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Chapter Author: Michael D. McCarthy, Carl Palash

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THE USE OF ALMON- AND OTHER DUMMY VARIABLE PROCEDURES TO INCREASE THE EFFICIENCY OF MAXIMIZATION ALGORITHMS IN ECONOMIC CONTROL

BY MICHAEL D. MCCARTHY AND CARL J. PALASH*

By constraining the maximizing policy instruments to lie along a polynomial over time, the Almon distributed lag technique can reduce the CPU time of a typical control solution. To illustrate its application, Almon and non-Almon techniques were used to maximize two objective functions, with the MPS quarterly econometric model serving as constraints. The imposition of Almon constraints yields improvements in computational efficiency, as well as highly acceptable solutions in an economic sense.

The past several years have witnessed substantial gains in the application of formal optimal control techniques to econometric models.[1] Maximization algorithms have proved sufficiently efficient and accurate for purposes of economic analysis. Nonetheless, the computer time (CPU minutes) required for solution of control problems has generally been so great as to make extensive application prohibitive for most users. This note proposes a simple technique that has the potential of reducing the CPU time of a typical control solution to about one-fifth to one-half of what is now normally the case. This technique consists of constraining the maximizing policy instruments to lie along a polynomial over time and thus taking advantage of the Almon distributed lag technique.[2] In section 1, the Almon technique is described and compared to those currently used. In section 2, the results of applying the Almon technique to objective functions constrained by the MPS econometric model are discussed.

I. THE ALMON TECHNIQUE

Most maximization algorithms that are now used iterate on the values of the policy instruments in each time unit (i.e. quarter) by the following formulae:

$$E_{itj} = E_{itj-1} + \alpha_j G_{itj} {}^* D_i$$

where E_{itj} = the value of the i^{th} policy instrument in the t^{th} quarter on the j^{th} iteration

*The views expressed here are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of New York or the Federal Reserve System, or the Council of Economic Advisers.

[1] See [2], [3], [4], [5], [6], [7], [8], [9].

[2] See [1].

α_j = optimal scale factor on the j^{th} iteration
G_{itj} = value of the function of the gradient of the objective function with respect to E_{itj}.[3]
D_i = constant weight associated with i^{th} policy instrument

Consequently, the number of gradients that must be calculated per iteration equals the product of policy instruments and quarters in the horizon.[4]

The Almon technique, which we propose, iterates on the values of policy instruments in each quarter according to the formulae:

$$E_{itj} = E_{itj-1} + \alpha_j{}^* \sum_{k=0}^{r} [(\overline{G}_{ikj}{}^*(t^k)^* D_{ik}]$$

where $\overline{G}_{ikj}$ = the function of the gradient of the objective function with respect to the k^{th} Almon coefficient for the i^{th} policy instrument on the j^{th} iteration.
D_{ik} = constant weight associated with the k^{th} Almon coefficient for the i^{th} policy instrument.
r = the degree of the polynomial

The number of gradients that must be calculated per iteration, therefore, equals the product of policy instruments and the degree of the polynomial plus one. The Almon technique will require fewer gradients if the degree of the polynomial plus one is less than the number of quarters in the horizon. We conjecture that the desired degree of the polynomial will possibly increase as the horizon is lengthened, although the maximum degree desired in most economic problems will be three.

The overall improvement in efficiency with the Almon technique depends not only on the reduction in the number of gradients per iteration but also on the number of iterations required to attain an optimum. It is not apparent whether the Almon technique will require more or less iterations, in general, then non-Almon techniques.

II. Comparison of Almon and Non-Almon Techniques

In this section we report our initial results of applying the Almon technique. Two objective functions were maximized both by Almon and non-Almon methods.[5] The MPS quarterly econometric model served as constraints.

[3]The function of the gradient varies across algorithms.

[4]The gradient can be calculated either by numerical perturbation methods or analytically. The former approach appears to be the most feasible for large econometric models. The analytic approach would require us to compute implicitly, at least, m^*T derivatives, where m is the number of policy instruments and T is the length of the horizon, and would conflict with our objective of reducing the number of derivatives.

[5]The gradient algorithm was used in the Almon case, while the conjugate gradient algorithm was used in the non-Almon case. It has been shown that the conjugate gradient

TABLE 1[a]

	Number of iterations	Value of the Objective Function	CPU Time[c]
I. Almon Technique			
a) Linear			
1) equal constant weights	25	−.084	6 min. 50 sec.
2) unequal constant weights[b]	20	−.073	4 min. 55 sec.
b) Quadratic	15	−.073	4 min. 53 sec.
II. Non-Almon Technique	37	−.0034	16 min. 55 sec.

(a) $$W = \sum_{t=1971.1}^{1972.3} [-(u_t - 5.3)^2]$$

where u = unemployment rate

(b) $D_{ik} = 10.^k. \nabla i.$

(c) Refers to CPU time on an IBM 370/155.

In the first example only one policy instrument, federal government spending in 1958$ (EGF), was used to maximize the objective function over a seven quarter horizon. Statement of the function and summary are given in Table 1. The polynomial was specified first as linear and then as quadratic when applying the Almon technique. In addition, we experimented using constant weights, in the linear case, to multiply the gradients on each iteration.

It is apparent that the tremendous saving in CPU time with the Almon technique was accomplished both through fewer calculations of gradients and through fewer iterations. The coincident lower optimal values of the objective function reflect the restriction of the space over which the objective function is maximized, implicit in the Almon constraints. However as the Almon technique smooths the sequence of government spending over time, its solution is more acceptable in an economic sense.[6] The smoothing feature of the Almon technique may be an added benefit to its time—saving feature.

The second problem is more complicated, consisting of three policy instruments, EGF, the Treasury bill rate (RTB) and the effective personal income tax rate (UTPF), with an horizon of seventeen quarters. The objective function was that used in the NBER Model Comparison Seminar.[7] In this example we experimented with both unequal constant weights

algorithm generally requires fewer iterations than the gradient algorithm in the non-Almon case: see [2], [8].

[6]In other words, if Almon constraints were not imposed, then constraints on EGF, either through the objective function or through inequality constraints, would have had to be specified to render the solution acceptable.

[7]Bounds on policy instruments were applied to guarantee an acceptable solution in both the Almon and non-Almon cases. Inequality constraints on the maximum change per quarter were applied in the non-Almon case, as well.

TABLE 2[a]

	Number of iterations	Value of Objective Function	CPU Time[d]
I. Almon Technique			
a) Linear	11	−542.7	13 min. 10 sec.
b) Quadratic			
1) Equal constant weights	4	−548.7	8 min. 13 sec.
2) Unequal weights across Almon coefficients[b]	4	−536.2	7 min. 32 sec.
3) Unequal weights across Almon coefficients and policy instruments[c]	12	−526.2	18 min. 44 sec.
II. Non-Almon Technique	12	−475.0	36 min. 39 sec.

(a) $$W = \sum_{t=1971.1}^{1975.1} [-(u_t \geq 4)^2 - (p_t \geq \bar{p}_t)^2 - (\text{GAP})^2 - (\text{TB})^2]$$

where u = unemployment rate

p = inflation rate

$$\bar{p} = \begin{cases} 3.0 & 1971.1 - 1973.4 \\ 7.0 & 1974.1 \quad 1975.1 \end{cases}$$

GAP = percentage of unused capacity in economy

TB = trade balance as a percent of GNP, current dollars

(b) $D_{ik} = 17^k, k = 0, \ldots, r \; \forall i$

(c) $D_{ik} = 17^k * 10^{\bar{i}}, k = 0, \ldots, r, \bar{i} = \begin{matrix} 1 \text{ for } i = \text{EGF} \\ 0 \text{ for } i = \text{RTB, UTPF} \end{matrix}$

(d) Refers to CPU time on an IBM 370/155.

across gradients and unequal weights across policy instruments. Summary statistics are given in Table 2. The small differences in the values of the objective function in conjunction with the enormous saving in CPU time suggests the application of Almon constraints on policy instruments over time may be highly acceptable.

It is obvious that smoothing procedures other than the Almon may also show promise. A sum of sinusoidal functions in time may be of particular interest. In addition, the use of zero-one dummy variable procedures in the case of policy instruments which typically are allowed to change only periodically, i.e. tax rates, may be attractive.

III. Conclusion

The results presented here are but a small sample. However, they suggest that the imposition of Almon constraints on policy instruments can lead to substantial improvement in efficiency while still yielding highly acceptable optimal solutions in an economic sense. Much work remains along these lines. Specifically, questions that must be addressed relate to the extent of lost welfare when Almon constraints are imposed, the desirable degree of the polynomial and the appropriate use of constant weights in the maximization algorithms.

Case Western Reserve and
Council of Economic Advisors
Federal Reserve Bank of New York

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