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Volume Author/Editor: John G. Cragg and Burton G. Malkiel

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Chapter Title: Empirical Connection of the Growth Forecasts with Share-Valuation Models

Chapter Author: John G. Cragg, Burton G. Malkiel

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4 Empirical Connection of the Growth Forecasts with Share-Valuation Models

We suggested in chapter 3 that a relationship should exist between the earnings growth expectations we have collected and the market values of the corresponding shares. The present chapter reports on our empirical investigation of this relationship. This investigation may be regarded in one of two ways. Assuming that growth-rate expectations are a major input used by investors to form expected security returns, our empirical work tests the validity of the valuation models. Conversely, if we maintain the validity of the valuation models, we may be regarded as testing the hypothesis that earnings growth expectations do play a major role, along with the other specified variables, in investors' evaluations of expected security returns.

We begin by investigating the expected rate of return measure suggested by equation (3.3-14) and obtained by using the averages of the long-term expected growth rates. We are particularly concerned with whether the relationship between expected return and the systematic risk variables represented by various regression coefficients holds when expected return is measured with our analysts' forecasts. First, in section 4.1 we specify more precisely exactly what measures of risk will be employed. Next, in section 4.2, we examine the prima facie evidence in favor of hypotheses suggested by the diversification model. Section 4.3 then adopts a more structural approach, which takes into account some econometric problems that were discussed in section 3.4. We switch in section 4.4 to the alternative specification (3.3-15), which we suggested might also give a good representation of the model. This price-earnings ratio formulation allows us to enquire whether other growth forecasts might give a closer explanation of valuation relationships than the expectations data we collected. Failure to find such improvement allows us to conclude that our growth measures are closest to the actual expectations that enter market valuation.

Having a model for prices also allows us to investigate whether knowledge of the model and access to the expectations data would have allowed superior stock selection. The fact that they would not comes as no surprise, but the reasons are of considerable interest. These are the subject of section 4.5. The various findings of these investigations are summarized in section 4.6.

4.1 The Risk Measures Used

It is not clear from the diversification model exactly what measures of risk would be most appropriate. We did provide, in section 3.4, a theoretical justification for the general approach that we shall take. Nevertheless, some empirical investigation is needed before we can ascertain what specific measures are most appropriate; that is, we need to select the exact form of the regression equation whose estimated coefficients will stand for the factor coefficients. We begin by exploring relationships between security returns and some economic variables that are of interest whatever valuation model is appropriate. Once we have established the variables to be used, we proceed to explore the valuation relationships suggested by the theory.

The first set of variables employed are measures of so-called market risk derived from the regressions of the realized rates of return on various market-wide variables.[1] We experimented with several market indicators including the Standard & Poor's 500 Stock Index, the Dow Jones Industrial Average (of 30 stocks), and the (value) weighted and unweighted indexes made available by the University of Chicago's Center for Research in Security Prices (CRSP). The realized rates of return were obtained from the CRSP. Our results turned out not to be sensitive to use of the alternative market indexes, so we report here only the results for the CRSP weighted index. This index tended to give results as strong as any in terms of r^2 for the regressions of company returns on the index and provided coefficients which were marginally stronger for the subsequent simple regressions reported in section 4.2.

Correlation with other types of variables may also yield needed risk measures whether the extended CAPM (involving nonmarketable income streams) or the diversification model is assumed. We selected three such additional variables. They are the rate of change of National Income (NI), the short-term interest rate measured by the ninety-day Treasury Bill rate, and the rate of inflation measured by the increase of the Consumer Price Index.[2] These may be considered typical measures of

1. These are the "beta" coefficients often calculated allegedly to give content to the CAPM.

2. We used alternatively the rate of change of GNP as opposed to NI; the long rate as opposed to the short; and the GNP deflator as opposed to the CPI. The alternative series were so highly correlated that it made little difference which we employed.

some risks to which investors are subject, stemming from variation in other sources of income, from changes in interest rates, and from changes in inflation.

The period over which the regression coefficients should be calculated is not clear a priori. It is not even clear that only past values should be used. The theory involves the covariances of returns with various quantities in the future. These parameters could safely be estimated from past data if they did not change or if investors did not perceive change. Such stability is unlikely. Changes in the nature and type of activities that corporations pursue and alterations in the structure of the economy make it likely that the appropriate regression coefficients change through time. Insofar as investors can perceive and even anticipate these changes, they are unlikely simply to extrapolate past betas into the future. Indeed, many of the popular "beta services" in the financial community explicitly adjust the betas calculated from past data, on the basis of changes that are known to have occurred in the structure of the business. Thus, in calculating the relevant betas at any time, it might be sensible to use values estimated with data following the time at which the valuation took place. Fortunately, our expectations data are not based on calculations using the realizations over the forecast period so we do not have to worry about spurious correlations being found between the expected return and these future values.

We adopted a compromise approach after some experimentation. The regression coefficients are calculated using quarterly observations over ten-year periods. The periods used covered the three years prior to the valuation date and the seven years following it. The results reported in the next section are not very sensitive to variations in the details of this procedure. Almost the same results were obtained, for example, when we took five years before and after the valuation date. Nevertheless, we did find that use of data entirely from past periods gave less satisfactory results than those obtained by including some future data. Extending the estimation period into the future improved the values of r^2 and was particularly important for obtaining some precision in evaluating the effect of inflation.

We also tried monthly rather than quarterly observations and shorter time periods over which to make the calculations of covariances with the market index. Again we found that the results improved when future data were included in the calculations, i.e., when some foresight regarding the future was assumed. However, the use of the shorter period made no substantial difference to the results. Since it is desirable to calculate all the regression coefficients over the same period so that the variance-covariance matrices of these estimates can be easily obtained for use in testing certain hypotheses, and since National Income is available only quarterly, we pursued the quarterly calculations.

4.2 Association of Expected Return and Risk

4.2.1 Strength of Individual Measures

The first question we investigate is the relationship between expected return and each of the various risk measures. The critical questions are whether the regression coefficients specified in the previous section are related to expected return and whether other types of risk measures (not suggested by the CAPM) are more important.

The expected return variable we use is suggested by equation (3.3-14). Let $\bar{g}_{jt}$ be the average of the long-term predicted (percentage) rates of growth available for company j at time t, D_{jt+1} be the dividends expected to be paid per share in the course of the next year (as estimated by the predictor which furnished data in all years), and P_{jt} be the end-of-year closing price (ex dividend where appropriate) for the shares of company j. Then the expected percentage rate of return, $\bar{\rho}_{jt}$, is calculated as

$$(4.2\text{-}1) \qquad \bar{\rho}_{jt} = \bar{g}_{jt} + 100(D_{jt+1}/P_{jt}).$$

Simple regressions of this expected return measure on the various risk proxies are summarized in table 4.1. The sort of cross-sectional data we are using makes us vulnerable to heteroscedasticity, which can produce some seriously misleading results from our data if the problem is ignored. To avoid the difficulties produced by heteroscedasticity, we calculated the standard errors of the coefficients in the way advocated by White (1980) that allows for any heteroscedasticity that may be present. We report in table 4.1 the asymptotic t-values for the regression coefficients calculated in this way. Because of the adjustment for heteroscedasticity, the coefficient of determination r^2 is not a monotonic transformation of these t-values. The values of r^2 did nevertheless tend to parallel the t-values.

The first risk measure is the regression coefficient of the (excess) rate of return of each security on the (excess) rate of return to the CRSP value-weighted market index. It is denoted by $\hat{\beta}_{Mj}$ and was obtained by estimating the equation

$$(4.2\text{-}2) \qquad \pi_{jt} - \rho_t = \beta_{Mj}(\pi_{Mt} - \rho_t) + u_{jt}$$

for each company j over forty quarters, that is, forty values of t. Here π_{jt} is the ex post return to company j, ρ_t is the short-term (ninety-day) Treasury Bill rate taken to represent the risk-free rate of interest, and π_{Mt} is the rate of return of the CRSP index. This β_{Mj} coefficient is, of course, the measure suggested by the CAPM if one ignores the problem that the market index must provide complete coverage of marketable securities. We then proceed to estimate the equation

$$(4.2\text{-}3) \qquad \bar{\rho}_{jt} = a_0 + a_1\hat{\beta}_{Mj} + v_{jt}.$$

Table 4.1 **Risk Measures and Naive Expected Return (asymptotic t-values adjusted for heteroscedasticity)**

A. Using Regression Coefficients

Year	$\hat{\beta}_M$	$\hat{\beta}_Y$	$\hat{\beta}_r$	$\hat{\beta}_p$
1961	4.04	2.37	−.59	−1.13
1962	2.01	1.82	.92	−.54
1963	1.74	.96	−.33	−.59
1964	2.21	.77	−1.45	−1.08
1965	1.92	1.48	−1.52	−1.40
1966	3.99	2.48	−4.04	−4.33
1967	3.11	2.93	−4.44	−3.83
1968	3.91	1.98	−4.27	−4.02

B. Using Variance Measures

Year	s_g^2	s_g	s_e	s_s^2
1961	1.90	.99	2.89	1.68
1962	3.63	3.63	1.56	−.32
1963	2.39	2.09	.52	1.51
1964	6.47	2.42	.83	−3.14
1965	4.76	3.30	1.21	−.91
1966	2.21	2.76	1.60	
1967	2.82	3.91	1.35	
1968	8.21	6.98	2.68	

$\hat{\beta}_M$ = coefficient of the CRSP value weighted index.
$\hat{\beta}_Y$ = coefficient of the rate of change of National Income.
$\hat{\beta}_r$ = coefficient of the Treasury Bill rate.
$\hat{\beta}_p$ = coefficient of the rate of change of prices.
s_g^2 = variance of the long-term growth predictions.
s_g = standard deviations of the long-term growth predictions.
s_e = standard error of regression of return on four variables.
s_s^2 = variance of the short-term growth predictions.

This equation is estimated separately for each year t on the basis of all companies j for which we had data in that year. The resulting t-values for a_1 appear in table 4.1.

The t-values obtained from estimating equation (4.2-3) are positive and usually significant. The strength of the association is not great, however: the value of r^2 corresponding to the highest t-value is only 0.16. The weakness of these associations could arise from the particular market index and periods used. However, as noted above, the results did not vary substantially if alternative indexes were used in place of the CRSP weighted index, and seemed more apt to be weaker than stronger. They also were not substantially changed by using the coefficient obtained by regressing individual returns on the market return rather than using excess returns in each case. Moreover, the results were not very sensitive to changing the period over which the coefficients were estimated, provided that at least some observations following the date at which the growth forecasts were made were included.

Although the regression coefficients with the CRSP index give significant results, strong t-values (and coefficients of determination) are sometimes obtained from using the regression coefficients of the securities' returns on the rate of change of National Income, indicated by $\hat{\beta}_Y$ in table 4.1, in place of $\hat{\beta}_M$ in estimating equation (4.2-3). These t-values are not, however, as strong as those for the coefficient of the CRSP index.

Our next risk measures come from estimating the regression of each security's rate of return on the rate of inflation ($\hat{\beta}_p$) and on the Treasury Bill rate ($\hat{\beta}_r$). Systematic relationships between security returns and inflation and interest rates are consistent with the wider specification of returns being associated with a variety of factors, as we argued in chapter 3. Table 4.1 indicates that these alternative risk measures do not do as well as the standard $\hat{\beta}_M$ measure during the early years. They do, however, tend to have a much stronger influence later in the 1960s when inflation rates and interest rates begin to soar. The signs of β_r and β_p can be expected to be negative if they do not also stand as proxies for other risk measures. A higher value of β_p indicates that a stock provides a better inflation hedge, which is a desirable attribute. Similarly, a positive value of β_r indicates that the stock does well when interest rates rise and hence is negatively correlated with realized returns from fixed income securities.

These results clearly indicate that the various regression coefficients are indeed related to expected return. The next question is whether other types of risk measure have still closer associations. Part B of table 4.1 summarizes the results obtained by using various variance measures for risk instead of regression coefficients.

The first of these alternative risk measures is the variance of the predictions of long-term growth, s_g^2. This quantity may possibly be interpreted as a measure of own variance and thus of specific risk. Nevertheless, the decomposition shown in equation (3.4-14) suggests that it may instead be a particularly good expectational proxy for systematic risk. For the years 1962 through 1965, when our sample was widest, s_g^2 gives stronger results than any of the regression risk measures. It also shows positive associations with expected rates of return in other years, which are clearly significant except in 1961.

Equation (3.4-14), which provides the basis for the possible interpretation of s_g^2 as representing systematic risk, also indicates that s_g^2 would be a quadratic rather than a linear combination of the factor coefficients γ_{jk}. This might suggest that the standard deviations of growth forecasts might be stronger measures of systematic risk than the variances. However, as the column of table 4.1 headed s_g shows, there was no reliable tendency for this to be the case.

If s_g^2 should represent specific risk rather than systematic risk, one might expect a better measure to be provided by the residual variances or standard errors of estimate of the regressions of the rates of return on the various systematic variables. Our findings do not, however, support this supposition. The standard errors from the regression of return on the four variables used to calculate the β coefficients produced weaker results than did s_g^2. They are shown in the column of table 4.1 headed s_e. The residual variances, that is, s_e^2, gave no stronger results.

The success of the variance of the long-term predictors makes one wonder whether the variance of the short-term growth predictions could also be used to provide a useful measure. This did not prove to be the case. The results, given in the final column of table 4.1, show mixed signs and are generally not significant. This risk measure quite clearly is weaker than the variance of the long-term predictions.

4.2.2 Use of Several Risk Measures

These results already have some interesting implications despite the simplistic approach used. There is, however, no reason to limit ourselves to only one risk measure. We now turn to the wider specification where in the first step the realized rate of return is regressed on all the suggested variables.[3] Before looking in the next section at the more structural aspects of this specification, we examine the prima facie case that all these variables are relevant to valuation, even though these inferences may turn out to be influenced by errors-in-variables difficulties.

The coefficients were obtained from the multiple regression of the rate of return of each security on the CRSP value-weighted index (M), on the rate of change of National Income (DY), on the Treasury Bill rate (r), and on the rate of inflation (DP). The equation fitted for each company is

$$(4.2\text{-}4) \qquad \pi_{jt} = \delta_{0j} + \delta_{Mj}M_t + \delta_{Yj}DY_t + \delta_{rj}r_t + \delta_{pj}DP_t + u_{jt},$$

and the estimated regression coefficient $\hat{\delta}_{ij}$ serves as risk measures. The cross-section specification for $\bar{\rho}_{jt}$ is expanded from (4.2-3) to

$$(4.2\text{-}5) \qquad \bar{\rho}_{jt} = a_0 + a_1\hat{\delta}_{Mj} + a_2\hat{\delta}_{Yj} + a_3\hat{\delta}_{rj} + a_4\hat{\delta}_{pj}.$$

Estimates of this equation are given in table 4.2.

A number of findings indicated by table 4.2 are worth emphasizing. Of most importance, each type of coefficient is significant in some years. In the first part of the period only the market coefficient is significant. However, toward the end of the period other coefficients tend to be important, especially those measuring systematic relationships with inflation and interest rates. When these results are taken at face value, two

3. These are the regressions from which the standard errors of estimate referred to in table 4.1 were obtained.

Table 4.2 **Regression Estimates for Extended Model for the Expected Rate of Return (asymptotic t-ratios adjusted for heteroscedasticity)**

Year	Constant	$\hat{\delta}_M$	$\hat{\delta}_Y$	$\hat{\delta}_r$	$\hat{\delta}_p$	R^2
1961	7.01	1.58	.23	−.02	−.03	.15
	(12.75)	(2.78)	(1.47)	(−.73)	(−.59)	
1962	7.82	1.19	.20	.02	−.02	.10
	(10.82)	(1.59)	(1.61)	(.54)	(−.29)	
1963	6.94	1.63	.05	−.03	−.01	.07
	(14.85)	(3.04)	(.44)	(−.46)	(−.12)	
1964	6.00	2.58	−.06	−.08	−.06	.12
	(10.22)	(3.80)	(−.74)	(−1.27)	(−.68)	
1965	8.31	.79	.11	−.07	−.10	.07
	(18.31)	(1.57)	(1.14)	(−1.25)	(−1.04)	
1966	9.85	.90	.17	−.09	−.19	.19
	(21.18)	(2.11)	(1.92)	(−3.25)	(−3.60)	
1967	9.82	1.26	.25	−.15	−.30	.26
	(17.46)	(2.11)	(2.55)	(−4.19)	(−3.67)	
1968	8.83	3.98	.42	−.24	−.52	.28
	(11.70)	(4.69)	(3.28)	(−4.19)	(−3.77)	

explanations for them come to mind. First, in the more stable early part of the period, estimates of the δ coefficients may be sufficiently imprecise that in the subsequent estimation of equation (4.2-5) the relatively greater errors of measurement lead to lack of significance. Second, investors may have become more concerned about the other sources of risk, such as inflation and interest-rate instability, as the decade proceeded.[4] Overall, the results suggest strongly that all influences play a role, though it is an open question whether this is because they act as proxies for other variables.

The signs of the coefficients tend to be the same across the different equations. Although with errors in variables we must be cautious in attaching much importance to the signs of particular coefficients, the patterns obtained do usually conform to the signs suggested by intuition. Positive association with either the market return or income raises the expected rate of return. Correspondingly, positive partial correlation with the rate of inflation, indicating that the stock tends to act as a hedge against inflation, lowers the expected rate of return. Finally, the coefficient for the Treasury Bill rate usually has the expected negative sign. There is, however, a good deal of correlation across securities (roughly about 0.6) between the coefficients for the Treasury Bill rate and for the rate of inflation so that one may be partly serving as an additional proxy for the other. This correlation is sufficiently low, however, that one cannot legitimately presume that variations in the rate of change of prices and in the short-term rate of interest necessarily represent the same

4. Inflation, as measured by the annual rate of change in the Consumer Price Index, remained below the 2 percent level through 1965. Later in the decade, inflation increased to the 6 percent level.

variable. Except for this fairly mild correlation, multicollinearity problems are small, making it less plausible that all the different measures serve as proxies for some single variable.

Inclusion of all these different regression coefficients does not account for the strength we found earlier for the variance of the predictions. When that variable was included in (4.2-5) along with the four $\hat{\delta}$ variables measuring various systematic risks, it usually was highly significant with a positive coefficient. The a coefficients for the four $\hat{\delta}$ variables tended to retain the same signs, though with lessened significance. The apparent importance of s_g^2 may in part result from errors-in-variables problems or misspecification. Nevertheless, it may also indicate that s_g^2 is a particularly useful expectational proxy for several of the systematic risk measures. What is important is that the values of R^2 are sufficiently high and so very highly significant that there is no question about there being some underlying systematic association among the variables included in the specification.

4.3 Structural Relations between Expected Return and Risk Coefficients

The results reported in the previous section may arise because the market actually takes a multifaceted approach to risk. In contrast, they may simply be the outcome of using poor data. To investigate this question, we proceed in two stages. First, we examine the extent to which our risk coefficients exhibit the linear structure that we indicated in section 3.4 would be found if there were fewer factors than the number of independent variables used in the regressions in which the $\hat{\delta}_j$ coefficients were calculated. Establishment of the number of factor coefficients is also needed in order to proceed to take account of the errors of estimation of the $\hat{\delta}$ coefficients. The second stage involves estimating the valuation model allowing for the presence of these errors.

4.3.1 The Number of Factor Coefficients

We showed in equation (3.4-12) that the variance-covariance matrix of the regression coefficients has a particular structure under the common-factor model for rates of return. Let $\hat{\delta}$ be the average of the $\hat{\delta}_j$ vectors, and let $\bar{\alpha}$ be the average of the α_j vectors whose elements α_{jk} are the coefficients of the common K factors in the (true) rate-of-return equation (3.2-16). Letting $\bar{h} = \Sigma_{j=1}^{J} h_j/J$, where h_j is the residual variance, we can rewrite equation (3.4-12) as

$$(4.3\text{-}1) \qquad V = E\left[\sum_{j=1}^{J} (\hat{\delta}_j - \bar{\delta})(\hat{\delta}_j - \bar{\delta})'/J\right]$$
$$= \Xi'\left[\sum_{j=1}^{J} (\alpha_j - \bar{\alpha})(\alpha_j - \bar{\alpha})'/J\right]\Xi + \bar{h}(X'X)^{-1}.$$

Table 4.3 **Significance Levels for the Hypothesis That More Than Specified Numbers of Factors Are Present in the Regression Coefficients**

	Number of Factors			
Years	0	1	2	3
1959–68	.000	.816	.594	.174
1960–69	.000	.134	.266	.126
1961–70	.000	.890	.784	.303
1962–71	.000	.935	.839	.951
1963–72	.000	.767	.789	.305
1964–73	.000	.001	.059	.694
1965–74	.000	.068	.196	.992
1966–75	.000	.005	.065	.398
1967–76	.000	.006	.053	.317

Since $(X'X)$, the cross-product matrix of the variables used to estimate the coefficients, is known,[5] we can investigate the hypothesis that this common-factor structure does apply[6] to the variance-covariance matrix of the estimated coefficients calculated for the different companies. Assuming that the coefficients are normally distributed across companies, we performed likelihood-ratio tests of a variety of hypotheses. In doing so we used the value of $\bar{h}$, the average of the estimates coming from the estimates of the individual regressions, rather than jointly estimating this parameter in the factor analysis. No substantial differences in results occur when instead $\bar{h}$ is estimated from the $\hat{\delta}$ data.

The regression coefficients used for different years are far from being independent, since thirty-six of the quarterly observations are the same in regressions for adjacent years. Nevertheless, the patterns that occur over time are of interest. When we tested the hypothesis that there are less than four factors represented by the four regression coefficients, the data strongly supported the hypothesis that there are fewer factors. These tests are summarized in table 4.3 in terms of the smallest significance levels at which one could reject the (null) hypothesis of only zero, one, two, and three factors over the alternative hypothesis of at least four different factors being present.[7]

The hypothesis of only one factor is very strongly indicated in the early part of the period. However, when observations from the 1970s begin to

5. Of course, when the $\hat{\delta}$ vector being investigated does not contain the constant term, the appropriate row and column are first removed from $(X'X)^{-1}$.

6. Specifically, the procedure involves the principal components of $\Sigma_{j=1}^{J} (\hat{\delta}_j - \bar{\delta})(\hat{\delta}_j - \bar{\delta})'/J$ in the metric of $(X'X)^{-1}$. See Anderson and Rubin (1956) for a discussion of maximum likelihood estimates of the model. The fact that $\bar{h}(X'X)^{-1}$ is known makes more factors identifiable than would usually be the case.

7. Qualitatively similar results are obtained when we test three versus four factors, two versus three, etc.

play an important part, the data indicate that at least two factors are present and would reject at the 0.10 level the hypothesis of two factors in favor of three factors for some of the estimations.

The reason for the success of a one common-factor model in the early estimates was not that the correlations of different quantities, which themselves all varied significantly, could be fully attributed to a single factor. Rather, it was the case that some of the estimated coefficients varied so little across companies, relative to their errors of estimation, that both the variances across companies of their true values, δ_{jk}, and their correlations with other coefficients could be treated as zero.

This problem is illustrated by the data from the 1960s shown in table 4.4. There we present the matrices

$$\sum_{j=1}^{J}(\hat{\delta}_j - \bar{\delta})(\hat{\delta}_j - \bar{\delta})'/J$$

and

$$\left[\sum_{j=1}^{J}(\hat{\delta}_j - \bar{\delta})(\hat{\delta}_j - \bar{\delta})'/J - \bar{h}(X'X)^{-1}\right].$$

All the variances of the $\hat{\delta}_r$ and $\hat{\delta}_p$ coefficients can be attributed to estimation errors, and the hypothesis that the variance across companies in the true coefficients was zero could not be rejected. Indeed, all the variance can be so attributed for $\hat{\delta}_p$, the coefficients of inflation. Later, as interest rates and inflation rates themselves showed more variation, this ceased to be the case and all coefficients showed variation across companies significant beyond the 0.05 level. As noted earlier, while short-term interest rates and inflation may primarily reflect the same factor (as might be the case if the real rate of interest is constant), the magnitude of measurement errors in each variable must then be very substantial since collinearity problems in the data were mild and do not clearly account for the

Table 4.4 **Covariance Matrices of the Regression Coefficients Fitted for 1960–69**

	$\hat{\delta}_M$	$\hat{\delta}_Y$	$\hat{\delta}_r$	$\hat{\delta}_p$
	A. Unadjusted			
$\hat{\delta}_M$	.09			
$\hat{\delta}_Y$	.17	4.46		
$\hat{\delta}_r$	.47	−3.78	58.9	
$\hat{\delta}_p$	−.03	1.27	−22.9	19.3
	B. After Subtraction of Estimation Error			
$\hat{\delta}_M$	.05			
$\hat{\delta}_Y$	.13	1.08		
$\hat{\delta}_r$	.61	1.63	5.13	
$\hat{\delta}_p$	−.28	−1.21	4.73	−4.25

difficulties. Furthermore, the results about the number of factors were repeated when we dropped the interest-rate variable from the original regressions. The 1964–73 period and later ones indicated the presence of at least two and possibly three factors. Prior to that period, the variance-covariance matrices suggest only a single factor.

Earlier investigations of the appropriateness of the common-factor model to security returns suggested that several factors would be found. King (1966) as well as Roll and Ross (1980) each found support for such a hypothesis. Hence one may suspect that our results for the early years reflect the peculiarities of the data on some of the independent variables in that period.

These tests have involved the variance-covariance matrices of the regression coefficients. This was appropriate in view of our desire to use the adjusted matrices subsequently in estimation where it is necessary to avoid using singular matrices. However, the original hypothesis applies also to the averages (across companies) of the coefficients, that is, to

$$E\left[\sum_{j=1}^{J} \hat{\delta}_j \hat{\delta}_j'/J - \bar{h}(X'X)^{-1}\right].$$

When we investigated the number of factors, recognizing that the means of the regression coefficients should have the same factor structure, we found evidence for two factors rather than only one in the early years. That is, the hypothesis of only one factor can be rejected well beyond the 0.05 level, but not that of there being only two factors. The results for the later years did not change appreciably. We can still conclude that there are certainly two, and possibly three, common factors.

4.3.2 Results Allowing for Estimation Error

The previous findings about the number of factor coefficients present in the rate of return regressions pose a dilemma for the next part of our investigation. We suspect that the reason for finding only one factor in the early years is that the other factors happened to have very little variation in the 1960s. However, if the risk was still present that they would vary, then their coefficients should still enter the valuation equation. Using a one-factor model would then involve misspecification. Testing the hypothesis that more than one factor is actually present does require that the data clearly involve more than one factor. A procedure developed in Cragg (1982) that allows for estimation errors in $\hat{\delta}$ involves the use of

$$\left[\sum_{j=1}^{J} (\hat{\delta}_j - \bar{\delta})(\hat{\delta}_j - \bar{\delta})'/J - h(X'X)^{-1}\right]^{-1}.$$

The procedure makes sense only if the matrix is clearly positive definite. When this is the case, we can allow for the estimation error to see what inferences stand up even when its effects are recognized. In doing so, we shall use the simplification, discussed in Cragg (1982), in which the u_{jt} of equation (4.2-4) are assumed to be normally distributed.

We resolve the dilemma posed by our findings about the structure of the $\hat{\delta}_j$ coefficients by fitting two types of model, allowing in each case for the estimation errors of the regression coefficients. First, we estimate the equations for the expected rate of return using only the regression coefficient for the market and the variance of the long-term predictors; i.e., we fit the equation

$$\bar{\rho}_{jt} = a_0 + a_1\beta_{Mj} + a_2 s_{gj}^2. \tag{4.3-2}$$

Here, the β_{Mj} are based on the three years before and the seven years after the valuation. Second, we use the coefficients for the 1966–75 period, estimated without the interest-rate variable; that is, we estimate

$$\bar{\rho}_{jt} = b_0 + b_1\gamma_{Mj} + b_2\gamma_{Yj} + b_3\gamma_{pj} + b_4 s_{gj}^2, \tag{4.3-3}$$

where the γ_j are calculated from the regression

$$\pi_{jt} = \gamma_{0j} + \gamma_{Mj}\pi_M + \gamma_{Yj}DY + \gamma_{pj}DP + v_{jt} \tag{4.3-4}$$

for the period 1966–75. As we noted, there γ coefficients do support (though not strongly) the conclusion that a three-factor model is appropriate.

The first approach does little to resolve the puzzle. In the early part of the period, β_M was not significant while s_g^2 was always stronger and usually significant. For 1966 and subsequent years, when the number of predictors available on which to base s_g^2 becomes small, β_M is highly significant, and positive, as is s_g^2 in the last two years. These results suggest that s_g^2 is not simply another proxy for the systematic risk measured with considerable estimation error by β_M. Instead, it suggests that a model with two or more factors is appropriate—or that there is another relevant risk concept proxied by s_g^2.

The results of the second approach shed quite a bit more light on the matter. When adjustment was made for errors in variables and allowance was made for heteroscedasticity, it usually turned out that none of the coefficients was significantly different from zero. At best, but one would be, and then only just at the 0.05 level. This was true whether s_g^2 was included or not. Overall, however, when s_g^2 was included in the equation, the hypothesis that all γ_j parameters had zero coefficients in equation (4.3-3) could be rejected beyond the 0.01 level, except in 1963 and 1965. When s_g^2 was not included, the hypothesis could sometimes be rejected at the 0.10 level and sometimes not.

Part of the difficulty stems from multicollinearity. As lack of certainty about the number of underlying factors indicated, the "corrected" $\hat{\gamma}_j$ coefficients are correlated with each other. Moreover, there is some correlation with s_g^2, though it is small. The technique used involves much more complicated standard errors than ordinary regression, and for a given covariance matrix of explanatory variables these standard errors are considerably larger. More coherent results were obtained when the γ_Y coefficient for National Income was eliminated from (4.3-3). A pattern then emerged in which the coefficient of inflation and the variance of the predictors were significant, but the coefficient for the market index was not. Eliminating this coefficient as well as the one for national income then produced the results shown in table 4.5.

The results shown in table 4.5 are similar in nature for the different years. The risk variable s_g^2 has a positive and usually significant effect. The notable change in its magnitude in 1966 corresponds to the change in the number of predictors from which the forecast data were collected. The sensitivity of the security's rate of return to the rate of inflation as measured by γ_p had a negative effect as we would expect.

These results suggest that at least two factors are relevant in valuation. One may be equated broadly to inflation and its associated effects. The other, possibly representing market risk, seems to be better represented by the variance of the predictions of long-term growth than by any of the regression coefficients. Its exact nature therefore remains a bit of a puzzle. The first factor has a negative sign and is usually significant at the 0.10 level. This was true even in the early years when the experienced variations in the inflation rate were very small. The second factor is very strongly positive and highly significant.

Table 4.5 **Equation for Expected Rates of Return Allowing for Estimation Error in $\hat{\gamma}_1$ (asymptotic t-values adjusted for heteroscedasticity)**

Year	Constant	γ_p	s_g^2	"r^2"*	r_ϵ†
1961	9.26	−1.13	.63	.56	—
	(12.62)	(−1.72)	(6.30)		
1962	8.40	−.46	.67	.38	.88
	(30.87)	(−1.70)	(4.96)		
1963	8.18	−.61	.72	.34	.89
	(32.30)	(−1.58)	(2.98)		
1964	8.55	−.74	.63	.54	.84
	(21.17)	(−1.92)	(18.77)		
1965	9.01	−.74	.62	.61	.90
	(24.20)	(−1.99)	(29.39)		
1966	10.72	−.20	.05	.08	.68
	(28.48)	(−.73)	(1.48)		
1967	11.35	−.53	.03	.25	.67
	(24.88)	(−1.65)	(2.05)		
1968	11.93	−.75	.05	.70	.48
	(17.74)	(−1.82)	(7.44)		

*"r^2" is 1 − (estimated residual variance)/(variance of ρ_{jt}).

†r_ϵ is correlation of residuals with previous year's residuals.

These results have been corrected for the errors of measurement in the regression coefficients, but errors in s_g^2 have been ignored. The interpretation we have been giving to that variable means that we cannot calculate the variance of errors in its measurement by assuming that it is simply the sampling variance of predictions which all have the same mean for each firm. We did, however, attempt to deal with this measurement error by the use of instrumental variables while continuing to allow for the estimation errors in the regression coefficients. To do so, we used as instruments the regression coefficients γ_M and γ_Y and the residual variances s_e^2, whose usefulness we explored earlier, in table 4.1.

The main difficulty with the instrumental-variable approach in this case was that the proposed instruments are not closely associated with s_g^2. The value of R^2 obtained from regressing s_g^2 on all the instruments and γ_p varied from 0.05 to 0.31. The main effect of this weakness on the estimates of the equations for expected return was to reduce the standard errors of the coefficients of s_g^2 sharply. These findings strengthen the impression that s_g^2 contains relevant information about risk not readily available in other forms. However, the significance levels of γ_p were not affected by the use of instrumental variables, and the results were qualitatively much the same as those shown in table 4.5 in terms of the signs and magnitudes of the coefficients.

4.3.3 Constancy over Time

One of the interesting questions about valuation equations is whether the coefficients remain the same each year or whether they change. There is nothing in the valuation theory to suggest that they should be constant. The opportunity sets faced by investors, extending beyond simply the financial securities available to them, probably change and so may their preferences and concerns about various types of risk. The results of tables 4.2 and 4.5 give an impression of considerable variation. We now test for variability explicitly.

The residuals from the equations shown in table 4.5 for different years are correlated even after allowance is made for the effects of estimation errors of $\hat{\gamma}_p$. Problems of missing observations mean that we can simultaneously calculate the equations for a common set of companies in all years only at the expense of losing a large number of companies. Pairwise comparisons indicated that the residuals for adjacent years are quite highly correlated. The correlations of these residuals are recorded in table 4.5 in the column headed r_ϵ. It gives the correlations of the residuals in one year with those of the year immediately preceding. The quantities tabulated are the correlations of residuals using a common set of companies to estimate the regression coefficients in the two years. The exact values of the coefficients used differ slightly from those shown in table 4.5 because of the reduced number of observations used in their calculation.

The correlations of residuals, which are highly significant, complicate the problem of inquiring into the stability of the regression coefficients over time. Zellner's (1962) "seemingly unrelated regression technique" can be adapted in a straightforward way to the estimation of our equations even when allowing for estimation error of the original regression coefficients as well as for heteroscedasticity. To avoid the extensive loss of observations involved when all equations are fitted simultaneously, only pairs of equations were fitted.

Pairwise estimation of the equations usually produced significant differences in the coefficients of the valuation equation for different years. The main exceptions, where rejection did not occur even at the 0.10 level, are the 1964–65 comparison and the 1962–63 one. The coefficient for 1963 did differ from that for 1964 significantly at the 0.01 level even though the values shown in table 4.5 indicate the same qualitative findings in the sense that the coefficients are of similar magnitude.

The different estimation procedure used in these tests, which involve estimating the coefficients of each of two years jointly, did not change the conclusions about risk that were derived from our regressions in section 4.3.2 for the individual years. Indeed, these estimates indicated stronger support than the ones in table 4.5 for the hypothesis that two types of risk measures are indicated by the data.

4.3.4 Average Realized Return and Risk

The constant term $\hat{\delta}_{0j}$ obtained when equation (4.2-4) was fitted to obtain the other $\hat{\delta}$ coefficient contains implicitly another estimate of the expected rate of return. It is the average rate of return realized over the period, which many empirical studies of valuation presume corresponds to the return expected *ex ante* by investors. We can use this estimate to investigate the *ex post* validity of the APT, or diversification model, which suggests that we should find the same number of factors in the $\hat{\delta}_j$ vector when $\hat{\delta}_{0j}$ is included as when it is not. This consideration induces us to repeat the investigations carried out in section 4.3.1 with the other coefficients, but now including the constant $\hat{\delta}_{0j}$ as well.[8]

The estimates for the earlier periods included in our investigation tend to confirm the model fully in the sense that exactly the same number of factors is significantly present in the covariance matrix including the constant as we found when only the regression coefficients were used. This support for the model is less than might appear to be the case, however. As was the case for some of the coefficients, significant variation across companies was not present in the average rates of return in the

8. All independent variables are measured as deviations from their averages, so the constant term is also the average quarterly rate of return in the period over which the regression coefficients are calculated.

early years. In the final two years, the wider covariance matrix indicated that at least five factors were needed to account for the covariances of the constants with the other coefficients.

With the companies altering their natures over time and with the market valuation of risk quite possibly changing substantially over the decade of the seventies, such a finding should not be surprising even if the common-factor model is a correct description of security returns. However, it does not seem feasible to use these "objective," *ex post* measures of returns to obtain comparisons with the very successful results obtained from the *ex ante* measures we have employed. These estimated average *ex post* returns are not closely correlated with the *ex ante* measures derived from using the long-term growth predictions. The strong and interesting results we have obtained with these *ex ante* measures of expected returns and the fact that the *ex post* ones are not closely related to them emphasize the importance of using genuinely *ex ante* expectations of returns for studying security valuation.

4.4 An Alternative Valuation Specification

The derivation of the valuation model in chapter 3 suggested that the expected return formulation we have been investigating is only one approximation to the underlying model and that an alternative model may also be usefully estimated. The alternative approximation produces a more traditional formulation in which the price-earnings ratio is the dependent variable and earnings (dividend) growth, the payout ratio, and our various risk measures are treated as explanatory variables. The expected return formulation is particularly convenient for focusing on the risk structure suggested by the diversification model. The alternative allows us to ask whether growth-rate expectations are more relevant for valuation than other measures. It also allows us to investigate the role of the short-term growth predictions as well as to examine again which risk measures appear to be strongest.

An empirical analysis of the price-earnings model is also desirable because of an ambiguity of interpretation of the expected return models we have been studying. The results of the return model indicate partly that predicted earnings growth is connected with the regression coefficients giving the associations of rates of return to various economic indicators. Recall, however, that we found evidence in chapter 2 that a common-factor model may fit the growth predictions of security analysts. Our findings for the expected rates of return may reflect this feature of the data, even though the expected rate of return includes the dividend yield as well as the expected growth rate. Thus it is not entirely clear that we have actually been investigating a valuation relationship.

Implementation of the alternative model involved dividing both end-

Table 4.6 **Risk Measures in Stock Price Regressions (asymptotic t-values for alternative risk variables in equation [4.4-1])**

Year	β_M	β_Y	β_r	β_p	s_g^2
1961	−.32	1.10	1.25	.01	−.41
1962	−4.54	−.71	.59	2.58	−5.79
1963	−.33	.74	−.43	.28	−2.37
1964	−2.38	−2.76	−.88	1.65	−9.75
1965	1.43	1.32	−.70	−.43	−1.24
1966	−1.49	−.87	.14	.33	−.19
1967	1.34	.74	−2.29	−2.34	−8.67
1968	2.29	−.22	−1.41	−2.74	−1.12

of-year prices (P) and the dividends projected to be paid (D) by average normalized earnings[9] ($\overline{NE}$) to give the equation

$$(4.4\text{-}1) \qquad P/\overline{NE} = a_0 + a_1\overline{g}_p + a_2 D/\overline{NE} + a_3 RISK,$$

where $RISK$ stands for the various risk variables used.

4.4.1 Risk Measures

We begin our investigation of equation (4.4-1) by treating each of the risk measures we have been using as alternatives, just as we did when considering equation (4.2-1). In these regressions, both the average expected five-year growth rate and the dividend payout ratio almost always had positive and significant coefficients throughout the sample period.

The pattern for the risk measures is more complicated than earlier. Table 4.6 corresponds to table 4.1. In these regressions, a negative sign should be expected for the risk measures based on covariance with the market index and with national income, since higher risk should, ceteris paribus, lower price-earnings multiples. Although both $\hat{\beta}$ measures have the correct negative values more often than not, the t-values indicate that they are only occasionally significant. Positive signs should be expected for the risk measures based on reported inflation and interest rates. As was found in the regressions in table 4.1, these risk measures are only significant toward the end of the period studied, but their signs are often incorrect in these valuation regressions.

These findings indicate the difficulties of using the simple regression coefficients as risk measures in a specification also containing several other variables. In contrast to these ambiguous results, the variance of

9. The "normalized" earnings were furnished by two of the forecasters and were described in chapter 1. When more than one forecaster's estimates of "normalized" earnings were available for a company, the estimates were averaged. The results are little different (but a bit poorer) if reported earnings over the most recent twelve-month period are substituted for "normalized" earnings.

Table 4.7 *P/NE* **Regression Estimates of Equation (4.4-1) (asymptotic *t*-ratios adjusted for heteroscedasticity)**

Year	Constant	$\bar{g}_p$	$D/\overline{NE}$	s_g^2	R^2
1961	1.88	3.91	1.22	−.57	.81
	(.64)	(7.51)	(.24)	(−.41)	
1962	3.30	2.23	8.41	−1.17	.75
	(1.75)	(16.69)	(2.91)	(−5.79)	
1963	2.85	2.70	6.71	−.59	.77
	(.94)	(11.20)	(1.75)	(−2.37)	
1964	2.53	2.15	13.16	−1.09	.77
	(1.73)	(23.94)	(6.13)	(−9.71)	
1965	1.76	2.82	4.73	−.66	.67
	(.62)	(6.98)	(1.14)	(−1.24)	
1966	.22	1.74	7.42	−.01	.57
	(.09)	(9.62)	(2.79)	(−.19)	
1967	1.88	2.35	−1.05	−.09	.69
	(.67)	(13.28)	(−.35)	(−8.67)	
1968	2.18	1.78	5.13	−.04	.52
	(.56)	(8.10)	(.99)	(−1.12)	

the predictions always has a negative sign. Its significance does vary considerably across years, primarily reflecting variation in the magnitude of its coefficient. The important point, which agrees with our previous results with the expected return measures, is that s_g^2 provided a better single risk proxy than the regression coefficients based on more objective calculations. It also provided a more significant and consistent measure than the residual variances of the regressions, s_e^2.

Table 4.7 shows the full estimates of equation (4.4-1) using s_g^2 as the risk variable. The growth-rate variable is highly significant in each of the years covered. The payout ratio has the expected sign except in one year but is usually insignificant.[10] As we have already noted, the risk variable always has the correct negative sign and is often significant.

4.4.2 Alternative Growth Measures

The extent to which using truly expectational data is important for valuation models is indicated in table 4.8. Here we show the values of R^2

10. The positive sign of the dividend coefficient should not be interpreted as evidence that dividend policy can affect the value of the shares. This coefficient indicates only that a ceteris paribus change in dividend payout will increase the price of the shares. Among the things held constant in this equation is the growth rate of earnings and dividends per share. A positive dividend coefficient thus indicates only that *given* the future growth rate in earnings and dividends, the price of a share should be higher, the higher is the current percentage of earnings that can be paid out. The famous "dividend irrelevancy" theorem of Miller and Modigliani (1961) says that an increase in dividend payout will tend to reduce the growth rate of earnings per share since new shares will now have to be sold to make up for the extra funds paid out in dividends. A positive dividend coefficient is thus in no way inconsistent with the dividend irrelevancy theorem.

Table 4.8 **Values of R^2 for Alternative Specifications of the Valuation Equation**

	Specification		
Year	1	2	3
1961	.42	.45	.81
1962	.50	.53	.75
1963	.49	.50	.77
1964	.37	.43	.77
1965	.29	.31	.67
1966	.31	.44	.57
1967	.32	.36	.69
1968	.33	.41	.52

NOTE. See text for specifications.

for various combinations of historical and expectational data. The first specification (column 2) involved regressing the price-earnings multiple on three historic figures: the past ten-year growth rate of cash earnings, the average (over the preceding seven years) historic dividend-payout rate, and $\hat{\beta}_M$, estimated using only previous data. The third column substitutes the expectational variable s_g^2 for the $\hat{\beta}_M$ coefficient. The fourth column repeats the specification of equation (4.4-1) with s_g^2 as the risk variables, $\bar{g}_p$ and $D/\overline{NE}$ in place of historic growth and payout, and $P/\overline{NE}$ as the dependent variable in place of P/E. These r^2 values are the same as in table 4.7.

The dramatic change in the value of r^2 for the valuation equation occurs when $\bar{g}_p$ is used for the growth rate. Other variations have comparatively minor effects. There are, of course, a large number of ways of calculating past growth. Our findings hold up for the wide variety of historical growth rate we tried as well as the one reported in table 4.8. Using the average predicted growth rates substantially improves the fit of the regression. It is therefore safe to conclude that insofar as the market does value growth, the growth rates involved are far better represented by actual predictions made by security analysts than by any mechanically calculated rate.

One may wonder whether we would have done better to use only one forecaster rather than the average we have employed. Problems of missing observations again hinder this investigation. One of the advantages of using the average is that it allows us to include most of the companies in the regressions. However, it is also the case that closer fits tended to be obtained by using the average growth rates of all predictors than by employing the forecasts of any single firm. This suggests that our survey was useful in getting closer to what might be considered the expectations of a "representative" investor.

4.4.3 Role of Short-Term Predictions

In addition to the long-term growth estimates, which have played such an important role in our empirical valuation work thus far, we also collected short-term predictions for earnings in the next year. These were described and analyzed in chapters 1 and 2. Given the long-term growth rate, a stock should sell for a higher price if more of that growth is expected to be realized earlier in the period. Therefore we augmented our valuation equation (4.4-1) to include the term $\tilde{E}_{t+1}/\overline{NE}$, the ratio of next year's average predicted earnings ($\tilde{E}_{t+1}$) to average normalized earnings (for the present period). Equation (4.4-1) then becomes

$$P/\overline{NE} = a_0 + a_1 g_p + a_2 \tilde{E}_{t+1}/\overline{NE} + a_3 D/\overline{NE} + a_4 s_g^2. \tag{4.4-2}$$

The results obtained with this specification are presented in table 4.9. The addition of a term for short-term growth does add some explanatory power to the regression, although the significant t-statistic for the coefficient of $\tilde{E}_{t+1}/\overline{NE}$ comes partly at the expense of the long-term growth coefficient. The dividend and risk terms generally retain their usual signs, though they are often not significant.

4.4.4 Variations of Specification

The success of the short-term growth variable raises the question whether more generally a nonlinear specification might be appropriate. As we noted in section 3.4, the linear form of the equation is only an approximation to some more complicated true form. To investigate this

Table 4.9 **$P/\overline{NE}$ Regression Estimates of Equation (4.4-2) (asymptotic t-values adjusted for heteroscedasticity)**

Year	Constant	$\bar{g}_p$	$\tilde{E}_{t+1}/\overline{NE}$	$D/\overline{NE}$	s_g^2	R^2
1961	−35.02	3.07	41.31	−1.58	−.71	.88
	(−4.16)	(11.94)	(4.78)	(−.35)	(−.75)	
1962	−3.36	1.99	8.57	6.96	−1.00	.75
	(−.82)	(14.05)	(2.15)	(1.97)	(−4.20)	
1963	−11.43	2.58	13.66	7.22	−.53	.81
	(−2.61)	(12.25)	(4.33)	(1.57)	(−2.16)	
1964	−7.21	2.13	8.56	13.19	−.84	.81
	(−2.46)	(18.67)	(3.80)	(5.41)	(−2.52)	
1965	−14.53	2.82	10.53	8.20	−1.09	.78
	(−1.89)	(7.12)	(1.73)	(1.82)	(.99)	
1966	−7.67	1.83	6.51	8.94	−.02	.58
	(−1.94)	(10.41)	(2.00)	(3.59)	(−.28)	
1967	−8.55	2.31	9.33	1.15	−.08	.72
	(−1.41)	(12.70)	(1.67)	(.33)	(−7.18)	
1968	−15.77	1.57	18.20	4.66	−.03	.55
	(2.54)	(6.74)	(3.12)	(.96)	(−.86)	

possibility, we used a quadratic specification for the growth and dividend-payout variables. That is, we added the squares of $\overline{g}_p$ and of $D/\overline{NE}$ and their cross-product to the specification (4.4-2).

Use of these nonlinear terms did little to improve the explanatory power of the equation, though in some instances they did have significant coefficients. Stability was found neither in which variables were significant nor in their signs. Since undoubtedly our variables have substantial measurement errors, these findings may well represent little more than the problems such errors produce.

It is not surprising in view of these findings that we sometimes found that breaking the sample into various groups produced significant differences between the groups. Thus, when the equation was run separately for low-dividend/high-growth and high-dividend/low-growth companies, (where the dividing lines are the medians of the variables), we did find some significant differences in coefficients. Similarly, fitting the equation for different industry groups produced some significant differences across industries in the coefficients (e.g., dividends were more highly valued in public utility companies). Since in each case the classifications tended to reduce the variances of the independent variables, the significant differences may arise simply from the changed importance of the variances of the measurement errors relative to the variances of the true underlying variables.

4.4.5 Measurement and Estimation Error

Allowing for errors of estimation in calculating the regression coefficients did not relieve the problems we encountered when we introduced the risk measures (based on regression coefficients) directly in estimating equation (4.4-1). Using either $\hat{\beta}_M$ or the $\hat{\gamma}$ coefficients defined in equation (4.3-4), whether alone or in conjunction with s_g^2, produced neither stable nor significant coefficients for these variables when they were added to (4.4-2). It is far from clear that the reason for this finding was that such risk terms do not also play a role in valuation; in other words, we cannot conclude that a model with only one factor is appropriate. Instead, we may ascribe the findings, at least partially, to multicollinearity, particularly with the payout ratio. When these regression coefficients were added to the specification, the coefficient of $D/\overline{NE}$ usually became completely insignificant and it was highly correlated with the coefficients for β_M or for the $\hat{\gamma}_j$ coefficients. As we noted earlier, the growth variable $\overline{g}_p$ is also somewhat correlated with these risk proxies. In this connection, it is interesting to note that Rosenberg and Guy (1976) have suggested that both dividend payout and growth potential are important systematic risk variables.

Measurement errors are far from being confined to the risk variables. Clearly our growth variables are subject to error and the payout variable also is only an approximation to what the market could perceive to be the payout rate. These errors may account for some of the problems we have encountered.

As was also the case when we sought instruments for s_g^2, finding good instruments for the growth rate and the payout variables was not easy. We have already seen that $\bar{g}_p$ contains useful information not available from mechanically calculated growth rates. As a result, satisfactory instruments for it are unlikely to be found. We tried using past four- and ten-year calculated growth rates as instruments for $\bar{g}_p$ and the lagged value of $D/\overline{NE}$ for the current value of this variable. When we used the specification (4.4-1), we also included $\tilde{E}_{t+1}/\overline{NE}$ as an instrumental variable. We could also take advantage of some of the correlations of risk with growth and payout by treating $\hat{\gamma}_m$ and $\hat{\gamma}_Y$ as additional instruments when only $\hat{\gamma}_p$ and s_g^2 were used as risk measures.

Using instrumental variables to deal with these measurement errors did not substantially alter our findings. What we obtained were equations qualitatively similar to those shown in tables 4.7 and 4.9, but with much larger standard errors for the coefficients. This finding may be taken to indicate, at least, that errors in variables have not produced seriously misleading results in those tables. When the problems of multicollinearity of the growth and dividend variables with the risk ones were combined with the complicated variances of the coefficients that were the result of making allowance for the estimation error of the risk parameters, it is small wonder that more precise results could not be obtained about the precise specification of risk.

4.4.6 Stability over Time

We found earlier that the coefficients of the expected return model varied over time. The question of the constancy of the valuation equation is particularly interesting in the present form, where prices are the dependent variable. Stability of the coefficients is also important to those who wish to make practical use of valuation equations in connection with assigned values of the independent variables to estimate the "intrinsic worth" of a security. Furthermore, constancy of the relationship is important if a firm is to seek to follow policies that will maximize the values of its shares, since it will find it hard to please investors if their desires are changing.

An inspection of tables 4.7 and 4.9 indicates that the coefficients of our equations do change considerably from year to year, and in a manner that is consistent with the changing standards of value in vogue at the different times. We may illustrate this finding by the regression results of table 4.9. At the end of 1961, "growth stocks" were in high favor, and it is not surprising to find that the coefficient of the growth rate (3.07) is highest in this year. During 1962, however, there was a conspicuous change in the structure of share prices that was popularly called "the revaluation of growth stocks." This revaluation is reflected in the decline of the growth-rate coefficient for 1962 to 1.99. At the same time, dividend payout became more highly valued in 1962 than it had been in 1961, the dividend coefficient rising from -1.58 to 6.96. Nineteen sixty-two was also the year when the coefficient of the risk measure was most strongly negative.

In order to test formally whether the coefficients of the valuation equation were the same over time, we again had to recognize that the residuals in different years were not independent. The correlations, which are shown in table 4.10, are somewhat smaller than those found in section 4.3 when we were investigating the expected rates of return, but they are significantly different from zero. They again raise the need to use an appropriate technique for assessing the stability of the coefficients and the problem that calculating all the equations simultaneously for a common set of companies entails the loss of a large proportion of the observations.

Using the seemingly unrelated regression technique for a pair of years, we could reject the hypothesis of equality of the coefficients in each pair of years at least at the 0.01 level. When all years were considered simultaneously, rejection occurred beyond the 0.0001 level despite the large loss of observations. Thus it seems clear that valuation relationships do change over time. While this finding may, of course, be due to problems with the data being used, it certainly lends no credence to the proposition that the parameters do not change.

4.5 Use of the Valuation Model for Security Selection

One of the most intriguing questions concerning empirical valuation models is whether they can be used to aid investors in security selection. The estimated valuation equation shows us, at a moment in time, the average way in which variables, such as growth, payout, and risk, influence market price-earnings multiples. Given the value of these vari-

Table 4.10 **Correlations of Residuals in Adjacent Years and with Subsequent Returns**

Year	Residuals from (4.4-1)	Residuals from (4.4-2)	Residuals of (4.4-2) with Future Returns
1961/62	.52	.62	−.20
1962/63	.56	.57	.09
1963/64	.41	.46	−.25
1964/65	.30	.39	−.06
1965/66	.37	.32	.06
1966/67	.50	.48	−.03
1967/68	.60	.64	−.10
1968/69	—	—	.20

ables applicable to any specific security, we can compute an estimated price-earnings ratio based on the empirical valuation equation. The next step is to compare the actual price-earnings multiple with that predicted by the valuation equation. If the actual multiple is greater than the predicted one, we might suppose that the security is temporarily overpriced and recommend sale. If the actual price-earnings multiple is less than the predicted multiple, we might designate the security as temporarily underpriced and recommend its purchase.

Even on a priori grounds, it is possible to think of many reasons why such a procedure would prove fruitless. For example, if high growth-rate stocks tended to be overpriced during one particular period, the estimated growth-rate coefficient would be larger (by assumption) than that which is warranted. However, the recommended procedure will not indicate that these stocks are overpriced because "normal" market-determined earnings multiples for these securities will be higher than is warranted. Nevertheless, in view of the popularity of these techniques with some practitioners, it seems worthwhile to try some experiments using our data.

The results of some of our experiments are shown in table 4.10. We measured the degree of "over-" or "underpricing" as the predicted ratio of the residual from the valuation equation (4.4-2) to the predicted earnings multiple, that is, as $(P/\overline{NE} - \hat{P}/\overline{NE})/(\hat{P}/\overline{NE})$. A percentage measure was chosen in view of the considerable variance in actual earnings multiples. If the model is useful in measuring underpricing, then underpriced securities, determined according to this criterion, ought to outperform overpriced issues over some subsequent period. We picked one year as the appropriate horizon and measured subsequent returns in the usual manner as

$$(4.5\text{-}1) \qquad P_{t+1} = (P_{t+1} - P_t + D_{t+1})/P_t.$$

If the empirical valuation model is successful in selecting securities for purchase, the percentage residual (degree of overvaluation) from the valuation equation ought to be negatively related to these subsequent returns. As the fourth column of table 4.10 indicates, in only five of the eight years for which this experiment was performed was the relationship negative, and the degree of association was low. There was a positive relationship for the other three years.[11] Two of these correlations are significant at the 0.05 level: the negative one in 1963/64 and the positive one for 1968/69. The 1961/62 correlation just misses significance at this level. We would not consider these significant correlations as representing forecasting success. As we argue below, we suspect strongly that we

11. We were no more successful at finding wrongly priced securities using expectations data for the individual predictors rather than the average expectations of the particular group.

have left out some common factors and that this omission could lead to correlations over particular periods of time. Unless one can forecast these changes in a way not already available to the general market participant, one can hardly exploit these changes. It is therefore particularly indicative that one of the significant correlations had the "wrong" sign.

Supplementary tests conducted by the type of equation or industry and other groupings produced similar results. For example, subsequent returns were still unrelated to the residuals when we first split the sample into high and low growth and dividend groupings. Similar results were obtained when the experiment was attempted for separate industries. We also found that the residuals from the equations employing historical data in place of our expectational data were no more successful in predicting subsequent performance. Moreover, these results were unaltered when the subsequent returns were measured over alternative time periods such as one-quarter ahead or two or more years ahead. The technique simply did not produce excess returns in any consistent or reliable fashion over any time period in the future. These findings are what we should expect in a reasonably efficient market.

Some statistics are presented in table 4.11 that may be helpful in interpreting the reason for our predictive failures. We note, using the 1963 valuation equation as an example, that the percentage degree of under- or overpricing is not highly correlated with subsequent returns, the coefficient of determination being only 0.06. It is possible to isolate four reasons for our lack of forecasting success.

1. The first reason is that the valuation relationship changes over time. We might be unable to select truly underpriced securities because by the next year the norms of valuation have been significantly altered. Thus what was cheap on the basis of the 1963 relationship may no longer represent good value on the basis of the 1964 equation. To test how important this change might be, we performed the following experiment: We assumed that investors knew at the end of 1963 exactly what the

Table 4.11 **Analysis of Lack of Forecasting Success**

Year	Description	r^{2*}
1963	Valuation equation with 1963 predictions	.06
1964	Valuation equation with 1963 data (assumes next year's valuation relationship is known)	.10
1963	Valuation equation with realized growth rates (assumes perfect foresight regarding future long-term growth and next year's earnings)	.14
1963	Valuation equation with 1964 predictions (assumes perfect foresight regarding market expectations next year)	.27

*Percent residuals versus 1964 return.

market valuation relationship would be for the end of 1964; that is, we assumed perfect foresight regarding next year's valuation equation. Then, on the basis of the 1964 valuation equation, we used the 1963 data to calculate warranted $P/\overline{NE}$ multiples, which could then be compared with actual multiples to determine whether each security was appropriately priced. Correlating the percentage residuals with subsequent returns, we found that the coefficient of determination nearly doubled, 10 percent of the variance in subsequent returns now being explained.

2. A second reason for lack of success might be the quality of the expectations data employed. As indicated in chapter 2, the growth-rate forecasts used in the present study were not accurate predictors of realized growth. To determine how much better off we would have been with more accurate forecasts, we assumed perfect foresight regarding the future long-term growth rate of the company. Thus the 1963 empirical valuation equation was used to determine "normal" value, but in place of $\overline{g}_p$ we substituted the realized long-term growth rate through 1968. Using these realized data to determine warranted price-earnings multiples, we correlated the percentage residuals therefrom with future returns. As expected, an even greater improvement in forecasting future returns was found. The r^2 rises to 0.14.

3. As a further experiment, perfect foresight was assumed not about the actual rate of growth of earnings but rather regarding what the market expectations of growth would be next year, that is, about $\overline{g}_p$ next year. Calculating the degree of overpricing as before, we find a much greater improvement in prediction of future returns. Twenty-seven percent of the variability of future returns is now explained, compared with only 6 percent in the original experiment. We conclude that if one wants to explain returns over a one-year horizon, it is far more important to know what the market will think the growth rate of earnings will be next year rather than to know the realized long-term growth rate. This observation brings us back to Keynes's celebrated newspaper contest. What matters is not one's personal criteria of beauty but what the average opinion will expect average opinion to think is beautiful at the close of the contest.

4. A final source of error is that the valuation model does not capture all the significant determinants of value for each individual company. Despite our success in accounting for approximately three-quarters of the variance in market price-earnings multiples, there are likely to be special features applicable to many individual companies that cannot be captured quantitatively. For example, it turned out that the stock of many tobacco companies always appeared to be underpriced. The reason for this is not difficult to conjecture. There is a risk of government sanctions against the tobacco industry that weighs heavily in the minds of investors, but that is not related to the risk measures we have employed. Such an explanation is not at variance with the underlying approach to risk

valuation that we have been using. The common susceptibility of the tobacco companies to an identifiable but ignored hazard is simply an important factor which we have omitted from our data.

This problem of omitted variables may account for the correlations of residuals which we found in the equations. If certain factors specific to individual companies were consistently missing, the residuals from the valuation equations could be expected to be positively correlated over time. This is exactly what we found in table 4.10. Thus, despite our success in using expectations data to estimate a valuation equation which has far more explanatory ability than those based on historic information, it is still quite clear that certain systematic valuation factors are missing from the analysis. Consequently, it cannot be said that all deviations of actual from predicted price-earnings ratios are simply manifestations of temporary over- or underpricing.

4.6 Conclusion

Our investigations of valuation models, while not without some ambiguous results, suggest several notable conclusions. These conclusions concern the role in market valuation of the sort of earnings forecasts we have collected, the nature of risk valuation, and the efficiency of the market.

4.6.1 Valuation of Expected Growth

One of our major findings is that the average of the expected long-term growth rates, together with the risk measure provided by the variance of the growth-rate predictions, gives a closer account of the valuation of common stocks than do alternatives. These growth rates were clearly superior in accounting for prices to any of the simple alternatives we considered. More closely fitting equations are the results that one would expect from smaller errors of measurement or from using data that contain more relevant information in place of less germane measures. Hence one can safely presume that our data are more similar to the expectations being valued in the market than are measures based on *ex post* realized growth or regression coefficients. This conclusion, based on the ability to "explain" prices, is buttressed by noticing that the overall risk-free expected rates of return suggested by the estimates of the expected return regressions are of plausible orders of magnitude.

The finding that prices reflect expected growth occurred in spite of the difficulties we encountered from the large variations in which companies were covered by each of the various predictors. Earlier we saw that there is a great deal of diversity of expectations among forecasters, an aspect of reality with which valuation models do not usually cope. We also found that, while hardly being strong predictions, the expectations data appear to yield forecasts at least as accurate as, and often better than, naive forecasts based on *ex post* realizations. Furthermore, we found that we could not calculate a linear combination of different types of forecasts whose superior forecasting performance continued over time.

Efficient market hypotheses suggest that valuation should reflect the information available to investors. Insofar as analysts' forecasts are more precise than other types we should therefore expect their differences from other measures to be reflected in the market. It is therefore noteworthy that our regression results do support the hypothesis that analysts' forecasts are needed even when calculated growth rates are available. As we noted when we described the data, security analysts do not use simple mechanical methods to obtain their evaluations of companies. The growth-rate figures we obtained were distilled from careful examination of all aspects of the companies' records, evaluation of contingencies to which they might be subject, and whatever information about their prospects the analysts could glean from the companies themselves or from other sources. It is therefore notable that the results of their efforts are found to be so much more relevant to the valuation than the various simpler and more "objective" alternatives that we tried.

We saw in section 3.2.3 that diversity of expectations together with market imperfections might invalidate the valuation model. However, we also argued that there were theoretical grounds for supposing that the model would still hold for the average of investors' expectations. It is therefore of particular interest that our empirical results do support the hypothesis that prices reflect average expectations.

It is no surprise that we found roles for both short- and long-term expected rates of growth. Models of valuation using only long-term growth rates are clearly only simplifications of the more complicated processes that earnings and dividends follow over time, and we would expect market valuation to reflect the more complicated processes.

4.6.2 Risk Measures and Valuation

The results did not provide wholly unambiguous support for the specific valuation models developed here. A number of aspects of our results about risk are particularly intriguing. It is clear from our results that expected returns do seem to be related to various systematic risk factors. Equally clearly, our results do not give straightforward support to the simple form of the CAPM. It would appear that systematic risk is not entirely captured by single measures of covariance with the market index. This has important implications for those who attempt to use the modern investment technology in practical problems of portfolio selection. One such suggestion, which had attracted a considerable following in the investment community by the 1980s, was the proposal for a yield-tilted index fund.

The reasoning behind the yield-tilted index fund seems appealingly plausible. Since dividends are generally taxed more highly than capital gains and since the market equilibrium is presumably achieved on the basis of after-tax returns, the equilibrium pretax returns for stocks that pay high dividends ought to be higher than for securities that produce lower dividends and correspondingly higher capital gains. Hence the tax-exempt investor is advised to buy a diversified portfolio of high-dividend-paying stocks. In order to avoid the assumption of any greater risk than is involved in buying the market index, the tax-exempt investor is also advised to purchase a yield-tilted index fund, that is, a very broadly diversified portfolio of high-dividend-paying stocks that mirrors the market index in the sense that it has a beta coefficient β_M precisely equal to unity.

Even on a priori grounds one might question the logic of the yield-tilted index fund. Many of the largest investors in the market are tax-exempt (such as pension and endowment funds) and others (such as corporations) actually pay a lower tax on capital gains than on dividend income.[12] Thus it is far from clear that the marginal investor in the stock market prefers to receive income through capital gains rather than through dividend payments. Our theoretical arguments in chapter 3 also indicated that great care must be taken with arguments involving "marginal" investors and pointed out that the diversification theory gives no presumption that dividends and capital gains will be valued differently. But apart from these a priori arguments, our empirical results can be interpreted as providing another argument against the yield-tilted index fund.

If the traditional beta calculation (β_M) does not provide a full description of systematic risk, the yield-tilted index fund may well fail to mirror the market index. Specifically, during periods when inflation and interest rates rise, it may well be the case that high-dividend stocks are particularly vulnerable; that is, they have high δ_P and δ_r coefficients. Public-utility common stocks are a good example. While they are known as "low-beta" stocks, they are likely to have high systematic risk with respect to interest rates and inflation. This is so not only because they are good substitutes for fixed-income securities, but also because public utilities are vulnerable to a profits squeeze during periods of rising inflation because of regulatory lags and increased borrowing costs. Hence the yield-tilted index fund with $\beta_M = 1$ may not mirror the market index when inflation accelerates.

The actual experience of yield-tilted index funds during the 1979–80 period shows that these funds did not live up to expectations and their performance was significantly worse than the market. Of course, we should not reject a model simply because of its failure over any specific short-term period. Nevertheless, we believe that an understanding of the wider aspects of systematic risk, such as those analyzed here, would have

12. For corporate investors, 85 percent of dividend income is excluded from taxable income while capital gains are taxed at normal gains rates.

helped prevent what turned out to be (at least over the short term) some serious investment errors.

Our findings on systematic risk still leave some major and intriguing perplexities. We found in both versions of the valuation model that the most important aspect of risk for valuation was that represented by the extent to which forecasters were not in agreement about the future growth of the company. Exactly what is the basis for this finding is not clear.

It might be quite reasonable to interpret s_g^2 as representing specific risk. In that case, the findings go against most recent models of valuation including both the CAPM and the APT. On the other hand, it may indirectly measure sensitivity to underlying common factors and thus serve as a very effective proxy for a variety of systematic risks. Finally, it may arise from technical difficulties having to do with undetected biases in our data. It seems unlikely that this would fully account for the strength we found for this variable, but it cannot be ruled out. Further investigation probably requires a data set less beset by problems of missing observations and an adequately specified model of earnings. Overall, our results do suggest that risk undoubtedly has dimensions not fully captured by the covariances with market indexes or other variables that have dominated recent work on valuation. They also suggest that the variance of analysts' forecasts may represent the most effective risk proxy available.

4.6.3 Efficient Markets

We find it encouraging that we were unable to use the expectations data to select securities with subsequent above- or below-average performance characteristics. We would not expect that analysts' forecasts would be sounder than those apparently used by the market or that they would be irrelevant to market valuations. Apparently, the expectations formed by Wall Street professionals get quickly and thoroughly impounded into the prices of securities. Implicitly, we have found that the evaluations of companies that analysts make are the sorts of ones on which market valuation is based. Thus, while our work raises questions about some currently popular valuation theories, it strongly supports the view that the market is reasonably efficient in incorporating into present prices whatever information there is about the future.