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Chapter Author: Nissan Liviatan

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5 On Equilibrium Wage Indexation and Neutrality of Indexation Policy

Nissan Liviatan

Recent literature on wage indexation[1] stresses its important role in the area of macroeconomic stability as reflected in the variability of aggregate output and the general price level. These papers compare economic stability under alternative wage indexation regimes: nonindexed, fully indexed, and partially indexed (according to some optimal criterion imposed externally). The results of these comparisons, as presented in Fischer (1977*a*), are that indexation tends to increase stability of output when the shocks are monetary while the reverse holds when the shocks originate from the real sector.

The approach taken in this paper is an entirely different one. The existing wage indexation regime shall not be treated as something imposed on the economy from the outside, but rather as an *endogenous* phenomenon which is part of the inflationary instability itself. Our approach is that an *equilibrium* degree of wage indexation exists which is determined by market forces. Therefore the question "How does indexation affect economic stability?" is, according to our approach, entirely meaningless if it refers to equilibrium wage indexation.

The only framework where the foregoing question can have meaning is when there is some sort of outside (say government) intervention policy to change the existing indexation arrangements. For example, given a formal fractional wage indexation of 50 percent for price inflation, the government may try to increase it to 80 percent. We would then argue that there are strong tendencies to offset the proposed policy and to leave the economic system effectively at its initial equilibrium.

The equilibrium degree of indexation in our analysis is a rather different concept from that of Gray's (1976) "optimal degree of indexation,"

Nissan Liviatan is a professor in the Department of Economics, Hebrew University, Jerusalem.

1. See especially Gray (1976, 1978), Fischer (1977*a*), and Cukierman (1980).

which is derived by minimizing some macroeconomic loss function based on differences between actual and spot equilibrium output. In particular, our concept of equilibrium degree of indexation depends on risk aversion of firms and workers. If, for example, firms are risk neutral, then the equilibrium solution is full indexation.

The idea of treating wage indexation as an endogenous phenomenon has been expressed in a paper by Shavell (1976). In fact, we follow his general analytic approach in deriving the Pareto-optimal degree of indexation, which we treat as corresponding to a market equilibrium. However, his formulation is a microeconomic one which does not utilize the macroeconomic framework in determining the equilibrium degree of indexation. The importance of macroeconomics in explaining the degree of indexation can be seen from our conclusion that an economy subject to purely monetary shocks tends to adopt full wage indexation. Also, Shavell did not consider the relation of indexation to labor input and economic stability which is a basic issue in our analysis.

Another paper which treats wage indexation as endogenous is Blanchard (1979). However, his indexing is not determined within the framework of a wage contract between parties of opposing interests. Instead, his concept is closely related to Gray's optimal degree of indexation mentioned earlier.

The conceptual framework for treating wage indexation which is closest to ours is that of Azariadis (1978). The details of the models are very different, however, and it is hoped that the endogeneity aspect of wage indexation and the determination of the indexing parameter will be presented in a much clearer fashion in our model.

The foregoing papers which treat the indexing parameter as an endogeneous phenomenon do not consider the effect of a government policy of changing the parameter directly, that is, through intervention in labor market negotiations. However, government often is in a position to affect this parameter. We shall therefore consider the consequences of direct government intervention in setting the indexing parameter, given that the behavior of the labor market is to a large extent regulated by labor contracts.

The other basic issue which will be considered in this paper is the interaction between wage indexing and asset indexing. This interaction has been pointed out by several writers (e.g., Blinder [1977] and Liviatan and Levhari [1977]), though not in the context of economic stability. We try to bring out the major significance of this interaction by proving a Modigliani-Miller-type theorem which states that under a perfect bond market the degree of wage indexing is indeterminate. In particular, any arbitrary change in the indexing parameter will generate offsetting forces in the capital market which will neutralize its effect on the real economy.

The neutrality property is also shown to hold under less stringent conditions.

5.1 Wage and Asset Indexation with a Perfect Bond Market

Let us consider an economy where all firms produce a single output with a variable labor input. The amount of labor is determined at the beginning of the production period, before uncertainty is resolved, and cannot be changed during the period. This is a case of ex ante determination of labor input. Later we shall extent the model to allow ex post adjustments in labor input. It is assumed that the only marketable assets are indexed and nonindexed bonds. There is no market for shares. As in the case of production, we assume that consumption takes place at the end of the period.

Wages to be paid at the end of the period can be considered as a payment by the employers to redeem their "bonds" held by the workers. We shall assume that these "employers' bonds" are not marketable. In addition, it will be assumed that the wage payments are perfectly safe (no risk of default) so that wages are equivalent to the redemption value of a nonmarketable bond. Although employers' bonds are nonmarketable, they are equivalent to ordinary (marketable) bonds if a perfect market exists for the latter.

Suppose that the workers would like to sell some of the employers' bonds in their possession to purchase some assets or to increase their money balances. Since they cannot do this directly, they can achieve the same thing indirectly by selling other bonds in their portfolios; this includes borrowing. If the workers can use their expected wages as collateral for borrowing, then it follows immediately that their employers' bonds are marketable indirectly. In our model, however, this may not be necessary since borrowing at the beginning of the period can be used only to acquire some assets, as consumption takes place only at the end of the period. Therefore the acquired assets may provide the main guarantee against default.

The production function of firms i is given by

$$y_i(s) = \gamma_i(s)\phi_i(n_{i1}L_{i1}, \ldots, n_{ij}L_{ij}),$$

where n_i denotes the number of workers of type j employed by firms of type i, L denotes hours of work, ϕ_i is an ordinary production function, and $\gamma_i(s)$ is a random factor which varies across states of nature s.

Wages are paid at the end of the period. The wage rate consists of the nominal base wage V and the cost of living allowance $V\theta[P(1/p^0) - 1]$, where P is the actual price level at the end of the period, and θ $(0 \leq \theta \leq 1)$ is a wage-indexing parameter. In the standard forms of wage indexing,

$(1/p^0) = (1/P_{t-1})$, so that $P(1/p^0) - 1$ is the relative rate of price inflation. For analytic purposes it is sometimes more convenient to define $(1/p^0)$ as the expected value of the purchasing power of money so that $(1/p^0) = E\pi$, where $\pi = (1/p)$. Using the notation $\bar{\pi} = (1/p^0)$, we may express the real wage rate as

$$\tilde{v}(s) = V[\theta\bar{\pi} + (1 - \theta)\pi(s)] \equiv V\tilde{\pi}(s).$$

We shall allow both V and θ to vary across firms and workers so that θ_{ij} is the indexing parameter for worker type j in firm type i.

The firms' profits are given by

$$R_i(s) = \gamma_i(s)\phi_i - \sum_j n_{ij}L_{ij}V_{ij}[\theta_{ij}\bar{\pi} + (1 - \theta_{ij})\pi(s)]. \tag{1}$$

We have noted that the total wage bill can be considered as a payment to redeem employers' bonds (B^w) held by the workers.[2] These bonds can be broken up into indexed and nonindexed components. From the point of view of the firms, the payments implied by these components are equivalent to holding negative amounts of ordinary bonds, the market values of which are given by

$$B^w_{Ii} = -\frac{1}{1+r}\sum_j n_{ij}L_{ij}V_{ij}\theta_{ij}\bar{\pi}; \tag{2}$$

$$B^w_{Ni} = -\frac{1}{1+i}\sum_j n_{ij}L_{ij}V_{ij}(1 - \theta_{ij}),$$

where I and N stand for indexed and nonindexed, r is the real interest rate on indexed bonds, and i is the nominal rate on nonindexed bonds.

In addition to the implicit liabilities in (2), the firms may invest in ordinary indexed and nonindexed bonds, denoted B^o_{Ii} and B^o_{Ni}, so that the employers' income (or resources) in state s is given by

$$Y_i(s) = \gamma_i(s)\phi_i + Q_{Ii}(1 + r) + Q_{Ni}(1 + i)\pi(s),$$

where $Q_{Ii} = B^w_{Ii} + B^o_{Ii}$ and $Q_{Ni} = B^w_{Ni} + B^o_{Ni}$.

The firms have an initial endowment of bonds equal to b_i. Since we do not consider a market for the firms' shares, it will be convenient to assume that the uncertain value of the firms' output cannot be capitalized. This means that the firms' future output can be used only as collateral to borrow from workers (i.e., to pay wages at the end of the period) and cannot be exchanged for ordinary bonds. This is, of course, an arbitrary assumption, but the exclusion of a market for shares is not essential for our analysis.

Under the foregoing assumptions and using the definition $w_i \equiv B^w_{Ii} + B^w_{Ni}$, we may write

2. The affinity of wage indexation to indexed bonds has also been pointed out in Brenner (1977).

(3) $$Q_{Ni} = b_i + w_i - Q_{Ii}.$$

We may then rewrite $Y_i(s)$ as

(4) $$Y_i(s) = \gamma_i(s)\phi_i + Q_{Ii}(1 + r) + (b_i + w_i - Q_{Ii})(1 + i)\pi(s).$$

For the workers, the indexed and nonindexed components of the wage bill constitute receipts which are equivalent to those originating from holding ordinary bonds, the market values of which are

(5) $$B^w_{Iij} = (L_{ij}V_{ij}\theta_{ij}\bar{\pi})\,\frac{1}{1 + r}\,;\; B^w_{Nij} = L_{ij}V_{ij}(1 - \theta_{ij})\,\frac{1}{1 + i}\,.$$

Since the workers may also invest in ordinary (nonwage) bonds, we may express their income in state s as

(6) $$Y_{ij}(s) = Q_{Iij}(1 + r) + Q_{Nij}(1 + i)\pi(s),$$

where

$$Q_{Iij} = B^w_{Iij} + B^o_{Iij}; \qquad Q_{Nij} = B^w_{Nij} + B^o_{Nij}.$$

Define

(7) $$w_{ij} \equiv B^w_{Iij} + B^w_{Nij} = L_{ij}V_{ij}\left(\frac{\theta_{ij}\bar{\pi}}{1 + r} + \frac{1 - \theta_{ij}}{1 + i}\right) \equiv L_{ij}X_{ij}.$$

In addition, if bonds are the only asset, we have a budget constraint on ordinary bonds of the form

(8) $$b_{ij} = B^o_{Iij} + B^o_{Nij},$$

where b_{ij} is an initial endowment. Adding w and b, we obtain

(9) $$Q_{Nij} = b_{ij} + w_{ij} - Q_{Iij}.$$

In these formulations we omit, for simplicity, other forms of wealth, such as real balances, held by economic agents. The workers' income can then be expressed as

(10) $$Y_{ij}(s) = Q_{Iij}(1 + r) + (b_{ij} + w_{ij} - Q_{Iij})(1 + i)\pi(s).$$

The expected utility of workers is given by

(11) $$\underset{s}{E}U_j[Y_{ij}(s), L_{ij}],$$

with partial derivatives $U_{j1} > 0$ and $U_{j2} < 0$. Similarly, the expected utility of firms is

(12) $$\underset{s}{E}U_i[Y_i(s)].$$

We may use these functions to formulate an efficient contract problem between workers and firms as follows:

(13) $$\text{maximize } EU_i \text{ subject to } EU_j = U^0_j \qquad (j = ,\ldots, J).$$

Using (7) and the fact that

(14) $$\sum_j n_{ij} w_{ij} = w_i\,,$$

we see that the maximization is carried out with respect to the variables n_{ij}, L_{ij}, Q_{Iij}, Q_{Ii}, and X_{ij}. (We shall denote the optimal values of these variables by asterisks.) The optimal values of Q_{Ni} and Q_{Nij} are derived from (3) and (9).

In a general equilibrium of the economy, we must have

(15) $$\sum_i Q^*_{Ii} + \sum_{ij} Q^*_{Iij} = 0\,; \qquad \sum_i Q^*_{Ni} + \sum_{ij} Q^*_{Nij} = 0\,.$$

This is so since $B^w_{Ii} + \sum_j B^w_{Iij} n_{ij} = 0$ for every firm and

(16) $$\sum_i B^o_{Ii} + \sum_{ij} n_{ij} B^o_{Iij} = 0$$

for the economy as a whole. Note that if the total number of workers type j in the economy is $\bar{n}_j$, then full employment requires.

(17) $$\sum_i n_{ij} = \bar{n}_j \qquad (j = 1, \ldots, J)\,.$$

These J conditions make it possible, in principle, to determine the values of U^o_j which are treated as given parameters on the level of individual agents, while (16) and the corresponding market clearing condition for B^o_N enable us to determine i and r.

It should be stressed that, given L_{ij} and X_{ij}, the workers determine their *overall* indexed position Q_{Iij} rather than the individual indexed components B^w_{Iij} and B^o_{Iij}, and the same is true for the firms. Note also that the contract determines X_{ij} (the present value of the real wage rate) rather than V_{ij} and θ_{ij} on which it is based.

Consider the following conditions resulting from efficient contracts:

(18) $$Q^*_{Iij} = L^*_{ij} V_{ij} \theta_{ij} \bar{\pi} \left(\frac{1}{1+r}\right) + B^o_{Iij}\,;$$

$$Q^*_{Ii} = -\bar{\pi} \sum_j n^*_{ij} L^*_{ij} V_{ij} \theta_{ij} \left(\frac{1}{1+r}\right) + B^o_{Ii}\,.$$

(19) $$Q^*_{Nij} = L^*_{ij} V_{ij} (1 - \theta_{ij}) \left(\frac{1}{1+i}\right) + B^o_{Nij}\,;$$

$$Q^*_{Ni} = \sum_j n^*_{ij} L^*_{ij} V_{ij} (1 - \theta_{ij}) \left(\frac{1}{1+i}\right) + B^o_{Ni}\,.$$

(20) $$X^*_{ij} = V_{ij} \left(\frac{\theta_{ij} \bar{\pi}}{1+r} + \frac{1 - \theta_{ij}}{1+i}\right).$$

In addition, we have the market clearing conditions in the bond market

(21) $\sum_{ij} n_{ij} B^o_{Iij} + \sum_i B^o_{Ii} = 0; \qquad \sum_{ij} n_{ij} B^o_{Nij} + \sum_i B^o_{Ni} = 0.$

These conditions are not sufficient to determine V_{ij} and θ_{ij} individually. However, given θ_{ij}, we see from (20) that V_{ij} is determined uniquely. The nondeterminacy of θ_{ij} can be seen as follows: Suppose we start with equilibrium values θ^*_{ij} and V^*_{ij}. Now, let all the θ^*_{ij} increase to θ'_{ij}. If we hold the left-hand side of (20) fixed, then $\Delta\theta_{ij} = \theta'_{ij} - \theta^*_{ij}$ implies a change in V_i, say $\Delta V_j = V'_{ij} - V^*_{ij}$. Specifically, we have

(22) $$\Delta V_{ij} = -\frac{[\bar{\pi}(1+i) - (1+r)]}{\pi(1+i)\theta_{ij} + (1-\theta_{ij})(1+r)}\,\Delta\theta_{ij}.$$

It is evident that, in general, $\Delta\theta_{ij}$ will cause a change in $V^*_{ij}\theta^*_{ij}$ so that to maintain (18) B^o_{Iij} will have to change by, say, ΔB^o_{Iij}. Similarly, B^o_{Ii} will have to change by ΔB^o_{Ii} to maintain (18). It is clear, however, that for each firm $\sum_j n_{ij}\Delta B^o_{Iij} + \Delta B^o_{Ii} = 0$, so that the market for indexed bonds remains in equilibrium. A similar argument shows that the market for nonindexed bonds remains in equilibrium as required by (21). It follows that the economy as a whole is invariant to an arbitrary change in the θ_{ij}'s.

The sign $\Delta V_{ij}/\Delta\theta_{ij}$ in (22) depends on the sign of $[(1+i)\bar{\pi} - (1+r)]$. This latter expression will be positive when nonindexed bonds carry a premium over indexed bonds as a result of the inflationary risk. It has been shown in a paper by Landskroner and Liviatan (1981) that this risk premium tends to be positive when real factors contribute significantly to the variability of π. If the risk premium of nonindexed bonds is positive, then $\Delta V_{ij}/\Delta\theta_{ij} < 0$, which is in line with intuition.

Moreover, if $\Delta V_{ij}/\Delta\theta_{ij} < 0$, then the elasticity of V_{ij} with respect to θ_{ij} is less than unitary (in absolute value), provided $0 < \theta_{ij} < 1$.[3] This implies by (18) that an increase in θ_{ij} will increase the demand for indexed bonds by employers and increase supply of indexed bonds by workers (by an equal amount). By the same argument, an opposite development will take place in the market for nonindexed bonds. Thus, the bond market enables employers to hedge fully against changes in the degree of wage indexing.

It may be pointed out that the existence of perfect bond markets is not essential for neutrality of wage indexing. It is sufficient to assume that workers can make transactions in bonds only with their own employers. Let A_{Iij} and A_{Nij} denote the net redemption value of ordinary indexed and nonindexed bonds of employer i held by worker j. The incomes of workers and employers can then be expressed as

3. This elasticity is given by

$$-\frac{\theta_{ij}}{V_{ij}}\frac{\Delta V_{ij}}{\Delta\theta_{ij}} = \frac{(1+i)\bar{\pi} - (1+r)}{(1+i)\bar{\pi} + (1+r)[(1-\theta_{ij})/\theta_{ij}]}.$$

$$Y_{ij}(s) = Z_{Iij} + Z_{Nij}\pi(s), \tag{23}$$
$$Y_i(s) = \gamma_i(s)\phi_i - \sum_j n_{ij} Z_{Iij} - \sum_j n_{ij} Z_{Nij}\pi(s),$$

where

$$Z_{Iij} = L_{ij} V_{ij} \theta_{ij} \bar{\pi} + A_{Iij},$$
$$Z_{Nij} = L_{ij} V_{ij}(1 - \theta_{ij}) + A_{Nij}.$$

An optimal contract will determine equilibrium values Z_{Iij} and Z_{Nij}. It then follows that an arbitrary change in θ_{ij} can be offset by an appropriate change in A_{Iij} and A_{Nij} even when V_{ij} is held constant. In this case there is no meaning to a trade-off between θ_{ij} and V_{ij}. If, however, we impose a constraint on capital transactions between workers and their employers, then a trade-off between θ and V may emerge.

For example, if we require that

$$A_{Iij} + \delta_{ij}\bar{\pi}A_{Nij} = 0, \tag{24}$$

then a change in θ_{ij} will require a specific change in V_{ij}. A simple calculation shows that in this case

$$\frac{dV_{ij}}{d\theta_{ij}} = -\frac{V_{ij}\left[\left(\frac{1}{\delta_{ij}}\right) - 1\right]}{\frac{1}{\delta_{ij}}\theta_{ij} + (1 - \theta_{ij})}, \tag{25}$$

where the optimal values of Z, L, and n are held constant. It follows that we obtain a trade-off (negative sign for [25]) if $0 < \delta_{ij} < 1$. The latter condition means that nonindexed bonds carry a positive risk premium over indexed ones. In this respect the result is similar to that obtained earlier.

The analysis in this section is based on the existence of some form of indexed bonds. In practice a market for indexed bonds rarely exists. An exception is the Israeli economy where a highly developed market for the government's indexed bonds has been operating for a long time. It should be noted, however, that close substitutes for indexed bonds have been developed recently in the form of "variable interest loans." In these loans the linkage is to the short-term interest rate rather than to the price level, but when the interest is computed over short intervals (as in fact it is) the two types of bonds become quite similar in nature. Our indexed bonds are, of course, also a proxy for many other assets (shares, commodities, etc.) which contain a high degree of insurance against inflationary risk.

5.2 The Case Where Wage Indexing Matters, and the Determination of a Fractional θ

In the foregoing analysis we have seen how wage indexing is completely offset by transactions in bonds. The case where an arbitrary change in the wage-indexing parameter will affect the real system is where capital transactions are sufficiently imperfect. To clarify this case let us take the extreme example where transactions in bonds, or other assets, are completely ruled out.

In this case we may set i, r, and all terms involving B^o equal to zero in (18)–(20). We may then see that V_{ij} and θ_{ij} are determined uniquely by (18) and (19) or by (18) and (20). Alternatively, the optimization leading to an efficient contract may be carried out directly with respect to θ_{ij} and V_{ij}, leading to unique optimal values of these variables. In this case it is clear that if the government enforces an arbitrary value of θ, say θ^g, then the general equilibrium solution will change.

In the present case, where no offsetting by capital transactions is possible, θ becomes a significant variable in wage negotiations. We shall now argue that, in general, one should expect to find a fractional θ rather than the extreme case of full wage indexation or no indexation at all.

Two arguments can be produced to rationalize a fractional θ in equilibrium. The first argument is concerned with holdings of nominal money balances by firms and workers. In the case of workers, this will only strengthen their desire to increase θ as a hedge against inflationary risks. In the case of firms, however, a nominal obligation to pay wages acts as a hedge against inflationary risk with respect to real balances. This argument is discussed in detail in Liviatan and Levhari (1977). This may lead to a fractional θ in equilibrium.

A second argument concerns the sources of inflationary uncertainty. Assume for simplicity that all workers are identical and all firms are identical. The profit function is then given by

$$(26)\qquad \begin{aligned} R(s) &= \gamma(s)\phi(nL) - nLV[\theta\bar{\pi} + (1-\theta)\pi(s) \\ &\equiv \gamma(s)\phi(nL) - nL\tilde{v}(s). \end{aligned}$$

If the random shocks in the system originate only in the monetary side, so that $\gamma \equiv 0$, then $\mathrm{var}(R) = (nL)^2\mathrm{var}(\tilde{v})$, which is also the expression for the variance of the workers' income. Suppose that the wage is fixed in nominal terms and is given by V^0. If $E(1/P)$ remains constant, then both parties can adopt full indexation to eliminate the variance of their incomes while maintaining the same expected value of incomes by setting $v = V^0E(1/P)$. The assumption that $E(1/P)$ remains constant can be justified by using a money market equilibrium equation of the form $M = PYk$, where Y is aggregate output and k is the "Cambridge k." Since Y is held constant, this yields $E(1/P) = YkE(1/M)$, which is independent of θ. The model of exclusively monetary shocks therefore tends to support the case for full indexation. In this respect it is similar to the behavior of Gray's optimal degree of wage indexation.

Consider now the other extreme case, where M is constant and where the entire variance of prices stems from the *real* disturbance, γ. Recall that the real wage is given by

(27) $$v = V[\theta\bar{\pi} + (1-\theta)\pi] \equiv V^0\tilde{\pi},$$

so that the real wage bill can be expressed as

$$w = vLn = nLV^0\tilde{\pi},$$

Then

(28) $$\text{var}(R) = \phi^2 \text{var}(\gamma) + \text{var}(w) - 2\,\text{cov}(\gamma, w)\phi.$$

Using (27), we obtain

(29) $$\text{cov}(\gamma, w) = V^0 nL(1-\theta)\,\text{cov}(\gamma, \pi),$$

and

(30) $$\text{var}(w) = [(1-\theta)nLV^0]^2\,\text{var}(\pi),$$

so that

(31) $$\text{var}(R) = \phi^2 \text{var}(\gamma) + \left\{[(1-\theta)nLV^0]^2 - 2\Phi nLV^0(1-\theta)\frac{\text{cov}(\gamma,\pi)}{\text{var}(\pi)}\right\}\text{var}(\pi).$$

While var (R) can be regarded as relating to an individual firm, we may compute cov (γ, π)/var (π) on a macroeconomic basis using the assumption that γ is identical for all firms. Expressing our variables on a *per firm* basis, we may use the money market equilibrium to write

(32) $$\gamma = \pi(M/\bar{y}k),$$

where $\bar{y} = \phi$. Then we have (assuming $E\gamma = 1$)

(33) $$\frac{\text{cov}(\gamma,\pi)}{\text{var}(\pi)} = \frac{M}{\bar{y}k} = \frac{1}{\bar{\pi}}.$$

Using this result we may express (28) as

(34) $$\text{var}(R) = \phi^2 \text{var}(\gamma) + \left\{[(1-\theta)nLV^0]^2 - 2\phi\frac{nLV^0(1-\theta)}{\bar{\pi}}\right\}\text{var}(\pi).$$

Differentiating (34) with respect to θ, we obtain

(35) $$\frac{\partial\,\text{var}(R)}{\partial\theta} = 2(nLV^0)^2\,\text{var}(\pi)[1/S_L - (1-\theta)],$$

where $S_L = \bar{\pi}V^0 nL/\phi$ is a concept of expected labor share which must be less than unity. It follows (assuming $\theta \le 1$) that the effect of indexation on the variance of R is negative. The reason for the difference compared with the monetary shocks assumption is that now we have a positive covariance between w and γ which tends to reduce the variance of $R = \gamma\phi - w$. In view of (35), a risk-averse firm will prefer less wage indexation.

The efficient wage contract is determined by

$$\max EF(R) \text{ subject to } EU(w, L) = U^0,$$

where F and U are the utility functions of the firm and workers, respectively. The maximization is carried out with respect to n, L, V, and θ. Let us concentrate on the partial indifference curves in the (V, θ) plane.

What can be said about the slopes of these curves? We can calculate[4]

$$\partial EU/\partial V^0 = LEU_1'\tilde{\pi} > 0, \tag{36}$$

$$\partial EU/\partial\theta = -V^0LEU'(\pi - \bar{\pi}) = -V^0L \operatorname{cov}(U_1', \pi) > 0, \tag{37}$$

where the last inequality follows from $\operatorname{cov}(U_1', \pi) < 0$ as a result of $U'' < 0$. Thus both V^0 and θ have a positive effect on EU.

Turning to the firm, we find

$$\partial EF/\partial V^0 = -nLEF'\tilde{\pi} < 0, \tag{38}$$

$$\partial EF/\partial\theta = V^0nL \operatorname{cov}(F', \pi). \tag{39}$$

Since F' is a function of $w = \gamma\phi - V^0L[\theta\bar{\pi} + (1-\theta)\pi]$, we have an ambiguous sign for $\operatorname{cov}(F', \pi)$. This is so since γ is positively correlated with π by macroeconomic considerations, while $-V^0Ln(1-\theta)\pi$ is negatively correlated with π if $1 - \theta > 0$. To have a fractional θ in equilibrium, we must assume $\partial EF/\partial\theta < 0$, which can be justified by assigning sufficient importance to real shocks in the economy (as explained earlier).

If firms are risk neutral, as is assumed in various studies of implicit contracts, then $\partial F/\partial\theta = 0$ and the equilibrium solution will be $\theta = 1$. This can be seen by noting that we can regard the equilibrium solution alternatively as being derived from the maximization of EU subject to a given EF. When both firms and workers are risk averse and $\partial EF/\partial\theta < 0$, the solution will tend to be one of partial indexation, $0 < \theta < 1$.

Since both V^0 and θ are desirable items for workers, it follows that they will be willing to trade, at an appropriate exchange rate, a higher θ for lower V^0. In equilibrium exactly the same trade-off will apply to firms where both V^0 and θ are undesirable items.

The foregoing analysis is illustrated in figure 5.1 which is based on a given level of L. When firms are risk averse and real disturbances are important, then the firms' indifference curves will look like FF, with EF increasing as we move toward the origin. For a given level of the workers' $EU = U^0$, equilibrium is determined at E. If the firms are risk neutral,

4. U_R' denotes the partial derivative of U with respect to the Rth argument.

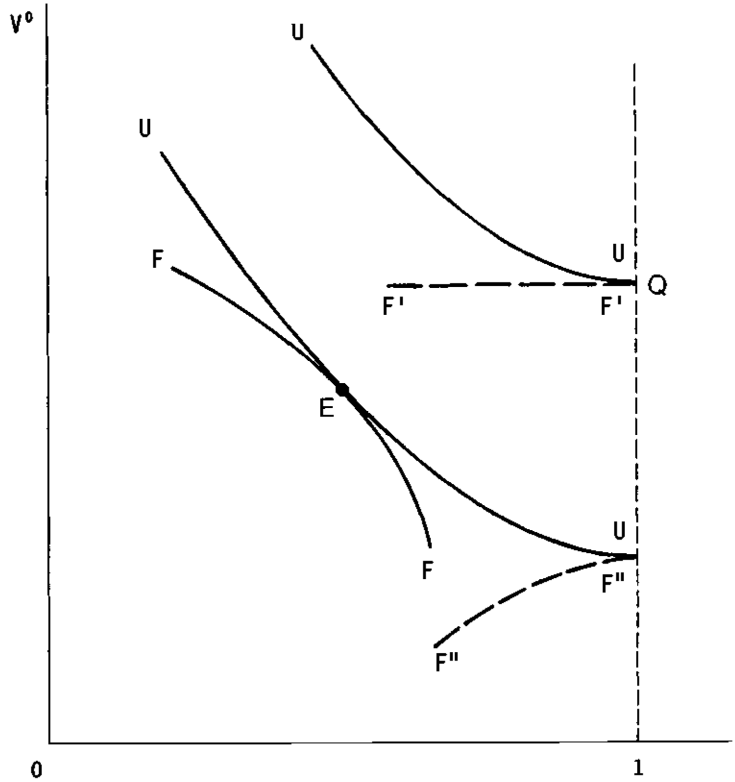

Fig. 5.1 It is assumed that all indifference curves have a zero slope at $\Theta = 1$. This is always true for the workers. For firm owners this is true under purely monetary shocks.

then their indifference curves will be horizontal, like $F'F'$, so that the solution will be at a point like Q with full indexation $\theta = 1$. Similarly, if firms are risk averse but real disturbances are insignificant, then θ is desirable for the firms so that their indifference curves will be upward sloping like $F''F''$. Clearly in this case the solution is again one of full indexation. (We assume that a solution with $\theta > 1$ is ruled out.)

Suppose that a fractional θ has been established in equilibrium, say θ^*, and let the government try to impose a higher θ, say θ^g. First, assume that the government passes a law which states that workers are entitled to require θ^g, and if they do so the firms must comply with their demand. If, however, we take into account that θ^* is the equilibrium rate, then the workers will ordinarily not be interested in using the option offered to them by the government.

The reason for this is that the option $\theta = \theta^g$ existed potentially when the negotiations leading to equilibrium took place. The reason that the workers did not insist on this option is because the firms would agree to it only if V^0 were reduced by an amount which exceeds the workers' subjective trade-off. There is therefore no reason to assume that the workers will opt to ask for θ^g after the law has been passed.

If the law takes a compulsory form, that is, firms are actually required to index wages by θ^g, then in general the equilibrium in terms of the real variables will be affected. This is a consequence of the restriction that no capital transactions are allowed. The implications of the nonneutrality of government wage indexation policy on economic stability will be discussed later.

5.3 Ex Post Adjustment of Labor Input and the Dual Labor Market

The foregoing analysis was concerned with the determination of equilibrium θ but had practically nothing to do with the problem of stability as analyzed in the recent Fischer-Gray models and related work. As we noted earlier, the foregoing works are concerned with the relation of indexation to the variability of output and prices during the period of production after the contract has been signed. Thus the conventional analysis of stability is concerned with ex post variability of output which has been ruled out in our earlier analysis.

The foregoing analysis allows, of course, ex post variability of prices. However, it is unlikely that the variability of the general price level will be significantly affected by indexation if labor input is determined on an ex ante basis. For if the money market equilibrium is $M\pi = \gamma\bar{y}k$, then the logarithmic variances satisfy var $(\log\pi) =$ var (γ/M) which is independent of $\bar{y}$.

In order to deal with the problem of stability let us turn to a model which permits ex post adjustments of labor input. In the Gray type of models the contract takes the form of setting V to conform with the "no-risk" competitive equilibrium. In addition, some value of θ is given externally. Then the workers are assumed to agree to supply, under the conditions of the contract, any quantity of services at the fixed terms when uncertainty is resolved.

This model of contracting is deficient in various respects. It is quite clear that we do not ordinarily observe this kind of contract in practice. Indeed, it seems to be an unreasonable arrangement from the point of view of the workers. The recent literature on implicit contracts stresses the risk aversion of workers, and it is clear that, quite apart from the question of indexation, the Gray contract will subject the workers' income to considerable variability resulting from the shifts of the firms' demand curves for labor. In a typical labor contract the workers will aim at stabilizing the amount of their employment at the cost of reducing their real wage (in fact, some implicit contract models lead to the result that employment will be fully stabilized).[5]

Another criticism concerning this type of model was raised by Barro (1977), who points out that the rigidity of the contract forces the parties to deviate considerably (ex post) from their best mutual interests as represented by the spot (or unconstrained) equilibrium. This implies that after the uncertainty is resolved both parties can benefit by moving toward the spot equilibrium.

The reply of Fischer (1979*b*) to this sort of criticism is that it is difficult to formulate a contract which will specify a formal procedure by which the ex post adjustment in employment à la Barro should be carried out, because there are obvious problems of moral hazard involved.

A sensible solution to these opposing views is to consider a dual system where part of the labor input is determined by ex ante contracts while another part is determined through a spot market. This is clearly the setup observed in many labor markets. There are always the long-term, wage-employment contracts which relate to the regular personnel, and there is the variable part which consists of temporary and part-time workers, ordinarily identified with special demographic groups. To the variable part we may often add overtime and special (unforeseen) assignments carried out by the regular personnel.

The employment conditions of the regular personnel which are determined in advance by a contract usually refer to a fixed amount of work determined by normal working hours. On the other hand, the conditions relating to wages and employment of the variable component are determined by something which resembles a spot market. If we consider the family as the basic decision-making unit with respect to labor supply, then we should consider the part-time work of one family member and the fixed labor input of another family member as originating from the same unit. We may therefore generally consider the ex ante determined labor input (by means of labor contracts) and the variable labor input as relating to the same optimization problem on the part of the workers. This is in fact the procedure which we shall adopt, although it is by no means essential for our conclusions.

We assume that the contractual part of the labor input is determined ex ante and fixed during the production period. This part will be denoted by L^0. In addition, the contract specifies the ex ante nominal wage, V^0, associated with L^0, and the degree of indexation, θ. The ex post amount of variable labor input supplied through the spot market will be denoted L_1^s and the corresponding real wage by v_1.

The workers' incentive to enter the contract is to reduce the risk of income fluctuations as a result of the variability of the real wage in the

5. See Sargent (1979, chap. 8).

spot market, which is in turn due to the variability of firms' demand for labor. This consideration holds quite independently of the problem of indexation. We shall begin the analysis of this model by ruling out the possibility of capital transactions and then consider the relaxation of this assumption.

When we deal with ex ante and ex post adjustment, both firms and workers face a two-stage optimization problem. The first-stage problem, which refers to the situation before uncertainty is resolved, must determine the contractual values of V^0, θ, n^0, and L^0 which are independent of the state of nature. In the second stage, uncertainty is resolved so that π and v_1 are known. At this stage, n_1 and L_1 (the number of workers and hours worked) are determined by firms and workers through an optimization process based on current market values of π and v_1. The second (ex post) stage of optimization depends, of course, on the predetermined values of the first-stage problem, but it is also clear that the first-stage optimization must take into account the distribution of π and v_1.

The firms' ex post profits are given by

$$R = \gamma\phi(n^0L^0 + n_1L_1) - n^0L^0V^0\tilde{\pi} - n_1L_1v_1. \tag{40}$$

Given the first-stage variables (n^0, L^0, V^0, and θ), the firms determine their spot market demand for labor by maximizing R with respect to the product n_1L_1. Since $n^0L^0V_0\tilde{\pi}$ can be considered (ex post) as a fixed-cost element, it will not affect the optimal solution as long as it can be covered by the firms' revenue. Hence the optimal n_1L_1 can be considered as a function of v_1, n_0L_0, and γ, say

$$n_1L_1 = L_1^d(v_1, n^0L^0, \gamma). \tag{41}$$

The workers' second-stage utility function is given by

$$U(V^0\tilde{\pi}L^0 + v_1L_1, L^0 + L_1);\ U_1' > 0, U_2' < 0, \tag{42}$$

which is maximized with respect to L_1. The second-stage supply of hours worked in the economy may then be considered as a function of $V^0\tilde{\pi}$, L_0, and v_1, say

$$\bar{n}L_1 = L_1^s(v_1, L^0, V^0\tilde{\pi}), \tag{43}$$

where $\bar{n}$ is the given number of workers in the population on a per firm basis. (This formulation assumes that all workers are employed in the spot market.) In equilibrium $n_1 = \bar{n}$ and $L^d = L^s$, so that v_1 and L_1 are determined. Thus for every set of first-stage variables a distribution of v_1 is determined.

Note also that through L_1 and L^0 the distribution of output and hence also that of prices is determined through the money market equation. In a full rational expectation equilibrium it is required that the distribution of π, determined in the foregoing manner, should be consistent with the a priori distribution of π on which the optimization problem is based.

In the first-stage problem the expected utility function of the firms is given by

$$(44)\qquad \underset{\gamma,\pi,v_1}{EF}\{\gamma\phi[L^0n^0 + L_1^d(v_1, n^0L^0, \gamma)] - V^0\tilde{\pi}L^0n^0 - v_1L_1^d(v_1, n^0L^0, \gamma)\} \equiv F^*(v^0, \theta, L^0n^0).$$

Similarly, the expected utility of workers is given by

$$(45)\qquad \underset{\pi,v_1}{EU}\left[V^0\tilde{\pi}L^0 + \frac{1}{\bar{n}}L_1^s(v_1, L^0, V^0\tilde{\pi})v_1, L^0 + \frac{1}{\bar{n}}L_1^s(v_1, L^0, v^0\tilde{\pi})\right] \equiv U^*(V^0, \theta, L^0).$$

Note that n^0 does not appear in U^* since it relates to an individual worker.

Unlike the second-stage equilibrium which is determined in a spot market, the first-stage equilibrium is determined through contracts between workers and firms which are assumed to be efficient. Consequently the firms are assumed to maximize F^* subject to U^* = constant, where the maximization is carried out with respect to n^0, L^0, V^0, and θ.

The maximization problem implies that, for internal solutions, V^0, θ, and L^0 should have opposite effects on F^* and U^*. The considerations leading to opposite effects of V^0 and θ were presented earlier (see [36]–[38]). As for L, it is assumed that the dominant factor is the workers' desire to ensure themselves against the real wage fluctuations in the spot market. In view of our earlier discussion in this section, we may assume that in equilibrium the effect of L^0 and U^* is positive while the effect on F^* is negative.

The structure of the first-stage problem has been reduced essentially to that of the model with ex ante determination of labor input which we have discussed in earlier sections. We may therefore apply some of the conclusions reached earlier. In particular, it is evident that when perfect bond markets exist, then θ will cease to be a relevant consideration for wage contracts. Consequently, an arbitrary imposition of θ^g will have no effect on L^0 nor on realized real wealth at the end of the period. As a result of this, the spot market supply and demand functions will be unaffected, and therefore the distribution of output will remain unchanged. Similarly, private loans between workers and firms, even when no general bond markets exist, will neutralize the effect of θ^g.

If follows from the foregoing remarks that external intervention in setting the degree of wage indexing can be effective only if the offsetting mechanism originating in the capital market is inoperative. A detailed analysis of the effect of a government's indexation policy under the latter conditions will not be undertaken in this paper.

As might be expected on the basis of earlier studies, the effect of indexation policy on economic stability is related to sources of random shocks to the economy. If, for example, the shocks are purely monetary, then, as we have seen, the equilibrium degree of indexing will be unitary. In this case, indexing will fully neutralize the effect of monetary shocks on the real system. If θ is reduced arbitrarily below unity, then part of the nominal shocks will be transferred to the real sectors. Thus reducing indexation is destabilizing. Alternatively, if the shocks originate entirely from the real sector, then an arbitrary increase in θ will tend to destabilize output. Thus, in the absence of capital transactions the results of our model tend to be in line with the Fischer-Gray analysis.

It should be stressed however that if an arbitrary change in θ^g increases stability of output (as is the case when θ is increased in a model where all shocks are "real"), it does not mean that this step is desirable. From the point of view of individual welfare, an arbitrary change in θ causes a deviation from the Pareto-optimal solution, which is undesirable. In particular, an increase in θ will ordinarily cause a reduction in the wage base V^0 (as we have seen when we dealt with perfect bond markets) to a greater extent than warranted by the workers' subjective trade-off between these variables.

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Comment Mario Henrique Simonsen

Wage indexation can be understood as a form of risk sharing between workers and firms. It only covers one single type of risk, the one corresponding to unanticipated changes in the general price level and, as such, it is a highly incomplete insurance scheme. Yet it is simple enough to be enforceable without problems of moral hazard and without the costs of describing and checking all the possible states of nature, as in the Arrow-Debreu (1971) model of general equilibrium under uncertainty.

Different views have been expressed recently on the effects of wage indexation based on different asumptions about how labor contracts are written. Let me summarize a few of them.

Gray's model (1976) accepts the degree of wage indexation as exogenously given. The wage basis is specified in the labor contract, being determined by a Walrasian auctioneer who clears the ex ante labor market under rational expectations. The level of employment is determined by the ex post labor demand curve after uncertainties have been realized. With such assumptions, one easily concludes that indexation protects output against demand shocks and monetary noises but overexposes it to supply shocks. In any case, the higher the degree of wage indexation, the higher the price instability is in face of either type of shock. Gray defines the optimal degree of indexation as the one which minimizes the variance of the difference between the actual and spot equilibrium output measured by their logs. A central planner aiming to stabilize employment would have strong reasons to choose such an indexation parameter. Yet there seems to be no reason why decentralized market forces would lead to such a result. According to Gray's analysis, the optimal degree of indexation is positive but less or equal to one. Full wage indexation is optimal in the above sense if and only if shocks are purely nominal.

Mario Henrique Simonsen is a professor at Fundacao Getulio Vargas, Rio de Janeiro.

Similar results obtained by Stanley Fischer (1977) were challenged by Barro (1977) who questioned why contracting parties should conform to Pareto-inefficient arrangements instead of moving to the welfare superior, spot market equilibrium. Barro's questions are of this type: "Why does the real world not behave like the Arrow-Debreu model with uncertainty?" or "Why does Coase's theorem not hold in so many cases?" But they stress an important point: the lack of consensus among economists about how labor contracts are actually written. Fischer's reply emphasizes that enforceable labor contracts cannot follow the Arrow-Debreu model of contingent claims because of moral hazard problems and because of the costs of specifying all the possible states of nature, and that labor contracts should conform to the simpler standards of efficiency described by Azariadis and Bailey. This is a sensible answer except that the Gray-Fischer indexation model assumes a Keynesian labor contract which is not efficient by the Azariadis (1976) standards. Problems of moral hazard and heterogeneous expectations can also oppose the enforcement of the Azariadis based contracts, and this is the only reason why wages and employment might behave according to the Gray-Fischer model.

Liviatan's paper takes a different look at the problem, treating wage indexation as an endogenous phenomenon. If a perfect capital market exists, with indexed and nominal bonds, the degree of wage indexation is irrelevant since any economic agent can choose his own indexation degree just as in the Modigliani-Miller theorem. If wages are not indexed and if a worker prefers to have them fully indexed, he just has to sell a nominal bond corresponding to his future pay and buy an indexed one, and so on. The case where the wage indexation degree does matter is the one where transactions with bonds are ruled out. In this case one should expect the degree of indexation to be determined by efficient labor contracting in the Azariadis sense. Workers are always assumed to be risk averse, and in two important cases efficient contracting will lead to full wage indexation: (a) when the firm is risk neutral; (b) when the firm is risk averse, but shocks are purely nominal. If both contracting parties are risk averse and if real shocks are brought onto the scene, the outcome may be fractional indexing.

According to Liviatan's analysis, a law which imposes a certain degree of wage indexation will be inoperative, provided firms can exchange indexed bonds for nominal bonds and vice versa with their own workers. (This is a much weaker assumption than the one of the existence of a perfect capital market.) Only if this escape valve is prevented from working will mandatory indexation produce meaningful economic results, the first of which would be to move the system to a Pareto-inferior equilibrium. Even in this case, the degree of wage indexation will not affect price and output stability if the labor input of each firm is determined before uncertainties are realized, as assumed by Liviatan in section 5.2 of his paper. To escape this neutrality theorem one must accept the possibility of ex post adjustment of the labor input. Liviatan's section 5.3 discusses this possibility through a dual labor market model, in which regular workers provide the ex ante labor input and where the variable labor force is supplied by temporary or part-time workers or by overtime assignments carried out by the regular personnel. Although the model fails to explain why regular workers may be laid off during recessions, its conclusions are in line with those of the Gray-Fischer analysis.

In the following comments I shall argue that the cases discussed by Liviatan which lead to full wage indexation exhibit strong efficiency properties which are absent in those corresponding to fractional indexation, which appears to be a poor hedge against supply shocks. This might help to explain why fractional indexation is not a popular clause in labor contracts which, at least implicitly, can adopt more efficient arrangements.

Throughout this discussion I will assume that labor contracts conform to the following hypotheses of Azariadis:

(a) An enforceable labor contract extends for N periods. In each period m different states of nature may occur, with probabilities g_1, $g_2, \ldots, g_m$. (The g_s are positive, and $g_1 + \ldots + g_m = 1$.) The possible states of nature and their probabilities are the same for the different periods covered by the contract.

(b) Expectations are homogeneous in the sense that the firm and its workers share the same description of the various states of nature and the same knowledge about their probabilities.

(c) The firm recruits N workers when the contract is signed and makes the commitment to employ N_s randomly chosen among then ($N_s \leq N$) at the real wage W_s in every period in which the state of nature s occurs; ($s = 1, \ldots, m$).

(d) No worker can contract his services with more than one firm, and firms are not allowed to employ individuals who were not assigned to them in the labor contract; any possibility of default is ruled out.

(e) There are only full-time jobs.

Let us indicate by $m_s f(N_s)$ the net real revenue of the firm, and hence by $R_s = m_s f(N_s) - W_s N_s$ its real profit in the state of nature s; $f(N_s)$ is assumed to be concave and nondecreasing in N_s; m_s represents the supply shock which may result from a proportional displacement of the production function or from a change in relative prices.

The firm will be assumed to be either risk averse or risk neutral with an utility function $F(R)$, where R indicates its real profit. Its expected utility will be given by:

$$EF = \sum_{s=1}^{m} g_s F(R_s) = \sum_{s=1}^{m} g_s F[m_s f(N_s) - W_s N_s]. \tag{1}$$

Let us assume that all workers are risk averse with the same utility function $V(Y, L)$, where Y indicates the individual real income, and L the leisure time per period. Since only full-time jobs are offered by the firm, there are only two possible values for L: L_0 for the unemployed individual and L_1 for the employed one ($L_1 < L_0$). Every worker is assumed to receive a capital income Y_0 and an unemployment compensation W_0 is paid by the government. (Of course, the possibilities $Y_0 = 0$ and $W_0 = 0$ are not ruled out.) This is to say that the utility of the unemployed individual in any state of nature is given by $V(Y_0 + W_0, L_0)$, moving to $V(Y_0 + W_s, L_1)$ when he is employed in the state of nature s.

Since the origin of a von Neumann-Morgenstern utility scale can be arbitrarily chosen, we shall take the utility of the unemployed individual $V(Y_0 + W_0, L_0) = 0$ and define $U(W_s) = V(Y_0 + W_s, L_1)$. $U(W_s)$ is the utility of the employed worker in state of nature s. Since the probability of being hired by the firm in that state is equal to N_s/N and since the utility of the unemployed is equal to zero, the expected utility of the individual in state of nature s is given by $(N_s/N)\ U(W_s)$, and his expected utility when the labor contract is signed is indicated by:

$$EU = \sum_{s=1}^{m} g_s \frac{N_s}{N} U(W_s). \tag{2}$$

We shall assume $U(W)$ to be twice differentiable, with $U'(W)>0$ (because of nonsatiation) and $U''(W)<0$ (because of risk aversion). A similar assumption will be made for the firm's utility function, except that now $F''(R) \leq 0$, since the firm may be either risk averse or risk neutral.

Let us indicate by U_0 the expected utility offered to the individuals by a competitive labor market. The problem of the firm is to choose the wage-employment plan $(W_1, W_2, \ldots, W_m)$, $(N, N_1, N_2, \ldots, N_m)$ which maximizes EF subject to $EU \geq U_0$. For any given employment program $(N, N_1, \ldots, N_m)$, the Kuhn and Tucker theorem yields:

$$\frac{F'(R_1)}{U'(W_1)} = \frac{F'(R_2)}{U'(W_2)} = \ldots = \frac{F'(R_m)}{U'(W_m)} = \lambda. \tag{3}$$

This is the well-known Arrow-Borch condition, according to which the ratio of the marginal utilities of the contracting parties should be independent of the state of nature in an efficient risk-sharing scheme.

Let us now analyze the two cases where full indexation is the outcome of efficient wage contracting: (a) when the firm is risk neutral; (b) when the firm is risk averse but shocks are purely nominal.

If the firm is risk neutral, the marginal utility of its income is a constant. Hence, by the Arrow-Borch condition, $U'(W_s)$ should be invariant to the state of nature. Since, because of risk aversion, $U'(W)$ is a decreasing function of W, the above result implies full wage indexation, that is, W_s should be independent of the state of nature. The theorem can be proved without differentiable hypotheses on the utility functions. It is enough to remember that when the firm is risk neutral, EF can be identified with the firm's expected profit. A wage-employment program $(W_1, \ldots, W_m)$, $(N, N_1, \ldots, N_m)$ with different real wages across the states of nature would be dominated by one which would make wages uniform at the unique real level W such that:

$$W \sum_{s=1}^{m} g_s N_s = \sum_{s=1}^{m} g_s W_s N_s .$$

In fact, making wages uniform by the above formula would not change the firm's expected profit and, because of strict concavity, would increase workers' utility.

If the firm is risk averse but shocks are purely nominal, then $m_s = 1$ for every state of nature. Let us assume that $(W_1, \ldots, W_m)$, $(N, N_1, \ldots, N_m)$ is a wage-employment program where real wages and employment are not invariant to the state of nature. We shall prove that this program is dominated by one with invariant wages $\bar{W}$ and employment $\bar{N}$ where:

$$\bar{N} = \sum_{s=1}^{m} g_s N_s ,$$

$$\bar{N}\bar{W} = \sum_{s=1}^{m} g_s W_s N_s .$$

In fact, since both the firm's utility function and its production function are concave and nondecreasing,

$$EF = \sum_{s=1}^{m} g_s F[f(N_s) - W_s N_s] \leq F[f(\bar{N}) - \bar{W}\bar{N}] .$$

Let us now observe that

$$\bar{N} \leq N ,$$

and that because of strict concavity in the workers' utility function,

$$\sum_{s=1}^{m} g_s N_s U(W_s) \leq \bar{N}\, U(\bar{W}) .$$

Since wages and employment are not uniform in the initial program, at least one of the above inequalities must hold with strict sign. Hence,

$$EU = \sum_{s=1}^{m} \frac{N_s}{N} U(W_s) < U(\bar{W}) .$$

Summing up, if shocks are purely nominal, the firm's expected utility will not decrease and the workers' utility will increase if wages and employment are made invariant to the state of nature at $\bar{W}$ and $\bar{N}$. This, once again, implies full wage indexation.

The latter result is by no means surprising. In this very special case, full indexation of all contracts solves the Arrow-Debreu model with uncertainty.

What about fractional indexation? The problem is that now W_s cannot be freely determined for each state of nature but must be expressed as a function of two single independent variables, the wage basis V_0 and the indexation degree θ. Using Liviatan's symbols, if $\bar{\pi}$ is the inverse of the general price level when the labor contract is signed, and π_s the corresponding value to state of nature s, then

$$W_s = V_0[\theta\bar{\pi} + (1-\theta)\pi_s].$$

This is to say, if there exist $m > 2$ possible states of nature, that $m - 2$ degrees of freedom will be lost by fractional indexation arrangements, and that these arrangements should not be expected to fulfill the Arrow-Borch condition. Of course, if fractional indexation is considered the only possible device to make real wages contingent on the state of nature, an optimal indexation degree will always be found, although not necessarily in the (0, 1) interval. Yet the strong efficiency properties of full indexation when the firm is risk neutral or when shocks are purely nominal will not hold in the fractional indexation schemes. This is why, I think, economic agents will always find some strong incentive to move to more efficient forms of wage contracting.

One possible scheme is full wage indexation combined with a profit-sharing plan. When both parties are risk averse, the Arrow-Borch condition implies that real wages should be an increasing function of the firm's real profits. A plausible additional assumption is that the firm is relatively less risk averse than its workers, in the Arrow-Pratt sense, that is,

$$A_F = -\frac{RF''(R)}{F'(R)} < \frac{WU''(W)}{U'(W)} = A_W.$$

Differentiation of the Arrow-Borch condition yields

$$A_F\frac{dR}{R} = A_W\frac{dW}{W},$$

which means that real wages should change in the same direction, although proportionally less than real profits. A fixed real wage plus a fixed share in profits, $W_s = W + aR_s$, might be an acceptable first order approximation to the efficient wage plan.

Moral hazard problems or inadequacy of first order approximations might make the idea of linking wages to profits undesirable. In this event all one can get from the Arrow-Borch equation is that wages should be fully indexed but adjusted for supply shocks. Of course, the measurement of supply shocks, especially at the microeconomic level which encompasses the firm's relative price position, would be too much of a nightmare to be included as an explicit clause of any feasible labor contract.

Yet, as an implicit clause it might work. Wages are settled in nominal terms but periodically revised. In an inflationary environment, the increase in the consumer price index appears to be the main determinant of nominal wage adjustments. Still, enough flexibility is kept so as to adapt real wages to supply shocks.

The present world is crowded with both real and nominal shocks. According to Liviatan's model, fractional indexation should be the rule, full indexing or no indexing at all being the exceptions. Empirical evidence suggests the opposite, namely, that fractional indexation is the exception. It is not even a necessary intermediate step between nominal wage contracting and full indexation when inflation rates escalate, as shown by the experience of so many countries. This can be explained by the assumption that economic agents will instinctively move to some approximate solution of the Arrow-Borch equation.

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