinga, and Obstfeld (1983) (in a single-equation setting) and by Hansen and Sargent (1982) permit one to weaken Hartley's assumptions while easing the computational burden encountered in ML estimation. In particular, the IV approach is appropriate even when $E(v_t \xi_t)$ is unknown.[7] Glaessner (1981) has used the IV approach to estimate Mussa's (1982) rational expectations exchange rate model. Here, I describe briefly how IV techniques might be applied in the present context.[8]

Return to the problem of estimating (4) and (7) jointly, subject to cross-equation restrictions. The simplifying assumption that $E(\xi_t | \phi_{t-1}) = 0$ implies that $E(x_{t-j} \xi_t) = 0$ for $j > 0$. The basic idea of the IV approach is to use *lagged* values of the forcing variables as instruments for the regressors in equation (7).

Rewrite equation (9) as

$$\hat{S}_t = \delta(1-\lambda) \sum_{i=0}^{\infty} \lambda^i E(x_{t+i} | \phi_{t-1}) + \delta\gamma(\lambda)^{-1} v_t + \xi_t. \tag{13}$$

As a consequence of results in Hansen and Sargent (1980),

7. The IV approach can also be applied when the exchange rate Granger-causes the money and income variables. That extension is not pursued here.

8. For another example of the IV approach, and a more detailed exposition, see the Hansen-Hodrick paper in this volume.

(14) $$\sum_{i=0}^{\infty} \lambda^i E(x_{t+i}|\phi_{t-1}) = \lambda^{-1}\{\gamma(\lambda)^{-1} \left[I + \sum_{i=1}^{r-1}\left(\sum_{k=i+1}^{r} \lambda^{k-i}\gamma_k\right)L^i\right]x_{t-1} - x_{t-1}\}.$$

It follows that (13) has the form

(15) $$\hat{S}_t = \delta \sum_{i=1}^{r} \psi_i x_{t-i} + \nu_t,$$

where $\nu_t \equiv \delta\gamma(\lambda)^{-1}v_t + \xi_t$, and

(16) $$\psi_1 = (1-\lambda)\lambda^{-1}[\gamma(\lambda)^{-1} - I],$$

(17) $$\psi_m = (1-\lambda)\gamma(\lambda)^{-1}[\gamma_m + \lambda\gamma_{m+1} + \ldots + \lambda^{r-m}\gamma_r], \ (2 \le m \le r).$$

Suppose that observations on x_t are available for $t = -K+1, -K+2, \ldots, T$ (where $K > r$). Let x_T denote the $4 \times T$ matrix $[x_1 \ldots x_T]$ and $\hat{S}_T$ the row vector $[\hat{S}_1 \ldots \hat{S}_T]$. Let θ denote the vector of free parameters of the model ($\alpha_1, \alpha_2, \alpha_3, \gamma_1, \ldots, \gamma_r$). Define a $5 \times T$ matrix $V(\theta)$ by

(18) $$V(\theta) = \begin{bmatrix} x_T \\ \hat{S}_T \end{bmatrix} - \sum_{i=1}^{r} \begin{bmatrix} \gamma_i \\ \delta\psi_i \end{bmatrix} x_{T-i}.$$

If $\mu(\theta) \equiv \text{vec}[V(\theta)']$,[9] then the true value $\tilde{\theta}$ of θ satisfies

(19) $$E[(I \otimes x_{T-j})\mu(\tilde{\theta})] = 0,$$

for $j > 0$. The foregoing orthogonality conditions imply that the $4K \times T$ matrix,

$$Z = \begin{bmatrix} x_{T-1} \\ \vdots \\ x_{T-K} \end{bmatrix},$$

can be used as the instrument matrix in estimating (4) and (15) jointly.

To write the IV estimator, let $W = I \otimes Z$ (where I is 5×5), let $\Omega = \lim_{T\to\infty} (1/T)E[W\mu(\tilde{\theta})\mu(\tilde{\theta})'W']$, and form the criterion function

(20) $$J(\theta) = \mu(\theta)'W'\Omega^{-1}W\mu(\theta).$$

The instrumental variables estimator $\hat{\theta}_{IV}$ is obtained by minimizing $J(\theta)$. The random variable $\sqrt{T}(\hat{\theta}_{IV} - \hat{\theta})$ has been shown by Hansen (1982) (under mild regularity conditions) to be asymptotically normal with covariance matrix

(21) $$p\lim T^2\left(\frac{\partial\mu}{\partial\theta}\bigg|_{\tilde{\theta}}' W'\Omega^{-1}W\frac{\partial\mu}{\partial\theta}\bigg|_{\tilde{\theta}}\right),$$

9. The operator vec(M) changes a $n \times m$ matrix M into a $nm \times 1$ column vector whose first n entries consist of the first column of M, whose second n entries consist of the second column of M, and so on.

where $\partial\mu/\partial\theta$ is a matrix whose ith column is $\partial\mu/\partial\theta_i$. In practice, an initial consistent estimate of Ω must be used in forming the function $J(\theta)$.

As noted above, the IV estimation strategy has computational advantages relative to ML and is consistent under a less stringent set of distributional assumptions. But the IV approach has another attractive feature, namely, that it takes account of the possible *conditional heteroscedasticity* of equation disturbances.[10] Even if the model's disturbances are jointly covariance stationary (so that unconditional covariances are constant over time), covariances conditional on past information may be nonconstant. Conditional heteroscedasticity poses several problems for estimation. In particular, it causes the standard covariance matrix estimators to be inconsistent.

Is conditional heteroscedasticity a problem of practical importance? Cumby and Obstfeld (1983) find strong evidence of conditional heteroscedasticity in time series of forward rate forecast errors. Hartley also encounters heteroscedasticity and attempts to deal with the problem through Glejser's linear time trend correction. I suspect that the time dependence of the disturbances in Hartley's model is much more complex than the Glejser correction suggests. The formulas (20) and (21) used in IV estimation do not assume conditional homoscedasticity, and they require no explicit specification of the form heteroscedasticity might take. The IV approach outlined here is therefore robust with respect to another important type of specification error.

My discussion has until now been based on the simplifying assumption that $E(\xi_t|\phi_{t-1}) = 0$. Hartley, of course, does not assume that ξ_t is white noise; but by estimating his model in innovation form, he obtains an exchange rate equation (equation [5′]) whose disturbance term is serially uncorrelated *by definition*. What happens to the IV approach when ξ_t is not serially uncorrelated? Provided $E(\xi_t|\phi_{t-j}) = 0$ for some j not "too large" relative to the sample size, the IV estimator described above remains consistent if the instrumental variables in Z are replaced by their own values lagged $j - 1$ periods. The point is that *some* assumption about the autocorrelation properties of ξ_t must be made if IV estimation is to be feasible. While this type of assumption is often made with little justification, I find it more palatable than the strong exogeneity assumptions needed to justify maximum likelihood methods.

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