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INFLATION AND PRICE CONTROLS IN A FLEXPRICE-FIXPRICE MODEL

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This paper develops a simple model of inflation for an economy characterized by both flexprice and fixprice output sectors. Inflation originates with excess demand in either sector, and it is transmitted across sectors. This transmission process together with an inflationary expectation hypothesis sustains the inflation even in the absence of any continuing excess demand. The inflation generated by this process is subjected to wage and price controls. Particular attention is given to the structure of the optimal control rule for the model and to the question of allocative efficiency. Equity and terminal concerns are also considered.

1. INTRODUCTION

This paper has two purposes. First it develops a simple analytically tractable model of inflation for an economy characterized by two output (flexprice and fixprice) markets and one input (labor) market. The model incorporates most of the major elements of short run inflation theory and demonstrates, at least in a stylized way, how inflation can be transmitted and sustained. Second, the study imposes a set of temporary wage and price controls on the model. The impact of these controls is examined to identify the circumstances under which controls might, and also might not, be appropriate, and also to suggest how controls might be designed to satisfy certain allocation, equity, and termination criteria.

2. THE INFLATION MODEL

Price and wage behavior is examined in a simple economy characterized by two output markets and a single labor (input) market. Prices in one output sector are determined primarily in auction-type markets, while prices in the other sector are largely administered. The first market-type is representative of agricultural and commodity markets in which prices are determined largely by excess demand considerations. The second market-type is typical of many manufacturing industries in which prices are largely determined by some mark up strategy. Gordon [3], Hicks [5] and Moroshima [6] refer to the former as the *flexprice* market and the latter as the *fixprice* market. Okun [7] prefers to call them the *auction* market and the *customer* market, respectively.

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In this simple economy the labor market is specified by a wage formation equation in which wage increases are assumed to respond to price increase expectations and to excess demand considerations in that market. Price increase expectations are assumed to be formed adaptively and are related to past price increases in both output markets. Since both output prices and wages influence costs, the postulated mark up strategy of the fixprice sector raises prices in that sector, and these increases in turn feedback into increased wage demands. Furthermore, the resulting changes in relative prices of the two sectors imply a shift in the composition of demand (even if one assumes that nominal aggregate demand is held constant or is determined exogenously), and this too feeds the inflation process. In this study aggregate demand and supply considerations are, in fact, assumed to be determined exogenously. However, the model can be readily adapted to allow for the impact of fiscal and/or monetary policies.

A. *The Flexprice (Auction) Market*

In this market it is assumed that the dominant determinant of price movements are output excess demand considerations. Prices are assumed to be essentially unrelated to *short-run* changes in costs; possibly because in many of these commodity or auction markets, labor costs are relatively unimportant.

The demand function appropriate to the flexprice (auction) market is given by

(1)

$$Q^D_{At} = Y_t^{\alpha} P_{At}^{\beta} P_{Ct}^{\gamma}$$

where α = income elasticity of demand

β = price elasticity of demand

γ = cross price elasticity of demand

The quantity supplied is assumed to be given exogenously and is controlled by the vagaries of the weather and by the OPEC ministers and/or other domestic and international cartel managers. Consequently, the supply function is of the form

(2)

$$Q^S_{At} = \bar{Q}^S_{At}.$$

Finally it is assumed that prices respond to excess demand as given by

(3)

$$P_{A(t+1)} = [Q^D_{At}/Q^S_{At}]^k \bar{P}_{At}.$$

where k = elasticity of price adjustment, and

$\bar{P}_{At} = P_{A0}(1 + \rho)^t$ = the trend flexprice which insures some long run price parity for the flexprice sector.

Combining (1), (2), and (3) yields

(4) $$P_{A(t+1)} = (Y_t^{\alpha} P_{At}^{-\beta} P_{Ct}^{\gamma} (\bar{Q}_{At}^{S})^{-1})^{k} \bar{P}_{At}.$$

Equation (4) can easily be rewritten as

(5) $$p_{A(t+1)} = k\gamma p_{Ct} - k\beta p_{At} + c_{1t}$$

where

$$c_{1t} = k\alpha y_t - kq_{At}^{S} + \rho$$

and $p_{A(t+1)}$ is the percentage change in $P_{A(t+1)}$, y_t is the percentage change in Y_t, etc.

It should be clear from (5) that inflation can be initiated either by a contraction in the quantity supplied in the flexprice market Q_{At}^{S}, or by an increase in demand resulting from an increase in national income Y_t.

B. *The Fixprice (Customer) Market*

The dominant market clearing mechanism in this sector is a quantity adjustment process. Accordingly, prices are essentially independent of *short run* excess demand considerations and are instead largely cost determined as firms employ a mark up strategy. The three inputs employed are customer goods, auction goods and labor. The mark up relationship can be written in lagged form as

(6)
$$P_{C(t+1)} = \eta[(\theta p_{Ct} + \psi p_{At} + \phi\tilde{w}_t) + (1-\eta)(\theta p_{C(t-1)} + \psi p_{A(t-1)} + \phi\tilde{w}_{t-1})$$
$$+ (1-\eta)^2(\theta p_{C(t-2)} + \psi p_{A(t-2} + \phi\tilde{w}_{t-2}) + \cdots]$$

where $\tilde{w}_t$ is the percentage change in unit labor costs, defined as the percentage change in nominal wages, w_t, minus the percentage change in long-run productivity, pr_t.

After a Koyck transformation this yields

(7) $$p_{C(t+1)} = (1 - \eta + \eta\theta)p_{Ct} + \eta\psi p_{At} + \eta\phi\tilde{w}_t$$

In this equation η represents the response coefficient, with a larger η implying a more rapid pass through of costs. The coefficients θ, ψ, and ϕ sum to one and represent the relative intensity of use of customer goods, auction goods and labor in the production process. It should be evident from (7) that price increases in the fixprice market are not *directly* apected by changes in short run aggregate demand. However these prices are indirectly influenced by aggregate demand through the corresponding tight labor market.

C. *The Labor Market*

The labor market is assumed to be characterized by a combination of 'administered' pricing and excess (input) demand considerations. In particular it is assumed that employees try to obtain a 'fair' money wage increase which covers both any productivity increase and any expected price increase. The actual increase is also influenced by excess demand in the labor market. This relationship is given by

$$\tilde{w}_{t+1} \equiv w_{t+1} - pr_{t+1} = p^e_{I(t+1)} + \delta(U_t - U_t^*). \tag{8}$$

where pr_t = the percentage change in labor productivity

U_t = the unemployment rate.

U_t^* = the natural unemployment rate, and

p^e_{It} = the expected percentage change in overall prices.

The actual percentage change in overall prices is a weighted sum of the percentage change in customer and auction prices. That is,

$$p_{It} = \nu p_{At} + (1 - \nu) p_{Ct} \tag{9}$$

where ν = the fraction of final output comprised of auction goods.

In addition we assume that expections are formed adaptively[1] so that

$$p^e_{I(t+1)} = \zeta[p_{It} + (1 - \zeta)p_{I(t-1)} + (1 - \zeta)^2 p_{I(t-2)} + \cdots]. \tag{10}$$

Substituting (9) and (10) into (8) and using a Koyck transformation yields

$$\tilde{w}_{t+1} = \zeta(1 - \nu)p_{Ct} + \zeta\nu p_{At} + (1 - \zeta)\tilde{w}_t + c_{3t} \tag{11}$$

where

$$c_{3t} = \{\delta(U_t - U_t^*) - \delta(1 - \zeta)(U_{t-1} - U_{t-1}^*)\}.$$

D. *The Complete Model*

The complete inflation model is thus specified by equations (5), (7), and (11). To illustrate the basic dynamics of the model, plausible parameter values were approximated from estimates reported in other related studies. These parameter estimates include: $k = .6$, $\alpha = 1$, $\beta = .7$, $\gamma = .55$, and $\rho = .01$ for equation (5); $\eta = .3$, $\theta = .2$, $\psi = .1$, and $\phi = .7$ for equation (7) and $\nu = .1$, $\zeta = .55$ and $\delta = .3$ for equation (11). As a check the three reduced form equations, (5), (7) and (11) were also estimated directly, and these estimates correspond surprisingly well to the parameter values obtained from other sources. The direct estimates include $k\gamma = .334$ and $k\beta = .430$ for equation (5); $(1 - \eta + \eta\theta) = .766$, $\eta\psi = .028$, and $\eta\phi = .205$ for equation (7); and $\zeta(1 - \nu) = .495$, $\zeta\nu = .055$ and $(1 - \zeta) = .450$ for equation (11).

Four simulation results for the inflation model are presented in

[1]In this context in which aggregate demand is determined exogenously the adaptive expectation hypothesis is probably superior to the rational expectation hypothesis. Furthermore, for the wage control option the wage increase formation equation (8) is overridden by the controller.

TABLE 1

FREE MARKET PRICE INCREASE TRAJECTORIES IN THE FIXPRICE-FLEXPRICE MODEL

Period	A. Equilibrium Conditions with $\eta = .3$				B. 10% Flexprice Supply Shortage with $\eta = .3$			
	p_c	p_A	w	p_I	p_c	p_A	w	p_I
1	1.500	3.000	1.650	1.650	1.500	3.000	1.650	1.650
2	1.572	.307	1.650	1.455	1.572	2.600	1.650	1.674
3	1.549	1.489	1.537	1.543	1.615	2.414	1.663	1.695
4	1.543	.973	1.540	1.468	1.646	2.126	1.681	1.695
5	1.524	1.193	1.510	1.491	1.665	1.878	1.688	1.686
6	1.510	1.092	1.500	1.468	1.674	1.608	1.687	1.667
7	1.494	1.130	1.482	1.458	1.673	1.345	1.676	1.650
8	1.479	1.180	1.469	1.442	1.662	1.076	1.656	1.603
9	1.465	1.113	1.454	1.430	1.642	1.188	1.627	1.596
10	1.451	1.106	1.441	1.416	1.624	1.133	1.610	1.575
11	1.437	1.104	1.427	1.404	1.605	1.151	1.591	1.559
12	1.424	1.101	1.414	1.391	1.587	1.137	1.574	1.542

Period	C. 10% Flexprice Shortage, 1% Excess Demand, $\eta = .3$				D. 10% Flexprice Supply Shortage with $\eta = .6$			
	p_c	p_A	w	p_I	p_c	p_A	w	p_I
1	1.500	3.000	1.650	1.650	1.500	3.000	1.650	1.650
2	1.572	2.600	1.950	1.674	1.653	2.600	1.650	1.747
3	1.676	2.414	1.963	1.750	1.708	2.441	1.703	1.781
4	1.754	2.147	2.011	1.793	1.750	2.145	1.746	1.790
5	1.816	1.905	2.056	1.825	1.772	1.904	1.770	1.785
6	1.865	1.647	2.094	1.843	1.779	1.632	1.778	1.765
7	1.903	1.392	1.821	1.852	1.770	1.370	1.771	1.730
8	1.869	1.132	1.838	1.796	1.746	1.098	1.748	1.681
9	1.840	1.233	1.815	1.779	1.708	1.207	1.712	1.658
10	1.815	1.180	1.795	1.751	1.680	1.147	1.682	1.626
11	1.794	1.194	1.771	1.731	1.649	1.163	1.651	1.600
12	1.767	1.180	1.749	1.708	1.621	1.460	1.623	1.573

table 1. All four simulations assume the same initial conditions, i.e., first period *quarterly* inflation rates of 1.5%, 3.0%, and 1.65% respectively in the fixprice, flexprice, and labor sectors. In the first simulation, no additional excess demand considerations are introduced. Nevertheless after 12 periods a quarterly inflation rate of approximately 1.4% persists. In the second simulation reported the only inflationary stimulus is a 10% shortfall in supply in the flexprice (auction) sector. This shortfall is assumed to persist for six quarters. A substantially higher rate of inflation obtains both in the short run and at the end of the planning horizon. In the third simulation in addition to the 10% supply shortfall in the flexprice sector a 1% increase in aggregate demand is introduced. The resulting inflationary trajectories are given in table 1C. The fourth simulation reported uses

the same economic environment as the second; however, the pass through coefficient η has been doubled from .3 to .6. The sensitivity of the model to this particular parameter has been explored because there appears to be some evidence of a *threshold effect* which dramatically increases the speed with which cost increases are recouped when the rate of inflation exceeds a certain level. One should not, however, infer from this sensitivity experiment that the parameter values employed in this study are offered as anything other than *plausible* estimates.

3. Wage and Price Controls

Income policies of any form are controversial and direct controls are particularly suspect. Two questions arise rather naturally in this context. Is it possible to identify any circumstances under which controls are appropriate? How should controls be designed to satisfy certain allocation and equity properties? These are not really separate questions because the economic arguments against controls are typically couched in terms of induced misallocation and of induced inequitable redistribution of income.

Proponents of controls acknowledge these shortcomings. However, they maintain that in some circumstances controls permit the use of stimulative monetary and/or fiscal measures whose positive impacts more than offset any negative impact of controls. This argument can not be fully tested in the model outlined above, because in it aggregate demand is assumed to be determined exogenously. However, the model can be used to estimate the potential for induced misallocation and income redistribution. It can also be used to derive control rules which minimize these negative features.

A. *The Criterion Function*

Four arguments appear to be germane to a criterion function appropriate to the simple model outlined above. These provide for (i) a reduction in price inflation to some acceptable level, (ii) an equitable distribution of any restraining impact on wages and profits, (iii) minimal interference with market allocation, and (iv) terminal characteristics which minimize the possibility of an explosive wage-price spiral following the suspension of controls. A criterion function constructed to achieve these objectives when it is minimized is given by

(12)

$$D = \sum_{t=1}^{T} u_1 \left| p_{It} - p^*_{It} \right| + \left| \tilde{w}_t - p_{It} \right| + u_2 \left| p_{Ct} - \frac{p^*_{Ct}}{\tilde{w}^*_t} \tilde{w}_t \right|$$

$$+ u_3 \left| p_{At} - \frac{p^*_{At}}{\tilde{w}^*_t} \tilde{w}_t \right| + u_4 \left| p_{At} - \frac{p^*_{At}}{p^*_{Ct}} p_{Ct} \right| + u_5 \left| p^c_{I(T+1)} - p^*_{I(T+1)} \right|.$$

The first term of this function, $u_1 | p_{It} - p^*_{It} |$, is designed to insure that the percentage price increase p_{It} does not differ substantially from the targeted price increase p^*_{It}, which may be set equal to some historical norm.

The second term of the welfare function, $| \tilde{w}_t - p_{It} |$, is designed to protect the purchasing power of the wage earner or salaried employee, and thus to guarantee that the burden of stopping the inflation is borne by both wage earner and property owner. It is evident from this term that whenever the increase in money wages adjusted for productivity gains fails to compensate for price changes, a penalty is incurred. Conversely, a penalty is imposed whenever the relative share of property income deteriorates.

The most severe critics of incomes policies typically focus on price controls' potential for disrupting the allocative function of the market. It is obvious that imposed uniform price and wage increases eliminate this function completely. Therefore, it is important to design policy measures so as to preserve some relative price flexibility in response to market pressures. The third, fourth and fifth terms of equation (12) are intended to achieve this objective. For example, the third term is given by

$$u_2 \left| p_{Ct} - \frac{p^*_{Ct}}{\tilde{w}^*_t} \tilde{w}_t \right|,$$

where the price ratio $p^*_{Ct}/\tilde{w}^*_t$ represents the relative price increase which would have prevailed in the absence of controls. Deviations from this ratio are penalized because the controlled price relatives may transmit erroneous signals.

A final important characteristic of any temporary control is that the termination of the control should be more or less automatic, and that this suspension should not induce a wage-price explosion such as occurred in 1973–74. Wage controls, for example, can only be readily abandoned whenever the restraining impact of the control is minimal, i.e., whenever the control rule grants most of the free market wage demand. Furthermore, whenever this condition prevails, no substantial pressure for a wage-price explosion exists.

It is evident from the analysis in the previous section and, in particular, from equation (8) that as $p^e_{I(t+1)}$ approaches p^*_I, adjusted wage increases $\tilde{w}_t$ induced by market forces will stabilize at p^*_I. From (7) it follows that market induced price increases p_{Ct} will also stabilize at p^*_I. Consequently a major objective of any control strategy is to reduce the price increase expectation $p^e_{I(T+1)}$ to $p^*_{I(T+1)}$. The last term of the criterion function, $u_5 | p^e_{I(T+1)} - p^*_{I(T+1)} |$ is designed to accomplish this objective.

The first and fifth terms of (12) can be combined using the relationship $u_t = u_1 + (1 - \zeta)^{(T-t)}u_5$. This relationship is derived from (10). The criterion function (12) can then be replaced by the more tractable quadratic structure of (13) given below.[2]

B. *The Control Theoretic Formulation of the Problem*

A formal statement of the wage-price control model is now possible. The objective is to minimize

$$(13) \qquad D = \frac{1}{2}\sum_{t=1}^{T} u_t(p_{It} - p_{It}^*)^2 + (w_t - p_{It})^2 + u_2(p_{Ct} - l_t^* \tilde{w}_t)^2 + u_3(p_{At} - m^* \tilde{w}_t)^2 + u_4(p_{At} - n_t^* p_{Ct})^2$$

subject to either one or two of the following price and wage increase equations

$$(5) \qquad p_{A(t+1)} = k\gamma p_{Ct} - k\beta p_{At} + c_{1t}$$

$$(7) \qquad p_{C(t+1)} = (1 - \eta + \eta\theta)p_{Ct} + \eta\psi p_{At} + \eta\phi\tilde{w}_t$$

$$(11) \qquad \tilde{w}_{t+1} = \zeta(1 - \nu)p_{Ct} + \zeta\nu p_{At} + (1 - \zeta)\tilde{w}_t + c_{3t}$$

with $p_{It} = \nu p_{At} + (1 - \nu)p_{Ct}$, $l_t^* = p_{Ct}^*/\tilde{w}_t^*$, $m_t^* = p_{At}^*/\tilde{w}_t^*$, and $n_t^* = p_{At}^*/p_{Ct}^*$.

It should be clear from (5), (7) and (11) that any inflation in this economy can be brought under control either by employing aggregate demand policies including monetary and fiscal measures which alter c_{1t} and c_{3t}, or by controlling either wage or price increases. If the policy option selected is to override the wage formation equation (11), i.e., to impose *direct wage controls*, the price formation equations (5) and (7) govern the inflationary process, and the control variable is $\tilde{w}_t$ of equation (7). This choice does *not* necessarily preclude price controls but prices must only be controlled at a level consistent with those generated by market forces.

[2]An alternative formulation of the criterion function would replace the sixth term of (12) with $\mu_5 |\tilde{w}_t - p_{It}^*|$. This formulation, appropriate to wage controls, has the advantage of focusing directly on the basic objective of the controls which is to align wage increases adjusted for productivity gains with the *targeted* rate of inflation. The trade off between this term and the second term of (12), which is designed to align adjusted wage increases with the *current* rate of inflation in order to preserve labor's share of national income, is immediately apparent. The inclusion of the term $\mu_5 |\tilde{w}_t - p_{It}^*|$ also obviates the need for the final period expectation term, because it forces $\tilde{w}_t$ and therefore p_{It} to approach p_I^* early in the planning horizon. This assures that $p_{I(T+1)}^e = p_{I(T+1)}^*$. The disadvantage of this formulation is that equating $\tilde{w}_t$ and p_{It}^* is an *intermediate* objective (much like a money supply target), which provides no *ultimate* utility for its attainment.

This alternative formulation does not materially alter the major findings of this study. For example, equations (16) and (17) below imply a control rule characterized by a variable policy coefficient. This finding also obtains for the alternative criterion function. A somewhat more substantial modification is required for equation (15) which must be rewritten as $\tilde{w}_t = 1/2(p_{It} + p_{It}^*) - (\eta\phi/2)\lambda_{C(t+1)}$. This implies a more vigorous and somewhat less equitable wage control rule.

i.e., by equations (5) and (7). Alternatively, the price increase trajectories generated by these equations can be interpreted as guidelines.

Conversely, the decision maker may elect to *control prices directly*. In this case he would either override (7) and use p_{Ct} as the control variable or possibly choose to control both p_{Ct} and p_{At}.

C. *Some Analytic Results with Direct Wage Controls*

The wage-price control problem outlined above with direct wage controls can be formulated in terms of a discrete time Pontryagin minimum principle problem with $\tilde{w}_t$ as the control variable. The necessary and sufficient conditions for an optimum are given by:

$$\text{(14i)} \quad \tilde{w}_t = \frac{1}{1 + u_2 l_T^{*2} + u_3 m_t^{*2}} [p_{lt} + u_2 l_t^* p_{Ct} + u_3 m_t^* p_{At} - \eta\phi\lambda_{C(t+1)}]$$

$$\text{(14ii)} \quad \lambda_{Ct} = \frac{\partial H}{\partial p_{Ct}} = u_t(1 - \nu)[p_t - p_{lt}^*] + (1 - \nu)[p_{lt} - \tilde{w}_t]$$
$$+ u_2(l_t^{*2} p_{Ct} - l_t^* \tilde{w}_t) + u_4(n_t^{*2} p_{Ct} - n_t^* p_{At})$$
$$+ (1 - \eta + \eta\theta)\lambda_{C(t+1)} + k\gamma_{A(t+1)}.$$

$$\text{(14iii)} \quad \lambda_{At} = \frac{\partial H}{\partial p_{At}} = u_t\nu[p_{lt} - p_{lt}^*] + \nu[p_{lt} - \tilde{w}_t] + u_3(p_{At} - 2m_t^* \tilde{w}_t)$$
$$+ u_4(p_{At} - n_t^* p_{Ct}) + \nu\psi\lambda_{C(t+1)} - k\beta\lambda_{A(t+1)}.$$

$$\text{(14iv)} \quad P_{C(t+1)} = \frac{\partial H}{\partial \lambda_{C(t+1)}} = (1 - \eta + \eta\theta)p_{Ct} + \eta\psi p_{At} + \eta\phi\tilde{w}_t.$$

and

$$\text{(14v)} \quad p_{A(t+1)} = \frac{\partial H}{\partial \lambda_{A(t+1)}} = k\gamma p_{Ct} - k\beta p_{At} + c_{1t}.$$

Given the proper boundary conditions these five difference equations can be solved for the optimal trajectories of the control variable $\tilde{w}_t$ and the corresponding trajectories of $p_{Ct}, p_{At}, \lambda_{Ct}$ and λ_{At}.

It is also instructive however to examine separately the individual equations. To facilitate this examination we set $u_2 = u_3 = 0$. Under these circumstances the wage control rule (14i) reduces to

$$\text{(15)} \quad \tilde{w}_t = p_{lt} - \eta\phi\lambda_{C(t+1)}.$$

The critical variable in this expression is the time varying shadow price $\lambda_{C(t+1)}$. Using (14i) through (14iii), it is possible to show that this shadow price is given by

$$(16) \qquad \lambda_{C(t+1)} = \frac{1}{1 - \eta\psi} \{\lambda_{Ct} - (1 - \nu)u_t[p_{lt} - p_{lt}^*] + \nu[p_{lt} - \tilde{w}_t]$$

$$+ u_4 n_t^*[p_{4t} - n_t^* p_{Ct}] - k_{\gamma}\lambda_{4(t+1)}\}.$$

For the *special case* in which the economy has no flexprice sector. i.e., for which $\nu = \psi = u_4 = \lambda_{4(t+1)} = 0$. equation (16) further reduces to

$$(17) \qquad \lambda_{C(t+1)} = \lambda_{Ct} - u_t(p_{lt} - p_{lt}^*).$$

It follows immediately from (17) that $\lambda_{C(t+1)} < \lambda_{Ct}$ whenever $p_{lt} > p_{lt}^*$. During a period of controls this would almost certainly be the case. This result together with (15) implies that $\tilde{w}_t$ approaches p_{lt} as $t \rightarrow T$, i.e.. the policy rule compensates the employee for an increasing fraction of any price increase over time. Alternatively the policy rule can be written as

$$(18) \qquad \tilde{w}_t = G_t p_{lt}$$

where the variable policy coefficient G_t increases over time and approaches 1 towards the end of the horizon. This is a very desirable property because it states that in the last few periods of the control program the employee is fully compensated for all price and productivity increases. Consequently, there should be minimum pressure for a wage-price explosion when the controls are terminated.[3]

For the more *general model* which includes the flexprice sector, the optimal wage control rule is governed by (15) and (16). As above, a smooth transition from wage controls back to the market requires, minimally, that $\tilde{w}_T \simeq p_{lT}$, i.e., that $\lambda_{C(t+1)}$ approaches zero as t approaches T. This obviously implies that $\lambda_{C(t+1)} < \lambda_{Ct}$. As can be seen from (16) this inequality holds when $\eta\psi$ is quite small, $p_{lt} > p_{lt}^*$, the third and fourth terms of the r.h.s. of (16) are also small, and $\lambda_{4(t+1)} > 0$. Typically these conditions all hold. However a separate examination of each condition is informative.

The first condition derives from the factor $1/(1 - \eta\psi)$ which applies to the entire r.h.s. of (16) and is therefore potentially very disruptive. The parameter η is the adjustment or pass through coefficient for any increased cost of producing fixprice goods. A large η could prove damaging because it implies that a high flexprice inflation rate would more quickly become imbedded in the price of fixprice commodities. However, it also implies that a wage increase controlled at a low level is also quickly passed on, and this serves to improve the performance of the controls. The net impact of increasing η from .3 to .6 is studied below.

The impact of this factor $1/(1 - \eta\psi)$ is also dependent on the parameter ψ which measures the input intensity of the flexprice good. The value

[3]It is, of course, true that at the beginning of the control period labor's share of income decreases. Consequently, an income tax rebate proportional to this loss would make wage controls more equitable and more palatable.

of ψ is approximately .1. If this should increase significantly either because of a higher relative cost for flexprice goods or because of a more flexprice intensive technology, the efficacy of wage-price controls would be undermined.

The second condition, that $p_{It} > p_{It}^*$, has been discussed previously. It imposes no real restriction since temporary controls will only be considered in periods characterized by an inflation which exceeds the targeted rate.

The smooth functioning of a wage control program also requires that the third term on the r.h.s. of (16), $\nu[p_{It} - \tilde{w}_t]$ be small. This obviously derives from equity considerations, but exists independently only for a non-zero flexprice market. The parameter ν measures the fraction of national product comprised of flexprice outputs. A plausible estimate here is .1. Again any substantial increase in this fraction would impede an orderly reduction in the level of the shadow price λ_{Ct}, and therefore obstruct the operation of a wage-price control program.[4]

Similarly, efficient operation of the control program requires that the fourth term of the r.h.s. of (16), $u_4 n_t^*[p_{At} - n_t^* p_{Ct}]$, be small. The bracketed term captures induced allocative inefficiency as measured by the deviation between the controlled and market determined relative price trajectories. From (16) it follows that any increase in the deviation interferes with the smooth dampening of λ_{Ct}. The extent of this interference also depends on the relative size of the allocation weight u_4. Too great an emphasis on allocation could prove so disruptive as to lead to a rejection of all controls, as is shown in the simulation studies reported in the next section.

In summary it should be clear from all of the above that the introduction of a flexprice sector into the model complicates any control program. The control rule is then governed by equations (14) and (16) instead of by the more simple equations (15) and (17). Nonetheless, in both cases the rule can be written in the general variable policy coefficient format of (18). Furthermore for the plausible parameter estimates used in this study, the controls for the more complex economy still appear to be relatively smooth and efficient. This is demonstrated by the simulation results presented in the next section.

4. Simulation Studies of Direct Wage Controls

To illustrate the basic arguments outlined above, eight simulation studies of direct wage controls are reported in this section. The first four focus on the allocation question and its implications for direct controls.

[4]We note that the implicit weight for this term in the criterion function is 1. Any increase in this relative weight would obviously also increase the negative influence of this term.

The next three experiments analyze different approaches to avoiding a wage-price explosion once controls are suspended. The final simulation tests the robustness of the earlier findings by varying the cost pass through coefficient η, which appears to be the most sensitive parameter in the system.

The model used in these simulations is the flexprice-fixprice model constructed in section 2. The economic environment is that appropriate to the price movements presented in table 1B. This implies an inherited inflation characterized by first period quarterly price changes of 3%, 1.5% and 1.65% respectively in the flexprice, fixprice and labor sectors. In addition a 10% shortfall in supply in the flexprice sector is posited for the first six quarters of the planning horizon.

A. *Misallocation and Controls*

In the first simulation study the criterion function is defined by $u_1 = .5$, $u_2 = u_3 = u_4 = 0$ and $u_5 = 100$. This implies that any induced misallocation is eostless. The results of this experiment are recorded in table 2A and figure 1 and are generally very satisfactory. By the end of the 12 quarter horizon the inflation rate in all three sectors approaches the targeted level of 1% per quarter. The shadow price $\lambda_{C(t+1)}$ of equation (15) diminishes over the horizon and approaches zero, and the policy coefficient G_t of equation (18) converges on one. These trajectories are consistent with the findings of section 3 that a variable coefficient policy rule is optimal. At the same time, they imply that wage earners and salaried employees are almost fully compensated for all price and productivity changes in the last few quarters of the control horizon. This near complete compensation reduces the pressure to recapture lost wages once the controls are suspended.[5]

This is an important finding and is one possible explanation for the failure of previous incomes policies. These policies have all been of the fixed policy coefficient variety. A fixed coefficient policy rule does not compensate fully for price and productivity increases even in the final periods. Consequently substantial latent pressure exists to right induced distributional inequities. This pressure is capable of triggering a wage-price explosion, similar to that of 1973–74, which can negate most of the gains of the controls.

The *second simulation* experiment recognizes the cost of induced misallocation. The criterion function is defined by $u_1 = .5$, $u_2 = u_3 = 0$, $u_4 = .3$ and $u_5 = 100$, and the control rule is governed by equation (16)

[5]This convergence is even more pronounced when either η is increased or v is decreased. The former implies that prices in the fixprice sector adapt more rapidly to cost increases, while the latter implies that the size of the flexprice sector is diminished. The more rapid convergence provides for a smoother reentry into the free market.

TABLE 2

ALLOCATIVE EFFICIENCY AND OPTIMAL PRICE INCREASE, SHADOW PRICE, AND POLICY COEFFICIENT TRAJECTORIES

A. Simulation 1: $u_1 = .5, u_2 = u_3 = u_4 = 0, u_5 = 100$

Period	p_C	p_A	p_I	w	λ_C	$G = w/p_I$
1	1.500	3.000	1.650	1.123	2.785	.680
2	1.465	2.600	1.578	1.088	2.582	.689
3	1.419	2.378	1.515	1.060	2.400	.699
4	1.371	2.076	1.442	1.019	2.231	.706
5	1.318	1.808	1.367	.978	2.072	.715
6	1.261	1.523	1.287	.936	1.908	.727
7	1.201	1.245	1.205	.900	1.722	.746
8	1.140	.962	1.122	.870	1.495	.775
9	1.078	1.063	1.076	.897	1.234	.834
10	1.039	.999	1.035	.926	.881	.891
11	1.014	1.014	1.014	.973	.525	.959
12	1.005	1.000	1.005	1.004	.203	.999

B. Simulation 2: $u_1 = .5, u_2 = u_3 = 0, u_4 = .3, u_5 = 100$

Period	p_C	p_A	p_I	w	λ_C	$G = w/p_I$
1	1.500	3.000	1.650	1.174	2.555	.712
2	1.475	2.600	1.587	1.138	2.333	.717
3	1.437	2.382	1.532	1.102	2.203	.718
4	1.394	2.081	1.463	1.050	2.106	.718
5	1.342	1.814	1.389	.997	2.020	.718
6	1.284	1.528	1.308	.944	1.923	.721
7	1.220	1.250	1.223	.899	1.785	.735
8	1.154	.966	1.135	.865	1.589	.762
9	1.088	1.066	1.085	.899	1.321	.828
10	1.046	1.001	1.042	.921	.964	.883
11	1.018	1.015	1.018	.970	.590	.952
12	1.007	1.000	1.006	1.005	.232	.999

C. Simulation 3: $u_1 = .5, u_2 = u_3 = u_4 = .1, u_5 = 100$

Period	p_C	p_A	p_I	w	λ_C	$G = w/p_I$
1	1.500	3.000	1.650	1.249	2.879	.756
2	1.490	2.600	1.601	1.197	2.780	.747
3	1.460	2.387	1.553	1.154	2.699	.743
4	1.423	2.087	1.489	1.093	2.625	.734
5	1.373	1.821	1.418	1.030	2.545	.726
6	1.314	1.536	1.360	.963	2.445	.708
7	1.207	1.257	1.248	.903	2.294	.723
8	1.176	.972	1.156	.854	2.073	.738
9	1.103	1.071	1.099	.878	1.767	.799
10	1.055	1.004	1.049	.910	1.345	.867
11	1.023	1.017	1.022	.979	.879	.946
12	1.010	1.000	1.009	1.024	.407	.985

D. Simulation 4: $u_1 = .5, u_2 = u_3 = u_4 = .3, u_5 = 100$

Period	p_C	p_A	p_I	w	λ_C	$G = w/p_I$
1	1.500	3.000	1.650	1.366	3.101	.827
2	1.514	2.600	1.623	1.307	3.122	.805
3	1.501	2.395	1.590	1.259	3.172	.791
4	1.476	2.097	1.538	1.186	3.235	.771
5	1.433	1.834	1.473	1.104	3.283	.749
6	1.376	1.550	1.393	1.012	3.298	.726
7	1.305	1.271	1.302	.923	3.229	.708
8	1.224	.985	1.200	.947	3.047	.705
9	1.139	1.081	1.133	.864	2.700	.763
10	1.080	1.012	1.073	.889	2.200	.828
11	1.038	1.022	1.036	.960	1.571	.829
12	1.020	1.003	1.018	1.055	.827	1.036

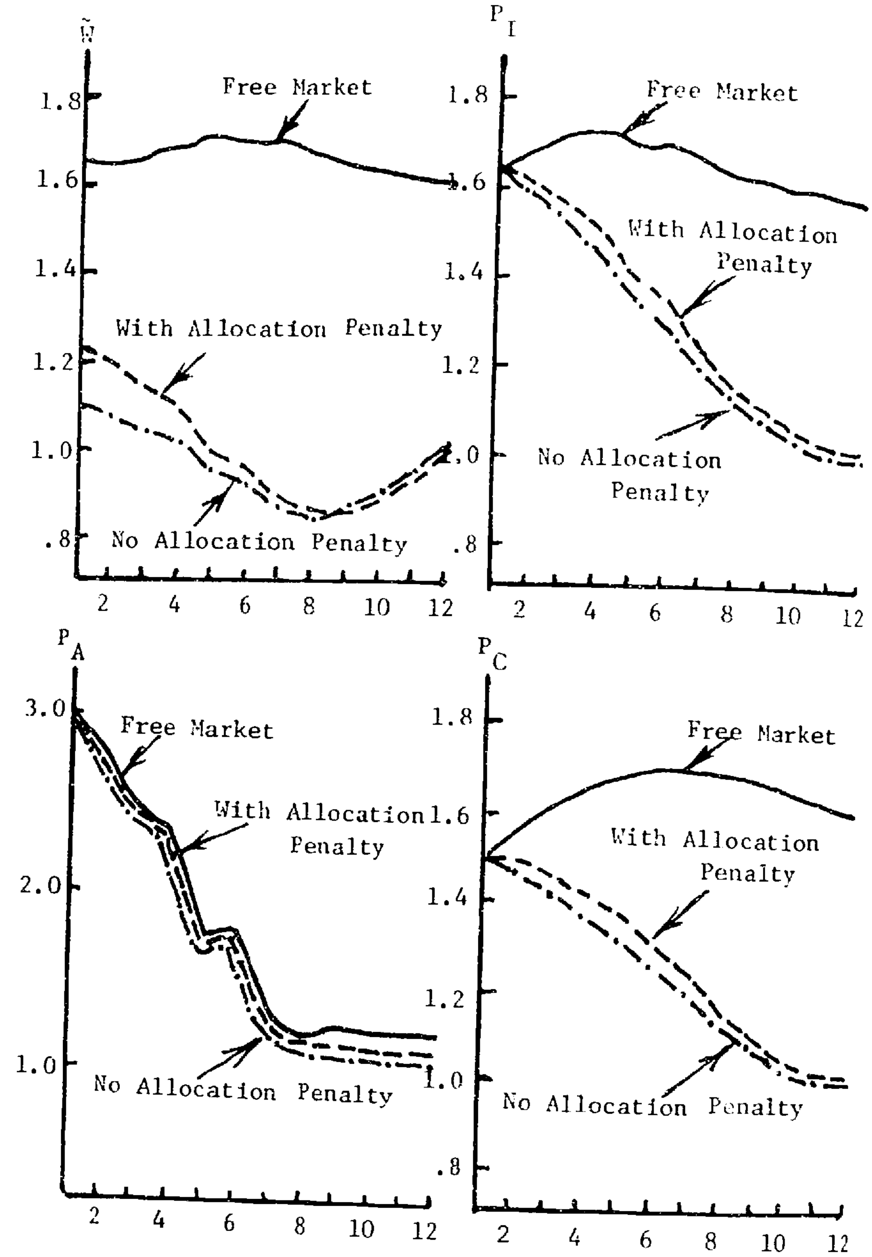

Figure 1: Inflation Paths with $\eta = .3$.

above. Because of the allocative pressure, wage controls are pursued somewhat less vigorously and the terminal characteristics are marginally less attractive. See table 2B. Nonetheless, the inflation is suppressed and good convergence is obtained in the policy coefficient.

The *third simulation* experiment also focuses on the allocative issue. While no increase in the aggregate penalty for misallocation is introduced, the criterion function has been modified to redistribute the penalty so as to penalize any deviant price relative. The loss function is defined by $u_1 = 5$, $u_2 = u_3 = u_4 = .1$ and $u_5 = 100$. The simulation results are presented in table 2C and figure 1. In this instance imposed wage constraints are markedly less severe at the beginning of the control horizon; still all price targets are approximately met by the twelfth period. The shadow price $\lambda_{C(t+1)}$ decreases monotonically, while the policy coefficient G_t declines from .756 to .708 in the sixth period before beginning its slow rise to one. This unevenness in the policy coefficient trajectory lessens the intuitive appeal of the control rule and therefore reduces its chance of adoption. However, the overall results of the policy appear reasonably satisfactory and closely parallel those of the first two simulations.

In the *fourth simulation* experiment, the weights on the allocative terms are tripled. The results are recorded in table 2D. The controls again meet the targeted inflation rates by the twelfth quarter. However, the shadow price and policy coefficient move so very slowly towards their respective targets that a successful reentry into the free market appears to be marginal, at best. A longer control period may be indicated; or *possibly* under these circumstances the inflation should be moderated with more conventional demand management policies, or with more selective measures.

B. *Variations on a Theme*

It should be clear from equation (17) that the variable coefficient policy rule is consistent with a time invariant u_t. However, any increase over time in u_t amplifies the variability, and a large u_5, the weight of the price expectation term in the criterion function, leads to such an increase. In all of the above simulations studies, u_5 is set equal to 100. In the *fifth* simulation, u_5 is reduced to 10, and the other weights are given by $u_1 = .5$ and $u_2 = u_3 = u_4 = 0$. The results of this experiment are presented in table 3A. These should be compared with those of the first simulation run recorded in table 2A. From this comparison it is evident that while the indicated controls of this fifth simulation are somewhat less vigorous and the targets slightly underachieved, all trajectories are quite similar to those which obtained using $u_5 = 100$. It follows that while the control results are indeed sensitive to the relative emphasis placed on the terminal expectation term, a wide range of emphases is quite acceptable.

TABLE 3

SENSITIVITY STUDIES AND OPTIMAL PRICE INCREASE, SHADOW PRICE, AND POLICY COEFFICIENT TRAJECTORIES

A. Simulation 5: $u_1 = .5, u_2 = u_3 = u_4 = 0, u_5 = 10$

Period	p_C	p_A	p_I	w	λ_C	$G = w/p_I$
1	1.500	3.000	1.650	1.215	2.346	.736
2	1.483	2.600	1.594	1.201	2.130	.753
3	1.456	2.384	1.548	1.193	1.931	.777
4	1.427	2.086	1.493	1.172	1.745	.785
5	1.392	1.823	1.435	1.145	1.577	.797
6	1.353	1.541	1.372	1.109	1.424	.808
7	1.307	1.267	1.303	1.065	1.289	.817
8	1.255	.988	1.228	1.012	1.167	.824
9	1.196	1.090	1.187	.998	1.057	.842
10	1.151	1.027	1.139	.986	.921	.865
11	1.113	1.039	1.105	1.009	.746	.913
12	1.088	1.021	1.081	1.074	.472	.993

B. Simulation 6: $u_1 = .5, u_2 = u_3 = u_4 = 0, u_5 = .5, u_6 = .5$

Period	p_C	p_A	p_I	w	λ_C	$G = w/p_I$
1	1.500	3.000	1.650	1.039	1.665	.629
2	1.447	2.600	1.560	1.042	1.404	.668
3	1.396	2.372	1.594	1.051	1.173	.703
4	1.352	2.071	1.424	1.054	.961	.740
5	1.310	1.804	1.359	1.056	.772	.777
6	1.271	1.522	1.296	1.054	.605	.813
7	1.233	1.248	1.235	1.046	.463	.846
8	1.193	.971	1.171	1.031	.349	.880
9	1.152	1.177	1.145	1.033	.267	.902
10	1.123	1.018	1.114	1.030	.193	.924
11	1.101	1.034	1.094	1.031	.132	.942
12	1.083	1.019	1.077	1.033	.078	.959

C. Simulation 7: $u_1 = .5, u_2 = u_3 = u_4 = .1, u_5 = .5\ u_6 = .5$

Period	p_C	p_A	p_I	w	λ_C	$G = w/p_I$
1	1.500	3.000	1.650	1.204	1.719	.729
2	1.481	2.600	1.593	1.182	1.493	.741
3	1.451	2.348	1.544	1.175	1.294	.761
4	1.419	2.085	1.486	1.160	1.106	.781
5	1.384	1.821	1.428	1.145	.928	.801
6	1.346	1.539	1.365	1.126	.762	.825
7	1.305	1.266	1.301	1.105	.607	.849
8	1.261	.988	1.234	1.080	.747	.875
9	1.215	1.093	1.202	1.083	.385	.901
10	1.183	1.032	1.168	1.077	.280	.922
11	1.156	1.047	1.145	1.079	.177	.942
12	1.136	1.032	1.125	1.084	.050	.964

D. Simulation 8: $u_1 = .5, u_2 = u_3 = u_4 = .1, u_5 = 100, \eta = .6$

Period	p_C	p_A	p_I	w	λ_C	$G = w/p_I$
1	1.500	3.000	1.650	1.274	1.381	.772
2	1.495	2.600	1.606	1.239	1.264	.772
3	1.454	2.388	1.574	1.206	1.177	.773
4	1.406	2.084	1.474	1.155	1.097	.783
5	1.341	1.817	1.388	1.098	1.012	.791
6	1.286	1.527	1.294	1.044	.911	.806
7	1.189	1.245	1.195	1.000	.733	.837
8	1.113	.958	1.098	.966	.595	.879
9	1.042	1.056	1.043	.979	.400	.939
10	1.017	.990	1.015	.988	.230	.973
11	1.003	1.010	1.004	1.001	.120	.997
12	1.002	1.000	1.002	1.002	.059	1.000

The two sets of simulation results given in tables 3B and 3C assume the use of the alternative criterion function described in footnote 2. In this criterion function, the fifth or terminal expectation term is replaced by a new term designed to align adjusted money wage increases in each period with the targeted inflation rate. This is a rather conventional approach to the design of incomes policy measures, and corresponds loosely to the structure of the Nixon controls of 1971–73. In the first of these simulations the criterion function is defined by $u_1 = .5$, $u_2 = u_3 = u_4 = 0$, $u_5 = .5$ and $u_6 = .5$. In the second simulation allocative criteria are considered and the function is defined by $u_1 = .5$, $u_2 = u_3 = u_4 = .1$, $u_5 = .5$ and u_6 .5. The coefficient u_6 is associated with the equity term which in all previous simulations had an implicit weight of one. The sum of the coefficient of the two equity terms in the alternative function, u_5 and u_6, also equals one. The results in tables 3B and 3C should be compared with those given in tables 2A and 2C respectively. As is evident from this comparison, the alternative specification is less attractive on all counts: inflation control, equity, and terminal or reentry considerations.

In the *final simulation*, the cost pass through coefficient η of equation (7) is doubled. This allows an analysis of the sensitivity of the control measures to variations in this parameter. From equation (16) it appears that an increase in η should reduce the effectiveness of the controls because this increase implies that prices in the fixprice sector adapt more quickly to the 'destabilizing' impact of volatile auction sector prices. This is true. However, this adverse impact is more than offset by the more rapid adaption of price in the fixprice sector to *controlled* wage increases. Consequently as shown in table 3D and figure 2, the simulated inflation responds very quickly and equitably to wage controls. The criterion function used in this simulation is given by $u_1 = .5$, $u_2 = u_3 = u_4 = .1$ and $u_5 = 100$, and corresponds to that of simulation 3 above. Table 3D should therefore be compared with 2C. The control results of this last simulation are significantly better on every count. This is particularly impressive since the model with $\eta = .6$ generates a somewhat higher rate of inflation on the free market. Compare tables 1B and 1D. This sensitivity test is obviously illustrative. Nevertheless, since it focuses on what was believed *a priori* to be the most sensitive parameter in the system, this experiment suggests that the control conclusions derived above may be rather robust.

5. Conclusion

An alternative to direct *wage* controls is direct *price* controls. No analytic or simulation results for direct price controls are presented in this paper. However, the results obtaining from direct price controls in the fixprice market are analogous to those of direct wage controls, with the qualification that severe fluctuations in auction market prices are more

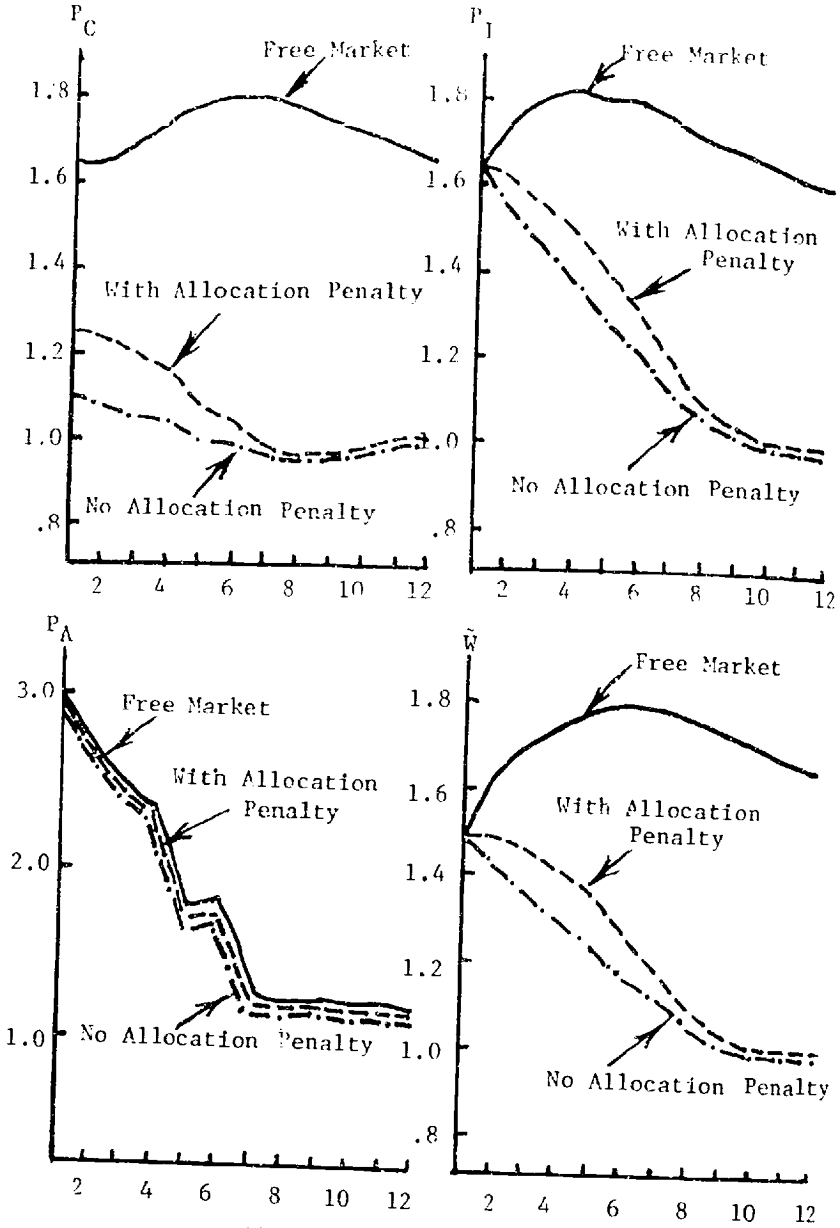

Figure 2: Inflation Paths with η = .6.

annoying for price controls than for wage controls. On the other hand, controlling prices in both the flexprice and fixprice sectors proved more or less unworkable. This is not surprising. Direct controls are most effective in combating inflation induced primarily by inflationary expectations in an economy characterized by mark-up pricing. Controls are least effective in dealing with excess demand inflation. Because of this, auction markets have historically been exempted from controls.

The study is obviously incomplete as several major questions remain unanswered. These include: How much relative price movement can be tolerated before wage controls must yield to direct intervention in sectoral markets? At what point do allocation considerations require the substitution of direct controls by restraining monetary and fiscal measures? How serious are the consequences of introducing a heterogeneous labor force? What is the implication of introducing a quasi rational expectation hypothesis? These are all subjects for a continuing study.

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