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11 Tax Asymmetries and Corporate Income Tax Reform

Saman Majd and Stewart C. Myers

11.1 Introduction

Under current (1985) law, corporate income is taxed only when positive. Although losses can be carried back to generate tax refunds up to the amount of taxes paid in the previous three years, losses must be carried forward once these tax refunds are used up. The present value per dollar carried forward is less than the statutory rate for two reasons: the firm may not earn enough to use the carryforwards before they expire, and carryforwards do not earn interest.

In previous work (Majd and Myers 1985), we showed that tax asymmetries can be modeled and valued as contingent claims, using option pricing theory combined with Monte Carlo simulation. Although the asymmetries' effects cannot be expressed in conventional summary measures such as effective tax rates, we did work out impacts on the after-tax net present values (NPVs) of incremental investment outlays. Tax asymmetries can dramatically reduce after-tax NPVs for high-risk investments, although the extent of reduction depends on the tax position of the investing firm. Tax asymmetries are irrelevant at the margin for a firm with sufficient other income that it always pays taxes on a marginal dollar of income or loss. Asymmetries may be the dominant tax effect for "stand-alone" projects, that is, for cases where the project and the firm are the same.

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This chapter develops previous work (Majd and Myers 1985). Comments on that paper from Alan Auerbach, Henry Jacoby, Michael Keen and Colin Mayer have significantly improved this chapter.

Here we focus on the *design* of the corporate income tax. We report the results of a series of numerical experiments comparing current (1985) tax law with a stylized tax "reform" proposal, with tax asymmetries of course emphasized. In doing so we have also improved the methods we used previously, notably by using more realistic and consistent numerical parameters.

We have also included an intelligent, although not fully optimal, project abandonment strategy in the simulations. The abandonment strategy links project life to *ex post* profitability. We constructed this link because fixing project life *ex ante* does not make sense under uncertainty, and because the extent to which tax-loss carryforwards can relieve tax asymmetries ought to depend on decisions about project life.

Section 11.2 briefly reviews prior work by others. Section 11.3 describes how option pricing concepts can be applied to value the government's tax claim on risky assets. Since no closed-form option-pricing formulas apply, values must be computed by numerical methods. Section 11.3, backed up by an appendix, also describes our calculations in more detail and presents after-tax values for a reasonably realistic "representative project" under various assumptions about project profitability, risk, and the tax position of the firm owning it. Section 11.4 investigates how the impact of tax asymmetries changes when a stylized, reformed tax system is substituted for the 1985 corporate tax law. Section 11.5 offers some concluding comments.

11.2 Prior Work

Formal analysis of the impacts of asymmetric taxation is just beginning to appear in the finance literature. For example, Cooper and Franks (1983) recognize that the firm's future tax rates are endogenous under asymmetric taxation with carryforward privileges. They use a linear programming framework to analyze the interaction between present and future investment and financing decisions induced by the tax system. They discuss some of the factors that limit financial transactions designed to offset tax losses and conclude that real investment by corporations can be distorted.

Ball and Bowers (1983), Galai (1983), Smith and Stultz (1983), Pitts and Franks (1984), and Green and Talmor (1985) all have noted the analogy between asymmetric taxes and call options. However, none of these papers has introduced realistic elements of the law, such as tax-loss carry provisions, nor have they obtained numerical estimates of the impact of tax asymmetries on asset values.

Building on earlier work in Auerbach (1983), Auerbach and Poterba (this volume) investigate the effects of tax asymmetries on corporate

investment incentives. They take the tax position of the firm as exogenous and use estimates of the tax losses carried forward by a sample of corporations to estimate the probability that the firm will pay taxes in the future. They compute effective marginal tax rates on new investment using these "transition probabilities."

The assumption that the future tax position of the firm is exogenous may be reasonable for incremental investment decisions that are small compared with the other assets of the firm. However, this approach cannot handle investment decisions when the project must "stand alone" or when it is a significant part of the assets of the firm. Moreover, as Auerbach and Poterba note, using past data on tax-loss carryforwards will not allow them to analyze proposed changes in tax law, since the change in tax regime will also change the transition probabilities they estimate.

By contrast, we take future pretax cash flows of the firm or project as completely exogenous and thus can allow the future tax position of the firm or project to be completely endogenous. This approach can shed light on effects of proposed changes corporate tax law.

11.3 Taxes as Contingent Claims

In the absence of tax loss carrybacks or carryforwards, the government's tax claim is equivalent to a portfolio of European call options, one on each year's operating cash flow. The heavy line in figure 11.1

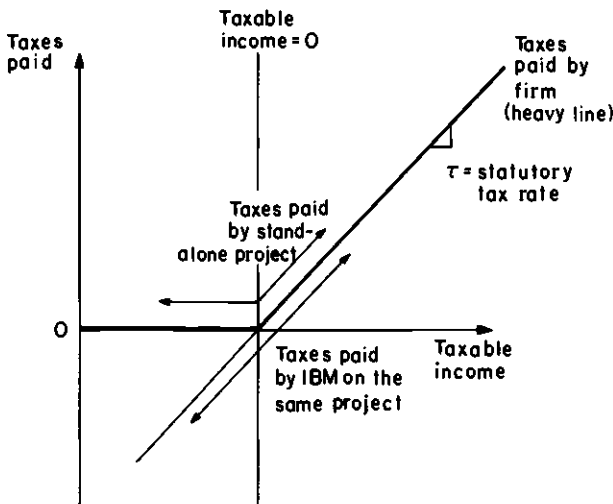


Fig. 11.1 Taxes paid as a function of taxable income.

shows taxes paid as a function of taxable income in a given year and has the same shape as a call option's payoff at exercise.

This option payoff function also describes the taxes paid on the income of a stand-alone project (i.e., taxes paid by a firm undertaking only one project). But in general the taxes paid on a project's income depend on the tax position of the firm owning it. Suppose the project is owned by IBM. It seems safe to say that IBM will not have tax-loss carryforwards at any time in the foreseeable future and will pay taxes at the margin at the full statutory rate. Thus any project losses can be offset against IBM's other income. The tax system is symmetrical for IBM when it considers an incremental capital investment project.

We can express these option analogies more formally. Consider a project that is the firm's only asset. Let the pretax operating cash flow and depreciation allowance at time t be y_t and d_t respectively. Ignore for now the investment tax credit (ITC) and assume that the project is all-equity financed. In the absence of tax-loss carryforwards or carrybacks, the project's after-tax cash flow at time t is:

$$c_t = y_t - \tau \max [y_t - d_t, 0].$$

The after-tax cash flow is the difference between the pretax operating cash flow and the government's claim on it. The government's claim is equivalent to τ European call options on the operating cash flow with exercise prices equal to the depreciation allowances.

Since the government taxes the firm's total income, the incremental effect of a new project on the value of the firm depends on the operating cash flows of the firm's existing assets and on their correlation with project cash flows. The after-tax cash flow for the firm and project is:

$$c_t = (y_t + z_t) - \tau \max [(y_t + z_t) - (d_t + d_{zt}), 0],$$

where z_t is the operating cash flow and d_{zt} the depreciation allowance for the firm without the new project. Because the government's claim is an option, the value of the tax claim on the sum of y_t and z_t is not the sum of the values of the tax claims on each taken separately.

Tax-loss carry privileges do not change the shape of the contingent tax payment drawn in figure 11.1. If the firm does not begin paying taxes until that year's taxable income exceeds cumulative tax losses carried forward from previous years, the vertical dividing line shifts to the right. Carrybacks shift the horizontal dividing line down from zero by the sum of taxes paid over the previous three years.

Again we can state this formally. Consider the case of unlimited carryforwards (but without any carrybacks). The tax loss carried forward to time t from the previous period is:

$$\theta_t = \max [\theta_{t-1} + d_{t-1} - y_{t-1}, 0].$$

The carryforward depends on the carryforward in the previous period, which in turn depends on the still-earlier carryforward, and so on. The carryforward at the beginning of the project (time zero) is given. The after-tax cash flow becomes:

$$c_t = y_t - \tau \max [y_t - d_t - \theta_t, 0].$$

Since θ_t depends on all realized incomes before time t , the payoff to the government (i.e., the tax paid) also depends on the realized incomes. It is straightforward to introduce the ITC and carrybacks and to limit the length of time allowed for carryforwards.

The carry privileges do not break the correspondence between the government's tax claims and a series of call options. The government holds a lottery across many possible options on y_t . Which option it ends up with depends on the firm's history of operating cash flow, y_0, y_1, \dots, y_{t-1} . This particular form of path dependency makes it infeasible to use closed-form option-valuation formulas.

Therefore, with carryforwards and carrybacks, developing comparative statics is a numerical rather than an analytical exercise. However, despite the complexity of the government's contingent claim, we have found no instances in which carry privileges reverse the normal properties of call options. For example, we have always found that the present value of the government's tax claim on a stand-alone project increases with project risk, defined as the variance rate at which cash flows evolve, and also with project life. The government is better off if nominal interest rates increase, even if tax depreciation is indexed to inflation. Of course all of these results can be shown analytically, using the Black and Scholes (1973) formula, if carry privileges are ignored and the government's tax claim is modeled as a series of non-interacting calls, one for each year's cash flow.

11.3.1 Valuing the Government's Tax Claim

This section describes the numerical procedure used to calculate the present value (to the corporation) of the taxes paid on a risky project. Our discussion will be restricted to finding the present value of the taxes paid on an operating cash flow in year t , y_t , in the presence of unlimited carryforwards. The present value of the taxes paid on the stream of operating cash flows is simply the sum of the values of the claims on each future year's operating income. Extension of the procedure to include carrybacks and to limit the carryforward period is straightforward.

We exploit a general property of options first explicitly noted by Cox and Ross (1976): if the payoff to the option can be replicated by a portfolio strategy using traded securities, the present value of the option is the expected payoff forecasted under a risk-neutral stochastic pro-

cess (conditional on the current values of the relevant state variables) and discounted at the risk-free rate. In other words, the option can be valued *as if* both it and the underlying asset are traded in a risk-neutral world.

The reason this risk-neutral valuation principle works is that options are not valued absolutely, but only relative to the underlying asset. For example, the Black and Scholes (1973) formula establishes only the *ratio* of call value to stock price. The stock price is marked down for risk because investors discount forecasted dividends at a risk-adjusted rate. The markdown of the stock to a current, certainty-equivalent value marks down the call value too, but not the ratio of call value to stock price.

The call is in fact riskier than the stock it is written on, and if at any instant investors demand an expected rate of return above the risk-free rate to hold the stock, they will demand a still higher return to hold the call. Suppose the required rate of return on the stock shifts up by enough to reduce the stock price by 1%. Then the call value will fall by more than 1%. However, the change in the stock's required return is not needed to calculate the fall in the call price. The change in stock price is a sufficient statistic.

Of course, the assets and options we are analyzing here are not explicitly traded. That may seem to violate a central assumption of the Black-Scholes model and its progeny. But we have actually taken only a small step away from the standard finance theory of capital investment under uncertainty. That theory assumes the firm maximizes market value, which in turn requires capital markets sufficiently complete that investors can find a security or portfolio of securities to "match" any investment project the firm may embark on. For every real asset, there must be a trading strategy using financial assets that generates a perfect substitute for the project in time pattern and risk characteristics of future cash returns. That assumption is routinely made for publicly traded firms. Incomplete markets are usually treated as a second- or third-order problem in light of the exceedingly rich menu of financial assets and trading strategies.

If investors can replicate real investment projects by trading in financial assets, they can also replicate options written on those projects by trading in the replicating assets and borrowing or lending. The heart of the classic Black and Scholes (1973) paper is the demonstration that a call's payoff can be exactly matched by a strategy of buying the underlying stock on margin, according to a hedging rule for the amount of stock held and the margin amount at each instant. Hedging rules can be written down for more complex or compound options. Thus, if markets are complete enough to support a market value standard for real assets, they are complete enough to support use of option-pricing theory.

There are other ways of justifying option-pricing techniques for options on nontraded assets. For example, the techniques also follow in the traditional capital asset pricing framework provided that asset returns are multivariate normal and there is a representative investor with constant absolute risk aversion (see, e.g., Brennan 1979 and Rubinstein 1976).

11.3.2 Solution Techniques

The path dependencies in our problem rule out closed-form solutions for the value of the government's tax claim. Moreover, these path dependencies would overwhelm the usual numerical option-valuation routines.

Fortunately, the rules for computing taxes are exogenous. Future tax payments are always unknown, but there are no *decisions* to be made about taxes. If the firm has the opportunity to use carrybacks or carryforwards, it does so at the first opportunity.¹ As far as taxes and carry privileges are concerned, the firm faces only an event tree, not a decision tree.

We can therefore employ a Monte Carlo simulation technique to approximate the distribution of the payoff conditional on the prior sequence of operating cash flow.² The rule determining the carryback or carryforward at any time (the path-dependent feature in this problem) is specified exogenously and depends only on past realizations of the operating cash flow. The Monte Carlo simulation technique exploits this feature of the problem by simulating the *sequence* of cash flows. Each time a value is generated for the cash flow y_t , the tax liability in t and any carryforward to period $t + 1$ are completely determined.

The simulation must also update the distributions of future cash flows every time a value is generated. Different assumptions about the stochastic process generating the time series of operating cash flow are possible. In our calculations, we break down operating cash flow as: operating cash flow = net revenues - fixed costs, $y_t = x_t - FC_t$, where "net revenues" means revenues less variable costs. We assume that FC_t is known with certainty, and that the stochastic processes generating each year's net revenue are perfectly correlated lognormal diffusions. That is, the forecast error in any one year's net revenue causes the same proportional change in the expectations of all future net revenues and the same proportional change in the present value of each year's future net revenues. If this assumption seems unduly restrictive, note that it is the usual justification for using a single risk-adjusted rate to discount a stream of cash flows. Thus it implicitly underlies standard practice (see Myers and Turnbull 1977 and Fama 1977).

The world of the simulation, however, is risk-neutral. Here is an example of how forecasting and discounting work in that world. Sup-

pose forecasted net revenues grow at a rate \hat{g} and are properly discounted at a rate $r + p$, where r is the risk-free interest rate and p is a risk premium:

$$V_0 = \int_0^H x_0 e^{-(r+p-\hat{g})t} dt = \frac{x_0}{r+p-\hat{g}} (1 - e^{-(r+p-\hat{g})H}).$$

In the risk-neutral world of the simulation, discounting is at r , but the growth rate is reduced to $g \equiv \hat{g} - p$. Note that this does not change the present value calculated just above: g could be interpreted as a certainty-equivalent growth rate. This rate g would be used in the simulation.

By generating a large number of simulated cash flows, an approximate distribution for the government's tax payment in each year is obtained. The expected value is computed and discounted at the riskless rate to obtain the present value of the payment. The present value of the government's claim on the project is the sum of the present value of the claims on individual cash flows.

11.3.3 Limitations of Monte Carlo Simulation for Tax Analysis

Our method is limited because it cannot capture possible links between the future tax position of the firm and its investment and financing decisions.

Our numerical procedure must take project and firm cash flows as exogenous. We do not consider whether a future tax loss on a project undertaken today will affect future investment decisions. We also rule out cases in which today's project is managed differently, depending on its (or the firm's) tax position. This is undoubtedly unrealistic. For example, an otherwise profitable firm might find it less painful to stick with a losing project in order to establish an immediate tax loss, for the same reason that investors in securities often find it worthwhile to realize capital losses before the end of the tax year.³

This is one of several ways a firm can react to tax asymmetries. Four additional examples are: (1) the firm may change its accounting policies to shift taxable income over time; (2) the firm may seek to acquire another firm that has taxable income; (3) the firm may choose to "sell" its tax shields to another firm by a leasing arrangement;⁴ (4) the firm may issue equity and buy bonds in order to generate taxable income.⁵

We admit that our results are uninteresting if firms can cash in tax losses at or near face value by these or other transactions. The transactions are not costless, however, and in many cases fall far short of exhausting the entire tax loss. Auerbach and Poterba (this volume) find that the percentages of a large sample of nonfinancial corporations with tax loss carryforwards ranged from about 7 (1981) to 14 (1984). In some

industries the percentages were substantially higher. For example, 30% of airline companies had loss carryforwards in 1981 and 40% had them in 1984. They also find that once a firm falls into loss carryforwards, there is less than a 10% chance of climbing out in the following year.⁶ If "selling tax losses" was feasible for these firms, the selling price was not attractive for 90% of them.

Our analysis, since it assumes operating cash flows are exogenous, gives a lower bound on after-tax project value and an upper bound on the impact of tax asymmetries.⁷ It shows the potential gain from changing financing or investment decisions to shift taxes over time or between firms. Since we do not analyze these tax-shifting decisions specifically, we cannot give point estimates of the effect of tax asymmetries under current law. We can make useful comparisons of corporate tax reform proposals, however. If the *potential* cost of tax asymmetries is reduced under a new tax law, that law is better than the old one, other things equal, because it reduces the real costs firms are willing to incur to sell carryforwards, and because tax asymmetries are less likely to distort real investment decisions.

11.3.4 Example of Numerical Results

Table 11.1 and figure 11.2 show results for the base-case project that is described in detail in the Appendix. The project offers exponentially decaying net revenues, moderate fixed costs, and under certainty would have an economic life of twelve years. Inflation is $i = .06$ and the nominal risk-free rate is $r = .08$. The standard deviations of annual forecast errors for project cash flow are $\sigma_x = .15, .10, \text{ and } .25$. In this section we discuss and plot NPVs only for the base case $\sigma_x = .15$.

Table 11.1 Project Net Present Value as a Percentage of Initial Investment

ZEROTAX	Reform (indexed)		
	SYMTAX	ASYMTAX	NOCARRY
-25.03	-17.46	-25.03	-25.03
-19.86	-14.00	-19.86	-19.86
-14.49	-10.45	-14.66	-14.71
-5.32	-5.02	-8.02	-8.79
7.97	3.21	1.05	-0.32
20.54	11.32	9.85	8.19
36.93	22.06	21.10	19.33
60.22	37.45	36.98	35.05

Note: Values are shown for symmetric tax (SYMTAX) and asymmetric tax with and without carry provisions (ASYMTAX and NOCARRY) for a range of pretax profitability (ZEROTAX). The parameters for the calculations correspond to the base case described in the Appendix.

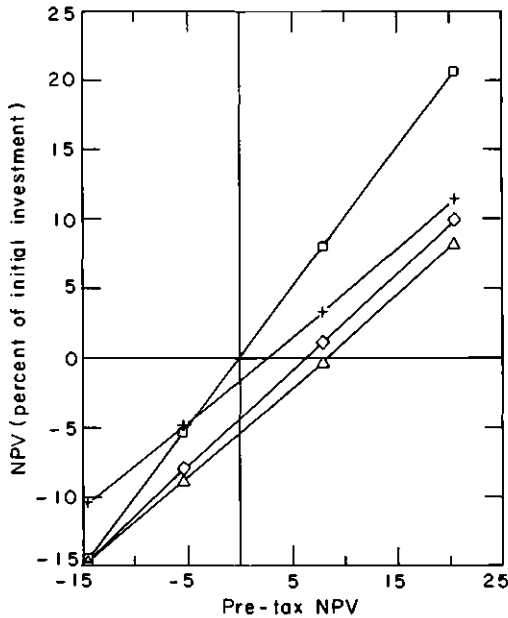


Fig. 11.2

Project NPV as a function of pretax NPV: reform with indexed depreciation. The parameters are for the base case described in the Appendix. Key: □ ZEROTAX + SYMTAX ◇ ASYMTAX △ NOCARRY.

Four sets of numbers are shown in the table and plotted in the figure. These correspond to various extreme assumptions about the firm undertaking the project.

Suppose the project is owned by a firm like Penn Central with such large tax loss carryforwards that we may assume a zero effective tax rate on new projects. We will use ZEROTAX as a label for this extreme case in which pretax and after-tax NPV are the same.

At the other extreme, we can imagine the standard project undertaken by a firm taxed symmetrically on marginal investment because it is sure to pay taxes at the margin at the full statutory rate. We label this case SYMTAX.

The NPVs in table 11.1 are calculated under a stylized tax-reform law, with indexed, exponential tax depreciation to scrap value at the end of the project's or asset's economic life. The tax rate is $\tau = .33$. There is no investment tax credit. The Appendix reviews tax and numerical assumptions in more detail.

The project's values under ZEROTAX and SYMTAX provide two extreme cases. A third extreme case occurs when the firm and the project are the same. Tax asymmetries have their maximum impact for

stand-alone projects. Of course, carrybacks and carryforwards mitigate the effects of the asymmetry. We assume three-year carrybacks and fifteen-year carryforwards (i.e., the current [1985] system). Results for stand-alone projects are labeled ASYMTAX. The ASYMTAX NPVs shown under "Reform" in table 11.1 are also plotted in figure 11.2.

The remaining numbers in table 11.1 and figure 11.2, labeled NO-CARRY, show the after-tax NPV of the stand-alone project with no carryforwards or carrybacks of losses allowed. Figure 11.2 shows that the NOCARRY NPVs are, as expected, somewhat worse than the ASYMTAX NPVs. Although carry privileges are valuable, they do not solve the tax asymmetry problem. We will not plot or comment on NOCARRY NPVs in the rest of the chapter.

Stand-alone project NPV (ASYMTAX) is always lower than either pretax NPV or NPV under a symmetric tax. A firm forced to take a negative NPV project would prefer a symmetric tax if it had the choice; second choice is no tax at all. A firm with a strongly positive NPV project would prefer no tax, but second choice is a symmetric tax. At some pretax NPV around zero, the firm is indifferent between no tax and symmetric tax. But the asymmetric tax is always in third place from the firm's point of view. It is furthest behind when pretax NPV is about zero.

In other words, if the firm must have unused tax loss carryforwards, it is better to have a lot of them, so that incremental investments effectively escape tax. The present value of the government's tax claim on a firm or stand-alone project is greatest when it is not known whether the firm or project will have to pay taxes.

Most of the following discussion focuses on experiments where NO-TAX or SYMTAX NPVs are not too far away from zero. Tax law is most likely to affect decisions about breakeven or near-breakeven investments. Investments with high positive or negative NPVs will be taken or rejected regardless of tax.

11.4 Tax Asymmetries and Tax Reform

So far we have confirmed the results of our prior work, that tax asymmetries can have a significant impact on the after-tax value of incremental investment. Now we arrive at the main goal of this chapter, which is to compare the potential impacts of tax asymmetries under current (1985) tax law with their impacts under a reformed law with lower marginal rates, exponential depreciation approximating economic depreciation, and no investment tax credit.

Compare the after-tax NPVs shown under "Reform" in table 11.2 with the after-tax NPVs under current law, shown on the right of the table under "ACRS." The comparisons are easier to grasp in figures

Table 11.2 Project Net Present Value as a Percentage of Initial Investment

ZEROTAX	Reform (indexed)			ACRS		
	SYMTAX	ASYMTAX	DIFF	SYMTAX	ASYMTAX	DIFF
<i>Base Case ($\sigma = .15$)</i>						
-25.03	-17.46	-25.03	7.57	-9.60	-25.03	15.43
-19.86	-14.00	-19.86	5.86	-6.81	-19.86	13.05
-14.49	-10.45	-14.66	4.21	-4.05	-14.74	10.69
-5.32	-5.02	-8.02	3.00	-0.89	-9.40	8.51
7.97	3.21	1.05	2.16	4.76	-2.17	6.93
20.54	11.32	9.85	1.47	11.01	5.16	5.85
36.93	22.06	21.10	0.96	19.48	14.78	4.70
60.22	37.45	36.98	0.47	31.83	28.46	3.37
<i>Low Risk ($\sigma = .10$)</i>						
-25.03	-17.46	-25.03	7.57	-9.60	-25.03	15.43
-19.86	-14.00	-19.86	5.86	-6.81	-19.86	13.05
-14.67	-10.53	-14.68	4.15	-4.01	-14.68	10.67
-8.94	-6.93	-9.46	2.53	-1.57	-9.81	8.24
-0.76	-2.31	-3.95	1.64	0.85	-5.82	6.67
9.63	4.07	3.06	1.01	5.27	-0.35	5.62
24.37	13.58	13.09	0.49	12.66	8.12	4.54
46.59	28.22	28.05	0.17	24.39	21.14	3.25
<i>High Risk ($\sigma = .25$)</i>						
-25.03	-17.46	-25.03	7.57	-9.60	-25.03	15.43
-19.59	-13.87	-19.82	5.95	-6.79	-19.93	13.14
-12.04	-9.25	-14.39	5.14	-3.93	-15.44	11.51
9.26	4.27	0.05	4.22	5.78	-3.74	9.52
30.81	18.35	15.01	3.34	16.73	8.76	7.97
46.17	28.48	25.82	2.66	24.74	18.02	6.72
66.76	42.14	40.05	2.09	35.67	30.11	5.56
93.14	59.67	58.19	1.48	49.74	45.51	4.23

Note: Values are shown for symmetric tax (SYMTAX), asymmetric tax with carry provisions (ASYMTAX), and their difference (DIFF) for a range of pretax profitability (ZEROTAX). Each panel corresponds to a different level of project risk (σ) and compares a stylized tax reform (with indexed depreciation) with current law (ACRS).

11.3, 11.4, and 11.5, which plot after-tax NPVs for SYMTAX and ASYMTAX against pretax project profitability measured by ZEROTAX NPV. Each figure shows NPVs for a different standard deviation of project cash flows.

For a firm facing symmetric taxation on marginal investments, reform reduces after-tax NPV when pretax NPV is negative or moderately positive. This reflects the loss of the investment tax credit and accelerated depreciation. Such a firm is better off when it finds projects with strong positive NPVs, however, because reform lowers the marginal tax rate.

Table 11.3 Project Net Present Value as a Percentage of Initial Investment

ZEROTAX	Reform (not indexed)			ACRS		
	SYMTAX	ASYMTAX	DIFF	SYMTAX	ASYMTAX	DIFF
<i>Base Case (i = .06)</i>						
-25.03	-19.32	-25.03	5.71	-9.60	-25.03	15.43
-19.86	-15.86	-19.86	4.00	-6.81	-19.86	13.05
-14.49	-12.42	-14.72	2.30	-4.05	-14.74	10.69
-5.32	-8.47	-9.30	0.83	-0.89	-9.40	8.51
7.97	-1.60	-1.93	0.33	4.76	-2.17	6.93
20.54	5.92	5.76	0.16	11.01	5.16	5.85
36.93	16.23	16.15	0.08	19.48	14.78	4.70
60.22	31.25	31.23	0.02	31.83	28.46	3.37
<i>No Inflation (i = 0)</i>						
-25.02	-17.46	-25.02	7.56	-6.79	-25.02	18.23
-19.86	-14.00	-19.86	5.86	-4.00	-19.86	15.86
-14.49	-10.46	-14.67	4.21	-1.14	-14.68	13.54
-5.21	-5.00	-8.00	3.00	3.32	-8.16	11.48
8.72	3.68	1.57	2.11	10.42	0.98	9.44
20.54	11.31	9.85	1.46	16.65	8.97	7.68
36.49	21.76	20.87	0.89	25.17	19.37	5.80
59.88	37.23	36.83	0.40	37.74	34.17	3.57
<i>High Inflation (i = .12)</i>						
-25.05	-21.09	-25.05	3.96	-12.26	-25.05	12.79
-19.86	-17.62	-19.86	2.24	-9.46	-19.86	10.40
-14.54	-14.27	-14.92	0.65	-6.79	-14.85	8.06
-5.25	-10.96	-11.28	0.32	-4.34	-10.57	6.23
9.29	-4.17	-4.40	0.23	1.10	-4.39	5.49
20.90	2.39	2.26	0.13	6.48	1.51	4.97
36.57	11.96	11.91	0.05	14.41	10.16	4.25
60.96	27.53	27.52	0.01	27.18	23.97	3.21

Note: Values are shown for symmetric tax (SYMTAX), asymmetric tax with carry provisions (ASYMTAX), and their difference (DIFF) for a range of pretax profitability (ZEROTAX). Each panel corresponds to a different level of inflation (i) and compares a stylized tax reform (without indexed depreciation) with current law (ACRS).

Reform decreases the present value of taxes on stand-alone projects, except at large negative pretax NPVs. In those cases the project is abandoned almost immediately, before any taxes are paid under either current or reformed tax rules. Notice that the ASYMTAX NPVs equal the NOTAX NPVs in the top row of the base-case and low-risk blocks of table 11.2.

SYMTAX and ASYMTAX NPVs are equal at very high pre-tax NPVs, not shown in table 11.2 and off-scale in figures 11.3–11.5. When the stand-alone project is so profitable that it always pays taxes, tax asymmetries are irrelevant.

Table 11.4 Project Net Present Value as a Percentage of Initial Investment

ZEROTAX	Reform (with interest)			Reform (without interest)		
	SYMTAX	ASYMTAX	DIFF	SYMTAX	ASYMTAX	DIFF
<i>Base Case (i = .06)</i>						
-25.03	-17.46	-25.03	7.57	-17.46	-25.03	7.57
-19.86	-14.00	-19.86	5.86	-14.00	-19.86	5.86
-14.49	-10.45	-14.65	4.20	-10.45	-14.66	4.21
-5.32	-5.02	-7.91	2.89	-5.02	-8.02	3.00
7.97	3.21	1.19	2.02	3.21	1.05	2.16
20.54	11.32	9.98	1.34	11.32	9.85	1.47
36.93	22.06	21.19	0.87	22.06	21.10	0.96
60.22	37.45	37.05	0.40	37.45	36.98	0.47
<i>No Inflation (i = 0)</i>						
-25.02	-17.46	-25.02	7.56	-17.46	-25.02	7.56
-19.86	-14.00	-19.86	5.86	-14.00	-19.86	5.86
-14.49	-10.46	-14.67	4.21	-10.46	-14.67	4.21
-5.21	-5.00	-7.97	2.97	-5.00	-8.00	3.00
8.72	3.68	1.62	2.06	3.68	1.57	2.11
20.54	11.31	9.89	1.42	11.31	9.85	1.46
36.49	21.76	20.90	0.86	21.76	20.87	0.89
59.88	37.23	36.85	0.38	37.23	36.83	0.40
<i>High Inflation (i = .12)</i>						
-25.05	-17.48	-25.05	7.57	-17.48	-25.05	7.57
-19.86	-14.00	-19.86	5.86	-14.00	-19.86	5.86
-14.54	-10.48	-14.68	4.20	-10.48	-14.68	4.20
-5.25	-4.92	-7.70	2.78	-4.92	-7.86	2.94
9.29	4.12	-2.23	1.89	4.12	2.01	2.11
20.90	11.60	10.26	1.34	11.60	10.07	1.53
36.57	21.84	21.04	0.80	21.84	20.90	0.94
60.96	37.96	37.57	0.39	37.96	37.47	0.49

Note: Values are shown for symmetric tax (SYMTAX), asymmetric tax with carry provisions (ASYMTAX), and their difference (DIFF) for a range of pretax profitability (ZEROTAX). Each panel corresponds to a different level of inflation (i) and compares the stylized tax reform (with indexed depreciation) with and without interest in carryforwards.

But in the interesting cases where pretax NPV is moderately positive or negative, stand-alone projects are worth more under reform despite the loss of the investment tax credit and accelerated depreciation. They are worth more *relative to* projects taxed symmetrically or not taxed at all.

These conclusions hold over a range of cash flow standard deviations, as figures 11.3–11.5 illustrate. We have also checked to confirm that they hold for projects with faster and slower tax and economic depreciation, and that they hold when the option to shorten or extend project life is “turned off” and project life is fixed at what it would be under certainty.

Table 11.5

Effects of Uncertainty and Abandonment Strategy on Pretax and After-Tax NPVs

σ_x	NOTAX ^b	Reform		ACRS	
		SYMTAX ^b	ASYMTAX	SYMTAX ^b	ASYMTAX
<i>A. NPVs with Project Life Fixed at Twelve Years</i>					
.0 ^a	-0.6	-3.7	-3.7	-1.2	-7.5
.15	0.3	-3.1	-8.6	-0.7	-17.1
.25	1.2	-2.4	-12.5	-0.2	-17.1
<i>B. NPVs with Option to Abandon before Year 12 or to Extend Life to Year 100</i>					
.0 ^a	-0.6	-3.7	-3.7	-1.2	-7.5
.15	26.8	15.4	14.1	14.2	8.8
.25	54.8	34.2	31.7	29.3	23.0

Note: NPV as percentage of project investment. Initial profitability and other project assumptions are given in table 11.A.1.

^aWe actually used $\sigma_x = .001$ in the Monte Carlo simulation. Note that the option to abandon early or extend project life become valueless as $\sigma_x \rightarrow 0$. Thus the figures in the first row of each panel are the same.

^bThe results in these columns should in principle be identical in Panel A. Differences reflect numerical errors introduced by the Monte Carlo simulation.

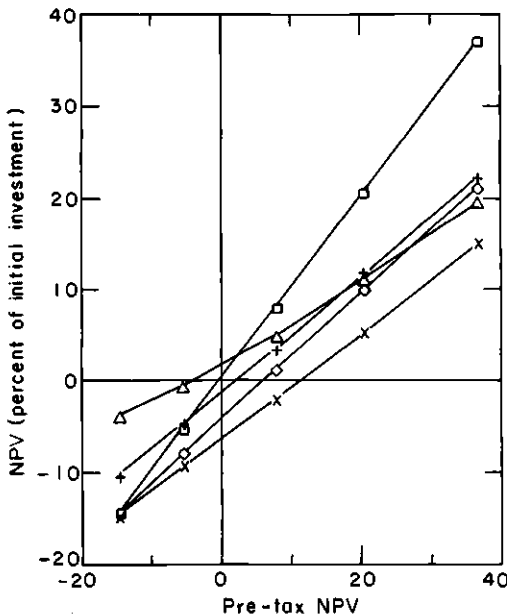


Fig. 11.3

Project NPV as a function of pretax NPV: reform with indexed depreciation versus ACRS. The parameters are for the base case described in the Appendix. Key: \square ZEROTAX + SYMTAX (reform) \diamond ASYMTAX (reform) Δ SYMTAX (ACRS) \times ASYMTAX (ACRS).

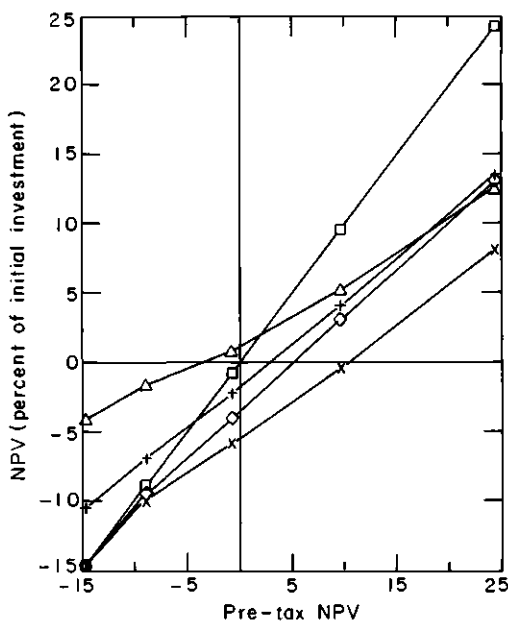


Fig. 11.4 Project NPV as a function of pretax NPV: reform with indexed depreciation versus ACRS, when cash flow volatility is 10% (other parameters are for the base case described in the Appendix). Key: □ ZEROTAX + SYMTAX (reform) ◇ ASYMTAX (reform) △ SYMTAX (ACRS) × ASYMTAX (ACRS).

11.4.1 Indexing Depreciation

Table 11.3 and figures 11.6, 11.7, and 11.8 show NPVs when reform does not include indexed tax depreciation. (The definitions of indexed and nonindexed depreciation are reviewed in the Appendix.) The format is identical to that of table 11.2 except that cash flow standard deviation is held at $\sigma_x = .15$ and the inflation rate is varied from .06 (the base case) to .12 and zero. Note that the "Reform" NPVs calculated under zero inflation match the base case NPVs in table 11.2, except for minor numerical errors introduced by the Monte Carlo simulation.

Without indexing higher inflation naturally means lower after-tax NPVs. Otherwise the patterns we noted in table 11.2 remain in table 11.3. Reform hurts symmetrically taxed projects when pretax NPV is below or around zero, but helps when pretax NPV is strongly positive. Stand-alone projects are uniformly helped, both absolutely and relative to symmetrically taxed projects.

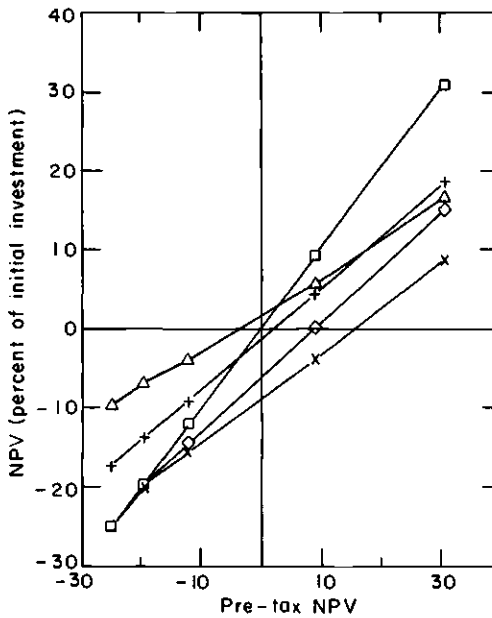


Fig. 11.5

Project NPV as a function of pretax NPV: reform with indexed depreciation versus ACRS, when cash flow volatility is 25% (other parameters are for the base case described in the Appendix). Key: □ ZEROTAX + SYMTAX (reform) ◇ ASYMTAX (reform) △ SYMTAX (ACRS) × ASYMTAX (ACRS).

11.4.2 Paying Interest on Tax-Loss Carryforwards

Paying interest on tax-loss carryforwards is a natural remedy for tax asymmetries. However, it is not necessarily a complete remedy. Paying interest on carryforwards works if the firm is sure to pay taxes eventually. If not, the government's option retains value, just as a call option does if the exercise price increases at the interest rate.

Table 11.4 and figure 11.9 show the extent to which the remedy works. Even with interest on carryforwards, there is a gap between ASYMTAX and SYMTAX NPVs. Consider the base-case project at a profitability level yielding a pretax NPV of 7.97 and an after-tax NPV under symmetric taxation of 3.21. (See the top block of numbers in table 11.4.) Allowing interest on carryforwards increases ASYMTAX NPV from 1.05 to 1.19. This represents an improvement but does not eliminate the effects of the asymmetry. Allowing interest on carryforwards makes less difference (compared with reform without interest) when pretax value is very low (ASYMTAX approaches ZEROTAX) or very high (ASYMTAX approaches SYMTAX).

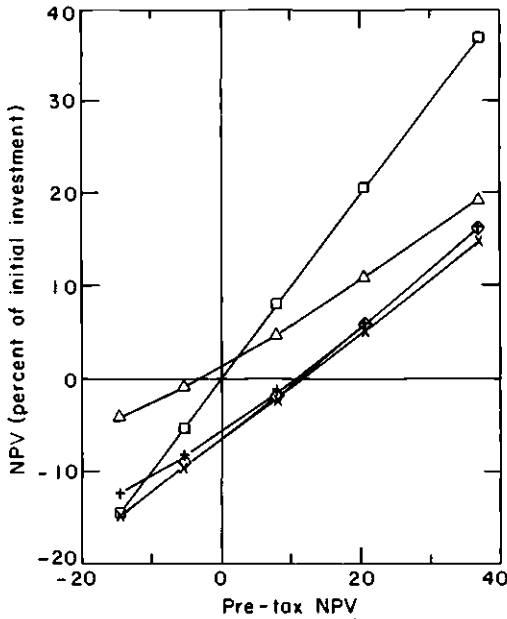


Fig. 11.6

Project NPV as a function of pretax NPV: reform without indexed depreciation versus ACRS. The parameters are for the base case described in the Appendix. Key: □ ZERO-TAX + SYMTAX (reform) ◇ ASYMTAX (reform) △ SYMTAX (ACRS) × ASYMTAX (ACRS).

The other two panels in table 11.4 show the effect of interest on carryforwards when inflation is zero or 12%: allowing interest makes a bigger difference to ASYMTAX NPV when inflation is high, but the effect of the asymmetries remains.

Allowing interest on loss carryforwards completely removes the burden of tax asymmetries only if the stand-alone firm or project is certain to regain tax-paying status sooner or later. But on this point full certainty requires immortality for the firm or project and no limit on the carryforward period. In our simulations the investment project may live to year 100—probably a good approximation of immortality—but it may be abandoned much earlier if its ex post performance is poor. The gap between SYMTAX and ASYMTAX NPVs with interest on carryforwards shows that carryforwards have no value to dead projects. Now if tax law allowed the firm to add a life insurance premium as well as interest to unused loss carryforwards, the potential extra burden of tax asymmetries would be essentially eliminated. The life insurance premium would equal the probability that the firm generating the carryforwards would pass away in the next tax year.

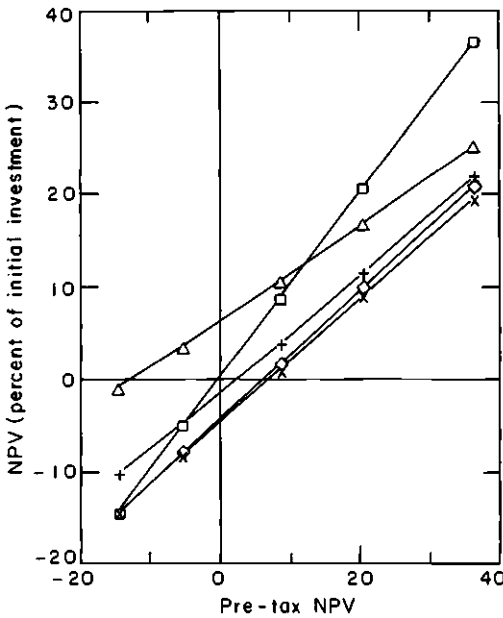


Fig. 11.7 Project NPV as a function of pretax NPV: reform without indexed depreciation versus ACRS, when inflation is 0% (other parameters are for the base case described in the Appendix). Key: □ ZEROTAX + SYMTAX (reform) ◇ ASYMTAX (reform) △ SYMTAX (ACRS) × ASYMTAX (ACRS).

11.4.3 Uncertainty and Abandonment

We conclude with a brief comment on the role of uncertainty and abandonment strategy in our simulation results.

Figures 11.3–11.5 confirm that the present value of the government's tax claim on a firm or stand-alone project increases with the risk (standard deviation) of the firm's or project's cash flows. But not all of the differences between SYMTAX and ASYMTAX NPVs can be attributed to risk. Some would persist under certainty, simply because the stand-alone project may not be profitable enough, at least in its early years, to use all the tax shields allotted to it.

Panel A of table 11.5 gives NPVs when risk disappears. First read across the row labeled $\sigma_x = 0$. The ZEROTAX NPV is effectively zero. Under Reform tax assumptions, NPV is about -4% of project investment for both a tax-paying firm (SYMTAX) and the stand-alone project. Now read down the columns under "Reform": as risk increases, there is no change in SYMTAX NPVs (the small differences reported are due to numerical errors in the Monte Carlo simulation),

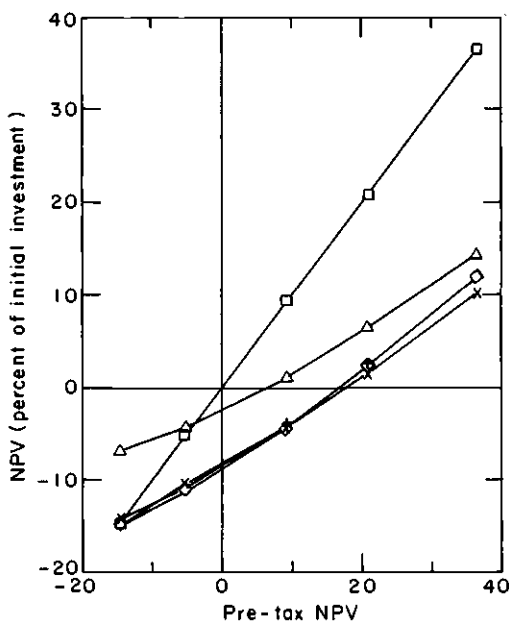


Fig. 11.8

Project NPV as a function of pretax NPV: reform without indexed depreciation versus ACRS, when inflation is 12% (Other parameters are for the base case described in the Appendix). Key: □ ZEROTAX + SYMTAX (reform)
 ◇ ASYMTAX (reform) △ SYMTAX (ACS)
 × ASYMTAX (ACRS).

but a steady decrease in after-tax NPVs for the stand-alone project. At least for projects like those examined in this chapter—projects with smooth downward trends in operating income—tax asymmetries have virtually no effect in the absence of risk. They increase the tax burden on incremental investment in risky assets but not on investment in safe assets.

The results grouped under “ACRS” in panel A tell a different story. The present value of the government’s tax claim on the stand-alone project is about -7% of project investment (-7.5 vs. -0.6). The present value of taxes increases further as risk increases, but clearly the largest part of the damage done to the ASYMTAX NPVs can be traced to *deferral* of the stand-alone project’s investment tax credit and ACRS write-offs.

The NPVs in panel A of table 11.5 were calculated after “turning off” the option to abandon the project early or to extend its life beyond its optimal life under certainty. We wanted to show how asymmetric taxation and risk interact with project life fixed.

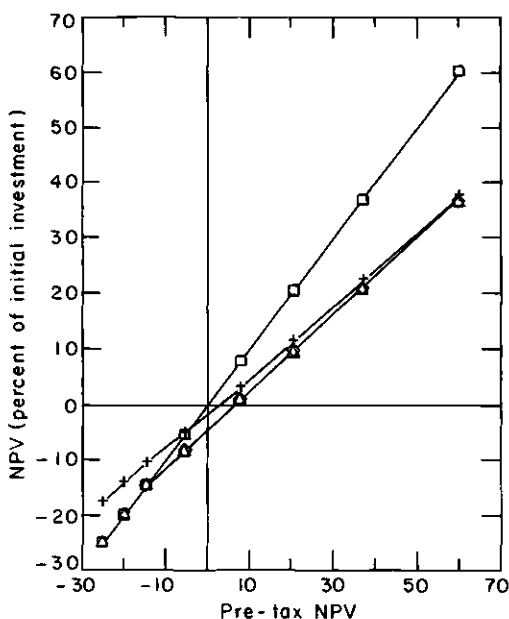


Fig. 11.9

Project NPV as a function of pretax NPV: reform (with indexed depreciation) with and without interest on carryforwards. The parameters are for the base case described in the Appendix. Key: □ ZEROTAX + SYMTAX (with interest) ◇ ASYMTAX (with interest) △ ASYMTAX (without interest).

Panel B shows what happens when the option is turned on again. The option sharply increases pretax NPVs as risk increases, because the firm can bail out of the project, recovering part of the initial capital outlay, if ex post performance is poor but continue almost indefinitely if performance is sufficiently good. The option likewise increases after-tax NPVs, even for the stand-alone project. In other words, additional risk adds more value to the option to abandon early or extend than it adds to the government's call options on project cash flows. The government's options still have significant value, however. For example, when $\sigma_x = .25$, they are worth 3% of project investment under tax reform (34.2 vs. 31.7) and 6.3% of investment under current law (29.3 vs. 23.0). Note that the latter difference is less than the comparable difference for $\sigma_x = 0$. Thus, under current law, the option to abandon or extend may interact with the government's call options to reduce the value of those options as risk increases. That does not, however, affect the main results of this chapter, which rest on comparisons of after-tax NPVs at given risk levels under current tax law and stylized

tax reform. The potential costs and distortions introduced by tax asymmetries depend on the differences between SYMTAX and ASYMTAX NPVs at given levels of investment risk. Under current tax law, these differences are dramatic regardless of risk and regardless of whether the option to abandon or extend project life is “turned on.” Under our stylized tax reform the differences are much smaller.

11.4.4 Investment in Intangible Assets

Our tax reform is too pure for real life. Many of the impurities of actual tax reform make the potential costs of tax reform worse. For example, the results presented so far overstate the difference reform might make because most reform proposals continue to allow corporations to expense investment in intangibles. Research and development (R&D) outlays are expensed, for example, as are most startup costs and advertising, which is sometimes intended to generate payoffs in the medium or long term.

Under current law, the present values of tax shields generated by investment in tangible and intangible assets are roughly the same. That is, the present value of ACRS write-offs plus the investment tax credit is roughly equal to the cost of the asset, and therefore roughly equivalent to writing off the asset when it is bought. High-tech companies that invest largely in R&D or other intangibles are not materially disadvantaged compared with smokestack companies that invest in tangible capital assets, providing both types of companies pay taxes year in and year out at the same marginal rate.

For stand-alone projects, however, shifting investment from tangible to intangible assets makes the burden of tax asymmetries worse. Moreover, that burden is carried over to tax reform proposals that allow intangible investments to be expensed.

In other words, tax reform that sets economically sensible tax depreciation schedules only for tangible assets will tend to slant investment toward R&D and other intangibles: high-tech companies will gain *relative to* smokestack companies as long as both types pay taxes regularly. However, the potential burden of tax asymmetries on high-tech projects or companies will remain substantial.

11.5 Summary

In this chapter we combine option-pricing theory with Monte Carlo simulation to derive numerical estimates of the potential effects of tax asymmetries. We confirm earlier results showing that asymmetries *can* have substantial effects on the after-tax NPVs of incremental investment projects. We go on to a more refined and detailed investigation, comparing current (1985) law with a stylized reform that eliminates the

investment tax credit and sets tax depreciation approximately equal to economic depreciation. The reformed marginal corporate tax rate is 33%.

This reform would increase the present value of taxes on incremental investments by firms that always pay taxes but *decrease* the present value of taxes on stand-alone projects. Reform dramatically reduces the potential burden of tax asymmetries.

The magnitude of these shifts in tax burden of course depends on numerical assumptions. However, the direction of the effects holds up over all our experiments. The experiments varied risk, the rate of economic depreciation, and the ratio of fixed to variable cost. We also generated results under reform with and without inflation indexing of tax depreciation, and with and without interest on tax-loss carryforwards. Although these measures help, they do not completely eliminate the effects of tax asymmetries.

There is more work to be done. For example, we would like to model uncertain inflation and develop a better understanding of its effects on value under asymmetric taxation. We expect our general conclusions to continue to hold, but this will enable us to make better recommendations regarding inflation indexing and its likely impact on asset values.

Although our methodology allows us to analyze a wide variety of tax codes in considerable detail, it requires that the pretax cash flows of the firm or project not be affected by the tax rules. There are interesting issues regarding the effect of the tax system on the distribution of future cash flows that we have not addressed.

Appendix

Numerical Assumptions and Design of Simulations

Virtually any a priori belief about the magnitude of effects of tax asymmetries might be confirmed by a cleverly constructed numerical example. Many of these examples would have at least one practical analogue somewhere in the corporate sector.

Because our examples are intended to bring out the general effects of tax reform on tax asymmetries, a "representative" investment project is called for. Therefore our numerical examples start with a base-case investment project reflecting the implicit assumptions of the stylized tax reform proposal we concentrate on. We want to avoid results that might be construed as reflecting our choice of an atypical base-case project. Our base project is therefore regular and unexceptional.

Project Life under Certainty

If tax and economic depreciation are exponentially declining, we want project cash flows to decline in the same way. Ignore taxes, and consider a project requiring an investment outlay of 1, with expected nominal cash inflows of $y_t = x_t - FC_t$. FC_t stands for "fixed cost," but for the moment we set $FC_t = 0$.

If both the asset value and "variable" cash flow x_t decay at the expected nominal rate $\hat{\delta}$, project NPV for economic life H^8 is:

$$NPV = -1 + \int_0^H x_0 e^{-(r+p+\hat{\delta})t} dt + e^{-(r+p+\hat{\delta})H},$$

where r is the nominal risk-free interest rate, p a risk premium, $r + p$ is the expected opportunity cost of capital, and $e^{-(r+p+\hat{\delta})H}$ is the present value of the proceeds from sale of assets at $t = H$.

Since our simulations take place in a hypothetical risk-neutral world, we may as well translate immediately to certainty equivalent flows. The decay rate of the certainty equivalents of x_t is $\delta \equiv \hat{\delta} + p$. Discounting at the risk-free rate r :

$$(A1) \quad NPV = -1 + \int_0^H x_0 e^{-(r+\delta)t} dt + e^{-(r+\delta)H},$$

Since this transformation does not affect NPV or decisions about project life H , we assume certainty in the following discussion.

The project summarized by equation (A1) is nicely consistent, because the value of the stream of cash flows x_t does decline at the assumed rate δ . However, project or asset life has only a bit part in the story. Remember that we assume certainty. If $NPV > 0$, the project would never be voluntarily shut down; H could only be a date of exogenous physical collapse. If $NPV = 0$, the natural base-case assumption in a competitive economy, then $dNPV/dH = 0$ for any H . In other words, the firm would be just as happy to shut down at $H = 1$ as at $H = 100$.

We can make project life a more interesting variable by introducing fixed costs, FC_t . Varying FC_t will allow us to examine how the tax system interacts with operating leverage. We also give variable cash flow x_t a possibly different decay rate λ .

$$(A2) \quad NPV = -1 + e^{-(r+\delta)H} + \int_0^H x_0 e^{-(r+\lambda)t} dt$$

$$\begin{aligned}
& - \int_0^H FC_0 e^{-(r-\delta)t} dt, \\
= & -1 + e^{-(r+\delta)H} + \frac{x_0}{r+\lambda} (1 - e^{-(r+\lambda)H}) \\
& - \frac{FC_0}{r-i} (1 - e^{-(r-i)H}).
\end{aligned}$$

Note that fixed costs are assumed to increase at the inflation rate i .

Assuming initial NPV > 0 at some life H , project life is determined by:⁹

$$dNPV/dH = -(r + \delta) e^{-(r+\delta)H} + x_0 e^{-(r+\lambda)H} - FC_0 e^{-(r-i)H} = 0.$$

If one multiplies through by e^{rH} and translates to future values x_H and FC_H ,

$$(A3) \quad dNPV/dH = x_H - FC_H - (r + \delta)SV_H = 0,$$

where SV_H is asset value at H . In other words, the project continues as long as the cash inflow x_t exceeds the fixed cost FC_t plus the opportunity cost of waiting a little longer for SV_t . The opportunity cost of waiting is the time value of money r plus the continuing depreciation rate δ .

Now imagine a tax czar who models firm's investment decisions as in (A2) and who wishes to assign economically sensible depreciation rates and depreciable lives to various asset classes. Asset lives depend on δ , λ , FC_0 , and x_0 ; x_0 is our index of profitability. The czar would take δ and λ as determined in product and factor markets. Competition would force profitability toward the level x_0 , at which NPV = 0. Then, given operating leverage (FC_0), asset life would be determined by (A3).¹⁰

The starting point for each of our numerical experiments is consistent with this story. We pick pairs of δ and depreciable life that roughly correspond to those in the initial Treasury tax reform proposal.¹¹ For each pair, various initial levels of fixed costs are assumed. For each fixed cost level, the initial level x_0 and decay rate λ of cash inflows are set so that NPV = 0 and optimal project life H equals the depreciable life originally assumed. A numerical example is given in table 11.6.

These base-case projects are only the starting points for our experiments, which calculate how the present value of the firm's tax liability depends on profitability levels, cash flow variances, the option to end the project early or late, and of course, on the specific tax rules.

Table 11.6 Numerical Example of a Base-Case Project

Variable Definitions

- y_t = pretax cash flow = revenue – variable cost – fixed cost
- x_t = revenue – variable cost, which decays at the nominal rate λ
- FC_t = fixed costs, which increase at the inflation rate i
- r = nominal risk-free interest rate
- SV_t = asset values; $SV_0 = 1$, the initial outlay; SV_t decays at the nominal rate δ
- σ_x^2 = variance rate of the realized cash flows \bar{x}_t
- H = optimal project life under certainty

Base-Case Values

x_0	= .259	λ	= .002
FC_0	= .1	δ	= .12
H	= 12	r	= .08
		i	= .06
		σ_x	= .15

Note: in real terms, cash flows decline at $\lambda + i = .062$ per year.

Calculate NPV and Check Project Life

$$\begin{aligned}
 \text{(A6)} \quad \text{NPV} &= -1 + e^{-(r+\delta)H} + \frac{x_0}{r + \lambda} (1 - e^{-(r+\lambda)H}) \\
 &\quad - \frac{FC_0}{r - i} (1 - e^{-(r-i)H}) \\
 &= -1 + .0907 + 1.9778 - 1.0669 = 0 \\
 d\text{NPV}/dH &= x_H - FC_H - (r + \delta)SV_H = 0 \\
 &= .2529 - .2054 - .0474 = 0
 \end{aligned}$$

Abandonment Value

With $\sigma_x = .15$, and $H = \infty$, the NPV of the project with *no* abandonment is -2.84 . The value of the abandonment put with last exercise date at $t = 100$, is $+3.13$. Thus adjusted NPV (APV) is:

$$\begin{aligned}
 \text{APV} &= \text{NPV} + \text{abandonment value} \\
 &= -2.84 + 3.13 = .29.
 \end{aligned}$$

NPV without abandonment and $H = 12$ is zero. Thus APV = $+.29$ is entirely due to the option to end the project before $t = 12$ or to extend it to $t = 13, 14, \dots$, or 100.

Taxes and Project Life

Suppose the firm always pays taxes at the marginal rate τ . Under stylized tax reform:

$$\begin{aligned}
 \text{(A4)} \quad \text{NPV} &= -1 + \int_0^H \tau(\delta + i)e^{-(r+\delta)t} dt + e^{-(r+\delta)H} \\
 &\quad + \int_0^H (1 - \tau)x_0 e^{-(r+\lambda)t} dt - \int_0^H (1 - \tau)FC_0 e^{-(r-i)t} dt,
 \end{aligned}$$

where:

$$e^{-(r+\delta)H}$$

Present value of asset at $t = H$. Since asset values equal tax book value throughout, no tax is paid at the end of project life.

$$+ \int_0^H \tau(\delta + i)e^{-(r+\delta)t} dt$$

The present value of tax depreciation.¹² The depreciation rate is expressed in real terms as $(\delta + i)$. Think of this as indexed depreciation: higher inflation would be reflected in a higher r and a smaller δ , that is, in slower, possibly negative, decay of nominal asset values. However, higher inflation should not reduce tax depreciation as a fraction of nominal asset value. Thus, we add inflation back to keep the depreciation rate in real terms.

By the way, the present value of *nonindexed* depreciation is:

$$\int_0^H \tau(\delta + i)e^{-(r+\delta+i)t} dt.$$

In this case tax depreciation charges decline at the real rate $(\delta + i)$ even though inflation is positive and reflected in the nominal discount rate.

The tax rules embodied in equation (A4) describe the “reformed” tax system to be compared with current (1985) law. The only rule not apparent from equation (A4) is the treatment of remaining book value at H ; we assume it is written off as a final, lump-sum depreciation allowance.

The NPV formula with taxes (A4) simplifies

$$(A5) \quad NPV = [1 - e^{-(r+\delta)H}][1 - \frac{\tau(\delta + i)}{r + \delta}] + \frac{(1 - \tau)x_0}{r + \lambda} [1 - e^{-(r+\lambda)H}] - \frac{(1 - \tau)FC_0}{r - i} [1 - e^{-(r-i)H}].$$

The condition for optimal project life is:

$$(A6) \quad dNPV/dH = x_H - FC_H - SV_H(r + \delta) \left[\frac{1 - \tau(\delta + i)/(r + \delta)}{1 - \tau} \right] = 0.$$

In this setup, the “tax term” in brackets tends to shorten project life.¹³ However, we do *not* assume that the tax czar takes tax effects such as this into account in setting depreciation rates or asset life classes. For our experiments, we define depreciation rates, asset lives, and so forth, in terms of pretax cash flows.

Optimal Abandonment

When a project description like that given in table 11.A.1 is handed to the Monte Carlo simulation, the assumption of a fixed project life H is left behind. Project life may be cut short if cash flows x_t are sufficiently bad or extended beyond H if they are sufficiently good. The maximum project life is set far beyond H , at $t = 100$.

The option to *choose* project life can be modeled as a long-lived American put, with varying exercise price, written on an asset with a varying dividend yield.¹⁴ The asset is the present value of future project cash flows x_t , *assuming* those cash flows will continue to evolve stochastically out to the far distant future. When the put is exercised, subsequent cash flows are given up in exchange for the exercise price. In our examples, exercise price at t equals SV_t , asset value at t , *plus* the present value at t of subsequent fixed costs, which are avoided by abandoning.

The optimal exercise strategy for the put gives the decision rule for choosing project life, and the value of the put, usually labeled “abandonment value,” is incorporated in adjusted project value.¹⁵

$$\text{Adjusted NPV} = \text{NPV with no abandonment} + \text{Abandonment (put) value}$$

Abandonment value and the optimal abandonment strategy are calculated numerically¹⁶ using *pretax* cash flows. It would be nice to explore how taxation affects the abandonment decision, but the computational problems seem overwhelming once tax-loss carrybacks and carryforwards are introduced. For example, including carry privileges in the put-valuation program would require at least two additional state variables: one for tax paid in the previous three periods and another for tax-loss carryforwards. Some partial analyses of how tax asymmetries interact with project life seem feasible, but we must leave them for further work.

Summary

The procedures used in our numerical experiments may thus be summed up as follows.

1. Choose an asset class described by a depreciation rate δ and a prespecified asset life. Assume an investment outlay of 1.

2. For various levels of operating leverage, measured by initial fixed cost FC_0 , pick the initial cash inflow x_0 and its decay rate λ so that project NPV = 0 and H , the optimal abandonment date under certainty, matches the prespecified life for the asset class. This step sets the decay rate λ and ensures that there is an initial cash flow level consistent with NPV = 0 at the assumed fixed life H .

3. Pick a variance rate σ_x^2 for the cash flow realizations \bar{x}_t and calculate optimal abandonment strategy and abandonment values. The abandonment strategy does not depend on the initial value x_0 , although the abandonment value does.¹⁷

4. Calculate the after-tax present value of the project for different levels of x_0 under whatever tax rules are being investigated, assuming that project life is terminated by the abandonment strategy calculated in step 3. (For a few runs step 3 was "turned off" to check that our qualitative results stand when project life is fixed.)

Notes

1. Since the value of a call option is convex in the exercise price (see Merton 1973), and no interest is paid on carry-forwards, it is always optimal to use tax losses as soon as possible.

2. Boyle (1977) first used a Monte Carlo simulation technique to value a European call option on a dividend-paying stock.

3. Constantinides (1983) sets forth the conditions for a tax-paying entity to realize tax losses immediately and to defer gains as long as possible.

4. Tax-loss carryforwards cannot be "sold" at face value via financial leases, for example. A firm with carryforwards can sell tax depreciation deductions to a taxable lessor, but the lessor has to pay taxes on the lease payments received. The net gain to lessee plus lessor occurs only because tax depreciation is accelerated relative to the lease payments. See Myers, Dill, and Bautista (1976). Even if the firm with carryforwards (lessee) captures the full net gain of the lease contract, it cannot capture what the depreciation tax shields would be worth to a taxable corporation.

5. Issuing equity to buy bonds will be effective only under certain assumptions about debt and taxes. See Cooper and Franks (1983) for a discussion of some of the financial transactions designed to exploit the firm's tax losses.

6. The percentages of firms with carryforwards is shown in Auerbach and Poterba's table 10.1 (this volume). The percentages are much smaller when weighted by the market value of equity, since firms with carryforwards tend to be small and poorly performing. The transition probabilities are from their table 10.7.

7. Our simulations of the stand-alone project show the maximum impact for tax asymmetries on incremental projects undertaken by a going concern. That is, the after-tax NPV of the stand-alone project is not reduced, and generally is increased, by adding it to other assets subject to corporate tax. We make this statement based on simulations in Majd and Myers (1985).

8. Interpret H as a precommitted shutdown date. Firms do not precommit, but present value calculations usually assume they do. We relax this assumption in the abandonment analysis described below.

9. Second-order conditions are satisfied in our examples.

10. The story now has some latent inconsistencies. First, as we will show, taxes may affect asset values and lives. Second, we have not shown that secondhand asset values would actually decline at a regular rate δ when $FC_t > 0$. They would do so only if we introduced intangible assets, or at least assets which are not depreciable for tax purposes.

11. U.S. Department of the Treasury (1984, vol. 2, chap. 8.01).

12. Depreciation tax shields should be discounted at $r(1 - \tau)$, the *after-tax* riskless rate, since they are safe nominal flows under symmetric taxation. See Ruback (1986). Thus, we have overstated the burden of a symmetric tax. We accept this bias to ensure comparability with the risk-neutral option valuation framework used in our simulation. The *pretax* risk-free rate is standard in that framework. We are not certain that it should be when a long or short position on an option is held directly by a corporation rather than by investors in its securities. For now, we can only note this as an open issue.

13. We would hardly claim this as a general result. For example, taxes would have no effect on project life providing depreciation is completed before H . (This is common under current law.) The present value of depreciation tax shields is then a "sunk" benefit and does not depend on H ; all tax terms cancel out of the derivative.

14. See Myers and Majd (1985). The dividend yield is just project cash flow x_t divided by the present value in t of expected subsequent cash flows x_{t+1}, x_{t+2}, \dots . As in the Myers-Majd paper, the assumption is that yield depends only on time, not on the outcomes $\bar{x}_0, \bar{x}_1, \dots, \bar{x}_t$.

15. NPV with no abandonment is calculated on an underlying asset that lives to $t = 100$, substantially greater than the optimal life under certainty. Thus NPV with no abandonment is less than NPV from equation (A2). Abandonment value more than makes up for this shortfall, so that adjusted NPV exceeds NPV at the fixed-life H . See table 11.6 for an example.

16. The numerical procedure differs in three ways from that used in Myers and Majd (1985). First, the uncertain cash flows \bar{x}_t are modeled as a process with monthly ($t/12$) binominal jumps. Second, the present value of remaining fixed costs is rolled into the exercise price. Myers and Majd (1985) ignored fixed costs. Third, abandonment is not allowed before month 13.

Accuracy of the abandonment value calculations was checked by comparing present results with results from the method used in Myers and Majd (1985), and by computing abandonment values numerically for special cases for which closed-form solutions are available.

17. The value of the option to extend project life is overstated in our simulations because we have not forced the firm to make replacement investments. The decision to finally bail out is determined solely by the downward trend of "variable" cash flow relative to fixed cost. However, this should not affect the relative sizes of pretax and after-tax NPVs holding initial profitability and risk constant.

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Comment Joseph E. Stiglitz

This is an extremely interesting chapter. I want to make a few comments suggesting some extensions to the authors' analysis and emphasizing certain general implications of their approach.

1. Their analysis makes clear the importance of "options" analysis for an understanding of tax consequences in the presence of uncertainty. There are a variety of circumstances in which the taxpayer has some discretion, for example, about the realization of a gain or a loss; in those circumstances, ignoring uncertainty—that is, looking at the tax consequences of the tax provision assuming there was no uncertainty—can be highly misleading. We encountered another instance of this in the analysis of churning. On average, it may not pay to churn. But there are circumstances in which it does, for example, if there is a large capital gain or if there is a change in the tax status of the owner, so that the tax imposed on the seller is less than the tax write-offs resulting from the step-up in basis for depreciation. (See J. E. Stiglitz, "General Theory of Tax Avoidance," *National Tax Journal*, September 1985.)

It should be emphasized that these options affect the desirability of investing in the given asset; that is, the appropriate way to value the tax consequences of the demand for different investment goods is not on the basis of an *ex post* analysis of actual taxes paid, but on the basis of an *ex ante* analysis of the value of the option.

2. The authors' analysis also makes clear the potential importance of disaggregating on the basis of tax status of firms. Earlier discussions have focused on the importance of disaggregation across classes of assets; one may get an inaccurate picture of the effect of the tax system by looking at its effect on an average asset. Similarly, one may get an inaccurate picture of the effect of the tax system by looking at its effect on an average firm.

I should add that the effect of a tax on a firm will depend not only on its tax status (whether it has loss carryforwards), but also on its financial position; that is, whether at the margin it is financing its investment by raising new equity or by borrowing or out of retained earnings.

3. A difficulty we have repeatedly encountered in the analysis of the effects of taxation is that we would like to use a sophisticated max-

imization model to describe the firm and the market and to evaluate the effects of a proposed tax change and the responses of the firm (investors). Yet we have considerable evidence that firms (investors) do not act in accordance with our theory. How reliable then are our inferences based on these models?

Here the issue is that there are numerous ways firms can seemingly mitigate the effects of limitations on loss deductibility, for example, through leasing. If these worked perfectly, then the tax status of the firm would make no difference. More generally, the effect of these provisions depends on how well these mechanisms work. These limitations, and how they affect the behavior of the firm, have not been modeled here; accordingly, how these limitations will operate in the presence of a change in the tax law has not been analyzed.

4. How concerned we should be about limitations on the deductibility of losses should also be affected by the extent to which the losses are the result of discretionary actions taken to realize capital losses. Thus, if a firm has an asset that has declined in value and sells that asset, it will record a capital loss. But it could have waited to sell the asset. It chooses to record the loss because of the tax advantages of doing so, just as it chooses not to recognize certain gains.

5. The approach taken here is based on the premise that in valuing risky streams there is a perfect capital market; the values calculated are those that would emerge, if in fact these options could be sold on the stock market. Two caveats are in order. First, the risk attitudes of the government may differ from those in the private sector; thus, these do not represent the value of these risky streams to the government. Second, this approach will have to be modified if a change in tax policy is contemplated that results in flows of funds from the government to firms in states when they have losses (are near bankruptcy). For it is precisely in those circumstances that the standard capital market models, assuming full information, are most inappropriate. For instance, with full loss deductibility the government is often pictured as a full equity partner; alternatively, the government is sometimes said to be lending money to the firm. But these metaphors, while suggestive, are not quite accurate: the government's claims, particularly in the presence of bankruptcy, may be quite different from the claims of another lender or another equity investor.

6. The approach taken here is based on a simulation model, in which an asset with a particular pattern of returns is investigated. The qualitative results that emerge are consistent with standard theory; for example, the greater the risk, the greater the option value. But simulation exercises are of particular use in assessing the quantitative significance of an effect; however, it is difficult to ascertain, on the basis

of this one exercise, how robust the results are. Would they, for instance, be modified if we considered a one-hoss-shay technology, or a technology in which output and labor requirements are fixed but the fixed lifetime of the asset is a result of rising real product wages?

7. Finally, it would be useful if the authors related their results to the Auerbach-Hines results given in chapter 5.