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A PREDICTIVE TEST FOR THE REDUCED FORM MODEL

BY W. A. JAYATISSA AND R. W. FAREBROTHER

In this paper we establish a test of whether two sets of observations come from the same reduced form model.

1. INTRODUCTION

In a recent issue of the *Annals of Economic and Social Measurement* Dhrymes *et al.* [2] extended the single equation stability testing procedure to the reduced form of the linear simultaneous equations model. However, as Jayatissa [4] has pointed out, their result is not correct when there are two or more additional observations. The purpose of the present paper is to obtain a result which is generally valid.

2. THEORY

Consider the reduced form model

$$Y = X\Pi + V \tag{1}$$

where Y is an $N \times G$ matrix of observations on the G endogenous variables of the model, X is an $N \times K$ matrix of observations on the K exogenous variables of the model, Π is a $K \times G$ matrix of unknown parameters and V is an $N \times G$ matrix of disturbances. We assume that X has rank K and that the rows of V are independently and identically normally distributed with zero mean and variance Σ, that is

$$EV_{i.} = 0 \qquad EV_{i.}'V_{j.} = \delta_{ij}\Sigma \tag{2}$$

or

$$EV_{.i} = 0 \qquad EV_{.i}V_{.j}' = \sigma_{ij}I_N \tag{3}$$

where δ_{ij} is the Kronecker delta.

Let Z be an $N \times (N - K)$ matrix satisfying

$$Z'X = 0 \quad \text{and} \quad Z'Z = I_{N-K} \tag{4}$$

and let $U = Z'Y = Z'V$, then

$$EU_{.i} = 0 \qquad EU_{.i}U_{.j}' = \sigma_{ij}I_{N-K} \tag{5}$$

whence

$$EU_{i.} = 0 \qquad EU_{i.}'U_{j.} = \delta_{ij}\Sigma \tag{6}$$

that is, the rows of U are independently and identically normally distributed with zero mean and variance Σ.

Let A be an $(N - n) \times m$ matrix satisfying $A'A = I_m$ where $n < N$ and $m \leq N - n$, let

(7)
$$w' = [U_1, U_2, \ldots U_{n-K}]$$

and let

(8)
$$u' = [U_{n-K+1}, \ldots U_{N-K}](A \otimes I_G)$$

Then

(9)
$$u \sim N(0, I_m \otimes \Sigma)$$

and

(10)
$$u'(I_m \otimes \Sigma^{-1})u \sim \chi^2(mG)$$

In practice Σ is not known and this statistic is not operational. However we may obtain an estimate of $I_m \otimes \Sigma$ from

(11)
$$w \sim N(0, I_{n-K} \otimes \Sigma)$$

Let r denote the largest integer less than, or equal to, $(n - K)/m$. Partition w into r groups of mG elements and let w_j denote the jth group. Then we have for $j = 1, 2, \ldots, r$

(12)
$$Ew_j = 0 \qquad Ew_jw_j' = I_m \otimes \Sigma$$

Applying the result of Anderson [1, p. 106] we have

(13)
$$\frac{u'\Lambda^{-1}u}{r} \cdot \frac{r - mG + 1}{mG} \sim F(mG, r - mG + 1)$$

provided that $r \geq mG$, where

(14)
$$\Lambda = \frac{1}{r} \sum_{j=1}^{r} w_j w_j'$$

3. Application

Let Y, X and V be partitioned by their first n rows and the remaining $N - n$ rows

(15)
$$Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}, \quad X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \quad \text{and} \quad V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

where X_1 has full column rank, K. And let Z be partitioned as

(16)
$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ 0 & Z_{22} \end{bmatrix}$$

where Z_{11}, Z_{12} and Z_{22} are $n \times (n - K)$, $n \times (N - n)$ and $(N - n) \times (N - n)$ matrices respectively. Then

(17)
$$U = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} Z'_{11} Y_1 \\ Z'_{12} Y_1 + Z'_{22} Y_2 \end{bmatrix}$$

where Z_{11} is defined by

(18)
$$Z'_{11} X_1 = 0 \quad \text{and} \quad Z'_{11} Z_{11} = I_{n-K}$$

Thus U_1, w and Λ depend only on X_1 and Y_1 and (13) may be used as a test of whether the observations X_2 and Y_2 were generated by the same model as generated X_1 and Y_1, large values of the statistic leading to rejection of the null hypothesis. Ideally we would like to choose $A = I_{N-n}$ but for this we require $n - K > G(N - n)^2$.

The remaining problem of obtaining a matrix Z satisfying (16) is most easily resolved by applying Givens' method to the matrix $[X\ Y]$. This procedure yields the matrix $U = Z'Y$ each of whose columns is a set of "recursive residuals" (see [3] and [5] for details).

4. Relation

We now indicate the relation between our test and that of Dhrymes *et al.* [2, eq. (12)] when $m = N - n$ and $A = I_m$. Let us rearrange the elements of

(19)
$$u' = [U_{1\cdot}^{(2)} U_{2\cdot}^{(2)} \ldots U_{m\cdot}^{(2)}]$$

as

(20)
$$u'_* = \text{col}\,\{U_{\cdot 1}^{(2)} U_{\cdot 2}^{(2)} \ldots U_{\cdot G}^{(2)}\}$$

then

(21)
$$u_*(\Sigma^{-1} \otimes I_m) u_* \sim \chi^2(MG)$$

since the left side of (21) is the same as the left side of (10). Let $M = I_N - X(X'X)^{-1}X'$, then we have from (4) that $M = ZZ'$, whence

(22)
$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ 0 & Z_{22} \end{bmatrix} \begin{bmatrix} Z'_{11} & 0 \\ Z'_{12} & Z'_{22} \end{bmatrix}$$

and

$$U^{(2)} = Z'_{\cdot 2}Y = Z_{22}^{-1}M_{2\cdot}Y = Z'_{22}P \tag{23}$$

where M is partitioned conformably with ZZ' and $P = M_{22}^{-1}M_{2\cdot}Y = Y_2 - X_2(X_1'X_1)^{-1}X_1'Y_1$. From (23) we have

$$u_* = (I_G \otimes Z'_{22})e \tag{24}$$

where

$$e = \text{col}\{P_{\cdot 1} P_{\cdot 2} \dots P_{\cdot G}\} \tag{25}$$

Thus equation (21) may be rewritten

$$e'(\Sigma^{-1} \otimes M_{22})e \sim \chi^2(mG) \tag{26}$$

where

$$M_{22} = Z_{22}Z'_{22} = I_m - X_2(X'X)^{-1}X_2' \tag{27}$$

and

$$M_{22}^{-1} = I_m + X_2(X_1'X_1)^{-1}X_2' \tag{28}$$

Finally if $m = 1$ we have from (14)

$$\begin{aligned}(n - K)\Lambda &= \sum_{j=1}^{n-K} U'_{j\cdot}U_{j\cdot} \\ &= U_1'U_1 \\ &= Y_1'Z_{11}Z'_{11}Y_1 \\ &= Y_1'[I_n - X_1(X_1'X_1)^{-1}X_1']Y_1\end{aligned} \tag{29}$$

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