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COMMENTS ON "IMPERFECT OBSERVATION AND SYSTEMATIC POLICY ERROR"

BY KENT WALL

The author of the paper investigates the adverse effects of observation errors on policy response in the context of a Linear-Quadratic-Gaussian (LQG) control problem. His principal conclusions are based on empirical findings arrived at by simulation of a macroeconomic model under the hypotheses of perfect and imperfect measurement of the states (current and lagged endogenous and exogenous variables). While there is no doubt as to the validity of the author's conclusions, it is suggested that the same conclusions could have been arrived at in a more efficient manner, by relying on some long established results given in the control theory literature.

The stated purpose of the paper is "...to assess the quantitative significance of the systematic tendency toward delayed policy response ..." This implies evaluating the effects of imperfect observations in terms of the only quantitative measure given in the paper, namely the optimal (minimum) expected value of the loss function, λ. If imperfect observations lead to a degredation in the quality of control due to the need to employ a state estimator, then an increase in the optimal expected value of λ will result. But such a finding can easily be obtained through examiniation of the representation for the optimal expected loss, without ever having to carry out any simulations (or Monte Carlo studies)!

Let us consider, for purposes of exposition, the quadratic regulator problem. This is related to the tracking problem of the present paper by fixing both $\hat{x}_i$ and $\hat{u}_i$ equal to zero over the planning horizon.* If J denotes the optimal expected value of λ then it is well known that (see for example Astrom [1970] pp. 269–283; or Bryson and Ho [1969], pp. 428–432)

$$J = \text{trace}\{KX_0 + \sum_{i=0}^{N-1} K_{i+1}W + \sum_{i=0}^{N-1} K_{i+1}BG_iT_iA'\}$$

where K_{i+1} represents the positive definite solution to the matrix Riccati equation, and X_0 is the covariance matrix associated with the expected initial state deviations from the desired initial condition ($x_0 = 0$). The first term in J expresses the loss due initial condition errors. The second term gives the loss due to random shocks in the state equations. The last

*No loss in generality is suffered by examining this special case; the results are the same for tracking.

term represents the loss due to control under uncertain state knowledge. It is this last term that is at the heart of the study of the article. Under perfect state knowledge $T_i = 0$. If there are observation errors, then it can be shown that

$$\sum_{i=0}^{N-1} K_{i+1} B G_i T_i A' \geq 0,$$

and thus the presence of imperfect observations will always tend to *increase* J. The expected increase in loss can always be precisely quantified by evaluation of the third term in J—without ever simulating the model.

Finally, note that the tendency for lagging response in the policy and the effects of the covariance structure of state estimation process could also have been ascertained a priori by employing the well known expressions for the mean-square trajectories of the controlled system. Examples of these equations can be found in Bryson and Ho [1969] pp. 431–432.

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