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IMPERFECT OBSERVATION AND SYSTEMATIC POLICY ERROR

BY WILLIAM E. CONRAD

This paper assesses the quantitative significance of the systematic policy errors resulting from optimal policy responses to imperfect information. The policy response mechanism is provided by the solution to the linear-quadratic control problem with additive model disturbances and noisy observations on the current state of the economy. The policy is implemented using a small quarterly U.S. macro model and "observation error" covariance matrix estimates from NIA revisions data. Pairs of simulations are run with one model equation at a time being shocked. In the first run of the pair, exact observations are used in determining the policy response. In the second run, exact observations again are used, but the Kalman filter is employed as if the observations were subject to error. Thus, we are able to focus on the systematic differences in the time paths of both policy and endogenous variables resulting from the differences between appropriate responses to exact and to imperfect data.

This paper assesses the quantitative significance of the systematic tendency toward delayed policy response to economic developments implied by an optimal response to imperfect data. In setting policy, the policymaker responds to incoming data reflecting economic developments, present and recently past. In fact, the incoming data provide an imperfect reflection of economic developments since sometimes the data are subject to substantial error.[1] The existence of error in the data on which he must rely creates a two-horned dilemma for the policymaker: on the one hand, if he ignores the possibility of observation error and responds fully and immediately to the incoming data as if it were perfect, he will frequently react to what turns out to be merely aberrations in the data; on the other hand, if he delays policy response until the situation clarifies, he will generally fail to react to economic disturbances when appropriate. If either course is followed, poor performance results.[2] Clearly, an appropriate response will involve a course somewhere in between the two extremes, leading to the presence of both types of error, but in moderation.

In the following section, the optimal policy response mechanism is sketched out and its implications for systematic policy error discussed. The succeeding three sections set forth the various elements needed to implement the policy response mechanism in order to assess the quantitative significance of systematic policy error. In the final sections, simulation exercises based on the optimal response framework are used to com-

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[1] A detailed examination of National Income Account revisions is available in Cole (4, chapter I).

[2] The implications of policy response to provisional data are examined by Conrad (5). Phillips (8) examined the effects of lags on the performance of stabilization policy.

pare the responses to identical disturbances with perfect data and with data known to be subject to error.

I. THE POLICY RESPONSE MECHANISM

The problem of policy formulation is modeled as a multiperiod dynamic minimization of the expected value of the policymaker's loss function over a finite planning horizon subject to the constraint of a linear macro-economic model and complicated by imperfect observation of the state of the economy. Thus, there are three distinct entities involved in the problem specification—the model, the loss function, and the observation process.

The economic model is a general vector first order linear difference equation of the form:

$$x_{i+1} = Ax_i + Bu_i + Cz_i + w_i$$

where x_i, u_i, and z_i are endogenous variables, policy variables, and exogenous variables respectively, and w_i is a disturbance vector assumed to be Gaussian with mean zero and independent over time.

$$E(w_i) = 0 \quad E(w_i w_i') = W \quad E(w_i w_j') = 0 \quad i \neq j$$

The loss function λ is assumed to be additive over time with each period's loss quadratic in endogenous and policy variables.

$$\lambda = \sum_{i=1}^{N} (x_i - \hat{x}_i)'Q(x_i - \hat{x}_i) + (u_{i-1} - \hat{u}_{i-1})'R(u_{i-1} - \hat{u}_{i-1})$$

where $\hat{x}_i$, $\hat{u}_{i-1}$ are the target values for the endogenous and policy variables respectively and Q and R are symmetric positive semidefinite penalty matrices. During the optimization horizon, observational information on the state of the economy comes from an observation process described by

$$y_i = x_i + v_i \quad i = 0,,,N - 1$$

where v_i is a Gaussian random variable of known density having mean zero, independent over time, and independent of all w_i

$$E(v_i) = 0 \quad E(v_i v_i') = V \quad E(v_i v_j') = 0 \quad i \neq j$$

It is assumed, however, that all exogenous variables for each time period are known exactly at the outset and that the values of all policy variables in prior periods are known exactly.

Problems of this type have been explored extensively in the engineering systems literature; a derivation of the solution to this specific problem is available elsewhere[3] and will not be given in detail. The solution involves two separable parts: 1) a feedback rule giving the optimal policy response to the estimated state of the economy and 2) a state estimation procedure involving the Kalman filter.[4]

The optimal policy in period i, u_i^*, can be represented by the response function

$$u_i^* = G_i s_i + g_i$$

where s_i is the state estimate, which is the expectation of x_i conditional upon past and current observations on the endogenous variables and the known actual values of past policy and exogenous variables.

The generation of the estimate of the current state of the economy, s_i, involves the combination of the current observation, y_i, and a projection using the model and an estimate of the state in the previous period. If we denote the projection of the current state of the economy, p_i, where

$$p_i = As_{i-1} + Bu_{i-1} + Cz_{i-1}$$

then

$$s_i = T_i N_i^{-1} p_i + T_i V^{-1} y_i$$

where T_i is the variance matrix of s_i

$$T_i = (N_i^{-1} + V^{-1})^{-1}$$

and N_i is the variance of the projection error[5]

$$N_i = W + AT_{i-1}A'$$

The projection error variance reflects (1) the inadequacy of the model in representing the economic process so that, even given the right starting point, the projection would diverge somewhat from the actual movement of the economy, and (2) error in the estimate of the past state of the economy leading to error in projecting the current state independent of the model's shortcomings.

Together, the state estimate, s_i, and the reaction function defining u_i^* given s_i constitute the optimal policy response in the face of imperfect observational information.

Given a stable model, both the policy response matrix G_i and the

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[3] For background in the engineering literature, the reader is referred to Aoki (1), Astrom (2) and Sage (9). This specific problem is treated in Conrad (5 pp. 31–53).

[4] The combination of multiple sources of information in generating forecasts has recently received attention (7). Also, the role of forecasts in stabilization policy has been reviewed (6).

[5] Due to possible matrix singularity, an alternative computational scheme based on Conrad (5, pp. 48–51) was used in the empirical work.

state estimation apparatus, N_i and T_i, will converge to constant values away from arbitrary and unrealistic end point assumptions, i.e., perfect knowledge of the starting point and no concern for policy effects beyond the horizon. Thus, it will be sufficient to examine the pattern of policy responses corresponding to these converged values.

II. POLICY RESPONSE CHARACTERISTICS RESULTING FROM IMPERFECT OBSERVATION

As was noted in the introduction, the existence of observation error leads to two types of policy errors—a failure to react with desirable speed and force to economic disturbances and a tendency to take undesirable policy action to offset what later are seen to have been data errors. The nature of the lag in policy response resulting from the optimal policy apparatus can readily be seen. Since $y_i = Ax_{i-1} + Bu_{i-1} + Cz_{i-1} + w_{i-1} + v_i$, any disturbance w_{i-1} appears in a term which gets only part of the weight in forming the state estimate. p_i will not reflect w_{i-1} at all. In fact, w_{i-1} will show up only partially in p_{i+1}—and then only to the degree that it has been incorporated in s_i. Thus, the policy response to a shock, w_i, is closely related to the speed with which the shock becomes incorporated in the state estimate s_i.

On the other hand, since y_i, which contains the observation error v_i, gets part of the weight in the state estimate, the policy adopted will include some response to the observation error. Error from this source will not lead to any *systematic* bias in policy response to economic developments.

In order to implement the policy response mechanism outlined above, we require a detailed specification of the three major elements of the problem statement, i.e., the model, the loss function, and the observation process. These elements are outlined in the next three sections.

III. THE MODEL

The simulation exercises are conducted using a small quarterly linear model of the U.S. economy estimated over the period 1957III through 1974II. The model has 13 endogenous variables and 11 stochastic equations. Consumer expenditures on durables are treated separately from consumption of nondurables and services. There are separate equations for each of the three main components of investment: inventory, residential construction and plant and equipment. Definitions of GNP and disposable income, together with an import equation, complete the expenditure sector. A single money market equation is used to generate the short term (Treasury bill) interest rate, and a term-structure equation supplies the long term (corporate bond) rate. The wage rate equation is based on

Kuh's marginal-value-product-of-labor approach and the price level is determined by a markup equation. Unemployment is the difference between the labor force adjusted to take into account the discouraged-worker-effect, and level of employment.

The policy apparatus consists of three variables—government expenditures, a lump sum tax surcharge, and the money supply. Taxes are defined as the sum of the surcharge and an endogenous component determined by a constant leakage rate used in defining disposable income analogous to an overall tax rate but including other net withdrawals from GNP as well.

The dynamic properties of this model are reasonable. The model is stable with the largest eigenvalue less than one in modulus, and the steady state multipliers are not out of line with those of larger models. The eventual response of real GNP to a \$1 sustained increase in real government expenditures is \$2.21. A sustained \$1 real tax surcharge increase leads to a \$1.93 decrease in GNP, while a \$1 increase in the money supply leads eventually to a \$.76 increase in GNP. The fiscal policy dynamic multipliers indicate substantial initial impact. The peak impact on GNP is reached after 6 quarters for a sustained increase in government expenditures and after 7 quarters for the tax surcharge. The peaks are followed by quite damped cycles. The response to monetary policy is much smoother and more gradual; the peak response to a sustained increase in the money supply is reached after 8 quarters.

IV. Loss Function and Target Paths

The general statement of the optimization problem presupposes a quadratic loss function. The remaining choices involve the relative weights to place on the departures on the various endogenous and policy variables from their target paths; the choice of target paths will not affect the results since they enter additively. After some experimentation, penalties of 300, 5, 100, and 30 were chosen for squared deviations in employment, GNP, the price level, and nonresidential fixed investment, respectively. In the simulations, only one policy variable was left relatively free at a time (penalty weight of .01 compared to 9999 for the other two). This provided a much clearer pattern of policy response to the disturbance by avoiding instances of "fine tuning" in which policy variables moved in opposing directions.

V. Data, Revisions and the Observation Process

Neither "measurement error" nor "observation error" captures the complexity of economic data problems. In the physical sciences it is the usual case that the magnitude of observation error can be obtained directly from knowledge of the precision of the instruments used to measure. However, economic data does not lend itself to such convenient specification of the error characteristics. In fact, the majority of numbers reported quarterly are hard to reconcile with the intuitive meaning of the word "observations," since they involve a combination of partially reported series with extrapolations (and, in revision, interpolations) of other series reported annually and even less frequently.[6]

The estimate of the observation error covariance matrix is based mainly on National Income Accounts data for the period 1965III through 1977II. For each data item, the most recent revision available on a compatible conceptual basis was subtracted from each earlier reported value of the particular item. The resulting differences—or observation errors—can be viewed as samplings from a stationary normal process. The sample covariance is used as an estimate of the covariance of the observation process in computing the optimal state estimate.

VI. SIMULATION OF POLICY RESPONSE

The multiplicity of stochastic elements in the problem—a disturbance vector and an observation error vector for each period of simulation—poses a potential difficulty in evaluating the characteristics of policy response resulting from the Kalman filter. Because we are working in a linear framework, however, the errors have additive impacts, and thus we may proceed to examine one source of error at a time. Also, because we are interested solely in the typical response pattern—which is embodied solely in the policy response mechanism—the observation errors themselves are not needed since they would not lead to any further *systematic* influence on policy responses. Thus, we need only concern ourselves with the disturbance vector, w_t. Since the disturbances also have additive impacts, it will be convenient to shock only one variable at a time. And because we use converged values of the policy response and filter mechanisms, it will be sufficient to shock only one period; the response in other periods would be the same. Finally, in order to facilitate focusing on the response to the shock, we first run a deterministic simulation without disturbances or observation errors to serve as a benchmark. The shocked simulations will be compared as deviations from this deterministic benchmark. This procedure has the advantage that our results are independent of the target paths chosen for the endogenous and policy variables, the time path of exogenous variables and the initial conditions.

In order to compare policy responses with exact observation to those

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[6] Cole (4) provides an introduction to some of the considerations along with references to the literature.

with imperfect observation, three series of eight pairs of simulations were run. In each series of simulations, only one policy variable was free to respond to disturbances while the other two policy variables were immobilized by large penalties for deviations from their respective target paths. The eight pairs of simulations in each series correspond to the eight imperfectly observed endogenous variables; in each pair of runs, a single equation was subjected to a shock having the size of the standard error of the regression residuals from the structural equations without the autoregressive correction. In the first run of each pair, it was assumed that the endogenous variables were observed exactly. Thus, policy could respond directly to x_i, rather than to the state estimates. In the second run of the pair, the policy was assumed to respond to imperfect observations so that the state estimation apparatus was used even though the observations were in fact exact. Thus, the pair of runs involve alternative responses to the same shock with the only difference being that in one case policy responses are based on the assumption that exact observations are available and in the other case the data is assumed to be subject to error typical of those encountered in the NIA revisions.

The simulation results show significantly different patterns of policy response between the two runs of each pair. (Table 1) In the case of the six expenditure variables, the policy response in the first period after the disturbance under the assumption of imperfect information is weaker than the response relying on exact information; this is true for each of the three policy instruments used.

In the case of EMPT, the policy response with imperfect information is perverse in that it moves in the opposite direction from the response to exact information. The response to the shock to PGNP is stronger under imperfect observation than otherwise. (Of course, this stronger response is not preferable. The response with exact observation assumed is the best possible response to the shock; any other response—stronger or weaker—will lead to a higher loss.)

An examination of the state estimates provides considerable insight into the nature of the policy response pattern evident in the case of EMPT and PGNP shocks. In the case of EMPT, it is clear that the typical covariances between EMPT and PGNP played an important role in the state estimate. Even though the increase in the level of EMPT is substantially underestimated, this increase leads to an estimated slight increase in PGNP, when in fact, PGNP actually fell slightly. Thus, the policy response to the state estimate was in a restrictive direction; while with exact observation, a slightly more expansionary stance was taken to accommodate the fortuitous increase in employment.

The situation is much the same with PGNP. Typical covariance patterns suggested that the increase in PGNP, implied an expansion in

TABLE 1

First Period Policy Responses to Disturbances

(Billions of Dollars)

Variable Shocked:	CN	ECD	INR	EH	IIN	EIM	EMPT	PGNP
Policy Instrument: Money Supply (M_1)								
Policy Response (M_1)								
Exact observation run	−17.6738	−24.0334	−18.3123	−8.3791	−9.1802	17.4517	0.1106	−0.1777
Imperfect observation run	−15.4556	−21.8928	−9.2014	−0.9426	−5.3474	9.5540	−0.1257	−1.6604
Policy Instrument: Government Expenditure (G)								
Policy Response (G)								
Exact observation run	−1.7385	−2.4526	−1.6053	−0.8226	−0.9030	1.6482	0.0039	−0.2429
Imperfect observation run	−1.5871	−2.1723	−0.8690	−0.1203	−0.5412	0.9166	−0.0144	−0.2782
Policy Instrument: Surcharge ($SURCHG$)								
Policy Response ($SURCHG$)								
Exact observation run	6.1123	8.6213	5.6438	2.8926	3.1748	−5.7971	−0.0149	0.8501
Imperfect observation run	5.5793	7.6360	3.0544	0.4224	1.9026	−3.2241	0.0498	0.9753

CN— consumption of nondurables and services
ECD— expenditure on consumer durable
EH— expenditures on residential construction
EIM— imports
EMPT—total employment (including armed forces)
IIN— inventory investment
INR— expenditures on producers' durables and structures
PGNP—implicit GNP deflator, (1958 = 100)
XGNP—gross national product

All expenditures and income variables are in billions of 1958 dollars, seasonally adjusted at an annual rate.

XGNP and its components. In this case, however, the appropriate policy direction to offset the PGNP shock was itself restrictive. Thus, the appearance of expansion given by the filter caused a larger move toward restraint even though the size of the increase in PGNP was itself underestimated. (See Table 2)

After the first period, it is more difficult to interpret the differences in the patterns of response because the differing first period responses have left different inherited conditions for the second period. Consequently, different responses are called for, in part as a result of the different inherited states of the economy and in part as a result of the differences in the policy formation mechanism. It is only in the first period that the results differ solely as a consequence of the assumption about the presence or absence of observation error, so that we can focus on this element alone. However, the first period results clearly show that the assumption of imperfect information typically builds in a systematic lag in recognition and policy response, although other varieties of systematic distortion are also encountered.

It would be desirable to separate the portion of the later period simulation differences due to different conditions inherited from prior periods from the portion due to different response rules in the current period. This can be done only indirectly.

It will be recalled that in the case of imperfect information the policy reaction function is composed of two separable parts—the response to the state estimate and the state estimation process. The response to perfect information, on the other hand, is comprised of only one part, since the state is known. However, the response function in the perfect information case, taking the state of the economy as the argument, is identical to the

TABLE 2

First Period Deviations in Several Endogenous Variables and Corresponding State Estimates

	Deviations in					
	XGNP (Billions of 1958 dollars)		EMPT (Millions of Persons)		PGNP (Index Level of 100 in 1958)	
Variable Shocked	Actual	Estimate	Actual	Estimate	Actual	Estimate
CN	8.8516	7.7026	.1855	.1626	.0484	.0677
ECD	4.3838	3.4136	.0919	.0677	.0240	−.0091
INR	2.2136	1.3293	.0464	.0312	.0121	.0199
EH	1.3762	.3152	.0288	.0158	.0075	.0137
IIN	4.5977	3.3010	.0964	.0727	.0251	.0395
EIM	−2.5608	−1.6575	−.0537	−.0361	−.0140	−.0065
EMPT	0	.0225	.2303	.0887	−.0123	.0014
PGNP	0	.4065	0	.0172	.2967	.1176

TABLE 3

UNDERESTIMATES OF STATE DEVIATIONS RESULTING FROM DISTURBANCES

Equation Shocked	Error in XGNP Estimate in Period (Billions of 1958 Dollars)				Error in EMPT Estimate in Period (Millions of Persons)				Error in PGNP Estimate in Period (Index Level of 100. in 1958)			
	1	2	3	4	1	2	3	4	1	2	3	4
CN	1.1490	−.0635	−.0442	−.0174	.0229	.0058	.0014	.0004	−.0193	−.0113	−.0039	−.0010
ECD	.9702	−.0476	−.0820	−.0280	.0242	.0068	.0009	.0001	.0331	.0211	.0078	.0024
INR	.8843	.3375	.1235	.0466	.0152	.0101	.0054	.0028	−.0078	−.0084	−.0057	−.0033
EH	1.0610	.5876	.2805	.1343	.0130	.0098	.0050	.0021	−.0062	−.0041	−.0015	−.0004
IIN	1.2967	.0435	−.0088	−.0098	.0237	.0051	.0004	−.0005	−.0144	−.0067	−.0014	.0002
EIM	−.9033	−.3418	−.1132	−.0395	−.0176	−.0104	−.0044	−.0017	−.0075	−.0056	−.0021	−.0004
EMPT	−.0225	.0000	−.0041	−.0013	.1416	.0645	.0295	.0135	−.0137	−.0113	−.0061	−.0027
PGNP	−.4065	−.2390	−.0603	−.0005	−.0172	−.0142	−.0060	−.0016	.1791	.0194	.0194	.0017

response function in the imperfect information case, taking the state estimate as the argument; the actual policy responses differ solely on account of differences between the state and state estimate. Thus, it is not until the state estimate and the actual state have converged that the effect of a disturbance is no longer leading to different responses under the two assumptions. Consequently, we can measure the degree to which the initial disturbance is still leading to differing policy values over time by observing the state estimate errors. In this way we can disentangle the two elements in later periods by focussing on the causal role of the state estimate.

Since the state estimation apparatus is linear, the time path of state estimation errors resulting from the shock is independent of the policy variable used. The time paths of state estimate errors for XGNP, PGNP, and EMPT are presented in Table 3. It is apparent that in several instances the state estimate errors in XGNP and EMPT tend to persist for several quarters, while in other cases they die out rapidly. The errors in the PGNP estimate are more generally persistent, with nearly one-half of the initial error remaining in the second period in each case. Clearly, in each of the instances of persisting state estimate error in any of these variables, we can be certain of systematic policy distortion and a delay in appropriate response.

VII. CONCLUSIONS

In summary, we have shown that an appropriate policy response to imperfect data necessarily means that economic disturbances will not receive full response immediately. In the case of shocks in expenditure variables, the simulations clearly showed a systematic weak initial policy response. However, covariance relationships used in estimating the current state of the economy can also lead to other types of systematic distortions in policy response. In a number of cases, the errors in the state estimates, and hence in policy response, persisted for several periods. The results presented show a caution in policy response, despite the fixed coefficient framework used. Thus, we have a source of caution in policy response complementary to that brought about by random coefficients and multiplier uncertainty, such as discussed by Brainard [3].

The results presented above have clear implications for the evaluation of the performance of policymakers. We should not assess their actions on the basis of the actual data available at the time, since what policy response is appropriate depends both on the quality of the preliminary data and on its actual value.

REFERENCES

[1] Aoki, M., 1967, *Optimization of Stochastic Systems*, Academic Press, New York.
[2] Astrom, K., 1970, *Introduction to Stochastic Control Theory*, Academic Press, New York.

[3] Brainard, W., 1967, "Uncertainty and the Effectiveness of Policy," *American Economic Review*, May.

[4] Cole, Rosanne, 1969, *Errors in Provisional Estimates of Gross National Product*, Columbia University Press, New York.

[5] Conrad, W. E., 1975, *Uncertainty and Optimal Stablization Policy*, unpublished doctoral dissertation, Massachusetts Institute of Technology.

[6] Johansen, L., 1972, "On the Optimal Use of Forecasts in Economic Policy Decisions," *Journal of Public Economics*, 1.

[7] Nelson, C., 1972, "The Prediction Performance of the FRB-MIT-PENN Model of the U.S. Economy," *American Economic Review*, December.

[8] Phillips, A. W., 1957, "Stabilization Policy and the Time-Form of Lagged Responses," *Economic Journal*, June.

[9] Sage, A. P., 1968, *Optimum Systems Control*, Prentice-Hall, Englewood Cliffs, New Jersey.

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