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# Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence

# Introduction

The study of aggregate consumption behavior was profoundly altered by the rational expectations revolution in macroeconomics. The first example in Robert Lucas's (1976) influential critique of econometric policy evaluation involved consumption. Lucas argued that traditional consumption functions, no matter how well they fit the data, were not useful for evaluating the effects of alternative policies. Soon thereafter, Robert Hall (1978) proposed a new approach to studying consumption that was firmly founded on the postulate of rational expectations and that was immune to the problems Lucas pointed out. Hall suggested that aggregate consumption should be modeled as obeying the first-order conditions for optimal choice of a single, fully rational, and forward-looking representative consumer. The new style of research based on this assumption—sometimes called the "Euler equation approach"—has dominated work on consumption during the past decade.

In this paper we appraise what has been learned about aggregate consumption from this approach. We propose a simple, alternative characterization of the time series data on consumption, income, and interest rates. We suggest that the data are best viewed as generated not by a single forward-looking consumer but by two types of consumers. Half the consumers are forward-looking and consume their permanent income, but are extremely reluctant to substitute consumption intertemporally in response to interest rate movements. Half the consumers follow the "rule of thumb" of consuming their current income. We document three empirical regularities that, we argue, are best explained by this model.

The first regularity is that expected changes in income are associated with expected changes in consumption. In contrast to the simplest version of the permanent income hypothesis, consumption is not a random walk: when income is expected to rise by 1 percent, consumption should be expected to rise by 0.5 percent. The strong connection between current income and consumption provides at least circumstantial evidence for "rule-of-thumb" behavior on the part of some consumers.

The second empirical regularity is that expected real interest rates are not associated with expected changes in consumption. This means that the predictable movements that we observe in consumption cannot be explained as a rational response to movements in real interest rates. It also means that forward-looking consumers do not adjust their consumption growth in response to interest rates, so their intertemporal elasticity of substitution in consumption must be close to zero. Hall (1988) also argues that the elasticity of substitution of permanent income consumers is small; but since he does not allow for current income consumers, he cannot explain the existence of any predictable movements in aggregate consumption.

The third empirical regularity is that periods in which consumption is high relative to income are typically followed by rapid growth in income. This finding suggests that at least some consumers are forward-looking: their knowledge of future income growth is reflected in current consumption. Yet we show that the magnitude of the association between consumption and future income growth is best explained by a model with both permanent income consumers and current income consumers.

Most of this paper is devoted to analyzing the data and documenting its consistency with the simple model we propose. In the final section, we briefly discuss the broader implications for economic policy and economic research.

# 1. Is Consumption a Random Walk?

In this section we reexamine the evidence on the simplest version of the permanent income hypothesis, according to which consumption should follow a random walk. We begin by reviewing the basic model and discuss how it can be tested. Our approach differs from the standard one in two ways. First, we emphasize a specific alternative hypothesis under which some consumers follow the "rule of thumb" of consuming their current income rather than their permanent income. Second, we argue that more structural estimation using instrumental variables should be preferred over the standard tests for a random walk using the reduced form of the

model. When we look at the data, we find that a substantial fraction of income accrues to rule-of-thumb consumers, indicating an economically important deviation from the permanent income hypothesis.<sup>1</sup>

# 1.1. THE PERMANENT INCOME HYPOETHESIS AND A RULE-OF-THUMB ALTERNATIVE

The permanent income hypothesis as usually formulated assumes that aggregate consumption can be modeled as the decisions of a representative consumer. The representative consumer maximizes

$$E_{t} \sum_{s=0}^{\infty} (1+\delta)^{-s} U(C_{t+s}) \qquad U' > 0, \qquad U'' < 0 \quad (1.1)$$

where C is consumption,  $\delta$  is the subjective rate of discount, and  $E_t$  is the expectation conditional on information available at time t. If the representative consumer can borrow and lend at the real interest rate r, then the first-order condition necessary for an optimum is

$$E_t U'(C_{t+1}) = \left(\frac{1+\delta}{1+r}\right) U'(C_t).$$
 (1.2)

This says that marginal utility today is, up to a constant multiple, the best forecast of marginal utility tomorrow.

If we assume that  $r = \delta$  and that marginal utility is linear, then we obtain the random walk result,  $E_t C_{t+1} = C_t$ . Consumption today is the optimal forecast of consumption tomorrow. This in turn implies

$$\Delta C_t = \epsilon_t \quad (1.3)$$

where  $\epsilon_i$  is a rational forecast error, the innovation in permanent income. Thus, according to this formulation of the permanent income hypothesis, the change in consumption is unforecastable.

In evaluating how well this model fits the data, it is useful to keep in mind an explicit alternative hypothesis. We nest the permanent income hypothesis in a more general model in which some fraction of income  $\lambda$ 

1. Obviously, these assumptions can be justified only as an approximation. One can obtain the random walk result with other sorts of approximations as well, e.g., the Taylor approximation in Mankiw (1981) or the log-normality assumption in Hansen and Singleton (1983). These other approximations may imply that the log of consumption, rather than the level, is a random walk—a more appealing specification. They also often introduce other terms, such as the difference between  $\delta$  and r and the variance of consumption growth; these other terms are usually included as part of the constant drift in consumption.

accrues to individuals to consume their current income, while the remainder  $(1-\lambda)$  accrues to individuals who consume their permanent income. If the incomes of the two groups are  $Y_{1t}$  and  $Y_{2t}$  respectively, then total income is  $Y_t = Y_{1t} + Y_{2t}$ . Since the first group receives  $\lambda$  of total income,  $Y_{1t} = \lambda Y_t$  and  $Y_{2t} = (1-\lambda)Y_t$ . Agents in the first group consume their current income, so  $C_{1t} = Y_{1t}$ , implying  $\Delta C_{1t} = \Delta Y_{1t} = \lambda \Delta Y_t$ . By contrast, agents in the second group obey the permanent income hypothesis, implying  $\Delta C_{2t} = (1 - \lambda)\epsilon_t$ .

The change in aggregate consumption can now be written as

$$\Delta C_t = \Delta C_{1t} + \Delta C_{2t} = \lambda \Delta Y_t + (1 - \lambda)\epsilon_t. \quad (1.4)$$

Under this alternative hypothesis, the change in consumption is a weighted average of the change in current income and the unforecastable innovation in permanent income. Equation (1.4) reduces to the permanent income hypothesis, equation (1.3), when  $\lambda = 0.2$ 

Having set up the permanent income hypothesis as the null hypothesis and the existence of these rule-of-thumb consumers as the alternative hypothesis, there are two approaches to estimation and testing. The approach we advocate is to estimate  $\lambda$  directly and test the hypothesis that  $\lambda = 0$ . It is important to note, however, that (1.4) cannot be estimated by Ordinary Least Squares, since the error term  $\epsilon_t$  may be correlated with  $\Delta Y_t$ . The solution is to estimate (1.4) by instrumental variables. Any lagged stationary variables are potentially valid instruments since they are orthogonal to  $\epsilon_t$ . Of course, good instruments must also be correlated with  $\Delta Y_t$ —therefore, one should choose lagged variables that can predict future income growth. Once such instruments are found, one can easily estimate the fraction of income accruing to the rule-of-thumb consumers.

The second approach to testing the permanent income hypothesis—used by Hall (1978) and in most of the subsequent literature—is to regress the change on consumption on lagged variables to see whether the change in consumption is forecastable. To see the relation between the two approaches, note that equation (1.4), estimated by instrumental variables, can be viewed as a restricted version of a more general two-equation system in which  $\Delta C_t$  and  $\Delta Y_t$  are regressed directly on the

<sup>2.</sup> This alternative model with some rule-of-thumb consumers is discussed briefly in Hall (1978). It is also a simpler version of the model proposed in Flavin (1981), in which the change in consumption responds not only to the contemporaneous change in current income, but also to lagged changes in current income. Flavin designs her model so that it is just-identified; by contrast, we view the over-identification of our model as one of its virtues. See also Bean (1986).

instruments. If we have K instruments,  $X_{1t}$  through  $X_{Kt}$ , then the general system is

$$\Delta C_{t} = \beta_{0} + \beta_{1} X_{1t} + \dots + \beta_{K} X_{Kt} + \eta_{Ct} = X_{t} \beta + \eta_{Ct}$$

$$\Delta Y_{t} = \gamma_{0} = \gamma_{1} X_{1t} + \dots + \gamma_{K} X_{Kt} + \eta_{Yt} = X_{t} \gamma + \eta_{Yt}. \quad (1.5)$$

The permanent income hypothesis implies that the vector  $\beta = 0$  (that is,  $\beta_1 = \ldots = \beta_K = 0$ ). This implication can be tested directly, without any need for considering the  $\Delta Y_t$  equation, by OLS estimation of the  $\Delta C_t$  equation. When there is more than a single instrument, however, equation (1.4) places over-identifying restrictions on the two equation system (1.5): predictable changes in consumption and income, and therefore the vectors  $\beta$  and  $\gamma$ , are proportional to one another ( $\beta = \lambda \gamma$ , or  $\beta_1/\gamma_1 = \ldots = \beta_K/\gamma_K = \lambda$ ). The instrumental variables test that  $\lambda = 0$  is in essence a test that  $\beta = 0$  under the maintained hypothesis that these overidentifying restrictions are true.

Although estimating the reduced form equation for  $\Delta C_t$  is more standard, there are compelling reasons to prefer the instrumental variables approach. One reason is power. Since there are many possible instruments, the instrumental variables procedure estimates far fewer parameters than are in the reduced form, thereby conserving on the degrees of freedom and providing a more powerful test of the null hypothesis.

Perhaps more important, estimation of  $\lambda$  provides a useful metric for judging whether an observed deviation from the null hypothesis is economically important. As Franklin Fisher (1961) emphasized long ago, an economic model can be approximately true even if the strict tests of overidentification fail. It is therefore hard to interpret a rejection of the permanent income hypothesis in the reduced form framework. Indeed, Hall (1978) concluded that the evidence favors the permanent income hypothesis even though he reported formal rejections using stock prices. An estimate of  $\lambda$  is more informative about the economic importance of deviations from the theory.<sup>3</sup> For example, if the estimate of  $\lambda$  is close to zero, then one can say the permanent income is approximately true—most income goes to consumers who obey the theory—even if the estimate of  $\lambda$  is statistically significant. Conversely, if the estimate of  $\lambda$  is large, then one must conclude that the evidence points away from the permanent income hypothesis.

One question that arises in interpreting a failure of the permanent

income hypothesis is whether our rule-of-thumb alternative adequately captures the reason for the failure. The best way to answer the question is to consider explicitly other alternative hypotheses.<sup>4</sup> Another way—more statistical and less economic—is to test the over-identifying restrictions that equation (1.4) imposes. This test is performed simply by regressing the residual from the instrumental variables regression on the instruments, and then to compare T times the  $R^2$  from this regression, where T is the sample size, with the  $\chi^2$  distribution with (K-1) degrees of freedom. We use this test below.

### 1.2. TWO SPECIFICATION ISSUES

Before we can estimate the model, we need to address two issues of specification that arise from the nature of the aggregate time series on consumption and income.

Our discussion so far has been couched in terms of levels and differences of the raw series  $C_t$  and  $Y_t$ . This is appropriate if these series follow homoskedastic linear processes in levels, with or without unit roots. Yet aggregate time series on consumption and income appear to be closer to log-linear than linear: the mean change and the innovation variance both grow with the level of the series. A correction of some sort appears necessary. The approach we take is simply to take logs of all variables. Although the parameter  $\lambda$  can no longer be precisely interpreted as the fraction of agents who consume their current income, one can view the model we estimate as the log-linear approximation to the true model. Thus, the interpretation of the results is not substantially affected. We use lower-case letters to denote log variables.

A second data problem is that consumption and income are measured as quarterly averages rather than at points in time. If the permanent income hypothesis holds in continuous time, then measured consumption is the time average of a random walk. Therefore, the change in consumption will have a first-order serial correlation of 0.25, which could lead us to reject the model even if it is true.<sup>6</sup> We deal with this problem by lagging the instruments more than one period, so there is at least a two-period time gap between the instruments and the variables in equation (1.4). The time average of a continuous-time random walk is uncorrelated with all variables lagged more than one period, so by using twice-lagged instruments we obtain a test of the model that is valid for time-averaged data.

6. See Working (1960).

<sup>4.</sup> For some examples see Campbell and Mankiw (1987).

<sup>5.</sup> An alternative scaling method is to divide  $\Delta C_i$  and  $\Delta Y_i$  by the lagged level of income,  $Y_{i-1}$ . In practice both scaling methods give very similar results.

### 1.3. ANOTHER LOOK AT U.S. DATA

To estimate our model, we use standard U.S. quarterly time series data, obtained from the Data Resources, Inc. data bank.  $Y_t$  is measured as disposable personal income per capita, in 1982 dollars.  $C_t$  is consumption of non-durables and services per capita, in 1982 dollars. The sample period is 1953:1 to 1986:4.<sup>7</sup>

Table 1, which reports the results, has six columns. The first gives the row number and the second the instruments used. The third and fourth columns give the adjusted  $R^2$  statistics for OLS regressions of  $\Delta c_i$  and  $\Delta y_i$ , respectively, on the instruments. In parentheses we report the p-value for a Wald test of the hypothesis that all coefficients except the intercept are zero. The fifth column gives the instrumental variables estimate of  $\lambda$ , with an asymptotic standard error. The final column gives the adjusted  $R^2$  statistic for an OLS regression of the residual from the instrumental variables regression on the instruments. In parentheses we report the p-value for the corresponding test of the over-identifying restrictions placed by equation (1.4) on the general system (1.5). For reference, the first row of Table 1 shows the coefficient obtained when we estimate equation (1.4) by OLS.

Rows 2 and 3 of the table use lagged income growth rates as instruments. These are not strongly jointly significant in predicting consumption or income growth; in row 3, for example, lags two through six of income growth are jointly significant at the 21% level for consumption growth and at the 14% level for income growth. It appears that the univariate time series process for disposable income is close enough to a random walk that income growth rates are not well forecast by lagged income growth rates. Our instrumental variables procedure estimates  $\lambda$  at 0.506 with an asymptotic standard error of 0.176 in row 3; this rejects the permanent income hypothesis that  $\lambda = 0$  at the 0.4% level. Yet instrumental variables procedures can be statistically unreliable when the instruments have only weak forecasting power for the right hand side variable. The rejection of the permanent income hypothesis in rows 2 and 3 should be interpreted cautiously. The rejection of the permanent income hypothesis in rows 2 and 3 should be interpreted cautiously.

<sup>7.</sup> In Campbell and Mankiw (1987) we discuss the importance of sample period and, in particular, the peculiar behavior of the first quarter of 1950, when there was a one-time National Service Life Insurance dividend payment to World War II veterans. The sample period of Table 1 extends the data used in Campbell and Mankiw (1987) by one year.

A constant term is always included as both an instrument and a regressor, but is not reported in the tables.

<sup>9.</sup> See Nelson and Startz (1988) for an analysis of this issue.

<sup>10.</sup> These findings confirm the conclusions of Mankiw and Shapiro (1985): since disposable income is so close to a random walk, modelling income as a univariate process (e.g., Flavin (1981) or Bernanke (1985)) leads to tests with little power.

We obtain stronger results in row 4 and 5 of the table, where we use lagged consumption growth rates as instruments. It is striking that lagged consumption forecasts income growth more strongly than lagged income itself does, and this enables us to estimate the parameter  $\lambda$  more precisely. This finding suggests that at least some consumers have better information on future income growth than is summarized in its past history and that they respond to this information by increasing their consumption. At the same time, however, the fraction of rule-of-thumb consumers is estimated at 0.523 in row 5 (and the estimate is significant at better than the 0.01% level). The OLS test also rejects the permanent income model in row 5.

Table 1 UNITED STATES 1953–1986  $\Delta c_v = \mu + \lambda \Delta y_t$ 

	Instruments	First-stage	regressions	λ estimate	Test of
Row		$\Delta c$ equation	$\Delta c$ equation $\Delta y$ equation		restrictions
1	None (OLS)	_	<del></del> -	0.316 (0.040)	
2	$\Delta y_{t-2}, \ldots, \Delta y_{t-4}$	-0.005 (0.500)	0.009 (0.239)	0.417 (0.235)	-0.022 (0.944)
3	$\Delta y_{t-2}, \ldots, \Delta y_{t-6}$	0.017 (0.209)	0.026 (0.137)	0.506 (0.176)	-0.034 (0.961)
4	$\Delta c_{t-2}, \ldots, \Delta c_{t-4}$	0.024 (0.101)	0.045 (0.028)	0.419 (0.161)	-0.009 $(0.409)$
5	$\Delta c_{t-2}, \ldots, \Delta c_{t-6}$	0.081 (0.007)	0.079 (0.007)	0.523 (0.131)	-0.016 (0.572)
6	$\Delta i_{t-2},\ldots,\Delta i_{t-4}$	0.061 (0.010)	0.028 (0.082)	0.698 (0.235)	-0.016 (0.660)
7	$\Delta i_{t-2},\ldots,\Delta i_{t-6}$	0.102 (0.002)	0.082 (0.006)	0.584 (0.137)	-0.025 (0.781)
8	$\Delta y_{t-2}, \ldots, \Delta y_{t-4}, \\ \Delta c_{t-2}, \ldots, \Delta c_{t-4}, \\ c_{t-2} - y_{t-2}$	0.007 (0.341)	0.068 (0.024)	0.351 (0.119)	-0.033 (0.840)
9	$\Delta y_{t-2}, \ldots, \Delta y_{t-4}$ $\Delta c_{t-2}, \ldots, \Delta c_{t-4}, \Delta c_{t-2}, \ldots, \Delta c_{t-4}, \Delta i_{t-2}, \ldots, \Delta i_{t-4}, c_{t-2} - y_{t-2}$	0.078 (0.026)	0.093 (0.013)	0.469 (0.106)	-0.029 (0.705)

Note: The columns labeled "First-stage regressions" report the adjusted  $R^2$  for the OLS regressions of the two variables on the instruments; in parentheses is the p-value for the null that all the coefficients except the constant are zero. The column labeled " $\lambda$  estimate" reports the IV estimate of  $\lambda$  and, in parentheses, its standard error. The column labeled "Test of restrictions" reports the adjusted  $R^2$  of the OLS regression of the residual on the instruments; in parenthesis is the p-value for the null that all the coefficients are zero.

We next consider using some financial variables as instruments. We tried using lagged changes in real stock prices (the quarterly percentage change in the real value of the Dow Jones Industrial Average), but found that this variable had no predictive power for consumption growth or income growth. Results using lagged changes in quarterly average three-month nominal Treasury bill rates ( $i_t$ ) were more successful, and we report these in rows 6 and 7 of Table 1. The instruments are jointly significant for consumption growth at the 1.0% and 0.2% levels. The parameter  $\lambda$  is estimated at 0.698 in row 6 (significant at the 0.3% level), and at 0.584 in row 7 (significant at better than the 0.01% level).  $^{12}$ 

The final two rows of the table report restricted error-correction models for consumption and income. Row 8 has lags of consumption growth, income growth, and the log consumption-income ratio as instruments; row 9 adds lagged interest rate changes. The results are broadly consistent with those in earlier rows.

Table 1 also tests the over-identifying restrictions of our model (1.4) on the unrestricted system (1.5). The test results are reported in the last column of the table. There is no evidence against our restrictions anywhere in this column.

Figures 1 and 2 illustrate what is going on in these instrumental variables estimates. Figure 1 is a scatterplot of ex post consumption growth against ex post income growth. The figure shows a positive relation, but not a tight one. Figure 2 is a scatterplot of expected consumption growth against expected income growth, where expectations were taken to be the fitted values from the reduced form equations estimated in row 9 of Table 1. Note that these points lie along a distinct line. In contrast to the permanent income hypothesis, expected increases in income are associated with expected increases in consumption.

The two lines shown in the figure are estimated by IV regression of  $\Delta c_t$  on  $\Delta y_t$ , as reported in Table 1, and by the reverse IV regression of  $\Delta y_t$  on  $\Delta c_t$ . It is apparent that the normalization of the IV regression makes little difference to the estimate of the slope  $\lambda$ ; this is what we would expect to

<sup>11.</sup> This finding contrasts with the positive results for stock prices reported by Hall (1978) and others. Yet close inspection of Hall's stock price regression (his equation (8), on p. 984) suggests that almost all the explanatory power comes from the first lagged stock price change. When we include the first lag, we also find strong predictive power from stock price changes; but for the reasons discussed above, we regard this as an illegitimate test of the permanent income model.

<sup>12.</sup> The spread between the yield on a long-term government bond and that on a three-month Treasury bill also provided a useful instrument. Using only the second lag of the yield spread, we obtained adjusted  $R^{2}$ 's of 0.094 for  $\Delta c$  and 0.048 for  $\Delta y$ , and an estimate of  $\lambda$  of 0.741 with a standard error of 0.235.

Figure 1 SCATTERPLOT OF CHANGES IN CONSUMPTION AND INCOME

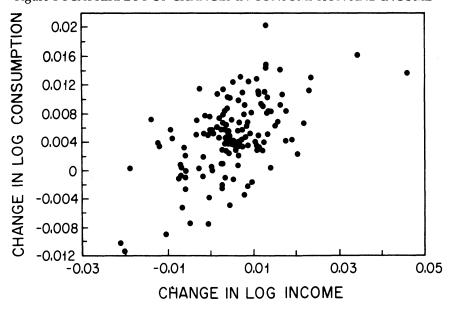
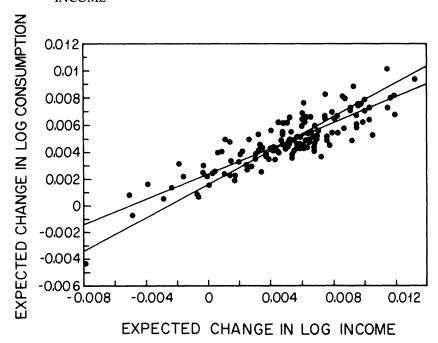


Figure 2 SCATTERPLOT OF EXPECTED CHANGES IN CONSUMPTION AND INCOME



find if our model is correctly specified and the true slope is not zero or infinite.<sup>13</sup>

While the results in Table 1 follow most of the literature by examining consumer spending on non-durables and services, we have also examined two measures of consumption that include consumer durable goods. The results are potentially sensitive to the treatment of durable goods, because spending on them is so volatile. We therefore estimated equation (1.4) both using total consumer spending and using the sum of spending on non-durables and services and the imputed rent on the stock of consumer durables. <sup>14</sup> The results obtained with these two measures turned out to be similar to those reported in Table 1.

In summary, we have found striking evidence against the permanent income hypothesis. The results from our instrumental variables test are particularly unfavorable to the permanent income model. When we use instruments that are jointly significant for predicting income growth at the 5% level or better, we get estimates of  $\lambda$ , the fraction of the population that consumes its current income, of about 0.5. The estimates are always strongly significant even though we have potentially lost some power by lagging the instruments two periods instead of one. The overidentifying restrictions of our model are not rejected at any reasonable significance level.

#### 1.4. EVIDENCE FROM ABROAD

To examine the robustness of our findings for the United States, we now turn to examining data for several other countries. From various DRI data banks, we obtained data on consumption and income to estimate equation (1.4) for the G-7 countries: Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States.<sup>15</sup>

Two data issues arise. First, we found that long time series of quarterly consumption data are often available only for total spending, which includes spending on durables. Assuming exponential depreciation, however, durability should merely lead to the change in consumer spending

- 13. Nelson and Startz (1988) point out that there are severe problems with the IV regression approach if the instruments do not forecast the right hand side variable. In our framework, this would occur in the IV regression of consumption growth on income growth if  $\lambda$  is infinite, and in the IV regression of income growth on consumption growth if  $\lambda$  is zero.
- 14. To calculate the stock of durables, we began with the Commerce Department's net stock of consumer durables for 1947 and then accumulated the spending flow assuming a depreciation rate of 5 percent per quarter. To calculate the imputed rent, we assumed a user cost of 6 percent per quarter.
- 15. Other studies that have used international data to test the permanent income hypothesis include Kormendi and LaHaye (1987) and Jappelli and Pagano (1988).

being a first-order moving average process rather than white noise. <sup>16</sup> Since we are using twice-lagged instruments, the inclusion of spending on durables does not change the implication of the permanent income hypothesis that forecastable changes in income should not lead to forecastable changes in consumption. We can therefore proceed as before.

The second data issue is that, for Canada, France, Italy, and Japan, we were unable to find a quarterly disposable personal income series and therefore used GDP as a proxy. The use of GDP to measure Y should still provide a valid test of the null hypothesis that the permanent income theory is correct. Yet real GDP is an imperfect proxy: in U.S. data, the correlation of real GDP growth and real disposable personal income growth is only 0.55. The use of this proxy can potentially reduce our test's power. It turns out, however, that loss of power appears not to be a problem.

Table 2 presents the estimates obtained for these seven countries. The results from six of these seven countries tell a simple and consistent story. For Canada, France, Germany, Italy, Japan, and the United States, the estimate of the fraction of income going to rule-of-thumb consumers is significantly different from zero and not significantly different from 0.5. Moreover, the over-identifying restrictions imposed by our model are not rejected. The only exception is the United Kingdom, where neither the permanent income hypothesis nor our more general model appear to describe the data adequately. Taken as a whole, these results confirm the failure of the simple random-walk model for consumption and the apparent rule-of-thumb behavior of many consumers.

# 2. Consumption and the Real Interest Rate

The "random walk" theorem for consumption rests crucially on the assumption that the real interest rate is constant. Here we examine the Euler equation that allows for a varying and uncertain real interest rate.

There are two reasons we look at this extension of the basic model. First, a rejection of the theory might be attributable to the failure of this assumption, rather than to an important deviation from the permanent income hypothesis. In particular, variation through time in the real interest rate can make consumption appear excessively sensitive to income, even though individuals intertemporally optimize in the absence of borrowing constraints.<sup>17</sup> We show, however, that the departure from the

<sup>16.</sup> See Mankiw (1982). Matters become more complicated, however, if one allows more complicated forms of depreciation or the possibility of adjustment costs; see Heaton (1988).

<sup>17.</sup> Michener (1984) makes this argument. See also Christiano (1987).

theory documented above—the apparent existence of rule-of-thumb consumers—is not an artifact of the assumed constancy of the real interest rate.

Second, we want to check whether Hall's (1988) conclusion that the intertemporal elasticity of substitution is close to zero is robust to the presence of current-income consumers. Hall assumes that the underlying permanent income theory is correct and uses the absence of a relation between consumption growth and real interest rates as evidence for a small elasticity. In contrast, we argue that the underlying theory is not empirically valid. Unless one is willing to admit that a substantial fraction of income goes to rule-of-thumb consumers, the data cannot yield an answer on the intertemporal elasticity of substitution.

#### 2.1. THE MODEL WITH ONLY PERMANENT INCOME CONSUMERS

We begin our examination of consumption and real interest rates by maintaining the hypothesis that the permanent income theory is correct. We will then go on to consider a more general model with some rule-of-thumb consumers.

The generalization of the consumer's Euler equation to allow for

Table 2 EVIDENCE FROM ABROAD  $\Delta c_t = \mu + \lambda \Delta y_t$ 

	Country	First-stage	regressions	λ estimate	Test of
	(sample period)	$\Delta c$ equation	$\Delta y$ equation	(s.e.)	restrictions
1	Canada	0.047	0.090	0.616	0.007
	(1963–1986)	(0.127)	(0.030)	(0.215)	(0.263)
2	France	0.083	0.166	1.095	$-0.055^{'}$
	(1970–1986)	(0.091)	(0.015)	(0.341)	(0.714)
3	Germany	0.028	0.086	0.646	-0.030
	(1962–1986)	(0.211)	(0.031)	(0.182)	(0.639)
4	Italy	0.195	0.356	0.400	-0.034
	(1973–1986)	(0.013)	(0.000)	(0.094)	(0.488)
5	Japan	0.087	0.205	0.553	0.018
	(1959–1986)	(0.020)	(0.000)	(0.096)	(0.178)
6	United Kingdom	0.092	0.127	0.221	0.086
	(1957–1986)	(0.012)	(0.002)	(0.153)	(0.010)
7	United States	0.040	0.079	$0.478^{'}$	0.004
	(1953–1986)	(0.092)	(0.014)	(0.158)	(0.269)

*Note:* For all countries, the consumption data are total spending. The set of instruments is:  $\Delta y_{t-2}$ , ...,  $\Delta y_{t-4}$ , ...,  $\Delta c_{t-2}$ , ...,  $\Delta c_{t-4}$ ,  $c_{t-2}$ ,  $y_{t-2}$ . Also see note, Table 1.

changes in the real interest rate is now well-known. The log-linear version of the Euler equation is <sup>18</sup>

$$\Delta c_t = \mu + \sigma r_t + \epsilon_t, \quad (2.1)$$

where  $r_i$  is the real interest rate contemporaneous with  $\Delta c_i$ , and as before the error term  $\epsilon_i$  may be correlated with  $r_i$  but is uncorrelated with lagged variables. According to (2.1), high ex ante real interest rates should be associated with rapid growth of consumption. The coefficient on the real interest rate,  $\sigma$ , is the intertemporal elasticity of substitution.<sup>19</sup>

Equation (2.1) can be estimated using instrumental variables, just in the way we estimated equation (1.4). The nominal interest rate we use is the average three-month treasury bill rate over the quarter. The price index is the deflator for consumer non-durables and services. We assume a marginal tax rate on interest of 30%.

We obtained the results in Table 3. We find fairly small values for the coefficient on the real interest rate. Hall interprets evidence of this sort as indicating that the intertemporal elasticity of substitution is close to zero—that is, consumers are extremely reluctant to substitute intertemporally.

In our view, however, the equation estimated in Table 3 is misspecified because it does not allow for the presence of rule-of-thumb consumers. This misspecification shows up in several ways in Table 3. First, the hypothesis that consumption growth is unpredictable is rejected at the 1% level or better in five out of eight rows of Table 3, and at the 5% level or better in seven rows. This is inconsistent with Hall's interpretation of the data: if the permanent income theory were true and  $\sigma$  were zero, consumption should be a random walk. Second, the over-identifying restrictions of equation (2.1) are rejected at the 5% level or better whenever lagged real interest rates are included in the set of instruments. Third, the estimates of  $\sigma$  are highly unstable; while they are generally small, they do exceed one when nominal interest rate changes are used as instruments.

Perhaps the most telling check on the specification comes from revers-

<sup>18.</sup> See, for example, Grossman and Shiller (1981), Mankiw (1981), Hansen and Singleton (1983), and Hall (1988). Note that in the process of log-linearizing the first-order condition, the variance of consumption growth has been included in the constant term. Hence, heteroskedasticity is one possible reason for rejection of the model; see Barsky (1985) for a preliminary exploration of this issue.

<sup>19.</sup> If the representative agent has power utility, then  $\sigma$  is the reciprocal of the coefficient of relative risk aversion. Epstein and Zin (1987a, 1987b) and Giovannini and Weil (1989) have shown that the same Euler equation can be obtained in a more general model in which risk aversion and the intertemporal elasticity of substitution are decoupled.

Table 3 UNITED STATES, 1953–1986  $\Delta c_t = \mu + \sigma r_t$ 

		First-stage 1	regressions	σ estimate	Test of	
Row	Instruments	$\Delta c$ equation r equation		(s.e.)	restrictions	
1	None (OLS)	<del></del>		0.276 (0.079)		
2	$r_{t-2},\ldots,r_{t-4}$	0.063 (0.009)	0.431 (0.000)	0.270 (0.118)	0.031 (0.029)	
3	$r_{t-2}, \ldots, r_{t-6}$	0.067 (0.014)	0.426 (0.000)	0.281 (0.118)	0.034 (0.050)	
4	$\Delta c_{t-2}, \ldots, \Delta c_{t-4}$	0.024 (0.101)	-0.021 (0.966)	-0.707 (2.586)	0.000 (0.215)	
5	$\Delta c_{t-2}, \ldots, \Delta c_{t-6}$	0.018 (0.007)	0.007 (0.316)	0.992 (0.478)	0.008 (0.189)	
6	$\Delta i_{t-2}, \ldots, \Delta i_{t-4}$	0.061 (0.010)	0.024 (0.105)	1.263 (0.545)	-0.021 (0.918)	
7	$\Delta i_{t-2},\ldots,\Delta i_{t-6}$	0.102 (0.002)	0.028 (0.119)	1.213 (0.445)	-0.022 (0.700)	
8	$r_{t-2}, \ldots, r_{t-4}, \\ \Delta c_{t-2}, \ldots, \Delta c_{t+4},$	0.062 (0.026)	0.455 (0.000)	0.204 (0.114)	0.047 (0.033)	
9	$r_{t-2}, \ldots, r_{t-4},$ $\Delta c_{t-2}, \ldots, \Delta c_{t-4},$ $\Delta i_{t-2}, \ldots, \Delta i_{t-4}$	0.103 (0.006)	0.476 (0.000)	0.150 (0.111)	0.100 (0.005)	

Note: See Table 1.

ing the Hall IV regression. Table 4 shows the IV regression of the real interest rate on the change in consumption. We do not find that the estimates of  $1/\sigma$  are extremely large, as would be predicted by the Hall hypothesis; instead, they cluster around one.<sup>20</sup>

Figure 3 shows graphically why the results are so sensitive to normalization. We regressed  $\Delta c$  and r on the instruments in row 9 of Table 3 and then plotted the fitted values as estimates of the expected change in consumption and the real interest rate. The figure shows that there is substantial variation in these two variables over time. Yet contrary to the predictions of the theory, the fitted values do not lie along a line. The two lines in this figure correspond to the two regressions estimated with the two normalizations. Because the fitted values are not highly correlated, the estimated regression is crucially dependent on which variable

<sup>20.</sup> This cannot be explained by small-sample problems of the Nelson and Startz (1988) variety, since consumption growth is fairly well predicted by the instruments in Table 3.

is on the left-hand side. Hence, this scatterplot does not imply that the elasticity of substitution is small. Instead, it suggests that the model underlying the Euler equation (2.1) should be rejected.

# 2.2. INCLUDING RULE-OF-THUMB CONSUMERS

We now reintroduce our rule-of-thumb consumers into the model. That is, we consider a more general model in which a fraction  $\lambda$  of income goes to individuals who consume their current income and the remainder goes to individuals who satisfy the general Euler equation (2.1). We estimate by instrumental variables

$$\Delta c_t = \mu + \lambda \Delta y_t + \theta r_t + \epsilon_t, \quad (2.2)$$

where  $\theta = (1 - \lambda)\sigma$ . We thus include actual income growth and the ex post real interest rate in the equation, but instrument using twice lagged variables. The results are in Table 5.

Table 4 UNITED STATES, 1953–1986  $r_t = \mu + 1/\sigma \Delta c_t$ 

		First-stage	regressions	$1/\sigma$ estimate	Test of	
Row	Instruments	$\Delta c$ equation	r equation	(s.e.)	restrictions	
1	None (OLS)			0.304 (0.087)		
2	$r_{t-2},\ldots,r_{t-4}$	0.063 (0.009)	0.431 (0.000)	1.581 (0.486)	0.086 (0.001)	
3	$r_{t-2},\ldots,r_{t-6}$	0.067 (0.014)	0.426 (0.000)	1.347 (0.390)	0.113 (0.001)	
4	$\Delta c_{t-2}, \ldots, \Delta c_{t-4}$	0.024 (0.101)	-0.021 (0.966)	-0.342 (0.428)	-0.021 (0.878)	
5	$\Delta c_{t-2}, \ldots, \Delta c_{t-6}$	0.018 (0.007)	0.007 (0.316)	0.419 (0.258)	-0.010 (0.440)	
6	$\Delta i_{t-2}, \ldots, \Delta i_{t-4}$	0.061 (0.010)	0.024 (0.105)	0.768 (0.334)	-0.021 (0.919)	
7	$\Delta i_{t-2},\ldots,\Delta i_{t-6}$	0.102 (0.002)	0.028 (0.119)	0.638 (0.249)	-0.024 (0.747)	
8	$r_{t-2}, \ldots, r_{t-4}, $ $\Delta c_{t-2}, \ldots, \Delta c_{t-4}$	0.062 (0.026)	0.455 (0.000)	1.034 (0.333)	0.236 (0.000)	
9	$r_{t-2}, \ldots, r_{t-4}, \\ \Delta c_{t-2}, \ldots, \Delta c_{t-4}, \\ \Delta i_{t-2}, \ldots, \Delta i_{t-4}$	0.103 (0.006)	0.476 (0.000)	0.521 (0.220)	0.455 (0.000)	

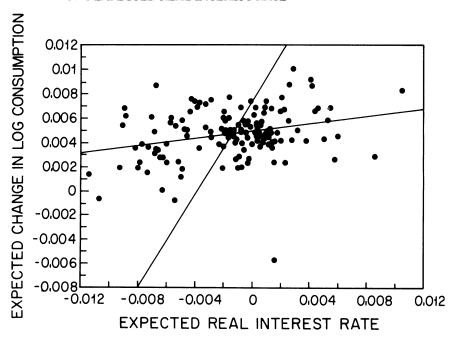
Note: See Table 1.

The first implication of the results is that the rule-of-thumb consumers cannot be explained away by allowing for fluctuations in the real interest rate. The coefficient on current income remains substantively and statistically significant.

The second implication of the results in Table 5 is that there is no evidence that the ex ante real interest rate is associated with the growth rate of consumption after allowing for the rule-of-thumb consumers. The coefficient on the real interest rate is consistently less than its standard error. The small estimated coefficients on the real interest rate indicate that the intertemporal elasticity of substitution for the permanent income consumers is very small. In addition, there is no evidence of any misspecification of the sort found when the rule-of-thumb consumers were excluded. The over-identifying restrictions are never close to being rejected.

Figure 4 illustrates the finding of a small elasticity of substitution by plotting the expected real interest rate and the expected change in consumption for the permanent income consumers assuming  $\lambda$ =0.5. This figure is exactly analogous to Figure 3, except that  $\Delta c$  has been replaced by  $\Delta c$ =0.5 $\Delta y$ . These fitted values lie almost along a horizontal line, as is required for an elasticity near zero. The figure also includes the regres-

Figure 3 SCATTERPLOT OF EXPECTED CHANGE IN CONSUMPTION AND THE EXPECTED REAL INTEREST RATE



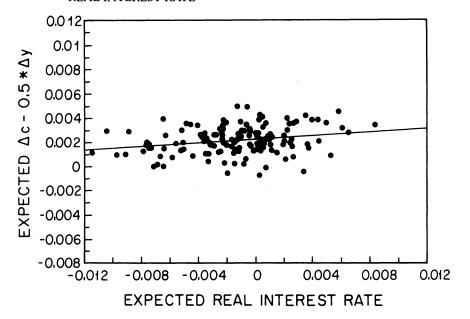
sion line of the expected consumption change on the expected real interest rate, and it is near horizontal. Note that we cannot estimate the reverse normalization: we have been unable to find any instruments that forecast  $\Delta c - 0.5\Delta y$  (as must be the case if  $\lambda = 0.5$  and  $\sigma = 0$ ).

Table 5 UNITED STATES, 1953–1986  $\Delta c_t = \mu + \lambda \Delta y_t + \theta r_t$ 

		First-stage regressions			λ	θ	Test of
Row	Instruments	Δc	$\Delta y$	r	(s.e.)	(s.e.)	restrictions
1	None (OLS)				0.294 (0.041)	0.150 (0.070)	
2	$\Delta y_{t-2}, \ldots, \Delta y_{t-4}$ $r_{t-2}, \ldots, r_{t-4}$	0.045 (0.061)	0.030 (0.125)	0.471 (0.000)	0.438 (0.189)	0.080 (0.123)	-0.010 (0.441)
3	$\Delta c_{t-2}, \ldots, \Delta c_{t-4}$ $r_{t-2}, \ldots, r_{t-4}$	0.062 (0.026)	0.046 (0.060)	0.455 (0.000)	0.467 (0.152)	0.089 (0.110)	-0.006 (0.391)
4	$ \Delta i_{t-2}, \ldots, \Delta i_{t-4}  r_{t-2}, \ldots, r_{t-4} $	0.092 (0.005)	0.034 (0.106)	0.431 (0.000)	0.657 (0.212)	0.016 (0.146)	-0.022 (0.665)

Note: See Table 1

Figure 4 SCATTERPLOT OF EXPECTED CHANGE IN CONSUMPTION FOR "PERMANENT INCOME" CONSUMERS AND THE EXPECTED REAL INTEREST RATE



In summary, the data show little or no correlation between expected changes in consumption and ex ante real interest rates. Yet this finding should not be interpreted as implying that the permanent income hypothesis holds with a small intertemporal elasticity of consumption: that hypothesis would require that expected changes in consumption are small and linearly dependent on the ex ante real interest rate. Instead, it seems that expected changes in consumption are dependent on expected changes in income, which can be explained by the existence of some rule-of-thumb consumers. Once these rule-of-thumb consumers are admitted into the model, the data become consistent with an elasticity of substitution near zero for the permanent income consumers.

# 3. From Euler Equation to Consumption Function

Modern empirical work on consumption behavior has focused almost exclusively on the Euler equations implied by optimizing models of intertemporal choice. Our own work is no exception. Yet is seems that something has been lost in this change of emphasis. The Euler equation determines only the level of consumption today, *relative* to the level of consumption tomorrow. We would like to be able to determine the absolute level of consumption, given either wealth and expected future interest rates, or expected future income flows and interest rates. For this we need a traditional consumption function, that is, a closed-form solution for consumption given exogenous variables.

Of course, there are considerable technical difficulties in deriving a consumption function from an optimizing model. In fact, closed-form solutions are available only in a very few special cases, the best-known being log utility or power utility with independently and identically distributed asset returns.<sup>21</sup> The problem is that a closed-form solution is obtained by combining an Euler equation with the intertemporal budget constraint. But even when the Euler equation is linear or log-linear, the budget constraint is always non-linear when asset returns are random. Consumption is *subtracted* from wealth to give the amount invested, and this amount is then *multiplied* by a random rate of return to give tomorrow's level of wealth.

In this section we explore a class of approximate consumption functions obtained by log-linearizing the intertemporal budget constraint. These approximate consumption functions give considerable insight into the implications of alternative models, and they offer an alternative way to confront the modesl with the data.<sup>22</sup>

#### 3.1. THE INTERTEMPORAL BUDGET CONSTRAINT

To see the way our approach works, consider the budget constraint of a consumer who invests his wealth in a single asset with a time-varying risky return  $R_t$ . We do not explicitly model income at this stage; this is legitimate provided that all the consumer's income flows (including his or her labor income) are capitalized into marketable wealth. The period-by-period budget constraint is

$$W_{t+1} = R_{t+1}(W_t - C_t)$$
. (3.1)

Solving forward with an infinite horizon and imposing the transversality condition that the limit of discounted future wealth is zero, we obtain

$$W_t = C_t + \sum_{i=1}^{\infty} C_{t+i} / \left( \prod_{j=1}^{i} R_{t+j} \right).$$
 (3.2)

This equation says that today's wealth equals the discounted value of all future consumption.

We would like to approximate the non-linear equations (3.1) and (3.2) in such a way that we obtain linear relationships between log wealth, log consumption, and log returns, measured at different points of time. To do this, we first divide equation (3.1) by  $W_t$ , take logs and rearrange. The resulting equation expresses the growth rate of wealth as a non-linear function of the log return on wealth and the log consumption-wealth ratio. In the appendix we show how to linearize this equation using a Taylor expansion. We obtain

$$\Delta w_{t+1} \approx k + r_{t+1} + (1 - 1/\rho)(c_t - w_t)$$
. (3.3)

In this equation lower-case letters are used to denote the logs of the corresponding upper-case letters. The parameter  $\rho$  is a number a little

22. Our log-linearization is similar to the one used by Campbell and Shiller (1988) to study stock prices, dividends, and discount rates. It differs slightly because we define wealth inclusive of today's consumption, which is analogous to a cum-dividend asset price. There is also an interesting parallel between our approach and the continuous-time model of Merton (1971). Merton was able to ignore the product of random returns and consumption flows, since this becomes negligible in continuous time. See also Hayashi (1982), who examines a similar model under the maintained assumption of a constant real interest rate.

less than one, and k is a constant.<sup>23</sup> This equation says that the growth rate of wealth is a constant, plus the log return on wealth, less a small fraction  $(1-1/\rho)$  of the log consumption-wealth ratio. In the appendix we solve equation (3.3) forward to obtain

$$c_t - w_t = \sum_{j=1}^{\infty} \rho^j (r_{t+j} - \Delta c_{t+j}) + \rho k/(1-\rho)$$
. (3.4)

Equation (3.4) is a log-linear version of the infinite-horizon budget constraint (3.2). It states that a high log consumption-wealth ratio today must be associated either with high future rates of return on invested wealth, or with low future consumption growth.

## 3.2. WEALTH-BASED AND INCOME-BASED CONSUMPTION FUNCTIONS

So far we have merely manipulated a budget constraint, without stating any behavioral restrictions on consumer behavior. We now assume that the consumer satisfies the log-linear Euler equation discussed earlier in Section 2:

$$E_{t}\Delta c_{t+1} = \mu + \sigma E_{t}r_{t+1}$$
. (3.5)

Equation (3.5) can be combined with equation (3.4) to give a consumption function relating consumption, wealth, and expected future returns on wealth. Take conditional expectations of equation (3.4), noting that the left-hand side is unchanged because it is in the consumer's information set at time t. Then substitute in for expected consumption growth from (3.5). The resulting expression is

$$c_t - w_t = (1 - \sigma) E_t \sum_{j=1}^{\infty} \rho^j r_{t+j} + \rho (k - \mu)/(1 - \rho)$$
. (3.6)

This equation generalizes Paul Samuelson's (1969) results for independently and identically distributed asset returns. It says that the log consumption-wealth ratio is a constant, plus  $(1-\sigma)$  times the expected present value of future interest rates, discounted at the rate  $\rho$ . When  $\sigma=1$ , the consumer has log utility and we get the well-known result that consumption is a constant fraction of wealth. When  $\sigma>1$ , an increase in

<sup>23.</sup> The parameter  $\rho$  can also be interpreted as the average ratio of invested wealth, W-C, to total wealth, W.

interest rates lowers the log consumption-wealth ratio because substitution effects outweigh income effects; when  $\sigma < 1$ , income effects are stronger and high interest rates increase consumption. Whatever the sign of the effect, persistent movements in interest rates have a stronger impact on the level of consumption than transitory movements do.

Traditional macroeconomic consumption functions usually determine consumption in relation to income flows rather than wealth. We can move from the wealth-based consumption function (3.6) to an income-based consumption function by expressing the market value of wealth in terms of future expected returns and the future expected income flows from wealth. A full derivation is given in the appendix. The resulting consumption function is

$$c_t - y_t = E_t \sum_{j=1}^{\infty} \rho^j (\Delta y_{t+j} - \sigma r_{t+j}) - \rho \mu / (1-\rho),$$
 (3.7)

where  $y_{t+j}$  is the income at time t+j generated by the wealth held at time t. The log consumption-income ratio depends on the expected present value of future income growth, less  $\sigma$  times the expected present value of future interest rates. As  $\sigma$  falls towards zero, interest rates have less and less effect on the consumption-income ratio and the model becomes a log-linear version of the standard permanent income model which ignores interest rate variation.

Two aspects of (3.7) are worthy of special mention. First, the interest rate terms in (3.7) capture the effects of changes in interest rates holding future income constant (while the market value of wealth is allowed to vary). By contrast, the interest rate terms in (3.4) capture the effects of changes in interest rates holding wealth constant (while future income is allowed to vary). When one holds future income constant, higher interest rates lower the market value of wealth; when one holds the market value of wealth constant, higher interest rates increase future income flows. As Lawrence Summers (1981) has emphasized, higher interest rates reduce consumption more when income flows are held fixed, since there is no positive income effect to offset the negative substitution effect of interest rates on consumption. With fixed income flows, the impact of interest rates on consumption approaches zero as  $\sigma$  approaches zero.

Second, the income growth terms in (3.7) represent the influence of expected growth in income on current wealth, that is, net of the effects of further wealth accumulation. This complicates the use of (3.7) in em-

pirical work, although the component of measured income growth that is due to wealth accumulation may be small in practice.<sup>24</sup>

The analysis of this section has so far ignored the possibility that some fraction  $\lambda$  of income accrues to individuals who consume their current income rather than obeying the consumption function (3.7). But it is straightforward to generalize (3.7) to allow for these consumers. We obtain

$$c_t - y_t = (1 - \lambda) E_t \sum_{j=1}^{\infty} \rho^j (\Delta y_{t+j} - \sigma r_{t+j}) - (1 - \lambda) \rho \mu / (1 - \rho) .$$
 (3.8)

The presence of current-income consumers reduces the variability of the log consumption-income ratio. The model of Hall (1988) sets  $\sigma = \lambda = 0$  and thus has the consumption-income ratio responding fully to expected income growth but not at all to expected interest rates. By contrast, our model with  $\lambda = 0.5$  has a reduced response of the consumption-income ratio to expected future income growth.

## 3.3. EMPIRICAL IMPLEMENTATION

Since equation (3.8) shows that both the permanent income model and our more general model with rule-of-thumb consumers can be written as a present value relation, all the econometric techniques available for examining present value relations can be used to test and estimate these models. Applying these techniques is beyond the scope of this paper. To see what such exercises are likely to find, however, we take an initial look at the data from the perspective of this present value relation.

If we assume the intertemporal elasticity of substitution is small and set  $\sigma = 0$ , equation (3.8) says that the log of the average propensity to consume (c-y) is the optimal forecast of the present value of future income growth. To see if in fact there is any relation between these variables, Figure 5 plots the log of the average propensity to consume (computed using spending on non-durables and services) and the present value of realized income growth (computed using personal disposable income per capita). We assume a quarterly discount factor of 0.99, and set the out-of-sample income growth rates at the sample mean. As the theory predicts, the figure shows a clear positive relationship between these variables. When consumption is high relative to current income, income will tend to grow faster than average. When consump-

tion is low relative to current income, income will tend to grow slower than average. <sup>25</sup>

We can obtain an estimate of  $\lambda$ , the fraction of income going to rule-ofthumb consumers, by regressing the present value of realized income growth on the log of the average propensity to consume. Since the error in this relationship is an expectations error, it should be uncorrelated with currently known variables—in particular, c-y. The coefficient on c-y is therefore a consistent estimate of  $1/(1-\lambda)$ . We can see from Figure 5 that the estimate is likely to be greater than one: the present value of future income growth seems to respond more than one-for-one to fluctuations in c-y, which suggests that  $\lambda$  is greater than zero.

Table 6 shows the regression results for three measures of consumption: spending on non-durables and services, total consumer spending, and the sum of spending on non-durables and services and the imputed rent on the stock of consumer durables. We present the results with and without a time trend. The implied estimates of  $\lambda$  in Table 6 vary from 0.233 to 0.496, which are similar to those obtained in Table 1. These findings lead us to believe that more sophisticated examinations of the present value relation will likely yield a conclusion similar to the one we reached examining the Euler equation: a model with some permanent income consumers and some rule-of-thumb consumers best fits the data.

# 4. Conclusions

We have argued that aggregate consumption is best viewed as generated not by a single representative consumer but rather by two groups of consumers—one consuming their permanent income and the other consuming their current income. We have estimated that each group of consumers receives about 50 percent of income and that the intertemporal elasticity of substitution for the permanent income consumers is close to zero. This alternative model can explain why expected growth in consumption accompanies expected growth in income, why expected

- 25. This figure thus confirms the findings using vector autoregressions in Campbell (1987).
- 26. We include a time trend to proxy for mismeasurement in the average propensity to consume attributable to the treatment of consumer durables. The ratio of spending on consumer durables to spending on consumer non-durables and services has grown over time. Therefore, a failure to include consumer durables or an incorrect imputation is likely to cause mismeasurement in *c-y* that is correlated with time. We confess that inclusion of a time trend is a crude correction at best.
- 27. We have somewhat more confidence in the estimates of  $\lambda$  obtained from Euler equation estimation. In Table 6, measurement error in consumption biases downward the estimate of  $\lambda$  (as does the inability to observe the out-of-sample values of future income growth.) Yet such measurement error does not affect the Euler equation estimates if this measurement error is uncorrelated with the instruments.

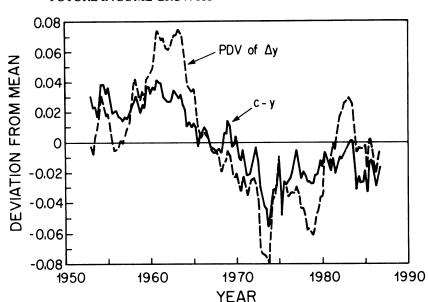


Figure 5 THE AVERAGE PROPENSITY TO CONSUME AS A FORECAST OF FUTURE INCOME GROWTH

Table 6 UNITED STATES, 1953–1986  $\Sigma_{j=1}^{\infty} \rho^{j} \Delta y_{t+j} = \mu + [1/(1-\lambda)](c_t - y_t)$ 

Consumption Measure	$1/(1-\lambda)$	time	$R^{-2}$	Implied λ
Non-durables and Services	1.306 (0.223)		0.690	0.234
Non-durables and Services	1.983 (0.221)	0.0005 (0.0001)	0.792	0.496
Total Consumer Spending	1.455 (0.408)		0.302	0.313
Total Consumer Spending	1.303 (0.256)	-0.0004 (0.0002)	0.463	0.233
Non-durables, Services, and Imputed Rent on Durables	1.576 (0.225)		0.740	0.366
Non-durables, Services, and Imputed Rent on Durables	1.937 (0.203)	0.0003 (0.0001)	0.776	0.484

*Note:* These regressions were estimated using Ordinary Least Squares. The present value of future growth was computed assuming  $\rho=.99$ ; out-of-sample growth rates were set at the sample mean. Standard errors in parentheses were computed using the Newey-West (1987) correction for serial correlation; these standard errors use a lag length of 20, although lag lengths of 10 and 30 yielded similar results.

growth in consumption is unrelated to the expected real interest rate, and why periods in which consumption is high relative to income are typically followed by high growth in income.

Our model also has the potential to explain the "excess smoothness" of aggregate consumption pointed out by Angus Deaton (1987). Deaton shows that if income follows a persistent time series process, then the variance of the innovation in permanent income exceeds the variance of the change in current income. According to the permanent income model, the change in consumption should then be more variable than the change in income; but in fact consumption is considerably smoother than income. Our model can resolve this puzzle because it makes the change in consumption a weighted average of the change in current income and the change in permanent income. If these two income changes are not perfectly correlated, then a weighted average of them can be less variable than either one considered in isolation. Aggregate consumption is smooth in our model because it is a "diversified portfolio" of the consumption of two groups of agents.<sup>29</sup>

Although our emphasis in this paper has been on characterizing the aggregate data rather than on analyzing economic policies, our findings are suggestive regarding the effects of policies. In particular, if current income plays as central a role in consumption as our alternative model suggests, economists should not turn so readily to the permanent income hypothesis for policy analysis. An important application of this conclusion is in the debate over the national debt. Since the Ricardian equivalence proposition relies on the permanent income hypothesis, the failure of the permanent income hypothesis casts doubt on this proposition's empirical validity. Rule-of-thumb consumers are unlikely to increase private saving and bequests in response to government deficits. The old-fashioned Keynesian consumption function may therefore provide a better benchmark for analyzing fiscal policy than does the model with infinitely-lived consumers.

Our alternative model with rule-of-thumb consumers is very different from the alternative models considered in much recent work on Ricardian equivalence.<sup>30</sup> Those alternatives are forward-looking, but in-

<sup>28.</sup> See also Campbell and Deaton (1989), Christiano (1987), Flavin (1988) and West (1988).

<sup>29.</sup> As an example, consider the case in which income is a random walk but is known one period in advance Flavin (1988). In this case, since the change in permanent income and the change in current income are contemporaneously uncorrelated, our model implies that the variance of the change in consumption will be one-half the variance of the change in income. For more discussion of excess smoothness in our model, see Flavin (1988) or the 1989 version of Campbell and Mankiw (1987).

<sup>30.</sup> For example, see Evans (1988), which tests Ricardian equivalence within the framework of Blanchard (1985).

volve finite horizons or wedges between the interest rates that appear in private sector and government budget constraints. We believe that such effects may be present, but are hard to detect because they are much more subtle than the rule-of-thumb behavior we document here. Thus, the tests in the literature may have low power.<sup>31</sup>

The failures of the representative consumer model documented here are in some ways unfortunate. This model held out the promise of an integrated framework for analyzing household behavior in financial markets and in goods markets. Yet the failures we have discussed are not unique. The model is also difficult to reconcile with the large size of the equity premium, the cross-sectional variation in asset returns, and time series fluctuations in the stock market.<sup>32</sup> The great promise of the representative consumer model has not been realized.

One possible response to these findings is that the representative consumer model examined here is too simple. Some researchers have been attempting to model the aggregate time series using a representative consumer model with more complicated preferences. Non-time-separabilities and departures from the von Neumann-Morgenstern axioms are currently receiving much attention.<sup>33</sup> It is also possible that there are non-separabilities between non-durables and services consumption and other contemporaneous variables.<sup>34</sup>

Alternatively, some have argued that random shocks to the representative consumer's utility function may be important.<sup>35</sup> This contrasts with the standard assumption in the consumption literature that fluctuations arise from shocks to other equations, such as productivity shocks or changes in monetary and fiscal policy. If there are shocks to the utility function and if they are serially correlated, then they enter the residual

- 31. An exception is the study by David Wilcox (1989) which reports that consumer spending rises when Social Security benefits are increased. This finding provides evidence against the infinite-horizon model of the consumer. Moreover, since these benefit increases were announced in advance, this finding also provides evidence against models with forward-looking, finite-horizon consumers.
- 32. See Mehra and Prescott (1985), Mankiw and Shapiro (1986), and Campbell and Shiller (1988).
- 33. For models with non-time-separability, see Constantinides (1988) and Heaton (1988). For departures from the von Neumann-Morgenstern axioms, see Epstein and Zin (1987a, 1987b) and Giovannini and Weil (1989).
- 34. In Campbell and Mankiw (1987), we looked at cross-effects with labor supply, government spending, and durable goods; we found no evidence for these types of non-separabilities. There is perhaps more evidence for non-separability with the stock of real money balances; see Koenig (1989). Nason (1988) proposes a model in which the marginal utility of consumption depends on current income. His model is observationally equivalent to ours, and has the same implications for policy; it is a way to describe the same facts in different terms.
- 35. See Garber and King (1983) and Hall (1986).

of the Euler equation and may be correlated with lagged instruments, invalidating standard test procedures.<sup>36</sup>

Unlike our model with rule-of-thumb consumers, these approaches remain in the spirit of the permanent income hypothesis by positing forward-looking consumers who do not face borrowing constraints. We believe that such modifications of the standard model are worth exploring, but we doubt that they will ultimately prove successful. We expect that the simple model presented here—half of income going to permanent income consumers and half going to current income consumers—will be hard to beat as a description of the aggregate data on consumption, income, and interest rates.

# Appendix: Derivation of Approximate Consumption Functions

We first divide equation (3.1) by  $W_t$  and take logs. The resulting equation is

$$w_{t+1} - w_t = r_{t+1} + \log(1 - C_t/W_t) = r_{t+1} + \log(1 - \exp(c_t - w_t)).$$
 (A.1)

The last term in equation (A.1) is a non-linear function of the log consumption-wealth ratio,  $c_t - w_t = x_t$ . The next step is to take a first-order Taylor expansion of this function,  $\log(1-\exp(x_t))$ , around the point  $x_t = x$ . The resulting approximation is

$$\log(1-\exp(c_t-w_t)) \approx k + (1-1/\rho)(c_t-w_t), \quad (A.2)$$

where the parameter  $\rho \equiv 1\text{-exp}(x)$ , a number a little less than one, and the constant  $k \equiv \log(\rho) - (1\text{-}1/\rho)\log(1-\rho)$ . The parameter  $\rho$  can also be interpreted as the average ratio of invested wealth, W - C, to total wealth, W. Substituting (A.2) into (A.1), we obtain (3.3).

The growth rate of wealth, which appears on the left-hand side of equation (3.3), can be written in terms of the growth rate of consumption and the change in the consumption-wealth ratio:

36. One response to this point is to try to find instruments that are uncorrelated with taste shocks. We have experimented with several instrument sets, including lagged growth of defense spending and political party dummies, but these did not have much predictive power for income. On the other hand, the change in the relative price of oil had significant predictive power two quarters ahead. When we used lags 2 through 6 as instruments, we estimated the fraction of current income consumers to be 0.28 with a standard error of 0.09. These instruments, however, did not have significant predictive power for real interest rates, so we were unable to estimate the more general Euler equation.

$$\Delta w_{t+1} = \Delta c_{t+1} + (c_t - w_t) - (c_{t+1} - w_{t+1}). \quad (A.3)$$

Substituting (A.3) into (3.3) and rearranging, we get a difference equation relating the log consumption-wealth ratio today to the interest rate, the consumption growth rate, and the log consumption-wealth ratio tomorrow:

$$c_t - w_t = \rho(r_{t+1} - \Delta c_{t+1}) + \rho(c_{t+1} - w_{t+1}) + \rho k.$$
 (A.4)

Solving forward, we obtain (3.4).

To obtain an income-based consumption function, we suppose that total wealth  $W_t$  consists of  $N_t$  shares, each with ex-dividend price  $P_t$  and dividend payment  $Y_t$  in period t:

$$W_t = N_t(P_t + Y_t)$$
. (A.5)

The return on wealth can be written as

$$R_{t+1} = (P_{t+1} + Y_{t+1})/P_t$$
. (A.6)

Combining (A.5) and (A.6) and rearranging, we get

$$W_{t+1}/N_{t+1} = R_{t+1}(W_t/N_t - Y_t), \quad (A.7)$$

where  $W_t/N_t = P_t + Y_t$  is the cum-divided share price at time t. This equation is in the same form as (3.1) and can be linearized in the same way. The log-linear model is

$$y_t - w_t = -n_t + E_t \sum_{j=1}^{\infty} \rho^j (r_{t+j} - \Delta y_{t+j}) + \rho k/(1-\rho).$$
 (A.8)

(Implicitly we are assuming that the mean dividend-price ratio equals the mean consumption-wealth ratio since the same parameter  $\rho$  appears in (A.8) and in (3.4)). Normalizing  $N_t$ =1 ( $n_t$ =0) and substituting (A.8) into (3.6), we obtain (3.7).

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# Comment

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# Introduction

Campbell and Mankiw report several empirical results that they feel warrant abandoning the representative agent model as an abstraction for thinking about aggregate consumption. The most important of these is that the predictable component of consumption growth is linearly related to the predictable component of income growth and the predictable component of the inflation-adjusted rate of interest. In this linear relation, the coefficient on income growth is around .5, while the coefficient on the interest rate is close to zero. Campbell and Mankiw argue that the most likely explanation of this result is that 50% of income goes to "rule-of-thumb" households who set consumption equal to income, and the other 50% goes to "representative agent" households whose consumption decisions are consistent with the choices of a representative agent with low intertemporal substitution in consumption. They claim that the representative agent model ought to be replaced with this hybrid model, saying that such a model "will be hard to beat as a description of the aggregate data on consumption, income, and interest rates." Unfortunately, it is impossible to evaluate the merits of this claim based on the evidence in the paper.

The reason for this is that their description of the model being criticized and of the model being proposed is not precise enough. The Campbell-Mankiw claim that introducing rule-of-thumb households into the representative agent environment helps it account for the comovements between predictable components in consumption growth, income growth, and interest rates seems plausible enough. But, without a more detailed description of the economic structure, it is impossible to say what the other empirical implications of introducing rule-of-thumb households might be. That there probably are other implications is suggested by the extensive cross-variable restrictions that characterize the typical fully specified representative agent model. To illustrate the possible quantitative significance of this observation, I have taken the liberty of filling in the missing details in both the representative agent model that Campbell and Mankiw criticize and their proposed alternative. I do so by drawing on the model specification in Christiano and Eichenbaum (1988). I find, consistent with the author's claim, that introducing ruleof-thumb households into my prototype representative agent model helps on the empirical dimension on which Campbell and Mankiw focus. At the same time, however, this modified model substantially overstates the volatility of consumption relative to income. Significantly, the representative agent model does very well on this dimension.

The relative smoothness of consumption versus income stands out as one of the most robust and well-documented empirical regularities in macroeconomic time series. Moreover, this fact has played a central role driving theoretical work on consumption. Initially, it inspired the permanent income hypothesis (PIH) and more recently it inspired further work when Deaton (1985) argued that the PIH has a hard time accounting for consumption smoothness when income is modeled as having a unit root. In the light of these considerations, it is not so clear that Campbell and Mankiw's rule-of-thumb household model beats the representative agent model. Conditional on the maintained assumptions of the experiment, the former model cannot account for a traditional concern of the consumption literature—the relative volatility of consumption—but can account for some facts about consumption that have (as yet) attracted relatively less interest. My prototype representative agent model, while not able to account for the Campbell and Mankiw facts, scores a bullseye on consumption smoothing.

Of course, the proposition that rule-of-thumb households *raise* the relative volatility of consumption cannot be general, and probably re-

<sup>1.</sup> For a review of the role of consumption smoothing in the construction of the PIH, see Sargent (Chapter XII, 1987).

flects the structure and parameter values of my prototype representative agent model. A feature of this example that probably is robust is the principle that introducing rule-of-thumb households can be expected to alter a variety of model implications. Any full evaluation of the Campbell-Mankiw recommendation—whether informal or formally, using a likelihood ratio statistic—would take into account an estimate of the quantitative magnitude of these implications.

My comments are divided into three parts. First, I document that the Christiano-Eichenbaum (C/E) version of the representative agent model does indeed have a difficult time accounting for the results in the second sentence. Before accepting the authors' conclusion on this point I first investigate several potential ways that the C/E model could be reconciled with the facts cited in the first sentence. The first is a simple model of measurement error. The second is motivated by the observation, associated with Mankiw and Shapiro (1985), that disposable income (the income measure used by the authors) is a random walk from a univariate perspective. This observation draws attention to the possibility that the forecastable component of income growth is also small in the present multivariate context. If it is too small, then Campbell and Mankiw's estimate that 50% percent of the population follows rule-of-thumb could be a statistical artifact.<sup>2</sup> Several Monte Carlo experiments are reported in this section which suggest that the empirical multivariate predictability in income growth is large enough to ensure the validity of Campbell and Mankiw's instrumental variables method. Since this kind of result may be somewhat model specific, it is comforting that Campbell and Mankiw (1987) reach the same conclusion in an earlier paper based on a Monte Carlo study that uses a different data generating mechanism from mine. Absent these kinds of considerations, it is perhaps not surprising that the C/E model is embarrassed by the Campbell-Mankiw observations, since it satisfies all the assumptions they place on the representative agent model.

Second, I document the claims made about the relative volatility of consumption above. Namely, I show that a version of the C/E model predicts exactly the amount of consumption smoothing observed in the data. However, introducing rule-of-thumb households into the C/E model in the manner advocated by Campbell and Mankiw substantially raises the model's implication for the relative volatility of consumption. I then point out the role played by time aggregation and interest rate movements in the C/E model's account of consumption smoothing. I argue there that it is by no means obvious what the appropriate empirical counterpart to the rate of return in the C/E model is. In any event, it

<sup>2.</sup> For another analysis of this point, see Nelson and Startz (1988).

seems clear that it is *not* the inflation adjusted return on three-month T-bills, used by Campbell and Mankiw. In all likelihood a more appropriate measure is one which aggregates over the returns on many assets. I examine several such crude measures and find some support for the proposition that the interest rate movements anticipated by the C/E model are present in the data. These calculations are meant to be suggestive only, however. More effort needs to be directed at finding a good empirical counterpart for the rate of return in the C/E model to see whether its account of consumption smoothing is supported. The final part of these comments offers some concluding remarks.

# 2. The Campbell-Mankiw Empirical Observations Reject the C/E Model

Campbell and Mankiw show that the forecast of consumption two periods ahead is linearly related to the forecast of disposable income growth two periods ahead and the forecast of the real rate of interest two periods ahead. Here, a variable's forecast two periods ahead is the fitted value in its regression on variables lagged two and more periods. In this relation, they show that the coefficient on income growth is around .5 and statistically significantly different from zero based on asymptotic sampling theory. In addition, the coefficient on the rate of interest is positive and close to zero. They argue that this result rejects a version of the representative agent model in which preferences for consumption are separable across time and other commodities. In such a model, one expects the coefficient on income to be zero and the coefficient on the interest rate to be the representative agent's elasticity of intertemporal substitution in consumption. Campbell and Mankiw speculate that this rejection is unlikely to be overturned by considering non-separabilities and other modifications to the utility function. Instead, they conclude that the most likely explanation for the failure is that roughly 50 percent of disposable personal income goes to households who simply set consumption equal to disposable income period by period, and the other 50 percent goes to households whose aggregate consumption decisions look as though they were selected by a representative agent with intertemporal substitution in consumption close to zero.

Before tentatively agreeing with Campbell and Mankiw that their evidence embarrasses their version of the representative agent model, I first carried out two Monte Carlo experiments. First, I investigate the possibility that their results are a statistical artifact and reflect the lack of predictability in disposable income growth. I then investigate the potential for measurement error in the rate of return to account for their results. Nei-

ther of these considerations seem to be able to be able to reconcile their results with the particular representative agent model studied in Christiano and Eichenbaum. Before reporting these experiments, I describe the versions of the C/E used to generate the data in the Monte Carlo studies.

#### FOUR VERSIONS OF THE C/E MODEL

According to the C/E model, a representative agent selects contingency plans for private consumption,  $c_t$ , capital,  $k_{t+1}$ , and hours worked,  $n_t$ , to maximize:

$$E_0 \sum_{t=0}^{\infty} (1.03)^{-t/4} \{ \ell n(c_t) + 6.98 \ell n(2190 - n_t) \}, \quad (1)$$

subject to the following resource constraint:

$$c_t + g_t + k_{t+1} - 0.9793k_t = (z_t n_t)^{0.65} k_t^{0.35}.$$
 (2)

The expression to the right of the equality in (2) is gross output, which is a function of  $n_i$ ,  $k_i$ , and a technology shock,  $z_i$ . It is assumed to have the following representation:

$$z_t = z_{t-1} \exp(\lambda_t), \ \lambda_t = .0047(1-\rho_{\lambda}) + \rho_{\lambda} \lambda_{t-1} + \epsilon_t, \ \epsilon_t \sim \text{IIN } (0,.018^2).$$
 (3)

where, as usual, IIN means independent (over time), identically and normally distributed. In C/E,  $\rho_{\lambda} = 0$ , but we shall find it useful to also consider other values of  $\rho_{\lambda}$ . In (2),  $g_t$  is government consumption, and it is assumed to have the following time series representation:

$$g_t = 199z_t \exp(x_t), x_t = 0.97x_{t-1} + \nu_t, \nu_t \sim \text{IIN } (0,.021^2).$$
 (4)

In addition, I defined disposable labor income as the wage bill (labor's share times gross output) minus government consumption. In defining disposable income as net of government consumption, I am implicitly assuming that the government balances its budget period by period by levying taxes on workers only. Thus, labor income,  $y_t$ , is as follows:

$$y_t = 0.65(z_t n_t)^{0.65} k_t^{0.35} - g_t.$$
 (5)

I define the interest rate,  $r_t$ , in this model as the return on investment in capital:

$$1 + r_t = 0.35(z_t n_t / k_t)^{0.65} + .9793 + .003254.$$
 (6)

Here, .9793 is one minus the rate of depreciation on a unit of capital. Also, .003254 is an estimate of the quarterly growth in population. All variables, including  $k_t$  and  $n_t$ , are measured in per capita terms so that without this adjustment,  $r_t$  would be the additional per capita output associated with a unit of per capita investment in  $k_t$  and would therefore not be comparable with empirical measures of returns, which are not in per capita terms. For details about the computation of the decision rules and the choice of parameter values (which have been rounded), see Christiano and Eichenbaum (1988).

The time period in the C/E model is quarterly. Campbell and Mankiw have in mind a situation in which agents' decision rule is finer than the data sampling interval. In order to be consistent with this I work with a time aggregated version of the above model. In that version, the time period is 1/8 of a quarter and all parameters with a time dimension are appropriately adjusted. In particular, the discount rate, one minus the rate of depreciation on capital (i.e., .9793 in [2] and [6]), all autoregressive coefficients and the discount rate are adjusted by raising them to the power 1/8. In addition, disturbance standard deviations and means (i.e., 199 in [4] and .0047 in [3]) are divided by 8. Finally, the time endowment in a quarter, 2190 in (1), is divided by 8. Prior to statistical analysis of data simulated from this fine time interval model, an 8 period moving sum of the data is taken and every 8th resulting observation is sampled. The resulting simulated "measured" data reflect the time aggregation properties emphasized by Campbell and Mankiw. In what follows I refer to this time aggregated model simply as the C/E model, without further qualification. Throughout, model parameters are always referred to in quarterly units.

Three other versions of the model are also considered. The first is the C/E model with serially correlated technology growth shocks, which is obtained by setting  $\rho_{\lambda}=.2$ . The second also adds measurement error to  $r_t$ . That is, the observed rate of return is  $r_t+\eta_t$ , where  $\eta_t$  has mean zero and is independent of all variables in the model. In addition,  $\eta_t$  is a first order autoregressive process with first order autocorrelation .8 and standard deviation .008. I call this the C/E model with serially correlated technology growth shocks and measurement error. This measurement error is assumed to hit  $r_t$  prior to summing and sampling the data. The third model introduces Campbell-Mankiw rule-of-thumb households into the second model. In this version of the model,  $c_t$  is replaced by  $c_t+y_t$  and  $y_t$  is replaced by  $2y_t$ . Thus, one-half of total disposable income goes to households who set consumption optimally while the other half goes to households who simply equate consumption and disposable income.

I call this the CM version of the C/E model with serially correlated technology growth shocks and measurement error.

Each of these four models was used to generate 100 data sets, each of length 136 observations on quarterly measured rates of return, disposable income and consumption. This was done by first generating  $8\times 136+100$  observations and then ignoring the first 100 in order to randomize initial conditions. The resulting  $8\times 136$  observations were then summed over the quarter and then skip-sampled to generate the 136 observations that were actually used. The results analyzed in this section are reported in Table A.

The first row in Table A reproduces the results in row 3 of Table 6 in Campbell and Mankiw's paper.  $R_{\Delta y}^2$  is the R-bar square of the regression of  $\Delta y_t$  on the instruments and measures the amount of information in the instruments for  $\Delta y_t$ . (Throughout,  $\Delta s_t$  denotes the first difference of log  $s_t$ .) The other rows report results of doing the same calculations on the 100 simulated data sets using the version of C/E model indicated in the first column. In each location, the number not in parentheses is the average, across 100 simulations. The number in ( ) is the standard devia-

Table A<sup>1</sup>  $\Delta c_t = \mu + \lambda \Delta y_t + \theta r_t$ INSTRUMENTS:  $\Delta c_{t-2}, \ldots, \Delta c_{t-4}, r_{t-2}, \ldots, r_{t-4}$ 

	λ	θ	$R_{\Delta y}^2$	Test of Restrictions <sup>3</sup>
CM Point Estimates	.0467	0.089	.046	-0.006
C/E Model	0.449	.972	.0025	026
	(.015)	(.046)	(.024)	(.019)
	[.55]	[1.00]	[.07]	[.09]
C/E Model with serially correlated technology growth	0093	.820	.098	017
	(.136)	(.485)	(.044)	(.022)
	[.00]	[.89]	[.92]	[.15]
C/E Model with serially correlated technology growth and measurement error	.163	.073	.074	0028
	(.165)	(.188)	(.052)	(.033)
	[.02]	[.53]	[.64]	[.38]
CM Version of C/E Model with serially correlated technology growth and measurement error	.594	.047	.046	0027
	(.114)	(.096)	(.053)	(.031)
	[.90]	[.35]	[.44]	[.40]

<sup>&</sup>lt;sup>1</sup>Results in the first row taken from row 3 in Campbell and Mankiw's Table 5. Results in subsequent rows based on Monte Carlo simulation of model indicated in left column. Numbers in those rows not in parentheses are averages across 100 simulations. Numbers in () are standard deviations and numbers in [] are the frequency of times that simulated results exceed the corresponding parameter value in row 1. 

<sup>2</sup>Adjusted  $R^2$  of regression of  $\Delta y_1$  on the instruments and corresponds to  $\Delta y$  column in the "First-Stage Regressions" section of Campbell and Mankiw's Table 5.

<sup>&</sup>lt;sup>3</sup>Corresponds to the "Test of Restrictions" column in Campbell and Mankiw's Table 5.

tion across 100 simulations. Finally, the number in  $[\ ]$  is the frequency of times that the simulated number exceeded the corresponding empirical point estimate in the first row. It is the p-value of the empirical point estimate under the null hypothesis that the data generating mechanism underlying the simulations is true.

IS THE CAMPBELL-MANKIW ESTIMATE OF THE NUMBER OF RULE-OF-THUMB HOUSEHOLDS A STATISTICAL ARTIFACT? NO.

The second row in Table A reports results of calculations on artificial data generated by the C/E model identical to those performed by Campbell and Mankiw on actual data and reported in the first row of Table A. The surprising feature of those results is that the simulated  $\lambda$ 's are very close to the estimated value of  $\lambda$ . Thus, though by construction there are no rule-of-thumb households in the C/E economy, Campbell and Mankiw's estimator would suggest that 44.9 percent of the households are liquidity constrained. The reason for this perverse result lies in the simulated  $R_{Au}^2$ 's, all but seven of which were less than .046. To see this, consider the results in the third row of Table A. It reports calculations using a modified version of the C/E economy in which  $\Delta y_t$  has been made more predictable by introducing some serial correlation into  $\lambda_t$ . Note that the simulated  $R_{\Delta v}^2$ 's for this model are much closer to its empirical value. Significantly, the simulated value of  $\lambda$  are now close to what one would expect: zero. This suggests that the C/E model's ability to account for Campbell and Mankiw's estimated number of rule-of-thumb households reflects the implausibly low degree of predictability implied for  $\Delta y_t$  in that model. When the model is modified so that it implies empirically plausible values for  $R_{\Delta v'}^2$  then it can no longer account for the high estimated value of  $\lambda$ , as asserted by Campbell and Mankiw.<sup>3</sup>

CAN A SIMPLE MEASUREMENT ERROR ARGUMENT BE USED TO DISMISS THE CAMPBELL-MANKIW ESTIMATES? APPARENTLY NOT.

Measurement error is another possible source of distortion to the Campbell-Mankiw estimates. For example, they use the inflation adjusted return on three-month Treasury bills as their measure of  $r_t$ . From the perspective of a highly aggregated representative agent model like the C/E model, this seems inappropriate since T-bills are the return on a

<sup>3.</sup> Evidently, the C/E model with serially correlated technology shocks generates  $R_{\Delta y}^2$ s which are somewhat *larger* than are observed in the data. I did another Monte Carlo simulation to make sure that the conclusion in the text—that Campbell and Mankiw's estimate of  $\lambda$  is not a statistical artifact—is robust to this. In the simulation I halved  $\rho_{\lambda}$ , setting it to .1. The results corresponding to  $\lambda$ ,  $\theta$ ,  $R_{\Delta y}^2$  and "Test of Restrictions" are .091 (.159) [0.0], .790 (.416) [.56], .036 (.037) [.31], and –.01 (.026) [.15], respectively. Evidently, the results are not much different from those reported in the second row of Table A. Moreover, now the simulated  $R_{\Delta y}^2$ 's are somewhat smaller than the estimated value.

single asset. Presumably, a better measure of  $r_t$  would be a weighted average of all asset returns. Such a measure would preserve symmetry with the way empirical estimates of other variables in the model are computed. For example, the empirical measure of consumption averages across many heterogeneous consumption goods. In any case, the C/E model has no hope of accounting even for the mean of three-month Treasury bills. Roughly, the average rate of return in the C/E model is 6% annually (3% discount rate + unit risk aversion  $\times$  1.88% per capita consumption growth + 1.31% population growth.) This exceeds by far the average return on three-month Treasury bills.

Another source of measurement error in  $r_t$  is more conventional, and centers on the calculation of the price index used to deflate  $r_t$ . In order to see how measurement error in  $r_t$  might affect the results, I simulated the C/E model with serially correlated technology shocks and measurement error. Results appear in the fourth row of Table A. The impact of measurement error can be seen by comparing these results with those in the third row. Doing so, we see that measurement error reduces  $\theta$  substantially, bringing it close to its estimated value of .089. It also moves the coefficient on disposable income in the right direction. However, that coefficient does not go up by very much, since the p-value of the estimated coefficient rises from 0% to only 2%. The other reported characteristics of the Campbell-Mankiw results are well accounted for by the C/E model with serially correlated shocks and measurement error. Apparently it is very hard for the C/E model to account for the high empirical estimate of  $\lambda$ .

Campbell and Mankiw posit the presence of rule-of-thumb house-holds in order to account for the large estimated value of  $\lambda$ . To see why, consider the results based on the CM version of the C/E model with serially correlated technology shocks and measurement errors. These are reported in row five in Table A. There we see that all features,

4. I investigated another possible modification of the C/E model which in principle could account for the large estimate of  $\lambda$ . In this modification the period utility function in (1) is replaced by  $\{\ell n(c_t + \alpha g_t) + 6.98\ell n(2190-n_t)\}$  for  $\alpha = \pm$  .5. (When  $\alpha < 0$ , a jump in  $g_t$  increases the marginal utility of private cosumption, and when  $\alpha > 0$ , it decreases the marginal utility of private consumption.) Permitting  $\alpha \neq 0$  raises the possibility that the statistical role of  $\Delta y_t$  in the Campbell-Mankiw regressions reflects the absence of  $g_t$  from the equation. However, it turns out that in practice this omitted variable effect is not quantitatively large. I simulated the C/E model with serially correlated technology shocks with these utility specifications. When  $\alpha = .5$ , the results corresponding to  $\lambda$ ,  $\theta$ ,  $R_{\lambda_0}^2$  and "Test of Restrictions" were -0.040 (.134) [0.0], .878 [.92], .103 (.043) [.93], and -0.0175 (.020) [.16], respectively. When  $\alpha = -.5$ , the results for  $\lambda$ ,  $\theta$ ,  $R_{\lambda_0}^2$  and "Test of Restrictions" were .020 (.139) [0.0], .767 (.472) [.86], .094 (.451) [.87], -0.016 (.023) [.13]. Evidently,  $\alpha$  negative moves the model in the direction of the empirical results. However, the effect is too small quantitatively to help.

including  $\lambda$ , of the Campbell-Mankiw results are reasonably well accounted for.

In sum, conditional on the model of measurement error, the key problem for the C/E model posed by Campbell and Mankiw's results is the high coefficient on disposable income growth, not the small coefficient on  $r_t$ . The measurement error added to  $r_t$  is very substantial. In particular, the standard deviation of  $r_t$  with and without measurement error is 3.10 (.272) and .665 (.153), respectively (numbers in parentheses are standard deviations across 100 replications.) These numbers—in contrast with all other quantities having a time dimension, which are reported in quarterly terms—are reported in annual terms. Thus the measurement error-ridden rate of return barely resembles  $r_t$ , the former having four times the standard deviation of the latter. I do not know whether this is empirically implausible. In any case, the estimated coefficient on  $\Delta y_t$  is too large to be accounted for by the C/E model, and this is enough to reject it.

# 3. So the C/E Model is False. But is the Campbell-Mankiw Model Any Better?

The first part of this section documents that a version of the C/E model accounts very well for the observed smoothness of consumption, while the introduction of rule-of-thumb households hurts. The second part acknowledges that the C/E's explanation for consumption smoothing rests on certain joint behavior of consumption and asset returns. Although, as suggested in the preceding section, it is by no means obvious how to measure the empirical counterpart of  $r_t$ , preliminary calculations reported below suggest the possibility that the joint behavior anticipated by the C/E model is present in the data.

## ACCOUNTING FOR LOW ORDER DYNAMICS OF CONSUMPTION OF INCOME DATA

Panel A of Table B reports several characteristics of the low order dynamics of  $\Delta c_t$  and  $\Delta y_t$  as implied by the four versions of the C/E model, as indicated in the first column. Panel B presents the corresponding empirical estimates. There, I use consumption of non-durables and services and disposable labor income. The data are quarterly, real, per capita, and seasonally adjusted, covering the period 1953Q2 to 1984Q4. They are the data used in Blinder and Deaton (1985) and Campbell (1987). In Table B,

5. I am grateful to John Campbell for supplying me with this data.

 $\sigma_s$ ,  $\rho(\tau)$  denote the standard deviation and  $\tau^{th}$  order autocorrelation of the variable,  $s_t$ , for  $\tau = 1,2$ .

We evaluate the performance of each model in relation to the empirical results, reported in Panel B of Table B. Note that the C/E model understates the relative volatility of consumption, measured by  $\sigma_{\Delta c}/\sigma_{\Delta y}$ . In each of the 100 artificial data sets generated by this model,  $\sigma_{\Delta c}/\sigma_{\Delta y}$  is less than its empirical counterpart. Also, in view of the discussion about  $R_{\Delta y}^2$  in the previous section, it is not surprising that the C/E model understates the persistence in  $\Delta y_t$ . Finally, the C/E model overstates the first order autocorrelation in  $\Delta c_t$ .

The second set of three rows shows that the C/E model with persistence in technology growth performs much better empirically. First, this model implies an empirically plausible degree of persistence in  $\Delta y_t$ , as can be seen by inspecting the p-values in the middle set of rows of Panel A, which correspond to  $\rho_{\Delta y}(1)$  and  $\rho_{\Delta y}(2)$  in Table B. The greater persistence in  $\Delta y_t$  implied by this version of the C/E model reflects the greater persistence in the technology shock in that model. This in turn implies that the wealth effect associated with an innovation in the technology

Table B<sup>1</sup> LOW ORDER DYNAMICS

	$\sigma_{\!\scriptscriptstyle \Delta y}$	$\sigma_{\!\scriptscriptstyle \Delta c}/\sigma_{\!\scriptscriptstyle \Delta y}$	$\rho_{\Delta y}(1)$	$\rho_{\Delta y}(2)$	$\rho_{\Delta c}(1)$	$\rho_{\Delta c}(2)$
Model		Pane	l A: Simi	ılated Da	ata <sup>2</sup>	
C/E Model with serially correlated technology growth	.0047 (.00031) [0.0]	.500 (.018) [0.0]	.248 (.062) [0.0]	.010 (.101) [.06]	.341 (.075) [.99]	.125 (.119) [.61]
C/E Model with serially correlated technology growth	.013 (.0008) [100.0]	.545 (.026) [.34]	.421 (.063) [.41]	.074 (.108) [.18]	.298 (.099) [.81]	.116 (.130) [.59]
CM Version of C/E Model with serially cor- related technology growth	.013 (.0008) [100.0]	.668 (.022) [100.0]	.421 (.063) [.41]	.074 (.108) [.18]	.539 (.060) [100.0]	.161 (.112) [.77]
	I	Panel B: U	.S. Data	. 1953Q3-	-1984Q4	
	.0088	.554	.443	.190	.220	.077

 $<sup>{}^{1}\</sup>Delta s$  is the first difference of log s.  $\sigma_{s}$  and  $\rho_{s}(\tau)$  are the standard deviation and  $\tau^{th}$  order autocorrelation of s,  $\tau=1,2$ . Results are not reported for the C/E model with serially correlated technology growth and measurement error because these coincide with the results in the middle set of rows.

<sup>&</sup>lt;sup>2</sup>Numbers not in parentheses are averages of the corresponding statistic across 100 artificial data sets generated by the model listed in the first column, while numbers in ( ) are the associated standard deviation. Numbers in [ ] are the frequency of times that simulated results exceed the corresponding empirical parameter value reported in the last row.

shock is greater, thus driving up the relative volatility of consumption. The distribution of  $\sigma_{\Delta c}/\sigma_{\Delta y}$  implied by the C/E model contains the empirical value of .554 very close to its central tendency. Inspection of the relevant *p*-values reveals that the serial persistence pattern for  $\Delta c_t$  implied by this model is also empirically plausible.

Next, we analyze the second moment implications of introducing rule-of-thumb households in the C/E model with serially correlated technology growth. Significantly, one effect is to substantially raise relative consumption volatility. As indicated by the p-value, every simulated value of  $\sigma_{\Delta c}/\sigma_{\Delta y}$  exceeds the empirical value of .554. Introduction of rule-of-thumb households also has the effect of driving  $\rho_{\Delta c}(1)$  implausibly high. In particular, every simulated value of  $\rho_{\Delta c}(1)$  exceeds the empirical value of .220. Of course, in this context rule-of-thumb households have no impact on the dynamics of  $\Delta y_t$ , since disposable income is double what it is in the C/E model with serially correlated technology growth. This doubling has no effect after logging and first differencing.

Note from the numbers in the column marked  $\sigma_{\Delta y}$  that the amount of volatility in output in each model economy differs substantially from its empirical counterpart. This may reflect problems with my method of parameterizing the time aggregated version of the C/E models. In any event, this should act like a scale effect and probably does not affect the remaining results in Table A and B.

## THE ROLE OF ASSET RETURNS AND TIME AGGREGATION IN THE C/E MODEL'S EXPLANATION OF CONSUMPTION SMOOTHING

The fact that the C/E model accounts so well for the observed smoothness of consumption may seem puzzling in light of the analysis of Deaton (1985). This is because the C/E model implies both that consumption is about half as volatile as income and that (the log of) measured income is approximately a first order autoregression in first differ-

6. To check the robustness of the result that rule-of-thumb households imply too much consumption volatility, I did one additional Monte Carlo simulation. Here I introduced the rule-of-thumb households into the C/E version of the model, i.e., the one in which  $\rho_{\lambda} = 0$ . I obtained the following results  $-\sigma_{\Delta y}$ : .0047 (.0003) [100.0],  $\sigma_{\Delta c}/\sigma_{\Delta y}$ : .730 (.0085) [100.0],  $\rho_{\Delta y}(1)$ : .248 (.062) [0.0],  $\rho_{\Delta y}(2)$ : .010 (.101) [.06],  $\rho_{\Delta c}(1)$ : .276 (.065) [.79],  $\rho_{\Delta c}(2)$ : .044 (.106) [.57]. Evidently, this model implies even more volatile consumption. Algebraically, this increased volatility must be due to an increase in  $\sigma_{\Delta c}$ , since  $\sigma_{\Delta y}$  is unaffected by the introduction of rule-of-thumb households. One factor that may account for the increased volatility as  $\rho_{\lambda}$  falls from .2 to .0 is that the correlation between representative agent households' consumption and disposable income rises with the fall in  $\rho_{\lambda}$ . In particular, in the C/E model the correlation between  $\Delta c_t$  and  $\Delta y_t$  averages .98 (.0085) across artificial data sets. On the other hand, in the C/E model with serially correlated technology growth the corresponding results are .53 (.069). (Numbers in parentheses are standard deviations.)

ences with autoregressive coefficient roughly .4. Indeed, with this time series representation for income, Deaton would predict that consumption is considerably more volatile than income. There are two reasons why consumption is instead predicted to be about half as volatile as income in this model. The first was described in Christiano (1987), and reflects that most of the fluctuations in income in the C/E model reflect the impact of technology shocks. It follows from this and the assumed positive autocorrelation in technology shocks, that jumps in income are typically associated with an increase in the prospective return on investment. The latter factor, which dampens the positive wealth effect of an income shock on consumption, is ignored in Deaton's analysis, which assumes a fixed rate of return on investment. The second reason the C/E model is able to account for the observed smoothness of consumption is that-consistent with Campbell and Mankiw's assumption—the timing interval of the C/E model is assumed to be much finer than the data sampling interval. The measured data simulated from this model, because they have been time averaged, display more persistence than do the data actually observed by the agents in the model.8

## THERE IS SOME EVIDENCE THAT THE ASSET RETURN MOVEMENTS ANTICIPATED BY THE C/E MODEL ARE PRESENT IN THE DATA

A particular pattern of co-movements between interest rates and consumption and income is at the heart of the C/E model's account of the relative smoothness of consumption. Obviously, the C/E model's explanation for consumption smoothing would be uninteresting if the co-movements it invokes are counterfactual. In addition to Campbell and Mankiw, Hall (1988) and Deaton (1985) argue that there is virtually no association between interest rates and consumption growth. However, each of these authors defines the interest rate as the real return on three-month T-bills. As I have suggested above, this may not be the appropri-

<sup>7.</sup> Deaton measures the relative volatility of consumption as the ratio of the standard deviation of changes in consumption to the standard deviation of the disturbance of a univariate model of income.

<sup>8.</sup> To see the role of time averaging of data here, consider the simple case in which labor income is a continuous time random walk. Point-in-time samples from this variable will also be a random walk and a Deaton-type analysis will conclude by predicting that consumption ought to be equally volatile as income. On the other hand, if the measured income data are sampled and averaged, then Working's (1960) result indicates that measured income changes will be a first order moving average with MA(1) coefficient roughly .265. A Deaton-type calculation based on these data would conclude that consumption ought to be 1.26 times as volatile as income. This reflects that time averaging a continuous time random walk imparts positive slope to the initial part of the impulse response function of the measured data.

ate empirical counterpart for  $r_t$  in a highly aggregated model. For this reason I investigated several alternative candidates.

Panel B of Table C reports the correlation between  $\Delta c_t$  and  $r_{t-\rho}$  for  $\tau = -2, \ldots, 2$  and several empirical measures of  $r_t$ , including the three-month T-bill. Apart from the last one, which measures the return on economy-wide capital, each is adjusted for inflation using the CPI. In

Table C RATE OF RETURN RESULTS

		Corr	$\cdot (\Delta c_t, r_{t-\tau})$	)1			Standard
	$\tau = -2$	$\tau = -1$	$\tau = 0$	$\tau = 1$	$\tau = 2$	Mean, $r_t$	Deviation, $r_t$
Model		Panel .	A: Resu	lts Based	d on Sin	nulated Da	ıta
C/E Model	.524 (.064)	.563 (.044)	.533 (.047)	.262 (.115)	.185 (.135)	6.07 (.075)	.213 (.047)
C/E Model with ser. corr. tech. growth	.559 (.057)	.538 (.057)	.348 (.109)	.213 (.141)	.210 (.135)	5.89 (.248)	.665 (.153)
C/E Model with ser. corr. tech. growth and meas. er- ror	.106 (.148)	.118 (.162)	.108 (.177)	.088 (.193)	.093 (.198)	5.20 (.494)	3.10 (.272)
CM version of C/E Model with ser. corr. tech. and meas. error	.075 (.152)	.076 (.162)	.079 (.176)	.063 (.190)	.060 (.205)	5.20 (.494)	3.10 (.272)
Return Data	Panel	B: Real, E	Ex Post I	Returns,	U.S. D	ata, 1953Q	3-1984Q4 <sup>2</sup>
S&P 500	075	010	.262	.244	.099	6.43	25.11
Industrial Bonds	046	.052	.177	.175	.151	2.79	3.13
3-Month T- Bills	054	.015	.095	.075	.054	1.04	2.55
Corporate Bonds	054	.044	.166	.164	.143	2.99	3.19
Economy- wide Capital Stock	.275	.280	.229	.155	.137	5.26	.528

<sup>&</sup>lt;sup>1</sup>In the simulated data,  $r_t$ , is the date t net marginal product of capital, plus measurement error as indicated. In the U.S. data,  $r_t$  is the real return on the indicated asset, inclusive of capital gains, adjusted for inflation using the consumer price index. The exception is the return on aggregate capital, which does not include capital gains.

<sup>&</sup>lt;sup>2</sup>The exception is the return on capital, for which data for the period 1953Q3–1984Q1 were used.

addition, the return on the S&P 500 includes the change in the S&P 500 price index to take into account capital gains. The last yield measure is the most comprehensive in coverage. It is the ratio of a measure of the earnings of capital to the stock of capital. Earnings of capital are measured as GNP minus compensation of employees and proprietor's income, all in real terms. The capital stock covers public and private residential housing, household durables, and public and private plant, equipment, and structures. This measure is documented in Christiano (1988). To place this measure on a net basis, I subtracted, .068, the quarterly measure of capital depreciation estimated in Christiano (1988). I did not adjust this measure of return for capital gains using, say, a measure of the change in the relative price of capital and consumption goods. This would be desirable. Without a doubt, this indicator of the return on capital has severe measurement error. For example, excluding proprietor's income from the numerator surely misses out some earnings to capital. Similarly, measurement problems with the stock of capital have been widely discussed. A measurement problem shared by all five asset returns is that they ignore tax effects. Despite these problems, results based on these measures of  $r_t$  are suggestive.

Four things in Panel B of Table C are notable. First, the correlation between  $\Delta c_t$  and  $r_{t-\tau}$  is close to zero for all reported values of  $\tau$  when  $r_t$  is measured by the inflation adjusted return on three-month T-bills. At least for  $\tau = 0$  the association between  $\Delta c_t$  and  $r_t$  is greater for all the other return measures. Second, the correlation between  $\Delta c_i$  and  $r_{i-r}$  is greater for  $\tau > 0$  than for  $\tau < 0$  for market measures of return, while the pattern is reversed in the case of the measure of return on capital. Third, the standard deviation of the return on capital is considerably lower than is the standard deviation of the other return measures. This is reported in the last column of Table C, and is expressed in terms of percent per annum. Fourth, it is roughly the case that an asset with a higher correlation with consumption growth also has a higher mean return. Grossman, Melino, and Shiller (1987), who also noted this pattern, interpreted this as qualitative evidence in favor of a representative agent model. This is because the relevant measure of the riskiness of an asset is its correlation with consumption. Greater correlation implies higher riskiness, which therefore requires a higher average return as compensation.

To see how well the four versions of the C/E model account for the empirical relation between  $\Delta c_t$  and  $r_t$ , one can compare the results in Panel A with those in Panel B. First note that—not surprisingly—the models with measurement error imply relatively little correlation between  $\Delta c_t$  and  $r_{t-\tau}$  for all reported values of  $\tau$ . They appear consistent with all the results in Panel B. Now consider the first two models in

Panel A, the ones without measurement error in  $r_t$ . Of these, it was seen earlier that the second performs better empirically in that it accounts best for the observed relative volatility in consumption and the serial correlation properties of  $\Delta c_t$  and  $\Delta y_t$ . Interestingly, this model also performs better in its implication for the correlation between  $\Delta c_t$  and  $r_t$ . For example, the contemporaneous correlation between these two variables is .348 with a large standard deviation: .109. Although all simulated correlations between  $\Delta c_t$  and  $r_t$  implied by this model exceed the empirical value of .095 obtained using the three-month T-bill, the other empirical correlations are much closer. In particular, the p-values of the correlation between  $\Delta c_t$  and  $r_t$  when the S&P 500, industrial bonds, corporate bonds, and economy-wide capital measures of return are used are .77, .93, .93, and .83, respectively.

Two other interesting features of these results are worth noting. First, the pattern of correlations between  $\Delta c_t$  and  $r_{t-\tau}$  follows that exhibited by the results in the last row in Panel B of Table C, with the correlations being larger for  $\tau < 0$  than for  $\tau > 0$ . Second, the standard deviation of the simulated  $r_t$  is on the same order of magnitude as that of the empirical return on capital, and much smaller than for the market rates of return.

In sum, the C/E model anticipates a positive association between rates of return and consumption growth. Several (admittedly crude) measures of rates of return suggest that that positive association may also be present in the data. This suggests the possibility that the interest rate argument implicit in the C/E's account for consumption smoothing may be on the mark. These results are obviously only suggestive at best and certainly far from definitive, since they use very crude empirical measures of  $r_t$ . Further research to develop better empirical measures of  $r_t$  is required. In addition a further study of these issues ought to consider variations in model parameters. For example, simulations in Christiano (Tables 5–7, 1989) suggest that increasing risk aversion reduces the correlation between consumption growth and the interest rate, while not substantially affecting the implications for the relative volatility of consumption.

### 4. Concluding Remarks

I have made two points. First, it is hard to make the case that the statistical relation between the forecastable components of consump-

<sup>9.</sup> Hansen and Singleton (1983) also find that a representative agent model with preferences like those used here performs better empirically when  $r_t$  is measured by the S&P 500 than by the three-month T-bill.

tion growth, income growth, and interest rates found by Campbell and Mankiw is spurious. I reach this conclusion after ruling out the possibility that the results reflect one kind of measurement error or bias in their econometric technique. Second, Campbell and Mankiw have not yet made a convincing case that this statistical relation warrants the inference that 50% of disposable income goes to rule-of-thumb consumers. One needs to have a sense of what the other implications of this assumption are first. Not enough detail is provided in the paper to make a judgment about this. I report calculations which suggest that the implications on other dimensions may be quantitatively large. I show that a version of the Christiano-Eichenbaum (1988) representative agent model accounts well for the observed smoothness of consumption relative to income. However, introducing rule-of-thumb households into that model raises its implied relative volatility of comsumption to a counterfactually high level.

There is another reason for being cautious about accepting the Campbell-Mankiw rule-of-thumb model. If one accepts their estimate that 50% of disposable income goes to rule-of-thumb consumers, then there is a puzzle as to why time series data imply so many rule-of-thumb households, while micro data studies (e.g., Hall and Mishkin [1982] and Runkle [1983] imply that the number is much smaller, if not zero. One possibility is that the Campbell-Mankiw rule-of-thumb model is misspecified. One particularly suspicious feature of that model is its assumption that the fraction of total disposable income going to rule-of thumb households is constant. An alternative model which does not have this property posits that a fraction of the population has no capital and is shut out of credit markets. Because of this they face a static consumption/leisure choice each period. They are rule-of-thumb households in the sense that they set consumption to disposable income period by period. The other part of the population, which owns the capital, faces a non-trivial dynamic optimization problem. (For details about a model like this, see Danthine and Donaldson [1989]). One expects that in this model the fraction of economy-wide disposable income going to rule-of-thumb households would vary in a systematic way. It would be of interest to see whether such an economy, with a relatively small fraction of rule-of-thumb households and with a reasonable amount of intertemporal substitution in consumption, could account for the Campbell-Mankiw empirical regularity.

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#### Comment

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#### 1. Lucas's Critique and the Euler Equation Approach

Before I comment on the substantive content of the paper by Campbell and Mankiw directly, I wish to say a few words about the so-called Euler equation approach to the study of savings by households.

As Campbell and Mankiw say in their paper, the development of this approach was in response to Lucas's critique of econometric policy evaluation. Lucas's critique emphasized the point that behavioral equations in most econometric models were decision rules of a group of economic agents, and usually contained explicitly or implicitly a specification of how expectations of future values for some critical variables are generated. Such procedure for the formation of expectations is, however, dependent on the characteristics of the environment, and in particular, it is subject to change when the policy rules of the government, which form a part of the environment in which economic agents must operate, are changed. Hence, any evaluation of the effects of policy changes without allowing for changes in the expectation formation procedures are subject to biases and not to be trusted.

In a narrow sense, the Euler equation approach is a proper response to Lucas's critique, since in this approach the rational expectations hypothesis is explicitly incorporated so that any significant changes in the environment are automatically reflected in the expectations formation procedure. On the other hand, so long as changes in the behavioral equations in question are very small in response to a change in the policy rule, the biases in the evaluation of policies pointed out by Lucas will also remain small (Sims, 1982 and 1986). In order to formulate the Euler equation approach, we must assume that the synthetic optimization behavior of a single, representative agent is a good approximation to the collective behavior of the whole population of households. In particular, we must assume that the collective preference ordering of all households over time can be represented by a time invariant utility function of a single representative agent. This is surely very unlikely to be the case, given the difficulties of aggregating preferences well known in the literature, unless the preference ordering of all households happens to be identical. If preferences are not identical, then the aggregate preference ordering (that is, the preference ordering of the representative agent) either cannot exist, or, if it exists at all, it will be subject to substantial changes over time, and therefore subject to Lucas's critique in the wider sense.

We can obtain some feel of how similar the consumption behavior of various groups is, and hence whether or not all groups can be presumed to be acting according to a common preference ordering. In Table A, I present the pattern of the net worth-permanent income ratio by age of the head of the household and by percentiles on the distribution of permanent income, based on the data from Survey of Consumer Finance conducted by the Board of Governors of the Federal Reserve System in 1983. A number of questions might be raised about the procedure followed in generating this table, especially in estimating "permanent income" for each household, but I do not believe that the basic conclusion for the purposes of the present discussion is dependent on such details. The pattern of savings and asset accumulation varies very significantly among age groups and also depends on the household's position in the distribution of permanent income. Therefore, the presumption of common preference ordering among all households cannot be maintained, and the description of the aggregate data based on a single representative consumer is of doubtful value. As the age structure of the population changes or the distribution of income changes over time, the Euler's equation for the representative agent must also change, and the procedure is subject to Lucas's critique as much as the consumption decision rule involving some fixed expectation formation procedure.

The advocate of the Euler equation approach may appeal to the "as if" methodology of Milton Friedman, and say that the empirical validity of the assumptions does not matter, and the test of the theory must be exclusively based on the empirical validity of its market implications. I do not accept this proposition. If we do not make some mistake in our derivation, the assumptions and the implications of a theory should be logically equivalent, and whichever are easier to check against data must be utilized. In the case under discussion, the assumptions are much easier to test than the implications.

## 2. Effects of Current Income

I now turn to specific results reported in the paper by Campbell and Mankiw. Given that we are working within the framework of the Euler equation approach, I like the formulation of the authors. The original formulation of Hall and most subsequent implementations do not specify the alternative hypothesis, so that when the simple version of the permanent income hypothesis is rejected, the rejection does not suggest where the difficulties are and what other possibilities should be investigated, while in the Campbell-Mankiw formulation, we have an alternative which can be elaborated and further investigated. Furthermore, I

Table A NET WORTH—PERMANENT INCOME RATIO'S BY AGE CLASS AND PERMANENT INCOME CLASS 1

Age Group 0 to 5  0.39 0 to 25 17 18.6				P	Fermunent Income Fercentile	ome rerent	211			
	5 5	6 to 10	11 to 25	26 to 50	51 to 75	76 to 90	91 to 95	96 to 99	100	all
	0.39	1.57	0.32	0.99	0.44	0.42	0.29	0.59	1.19	0.62
	17	18	55	90	90	53	17	14	4	358
	18.6	19.9	58.3	98.3	97.9	58.4	19.5	15.6	4.5	390.9
3.13	.13	1.59	1.79	1.70	1.60	1.47	1.52	2.18	1.76	1.73
26 to 35 44		45	132	218	216	126	43	37	14	875
46.7		46.5	140.9	235.3	234.3	140.4	48.0	37.0	10.0	939.1
5.78	.78	3.05	3.42	2.55	3.45	3.56	3.29	4.97	7.86	3.43
36 to 45 35		35	104	169	171	102	36	53	40	745
37.4		38.3	113.8	190.1	187.7	114.0	37.5	31.4	7.6	759.7
16.64	<b>4</b> 0.	5.07	4.63	5.71	4.05	7.01	7.06	16.81	18.97	6.48
46 to 55 29		30	88	141	149	86	36	76	40	675
31.0		32.9	96.0	159.6	160.5	95.4	32.6	26.1	6.5	640.6
15.56 56 to 60 14 14.8	.56 8	3.33 14 14.5	8.64 41 44.8	6.98 65 74.0	7.25 66 74.0	9.73 42 44.3	10.10 18 15.0	15.33 43 12.9	18.83 24 3.2	8.59 327 298.5

61 to 65	52.11	10.68	7.21	7.22	8.20	9.19	14.02	26.38	18.60
	15	13	40	67	68	41	18	48	23
	15.1	14.9	45.3	75.2	75.7	45.1	15.1	12.1	3.2
66 to 70	34.42	14.85	7.06	8.43	7.40	9.81	15.27	18.55	30.29
	11	11	36	60	59	33	18	34	15
	12.4	12.2	39.2	63.9	64.9	38.2	13.1	10.6	2.6
71 to 75	31.61	17.64	4.36	9.39	9.46	8.63	12.27	5.36	27.96
	8	10	25	42	43	26	11	11	15
	8.6	9.9	27.7	46.8	46.5	27.5	10.1	7.6	1.9
75 and over	4.68 11 11.4	26.88 11 12.4	5.34 33 36.2	7.99 52 59.4	6.48 52 60.6	5.61 32 36.0	4.37 11 12.8	10.40 13 9.5	32.33 19 2.6
All	13.53	6.37	4.00	4.40	4.17	4.95	5.77	9.45	12.87
	184	187	554	904	914	541	208	329	194
	196.0	201.6	602.3	1003.4	1004.1	599.2	203.7	162.9	42.0
Top: Wealth to permanent income ratio Middle: Sample size. Bottom: Weighted sample size. Data from Survey of Consumer Finance, F	permanent in size. ed sample siz ey of Consume	come ratio. .e. r Finance, Fed	eral Reserve B	r. Federal Reserve Board, 1983, and estimates prepared by Scott Hoyt	estimates prep	ared by Scott 1	Hoyt.		

11.40 333 301.7 10.71 277 257.1 10.19 191 186.7 7.83 234 240.8 5.27 4015 4015

find the basic result obtained by Campbell and Mankiw to be broadly consistent with results that some of us often encounter working with micro data; namely, that only one-half to two-thirds of households behave according to the permanent income hypothesis, while the remaining one-third to one-half respond to current income.

We must, however, be cautious in interpreting the results like the ones reported in Table 1 of their paper. The authors are saying that equation (1.4) is obtained by summing (1.3) and the equation given on the second line at the top of page 188 of their paper, and hence the estimated coefficient  $\lambda$  in equation (1.4) must be the properly weighted average of the coefficients applicable to the two groups, namely, zero and unity. There are a number of fairly strict conditions under which the expected value of the estimated parameter using aggregated data would in fact turn out to be such a weighted average, and we must pay careful attention to such conditions (Theil, 1954).

In order to assess how robust the results reported in their Table 1 may be, we may ask ourselves what mechanisms may be present that would make current consumption a function of past events such as  $\Delta C_{t-i}$ ,  $i \geq 2$ , given  $C_{t-1}$ . Any gradual adjustment process may cause such a correlation, and even though the authors are dealing with non-durables and services, there are prime examples of slowly adjusting items among consumption goods. Income contains many different components. When the weight for some income component, such as social security benefits, increases over time during the sample period, some biases in the estimate of  $\lambda$  can easily be introduced, especially if this component behaves differently from the rest.

I wish to deal explicitly with one possible mechanism that may create biases in the estimate of  $\lambda$ . According to the life cycle theory as distinct from the permanent income hypothesis, the consumption needs of families are critically dependent on the age of the family. The earnings pattern over life is also known to be a significantly dependent on age. Therefore, both aggregate consumption and aggregate income are dependent on the age distribution of population, and hence, if the age distribution has been changing over time during the sample period, this may generate the positive correlation between  $\Delta C$  and  $\Delta Y$  even when the instrumental variables procedure is used.

I have conducted a quick experiment to see if there is any indication suggesting that this consideration is significant. In Table B, I report a slight modification of one of the estimates reported in Table 1 of the Campbell-Mankiw paper. Row 1 of Table B corresponds to Row 8 of Campbell-Mankiw, except that I drop  $c_{t-2} - y_{t-2}$  from the list of instruments. Actually, this was an oversight on my part, but it makes little

difference to the point that I wish to make. For Row 2 of Table B, I introduce a set of age compositions variables, both as instruments and as regressors. The estimate of the weight  $\lambda$  is reduced substantially, although none of the coefficients for the population composition variables is significant. The lack of significance is not surprising in view of the fact that the linear introduction of the age composition variables is not really appropriate, but the result is suggestive in that the presence of these variables even in this crude form appears to have an important effect in the coefficient of  $\Delta Y$ .

This result is more or less consistent with Table A and suggestive of the significance of the age composition of the population. In order to estimate the effect of age composition, a much more precise formulation must be undertaken.

#### 3. Consumption Income Ratio and the Expected Growth of Income

I now turn to the novel attempt by Campbell and Mankiw to look at the consumption decision rule rather than the Euler equation. The basic non-linearity of the budget constraint that they refer to arises because they focus their attention on the random character of the rate of return. There is little question that the rate of return in reality is a random variable. Does a typical consumer, however, really optimize in the context of such a complex formulation of his environment? And, if so, can such a sophisticated consumer really be characterized by an infinite horizon, symmetric and separable utility functions?

Modigliani thought otherwise. He thought that the savings-income ratio was positively related to the rate of growth of income. His reason-

Table B	EFFECTS OF INTRODUCING SHIFTS IN AGE COMPOSITION OF
	POPULATION ADDENDUM TO CAMPBELL-MANKIW TABLE 1

				Coeffic	cients of	
	Instrument	λEstimate	N20	N25	N45	N65
Row 1	$\Delta y_{t-2}, \ \Delta y_{t-3}, \ \Delta y_{t-4} \ \Delta c_{t-2}, \ \Delta c_{t-3}, \ \Delta c_{t-4}$	.455 (.123)				
Row 2	$\Delta y_{t-2}$ , $\Delta y_{t-3}$ , $\Delta y_{t-3}$ , $\Delta y_{t-4}$ $\Delta c_{t-2}$ , $\Delta c_{t-3}$ , $\Delta c_{t-4}$ N20, N25, N45, N65	.386 (.131)	.025 (.221)	+.020 (.116)	+.041 (.203)	+.110 (.226)

N20: The ratio of population aged 20-24 to population 16 and over

N25: The ratio of population aged 25-44 to population 16 and over

N45: The ratio of population aged 45-64 to population 16 and over

N65: The ratio of population aged 65 and over to the population 16 and over

ing was based on the assumption that the relative age pattern of consumption observed in the micro data represented, to a large extent, the preferred pattern of consumption, independent of the size or the life pattern of income, including the dissavings by retired families (Modigliani, 1966, 1970, and 1980).

For a few countries for which there are data covering long periods of time, the savings-income ratio tends to be very stable. In one case, Japan, the savings rate during the 1950–85 period when the growth rate was very high was distinctly higher than the years before World War II when Japan's growth rate was lower. The cross country correlation between the savings rate and the rate of growth of output appears to be very strong and positive (Modigliani 1970). Thus, the finding by Campbell and Mankiw that these two ratios are actually negatively correlated in the U.S. came as a surprise to me.

I then realized that they are working with the NIA definition of disposable income during a period when the rate of inflation varied quite significantly. Since the NIA definition of disposable income includes nominal interest flows while it does not adjust for real capital gains or losses in nominally fixed assets and liabilities due to inflation, it contains an inflation bias. One may argue exactly which assets and liabilities may be subject to this bias, but my experience with this subject suggests that the results of the correction do not depend on the choice of assets within reason. I have supposed that corporation and financial institutions are a veil for this purpose, and taken government debt outside the government (alternatively, government debt in private hands plus currency plus reserves at the FRB) as the quantity subject to real capital loss by households, and made a rough correction based on this assumption. The resulting changes in the savings-income ratio is shown in Table C. Column (3) is the savings-income ratio before the correction, and column (7) is the ratio after the correction. We can see that the savings rate during the period between the 1950s and 1980s is virtually constant for the corrected ratio except for the very low rate for the 1980s. It is unlikely that we get any relationship between column (7) and the rate of growth of income.

It is also useful to remember the accounting identity. For the household sector of the economy, we have

 $s \equiv g_a a$ 

where s is the savings-income ratio,  $g_a$  is the rate of growth of net worth, and a is the ratio of net worth to income. For the U.S., a is very stable over time so that, except for very short-run fluctuations, the rate of

growth of net worth,  $g_a$ , is very close to the rate of growth of income, g. Therefore, in order for s to be negatively related to g, in view of the above identity, the net worth-income ratio must move inversely with the rate of growth of income very sharply. That is, when the growth rate rises by 20% from .015 to .018 per year, the net worth-income ratio must decline much more than 20% in order for the saving-income ratio to decline, except in very short-run fluctuations of one or two years. This seems very implausible to me.

### 4. Stability of the Relationship Between Consumption and Income

I began this note by suggesting that Lucas's critique should be more broadly understood and that the basic question is how stable and reliable the critical macro relationships are over time, especially when some conditions in the economy including major policy rules of the government change. I suggested that this question must be an empirical one. In the case of consumption-savings behavior of the household, I expressed my skepticism of a single representative agent model on the basis of micro data indicating that the behavior of different groups of households, for example, age groups and groups defined by relative positions in the income distribution, appears to be very different from each other.

In the older literature, a number of investigators found that the relationship between consumption and some combination of income and wealth seemed to be quite stable over time. We have always known that

Table C	AGGREGATE SAVINGS/INCOME RATIO FOR U.S. HOUSEHOLDS
	NIA DEFINITION AND INFLATION ADJUSTMENTS

	(1) YD\$	(2) S\$	(3) S\$/YD\$	(4) Inflation Adjustment	(5) (1)–(4)	(6) (2)–(4)	(7) (6)/(5)
1953	255.1	18.4	7.2	4.4	250.7	14.0	5.6
1954	260.5	16.4	6.3	1.2	259.3	15.2	5.6
1955	278.8	16.0	5.8	4.5	274.3	11.5	4.2
1960	358.9	20.8	5.8	6.7	252.2	14.1	5.6
1965	486.8	34.3	7.0	7.2	479.6	27.1	5.7
1970	715.6	57.7	8.1	16.7	698.9	41.0	5.8
1975	1142.8	104.6	9.2	44.5	1098.3	60.1	5.5
1980	1918.0	136.9	7.1	88.1	1829.9	48.7	2.7
1985	2838.7	125.4	4.4	64.3	2774.4	61.1	2.2

<sup>(1):</sup> NIA Table 2-1, Line 25

<sup>(2):</sup> NIA Table 2-1, Line 30

<sup>(3):</sup> MPS Model Data File, (Government Dept Outside Government and Outside Fed + Currency + Reserves) Inflation Rate (Consumption Component of GNP Deflator).

such an empirical relationship is subject to serious questions, and the causality may be running from consumption to income rather than income to consumption. In recent years, we have not paid attention to this formulation, but I have taken this occasion to quickly review the history of this type of relationship. I am rather impressed that the stability of this relationship appears to persist for a very long time. In Table D, I reproduce some of this history, covering the period from 1900 to 1987 divided into three segments and excluding the major war years.

First, the results of the regression in level form are almost identical for all three sub-periods, in spite of the differences in the quality of the data and the fact that for the two earlier periods, income is represented by labor income after taxes while for the last period it is total income after taxes (the coefficient of Y for the last period is therefore somewhat smaller), and for the earlier two periods annual average data were used

#### Table D RELATION BETWEEN CONSUMPTION AND INCOME

1. OLS Estimate of Consumption - Income + Wealth Relation 1953-II—1987-I

$$C = .714 \ Y + .055 \ W - .589$$
  
(.020) (.005) (.078)  
 $R^2 = .995 \ DW = .25$ 

1b. Regression of 1st difference in logs

$$\Delta ln C = .307 \ \Delta lnY + .094 \ \Delta lnW + .003$$
 $(.041) \ (.026) \ (.0004)$ 
 $R^2 = .43 \ DW = 1.92$ 
 $R^2 = .43 \ DW = 1.92$ 

- 2. Ando-Modigliani Estimates (1963)
  - 2a. Annual Data for 1929-59 excluding 1941-46

$$C = .75 \ YL + .042 \ W + 8.1$$
  
 $(.05) \ (.009) \ (1.0)$   
 $R^2 = .948 \ DW = 1.26$   
 $\Delta C = .52 \ \Delta YL + .072\Delta W$   
 $(.16) \ (.018)$   
 $R^2 = .929 \ DW = 1.85$ 

2b. Annual Data for 1900-1928 excluding 1917-19

$$C = .76 YL + .073 W$$
 $(.13) (.020)$ 
 $R^2 = .995 DW = 1.63$ 
 $\Delta C = .73 \Delta YL + .047 \Delta W$ 
 $(1.8) (.037)$ 
 $R^2 = .44 DW = 2.48$ 

while for the last period quarterly data were used. If annual data were used for the last period, estimates would have been about the same, but the DW statistics would have been considerably larger.

It also turns out that the results using data in first difference form are very similar to the level regression for the first two periods. For the most recent period, I present the result using data in the form of the first difference of logarithms, but if the appropriate transformation is carried out to get an approximate linear form, the result in the level and the result in the first differences are similar.

These results are subject to all the well known objections to the naive formulation and estimation procedure, and hence we must view them as merely suggestive rather than as a strong evidence for any well formulated hypothesis. See, however, the proximity theory of Wold (1953) and Fisher (1961). We can improve the quality of the result and strengthen the stability of the result over time by recognizing that income and wealth both contain a number of different components and they should be treated somewhat differently, by smoothing short-term fluctuations of income by some filtering procedure to approximate a longer-term normal income, and by recognizing that the coefficients are functions of the age distribution of the population and hence they should be allowed to change in response to the changing age distribution over time. The proximity theorem would then apply to these results with even more force.

Some of us thought that the formulation like the one presented in Table D was a unique implication of the life cycle theory. It turns out, however, that they can be derived almost equally well from very different theories, so in this context I am reporting them merely as a surprisingly stable empirical relationship, not as an implication of any particular theory. On the other hand, I should point out that the stability of the result persisted over a long period in which very radical changes in government policies toward households took place. At the beginning of the period, there was no income tax and the Federal Reserve System did not exist. Given that the relationship retained its stability in spite of all these changes, if this relation formed a part of the model used to analyze policy changes that did take place during this period, this relationship would not have caused any apparent bias in the results.

In an ideal world, we should begin with a description of the individual household's behavior based on micro data, allowing for critical and significant differences among various groups, and go through the detailed aggregation process to arrive at aggregate behavioral functions. In the process, we have some knowledge of properties that aggregate relation-

ships must satisfy, such as the one described in this section and perhaps the one Campbell and Mankiw described in their Table 1. We then have a much better understanding of the source of these relationships that persist over time, and we can judge with more confidence under what conditions persistent relationships will remain stable.

In such an effort to understand the behavior of households combining information from the micro and macro data together, on the macro side, we have come to focus our attention completely on the result obtained from the Euler equation approach to the exclusion of the type of information reported in this section, quoting Lucas's critique as the authority. I believe that we have gone too far, and that judicious attention to all information, especially to those relationships that have survived over very long periods of time under a number of different conditions in several countries, would be essential if we are to make really significant progress in our attempt to improve our knowledge of household behavior.

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### Discussion

Mankiw noted that Christiano's implied regressions yielded much poorer first stage regressions than those found by the authors and that interest rate mismeasurement does not matter for the results. He also questioned whether "rule-of-thumb" consumers make Ando's inflation-adjustment to income.

Bob Hall objected that Campbell and Mankiw had set up a "straw man" version of the random walk hypothesis by not taking into account the effects of liquidity constraints. He further stated that Campbell and Mankiw had used the identifying restriction that there are no random consumption components. He argued that if such components exists, they cause spontaneous movement in output, which would be correlated with the instruments used by Campbell and Mankiw. They do not, he argued, establish the direction of causation and yet take a strong stand on the results. In addition, Hall suggested that Campbell and Mankiw should use additional measures of rates of return.

Mankiw responded that there is large variation in post-war real interest rates. Further, the authors had tried "truly exogenous" instruments to account for taste shocks, but the results were insignificant. Theory suggests that such instruments may be poor in small samples.

Kevin Murphy asked whether the estimated coefficient of zero on the interest rate was evidence of bad instrumental variables or zero intertemporal substitution. Bill Nordhaus questioned whether there is measurement error in consumption since the theory applies to utility. Consumption ignores durables, such as housing services consumed. Further, lagged variables would not be good instruments if durables are included. Mankiw responded that since durable goods follow a random walk, that would not affect the estimate.

