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INTERTEMPORAL EFFICIENCY AND THE WORLD PRICE OF OIL: AN EMPIRICAL MODEL

BY ROBERT A. MARSHALLA*

This article presents an empirical depletable resources model which calculates the efficient time paths of world oil price and supply. Besides using dynamic demand functions, the model is calibrated to the results of large scale market equilibrium models based on econometric studies and engineering cost data. Sensitivity tests are made for assumptions regarding interest rates, extraction costs, variations in OPEC undiscovered recoverable resources, and variations in sources, prices, and timing of synthetic liquids. A key conclusion is that monopoly rents, rather than scarcity rents are accruing to OPEC nations.

I. INTRODUCTION

In recent years the adequacy of the world resource base relative to historical growth patterns is a matter that has been thrust into the public's attention. No aspect of this issue has commanded a greater sense of urgency than the ultimate depletability of world oil supplies and the necessity of eventually converting world energy demand from reliance on oil to other fuel sources. The nature and timing of this conversion will be affected in large part by the time path of world oil price through its effects on demand, oil extraction rates and the development of substitute source industries. Should oil be priced too low, oil supplies could be essentially depleted before alternative sources were competitively available. However, inefficiency would also result from prices which were too high. Development of alternative energy technologies would be over-accelerated, leading to transition to expensive new energy sources while lower cost oil remains forever unused, and to inefficient allocation of capital and other resources.

The theory of depletable resources characterizes a time path of *scarcity rents* which, if included in the price of a depletable energy resource, would lead to efficient consumption patterns, and a "smooth" transition to alternative energy sources (i.e, no abrupt price increases or shortages occur when the given resource is exhausted). The basic result is, of course, well known, dating back to Hotelling [3]. The scarcity rents must grow at the rate of return to capital in the same risk class as the oil resources. Only then would competitive suppliers be at the point of indifference between selling oil or holding it in the ground as an investment. For efficiency, the initial scarcity rent must be just large enough so that

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acting through the demand function, the resource is intertemporally rationed so as to exhaust it precisely at the time and price at which world demand for it reaches zero. (A survey of the literature on depletable resources can be found in Peterson and Fisher [10].)

It is to be noted, however, that scarcity rents are entirely distinct from *monopoly rents* which derive from the market power of resource producers. The potential existence of rents other than true scarcity rents makes it difficult to evaluate the efficiency of actual numerical prices on the basis of the theoretical results alone.

This study attempts to calculate quantitatively the intemporally efficient time paths of world oil price and supply by using an empirical depletable resources model. A prominent feature of the model is that it employs dynamic demand functions which are calibrated to the "pseudo-data" results of large scale comprehensive energy market equilibrium models. The model explicitly represents world demand for and supply of liquid fuel sources, namely crude oil and synthetic liquids from coal and/or shale. While the economic interactions between liquids supply and demand and other types of fuels—gaseous, solid, electricity—are not explicitly represented, the advantages of a fuller equilibrium treatment are not entirely lost because of the links to the larger models. The change in demands for and supplies of a liquid fuel source due to a change in the price of oil is not here a *ceteris paribus* effect; rather, it is the change after the prices of all other energy sources and products have shifted so as to restore equilibrium. The procedure of calibrating the model to the larger models' "pseudo-data" is explained in Section III.

The model is actually used to compute the competitive oil price, which under certain assumptions would be equivalent to the efficient price. While supply decisions are modeled normatively (*as if* OPEC were composed of ideal competitive suppliers), demand is econometrically based; ie., consumers are modeled as they are, not as they "should" be. The pure competitive assumption of perfect foresight, therefore, does not hold. The implications are discussed in Section IV.

The key empirical result of this study is that the intertemporally efficient world oil price is substantially below the prevailing level; i.e., the greatest part of rents accruing to the oil producing countries of OPEC can be interpreted as monopoly rents, as opposed to scarcity rents. The same qualitative result has been attained before in various studies (e.g., Adeleman [1], Nordhaus [8], Cremer and Weitzman [2], Pindyck [11]), although quantitatively the results do differ for a multitude of reasons. Due to space limitations a brief statement of the current study's merits will have to take the place of a comparative discussion. On empirical grounds the parameters of this model, unlike the others, are calibrated to the results of large scale energy market equilibrium models based on econometric and engineering cost data. On structural grounds, the advantages of the present model are that it includes a much more realistic treatment of demand and the development and phase-in of substitute liquid fuel sources in the sense of modeling dynamics and price effects.

II. The Depletable Resource Model.

The normative part of the model concerns only the OPEC crude oil producers. OPEC demand is modeled as the residual of several world regional demands for liquid fuel sources, minus non-OPEC oil supplies and liquid synthetic fuel supplies (which eventually fill the demand for liquid fuels when oil supplies are exhausted). The regional demands and non-OPEC supplies are determined by dynamic functions which depend on current and all past prices. Given this demand function OPEC is modeled as if it were a *price-taking* profit maximizing supplier. Marginal extraction cost is presumed to be an increasing function of cumulative extraction.

To the OPEC supplier, for a given time path of prices, $(p_1, p_2, \ldots)$ (denoted $\{p_t\}$), the problem is:

$$\operatorname*{Max}_{\{q_t\}} \sum_{t=1}^{\infty} [p_t q_t - C(Q_t) q_t](1 + r)^{-t}$$

(Problem C)

$$\text{subject to: } \sum_{t=1}^{\infty} q_t \leq \bar{Q} \qquad q_t \geq 0 \qquad t = 1, 2, \ldots, T$$

where:
q_t = OPEC extraction, period t
$Q_t = \sum_{i=1}^{t} q_i$ = cumulative extraction through time t
$C(Q_t)$ = unit extraction cost
r = interest rate
$\bar{Q}$ = OPEC reserves plus undiscovered recoverable resources

If the availability constraint is active there will usually be some finite T such that $q_t = 0$ for $t > T$ and $\sum_{i=1}^{T} q_i = \bar{Q}$. T is called the exhaustion date.

Meanwhile, OPEC demand is modeled as a function of prices. Again, for a given time path of prices, $\{p_t\}$, demand is calculated in each period as:

$$x_t = x_t(p_1, p_2, \ldots, p_t) \quad t = 1, 2, 3, \ldots \tag{1}$$

To solve the problem, one must find a time path of prices so that the systems in problem (C) and equations (1) generate identical quantities in years when OPEC supplies are positive; that is,

$$(2) \qquad \begin{cases} q_t = x_t & \text{if } q_t > 0 \\ x_t \leq 0 & \text{if } q_t = 0 \end{cases}$$

The well known form of solution to a problem of this generic type is that the price when $q_t > 0$ should equal the marginal extraction cost, y_t, plus a scarcity rent which rises at the rate of interest,[1]

$$(3) \qquad p_t = y_t + \lambda(1 + r)^t$$

where λ = initial scarcity rent
= availability constraint multiplier from problem (C).

What makes this particular formulation interesting is (a) *the method of empirically calibrating OPEC demand*, to be discussed in the next section; and, (b) *the dynamics*. The dynamics of interest are in the OPEC demand and extraction cost function. The linkages between periods there make it impossible to calculate anything recursively. The entire time paths need to be considered simultaneously. (An appendix describing the computational algorithm devised to solve the problem is available on request from the author.)

Two interesting implications of the dynamic forms are worth noting. First, since extraction costs increase with cumulative depletion, the "full" marginal cost, y_t in (3), is greater than current unit extraction cost, $C(Q_t)$:

$$(4) \qquad y_t = C(Q_t) + \sum_{j=t}^{T} C'(Q_j)q_j(1 + r)^{t-j}$$

The second term on the right hand side is a user cost which accounts for the fact that current extraction, q_t, makes all future extraction more costly. (See Weinstein and Zeckhauser [14].)

The second point concerns the *dynamics of determining the time and price of resource exhaustion*. In the simplest case, one could consider that a perfect substitute for oil, or "backstop technology," is available at some cost, p_s. The substitute, then, has an infinitely elastic supply curve at price p_s. One knows in advance that the price of oil can never exceed p_s, that $p_T = p_s$, and further that oil's scarcity rent at any time t before T is $[p_T - C(Q_T)](1 + r)^{(t-T)}$. One (of several) reasons this is unrealistic is that it takes no account of the substantial time lags required for development of new technologies and new industries. Even if the price of oil

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[1] The scarcity rent will turn out to be zero in the solution if the availability constraint is inactive, in which case the resource is not intertemporally scarce relative to demand. Sand may be an example of a depletable resource for which this is true.

actually increased suddenly to twice the price necessary to make some new energy process economic, it might take several decades to perfect the process, establish commercial scale plants, and finally totally displace the demand for oil. This problem can be partly solved by specifying some date T_s before which the substitute is unavailable, and possibly some time varying "required price" path after T_s representing learning effects in the new industry. Then at least the price of oil would not be constrained to stay below the substitute cost before it is available. Yet, this is still unrealistic because the pace of development of the new industry is still an exogenous time schedule. In fact, the past history of the price of oil, and future price expectations based upon it, will have major influence on the pace of development of the new industry.

The current model makes an attempt to represent these effects. To see how consider the price path, $\{p_t^*\}$, where:

$$p_t^* = \text{Min}\{p : x_t(p_1, p_2, \ldots, p_{t-1}, p) = 0\} \tag{5}$$

p_t^*, to be called here the "self-sufficiency" price, is the price above which there is zero demand for OPEC oil. p_t^* will always exist in this particular model by virtue of the form x_t takes. One can conceive of the problem (C) as one of finding the initial scarcity rent λ such that the price path $\{p_t\}$ from (3) just meets the price path $\{p_t^*\}$ from (5) at the time when $\{p_t\}$, acting through the demand function, exhausts all resources. This is represented graphically in Figure II-1.

The $\{p_t^*\}$ path serves the function of the more simplistic cost-of substitute or backstop technology price but with the added feature that it varies not only with time, but is dependent on the whole past history of oil prices. Changing $\{p_t\}$ also changes $\{p_t^*\}$. The time and price at which $\{p_t\}$ meets $\{p_t^*\}$ is, in our model, the point of full transition from oil to synthetic fuels. However, synthetic fuels begin being phased in long before that point is actually reached, as will be clear after discussion of how synthetic supplies enter into (1).

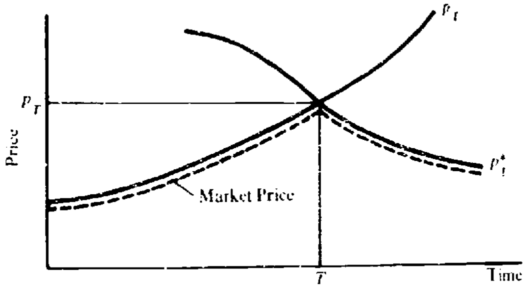

Figure II-1

III. DATA AND CALIBRATION OF FUNCTIONAL FORMS

The first three subsections below describe the functional forms and calibration procedures used to construct OPEC demand. Subsections A, B and C discuss, respectively, demand for liquid fuel sources, supply of non-OPEC oil and supply of synthetic liquid fuels. Subsection D describes the OPEC extraction cost function and other parameters in the OPEC supply model.

A. Demand

The "pseudo data" to which the present model's demand functions were calibrated are the results of two much larger energy models. These are the Federal Energy Administration's International Energy Evaluation System (IEEES) [5] and the Stanford Research Institute-Gulf Energy Model (SRI-GEM) [13]. These models are integrated systems of submodels which trace energy prices and flows from resource extraction through logistical systems and conversion processes to final products, and ultimately to various final demand sectors. The IEES model covers some 20 world regions comprising the non-Communist world, but has only been run to give results through 1985. The SRI-GEM model covers only the United States, but results are available over a 50 year time period, and it includes the most substantial modeling of the development of the synthetic fuels industry currently available. A "result" from either of these models is a snapshot of a competitive spatial energy market equilibrium. Each such "snapshot" result presumes some particular exogenous time path of world crude oil price. The idea here is to extract from these results the information needed to construct a much smaller, but dynamic, model in which crude oil prices are endogenous.

Liquid fuel source demand functions were constructed for four world[2] regions: North America, other OECD, OPEC and the rest of the non-Communist world. These were calibrated to the IEES results[3] through 1985 and the SRI-GEM[4] results from 1986 through 2025. Non-US regional demands after 1985 were devised by preserving the 1985 ratios to US demand, and all demands after 2025 were based on assumed geometric growth rates. A log-linear Koick-lag form was used for each region:

$$d_t = f_t(d_{t-1})^{\alpha}(p_t)^{\beta} \tag{6}$$

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[2] The Communist bloc is treated only as a net exporter by assuming an exogenous time path of exports to the non-Communist world. See Table III-2.

[3] Essentially, the IEES Jan. 19, 1976 "Business as Usual" results for 1980 and 1985 were used, except for modifications to account for expected effects of recent legislation in the U.S.

[4] The calibration to SRI-GEM results is based on results of that model published in Volume II of "Recommendations for a Synthetic Fuels Commercialization Program" by the Synfuels Interagency Task Force, a report prepared for the U.S. President's Energy Resources Council in November, 1975.

where d_t is demand in period t and α, β, and f_t are parameters chosen to best fit the results of the larger models. The lagged demand factor captures the influence of past prices and quantities on the present demand. (In fact, it is easily seen that (6) can be translated to a function of the form $d_t = d_t(p_1, \ldots, p_t)$.) The full adjustment of demand resulting from a lasting price change evolves gradually over time, the speed of adjustment being determined by the exponent, α. This adjustment process can be interpreted as representing the time it takes consumers and producers to change consumption patterns, alter capital stocks and possibly redesign technologies in response to a long term change in oil price.

The $\{f_t\}$ time paths were determined by observing the time paths of liquid fuel demands forecasted by the larger models under a particular time path of crude oil price. This price path and its associated demand paths are hereinafter called "reference paths," and are denoted by a superscript "0." (6) can be reduced to the following equivalent form:

$$(6') \qquad \left(\frac{d_t}{d_t^0}\right) = \left(\frac{d_{t-1}}{d_{t-1}^0}\right)^{\alpha} \left(\frac{p_t}{p_t^0}\right)^{\beta}$$

Thus, the $\{f_t\}$ parameters are simply represented in terms of the reference path variables.

The α and β parameters, which represent a dynamic price sensitivity, were fitted to IEES results. Each of several alternative IEES constant price paths can be considered as a price change at time zero from the designated reference price path, which is maintained for n years. If the set of resultant demands in year n are fitted to a constant elasticity functional form, its elasticity, ϵ_n, can be interpreted as the n-year price elasticity of demand:

$$\epsilon_n = \beta(1 + \alpha + \alpha^2 + \cdots + \alpha^{n-1})$$

By calculating ϵ_n for two (or more) values of n one obtains the simultaneous equations:

$$\epsilon_{n_1} = \beta(1 + \alpha + \cdots + \alpha^{n_1-1})$$

$$\epsilon_{n_2} = \beta(1 + \alpha + \cdots + \alpha^{n_2-1})$$

which can then be solved for α and β. This process is repeated for each demand region.

The "elasticities" represented by the ϵ_n's are not actually conventional own-price elasticities because they give the change in demand due to a change in own price *while allowing other energy prices to adjust as needed to restore equilibrium.* Thus, while the model only explicitly represents liquid fuel sources, which greatly simplifies it, it is a clear step ahead of those which merely presume some exogenous behavior in related markets.

TABLE III-1

LIQUIDS DEMAND DATA

$$\left(\frac{d_t}{d_t^0}\right) = \left(\frac{d_{t-1}}{d_{t-1}^0}\right)^{\alpha} \left(\frac{p_t}{p_t^0}\right)^{\beta}$$

d_t = demand
p_t = price
$d_t^0 p_t^0$ = reference demand paths

	Exponents		
	North America	Other OECD	Other Non-OPEC, Non-OECD
Speed of Adjustment, α (for one year period)	.760	.834	.800
One Year Elasticity, β	−.0877	−.0425	−.0300
Long Run Elasticity, $\beta/1-\alpha$	−.3654	−.2560	−.1500

*If n-year periods are used, the exponents have to be modified from α and β to α^n and $\beta(1+\alpha+\ldots+\alpha^{n-1})$ respectively.

Reference Demand Paths[a]

Time Period	North America	Other OECD	Other Non-OPEC, Non-OECD	OPEC
74–75[b]	12.99	13.21	4.96	
76–77	12.97	14.53	5.40	1.42
78–79	12.94	16.28	5.99	1.72
80–81	13.11	17.94	6.59	1.97
82–83	13.86	19.35	7.26	2.41
84–85	14.63	20.75	7.94	2.85
86–90	39.45[c]	56.16[d]	21.57[d]	8.03[e]
91–95	44.21	62.94	24.17	9.53
96–00	49.47	70.43	27.05	11.33
01–05	54.45	77.52	29.77	13.45
06–10	62.06	88.35	33.93	15.94
11–15	70.38	100.20	38.48	18.48
16–20	78.83	112.23	43.10	22.54
21–25	88.55	126.07	48.12	26.77
26–30[f]	100.19	142.64	54.78	31.79
31–35	113.36	161.39	61.98	37.76
36–40	128.26	182.60	70.13	44.86
41–45	145.11	206.60	79.35	53.27
46–50	164.18	233.74	89.77	63.27
51–55	185.75	264.46	101.57	75.14

[a] Units: 10^9 bbl/time period. Reference Prices: $13/bbl c.i.f. U.S. in all periods.

[b] All regions are based on IEES BAU Case results through 1985.

[c] U.S. demand, 1986–2025, is based on SRI-GEM "Nominal Case" results as adjusted to conform to the price path used in this study.

[d] Demand after 1985, is based on the assumption that the ratio to U.S. demand will be constant.

[e] OPEC demands increase at 3.5% per year from 1986–2025 irrespective of price.

[f] All region's demands are based on a 2.5% per year growth rate after 2025.

Also, note that the elasticities are defined in terms of a reference, or autonomous, time path of demand. For example, if the price were increased by one percent from the reference price path and held one percent higher for n periods, then demand n periods hence would be $\epsilon_n = \beta(1 + \alpha + \cdots + \alpha^{n-1})$ percent lower *than the reference demand* in that period. The data giving the reference demand paths and α and β exponents for equation (6) are contained in Table III-1.

B. Non-OPEC Oil Supply

Non-OPEC crude oil supply functions were constructed for North America, the North Sea, and the rest of the non-Communist world. The Communist sector was represented by price invariant rates of exports to the non-Communist world.

The same type of functional form was used for the supply functions as the demand functions:

$$\left(\frac{s_t}{s_t^0}\right) = \left(\frac{s_{t-1}}{s_{t-1}^0}\right)^a \left(\frac{p_t}{p_t^0}\right)^b \tag{7}$$

where s_t = supply in year t

p_t = price in year t

s_t^0, p_t^0 = reference path supplies and prices

a = speed of adjustment of supply to price

b = one-year price elasticity

The reference supply paths for the non-Communist crude oil supply regions through 1985 were those generated by IEES. After 1985 the reference paths were based on the primary constraint that the total cumulative extraction, given a constant \$13 reference price path, should be equal to best estimates of reserves plus undiscovered recoverable resources. The time of ultimate depletion in each region was chosen (by assumption) so as to allow reasonably shaped supply trajectories. Note that because the supply function in equation (7) is used, if the actual price path were higher (lower) than the reference path, then cumulative resources depleted would be higher (lower) than in the reference case. This represents, although admittedly in an indirect way, the concept that the availability of reserves is a function of price. (The maximum variation in the model runs made above the base estimate of 1.28 trillion barrels was 10%.)

The exponents in (7) were fitted to the IEES results by the same procedure as in the case of demand.

The data giving the reference crude oil supply paths, the a and b parameters for equation (7), and the resource availability estimates and their sources are contained in Table III-2.

TABLE III-2

NON-OPEC SUPPLY DATA

$$\frac{s_t}{s_t^0} = \left(\frac{s_{t-1}}{s_{t-1}^0}\right)^a \left(\frac{p_t}{p_t^0}\right)^b$$

s_t = supply
p_t = price
s_t^0, p_t^0 = reference trajecteries

Exponents

	North America	North Sea	Other Non-OPEC, Non-OECD
Speed of Adjustment, *a* (one year period)*	.886	.600	.800
One Year Elasticity, *b*	.0532	.0800	.062
Long Run Elasticity, $a/1-b$	.467	.200	.310
Recoverable Resources[i/ii]	250	67	206

*If *n*-year periods are used, the exponents have to be modified from a and b to a^n and $b(1 + a \ldots + a^{n-1})$ respectively.

Reference Crude Oil Supply Paths[i]

Time Period	North America	North Sea	Other Non-OPEC, Non-OECD	Communist Exports
73 75	8.54	.07	3.14	.80
75 77	9.15	.81	3.69	.85
77 79	9.97	1.99	4.42	.91
79 81	10.62	3.83	5.11	.95
81 83	10.77	4.97	5.69	.95
83 85	10.91	5.41	6.48	.95
85 90	26.61	14.97	16.94	2.37
90 95	26.10	15.70	18.62	2.37
95 00	26.10	12.92	20.44	2.37
00 05	26.10	8.30	22.27	2.37
05 10	25.92	3.69	24.09	2.37
10 15	22.45	0.33	24.82	2.37
15 20	17.89	0.18	21.54	2.37
20 25	13.32	0.18	15.97	2.37
26 30	8.76	0.02	10.40	2.37
31 35	4.20	0.02	4.85	2.37
36 40	0.40	0.02	1.93	2.37
41 45	0.02	0.02	0.02	2.37
46 50	0.02	0.02	0.02	2.37
51 55	0.02	0.02	0.02	2.37

[i] Units: 10^9 bbls/time period. Reference Prices: $13/bbl c.i.f. U.S. in all periods.

[ii] All figures are reserves plus undiscovered recoverable resources. The United States estimate, including NGL's, is from [2]. The Canadian and other world regions' estimates are from [7].

C. Synthetic Liquids Supply

Liquid synthetic fuels from coal and/or shale will eventually fill the demand for liquid fuels when oil supplies are exhausted in the present model. The liquid synthetic supply function is based on the SRI-GEM results reported in [11], a report specifically designed to model and forecast the future development of the synthetic fuels industry.

The synthetic liquids supply function is represented by a function of the same form as (7). The reference supply path comes from an SRI-GEM scenario entitled "High Oil Shale Price." This scenario was selected to construct the base case reference supply path because it projects virtually all synthetic liquids production to be coal based; shale oil is effectively eliminated. The SRI-GEM nominal case projects a dominant role for shale in the synthetic industry after the turn of the century. The more conservative case has been chosen; that is, the one which tends to bias price estimates upward. Sensitivity runs have been made, however, using the SRI-GEM nominal case for the reference synthetics supply trajectory. Since the SRI-GEM results pertain only to the U.S., and since 70-75% of the world's coal resources lie outside the U.S., it was assumed that coal syncrude would be produced outside the U.S. as well. In the reference case, U.S. synthetic liquids supply was augumented by a conservative factor of 0.75 to estimate production in, and Communist country exports to, the rest of the world. Sensitivity runs were made using factors of 0.50 and 1.00 as well. No non-U.S. production was assumed in the sensitivity run based on shale oil availability.

The exponents in the supply function were also calibrated from the SRI-GEM results using the same procedure as for oil supply and demand.[5] All relevant data on synthetic liquids supply are contained in Table III-3.

D. OPEC Supply

The remaining data needed for the model of OPEC supply include the extraction cost function $C(Q_t)$, the total resource availability $\bar{Q}$, and the discount rate r. The OPEC extraction cost function is defined so that unit extraction costs are an increasing function of cumulative depletion. Two functional forms are reported here, a low and a high cost case. The two forms are:

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[5] The alternative price paths reported in SRI-GEM were not constant in time, but maintained the same ratio over time, one to another, so that the same procedure reported in Section III-A for fitting the exponent parameters can still be used.

TABLE III-3

SYNTHETICS SUPPLY DATA

$$\frac{s_t}{s_t^0} = \left(\frac{s_{t-1}}{s_{t-1}^0}\right)^a \left(\frac{p_i}{p_t^0}\right)^b$$

s_t = supply
p_t = price
s_t^0, p_t^0 = reference trajectories

Exponents

	Synthetic Liquids
Speed of Adjustment, *a* (one year period)	.872
One Year Elasticity, *b*	.433
Long Run Elasticity, *b*/1 − *a*	3.380

Reference Synthetic Liquids Supply and Price Paths[a]

Time Period	Reference Price	Synthetic Liquids Supply: Base Case[b] (Coal liquids only)	Synthetic Liquids Supply: Sensitivity Case[c] (Coal and shale liquids)
74 75	11.00	0.00	0.00
76 77	11.42	0.01	0.01
78 79	12.19	0.02	0.01
80 81	12.86	0.05	0.03
81 83	13.46	0.13	0.07
84 85	13.99	0.18	0.18
86 90	14.87	0.46	1.25
91 95	15.70	1.33	3.18
96 00	16.31	2.31	6.62
01 05	16.76	5.78	12.71
06 10	17.09	12.64	22.02
11 15	17.33	21.68	23.03
16 20	17.51	23.71	43.83
21 25	17.64	27.84	54.49
26 30	17.73	33.88	66.30
31 35	17.80	41.21	80.66
36 40	17.85	58.14	98.13
41 45	17.89	61.01	119.39
46 50	17.92	74.22	145.26
51 55	17.94	90.30	176.73

[a] Units: Reference prices: 1975 U.S. \$/bbl. Supplies: 10^9 bbls/period.

[b] Based on SRI-GEM "High Shale Oil Prices" Scenario. Figures here are obtained by multiplying liquid synfuel quantities for U.S. by 1.75 to account for supplies from non-U.S. sources.

[c] Based on SRI-GEM "Nominal Case." Figures are not multiplied to account for non-U.S. production in this case.

Low Cost:

$$C(Q_t) = \frac{K_0}{(\bar{Q} - Q_t + \epsilon)}$$

$\bar{Q}$ = available resources

= 830 (billion bbls)

K_0 = constant = 207.5

ϵ = constant = 20

High Cost Case:

$$C(Q_t) = K_1 + K_2 Q_t^2$$

K_1 = constant = .25

K_2 = constant = $(1.84)(10^{-5})$

The low cost form makes cost almost inversely proportional to resources remaining. (The "ϵ" constant in the denominator is to prevent cost from becoming infinite). The high cost case makes cost a quadratic function of cumulative supply. It can be seen in the table below that while both functions start and end at roughly equivalent costs, the quadratic form has much higher costs over the mid ranges of depletion:

% Depletion of OPEC Resources	Unit Cost of Extraction Low Cost (Inverse Form)	 High Cost (Quadratic Form)
0%	$0.24	$0.25
25%	0.32	1.04
50%	0.53	3.42
75%	1.11	7.38
100%	10.38	12.93

OPEC resource availability, $\bar{Q}$, was estimated at 830 billion barrels. This is the sum of proved plus prospective reserves, plus undiscovered, recoverable resources cited in a recent published estimate.[6] Sensitivity runs were made in which the expected figure for undiscovered recoverable resources is shifted up and down by 50%, giving overall resource availability figures of 969 and 692 billion barrels respectively.

Discount rates were handled parametrically. Rates of 4%, 6% and 8% were used. The rate which should be used to be efficient is the real marginal productivity of capital.

[6] "World Crude Resource May Exceed 1,500 Billion Barrels." *World Oil*. September. 1975; pp. 48 The figure cited above is the sum of estimates for the Mideast. North Africa. the Gulf of Guinea. Northwest South America and Southeast Asia in the article.

IV. COMPETITION AND INTERTEMPORAL EFFICIENCY IN THE EMPRICAL MODEL

In section I it was mentioned that the competitive solution to this model would in general diverge from a true intertemporally efficient solution because consumers do not possess perfect foresight. That is, demand in period t depends on present and past, but not future prices. In another study (Marshalla [7]) it has been concluded that the quantitative difference between competitive and "true efficient" prices is small. Therefore, for reasons of computational simplicity the results reported in this study are those derived from the competitive model.

The approach in the above study is to maximize directly the social surplus derived from usage of the resource. Thus, problem (C) of section II is replaced by problem (S) below:

$$\underset{\{q_t^*\}}{\text{Max}} \sum_{t=1}^{x} \left[\int_0^{q_t^*} p_t(q_t, q_{t-1}^*, \ldots, q_1^*)dq_t - C(Q_t^*)q_t^* \right](1+r)^{-t}$$

(Problem S)

$$\text{Subject to: } \sum_{t=1}^{x} q_t \leq \bar{Q} \qquad q_t \geq 0$$

Equation (1) is still used to compute demand and (2) is still required to hold as in (C). The difference is that the condition in (3) which gives the price when $q_t > 0$ is replaced by:

(3′) $$p_t = y_t + \lambda(1+r)^t - s_t$$

where

$$s_t = \sum_{i=t+1}^{T} \int_0^{q_i^*} \frac{\partial p_i}{\partial q_t}(q_i, q_{i-1}^*, \ldots, q_1^*)dq_i(1+r)^{t-i}$$

s_t is particularly difficult to evaluate because the functions

$$p_t = p_t(q_t, q_{t-1}, \ldots, q_1)$$

cannot be found in closed form; and, a change of variable is necessary prior to integration. Also, the integral in s_t is path dependent so that the particular path used must be given an economic interpretation.[7] (See Willig [15])

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[7] There exists no continuous function W defined on intertemporal allocations such that the demand functions used could be interpreted as being the outcome of maximization of W. Therefore, we are forced to define the social surplus integral itself to be the benefit function in line with the "willingness-to-pay" criterion of value. The particular path selected, namely moving in order of subscripts each component of $(q_1, \ldots q_T)$ from 0 to its ultimate value q_i^*, is the only one which makes sense in this context.

Subject to some caveats from the above issues, the conclusion of the cited study was that the differences in the price solutions of problem S to problem C are at most ±5% over the next fifteen years, and −15% to +8% over the lifetime of OPEC oil resources.

V. RESULTS

The principal finding of the study is that the efficient world price of oil is far below the prevailing level for at least the next fifteen years. The results of the empirical model are that over a wide range of variations of key parameters and assumptions the efficient real price[8] of oil is bounded inside the range:

\$2-\$7	in 1976-77
\$5-\$12	in 1986-90
\$26-\$60	at the time of OPEC exhaustion

OPEC resources last between 45 and 60 years, by which time synthetic liquid fuels from coal and possibly, but not necessarily, shale have come to dominate the liquid fuels market. In no case, however, is the synfuels industry required to supply over 10 MMBD of liquid synfuels sooner than 30 years into the future or at a real price below \$18 per barrel.

These results hold:

1. for real interest rates in the range 4%-8%,
2. for extraction cost assumptions that vary widely with respect to rate of increase of cost,
3. for ±50% variations in the expected level of OPEC undiscovered, recoverable resources,
4. for large variations in assumptions on the price response, timing, location and sources (coal, shale) of synthetic liquids,

and also for combinations of the above changes.

An attempt was made throughout the study to make assumptions and define ranges of parameters conservatively relative to the principal conclusion. That is, when in doubt, choices were made which could tend to bias the consequent price path upwards in early years.

Since a great deal of uncertainty is associated with several of the model's parameters, a series of sensitivity runs have been performed. The results of these tests are those alluded to in the opening paragraph of this section. A sample of information from the sensitivity runs is contained in Table V-1. The first column in Table V-1 contains an index by which the various runs can be referenced. The next four columns specify the parameter settings for each run (explained in the table footnotes). Columns 6-8

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[8] 1975 U.S. dollars, f.o.b. Persian Gulf.

TABLE V-1

SENSITIVITY RUNS

		Extraction Cost Function	Discount Rate	OPEC Resource Availability	Synthetics Assumptions	Initial Price (1976-7)	Mid-Term Price (1985 90)	Maximum Price	Time of OPEC Resource Exhaustion
Sensitivity Run Index		Low or High	4%, 6% or 8%	691.5, 830 or 968.5	See test and footnote*	1975 U.S. $/bbl f.o.b. Persian Gulf			Year
(Base Case)	A	(Low)	(4)	830	Base	5.80	8.98	32.97	2030
	B	(Low)	6	830	Base	3.41	6.43	33.64	2025
	C	(Low)	(8)	830	Base	2.17	4.91	34.62	2020
	D	High	(4)	830	Base	6.68	10.62	31.62	2030
	E	High	6	830	Base	4.29	8.45	31.85	2025
	F	High	(8)	830	Base	3.00	7.17	34.76	2025
	G	(Low)	(4)	(691.5)	Base	6.51	10.06	31.91	2025
	H	(Low)	6	(691.5)	Base	4.01	7.57	32.77	2020
	I	(Low)	(8)	(691.5)	Base	2.68	5.08	32.98	2020
	J	High	(4)	(691.5)	Base	7.37	11.77	32.44	2025
	K	High	6	(691.5)	Base	4.89	9.67	33.31	2025
	L	High	(8)	(691.5)	Base	3.51	8.46	32.34	2020
	M	(Low)	(4)	(968.5)	Base	5.25	8.12	32.72	2035
	K	(Low)	6	(968.5)	Base	2.95	5.57	34.55	2025
	O	(Low)	(8)	(968.5)	Base	1.82	4.09	35.83	2025

O	(Low)	(8)						
P	High	(4)	(968.5)	Base	6.11	9.67	32.54	2035
Q	High	6	(968.5)	Base	3.82	7.47	33.75	2030
R	High	(8)	(968.5)	Base	2.63	6.24	32.94	2025
S	(Low)	6	830	(Augment Factor) = 1.5	3.48	6.57	34.97	2025
T	High	6	830	(Augment Factor) = 1.5	4.36	8.58	33.20	2025
U	(Low)	6	830	(Augment Factor) = 2.0	3.34	6.31	32.54	2025
V	High	6	830	(Augment Factor) = 2.0	4.23	8.33	30.61	2025
W	High	6	830	(Elas. ÷ 2)	4.84	9.47	54.17	2030
X	High	6	830	(Elas. ÷ 2 & 5 yr. delay)	5.07	9.91	59.58	2030
Y ·	(Low)	6	830	(Shale Avail.)	3.14	5.91	28.25	2025
Z	High	6	830	(Shale Avail.)	4.02	7.94	26.20	2025

Key to notation in "Synthetics Assumptions" column"

"Base"	Uses base case reference trajectory which depends entirely on coal based liquid synfuels.
"Augment Factor = 1.5" "Augment Factor = 2.0"	The projected U.S. supply from SRI-GEM was multiplied by this factor to get total non-Communist usage. The factor 1.75 was used in the base case.
"Elas. ÷ 2"	Uses base case trajectory but supply elasticity is halved.
"Elas. ÷ 2" & 5 yr. delay	Same as above except the reference trajectory is delayed 5 years at 1990.
"Shale Available"	Reference supply trajectory is based on the SRI-GEM nominal case where shale oil is available for use. No augment factor is applied.

TABLE V-2

6% DISCOUNT RATE-HIGH COST FUNCTION

(RUN E)

Units: Price. Costs 1975 U.S. $/bbl
Cumulative Production 10^9 bbls
Supply and Demand Rates MMBD

Time Period	F.O.B. Price	Conventional Marginal Cost	User Cost	Scarcity Rent	OPEC Supply Rate	OPEC Cumulative Production
76-77	4.29	0.26	2.86	1.18	33.06	24.13
78-79	4.81	0.30	3.19	1.32	36.95	51.11
80-81	5.37	0.37	3.52	1.49	39.04	79.61
82-83	6.01	0.47	3.86	1.67	42.37	110.54
84-85	6.71	0.63	4.20	1.88	45.42	143.69
86-90	8.45	1.22	4.94	2.30	46.75	229.01
91-95	10.84	2.12	5.64	3.09	49.02	318.47
96-00	13.67	3.40	6.15	4.12	52.36	414.03
01-05	16.89	5.10	6.28	5.51	54.43	513.36
06-10	20.48	7.21	5.89	7.38	55.72	615.05
11-15	24.10	9.43	4.90	9.87	49.94	706.20
16-20	28.05	11.59	3.26	13.21	43.17	784.99
21-25	31.85	12.87	1.31	17.67	23.59	828.04

Demand

Time Period	North America	Other OECD	Other N-OPEC N-OECD	OPEC	N-Cmn. World Total
76-77	20.12	21.35	7.76	1.95	51.18
78-79	21.29	24.93	8.84	2.36	57.42
80-81	22.04	23.07	9.84	2.70	62.65
82-83	23.28	30.51	10.87	3.30	67.96
84-85	24.24	32.65	11.84	3.90	72.64
86-90	24.75	34.17	12.55	4.40	75.87
91-95	25.78	36.49	13.64	5.22	81.07
96-00	26.63	38.76	14.78	6.21	86.38
01-05	27.22	40.53	15.78	7.37	90.90
06-10	28.97	44.00	17.48	8.76	99.21
11-15	30.94	47.78	19.34	10.40	108.46
16-20	32.80	51.43	21.17	12.35	117.75
21-25	35.13	55.79	23.31	14.67	128.91

Supply

Time Period	North America	North Sea	Other N-OPEC	Synthetic Liquids	Cmn. Exports	Cumulative N-OPEC Oil Production
76-77	11.45	0.99	4.57	0.01	1.16	12.43
78-79	11.74	2.36	5.19	0.01	1.24	26.50
80-81	12.02	4.55	5.86	0.02	1.30	42.88

(*Continued*)

TABLE V-2 (*Continued*)

Time Period	North America	North Sea	Other N-OPEC	Synthetic Liquids	Cmn. Exports	Cumulative N-OPEC Oil Production
82-83	11.94	5.97	6.49	0.04	1.30	60.69
84-85	12.01	6.61	7.45	0.05	1.30	79.73
86-90	12.10	7.67	8.20	0.05	1.30	130.75
91-95	12.66	8.41	9.61	0.21	1.30	186.74
96-00	13.73	7.23	11.26	0.55	1.30	245.53
01-05	14.96	4.83	13.07	2.32	1.30	305.50
06-10	16.19	2.23	14.99	8.74	1.30	366.47
11-15	15.19	0.21	16.26	25.11	1.30	424.25
16-20	13.04	0.12	14.80	44.90	1.30	475.37
21-25	10.38	0.12	11.43	82.23	1.30	515.30

show, respectively, the initial price (1976 1977), the price in the 1986-90 period, and the highest price ever attained. The last column tells when OPEC runs out of oil.

To provide a point of reference, a particular intermediate setting of parameters, run E in the table, has been designated the "base case" run, although we offer no claims as to its being "most likely." Parameters in other runs which have been varied from their setting in run E are in parentheses in the table. The base case, run E, uses the quadratic (high) extraction cost function, a 6% discount rate, 830 billion barrels for OPEC resource availability and the coal-based synthetics reference supply path. Table V-2 presents the detailed results for the base case run. The most striking result, which can be observed in the second column, is that price over the next 15 years is very low compared to current prices. The price for the initial period, 1976 to 1977, is seen to be only $4.29 per barrel, and it rises to only $8.45 by 1990. Thereafter, price does continue to rise steadily for 35 more years, reaching a high of $31.85 per barrel when OPEC runs out of oil in the year 2025.[9]

It is interesting to observe the marginal cost and scarcity rent components of the optimal prices listed in columns (3-5) of the table. The full marginal cost is seen to be dominated by the user costs (the second term of equation (4)) over most of the time horizon. User cost is calculated at $2.86 in the initial period compared to only $0.26 for current marginal cost, the actual out-of-pocket unit cost of extraction. The current marginal cost rises continually with increasing cumulative depletion (by definition of the extraction-cost function), but the user cost can be seen to level off and decline eventually.

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[9] The table does not list any data for years after OPEC has exhausted its oil resources, although the model is solved over its full 80 year time horizon in all cases.

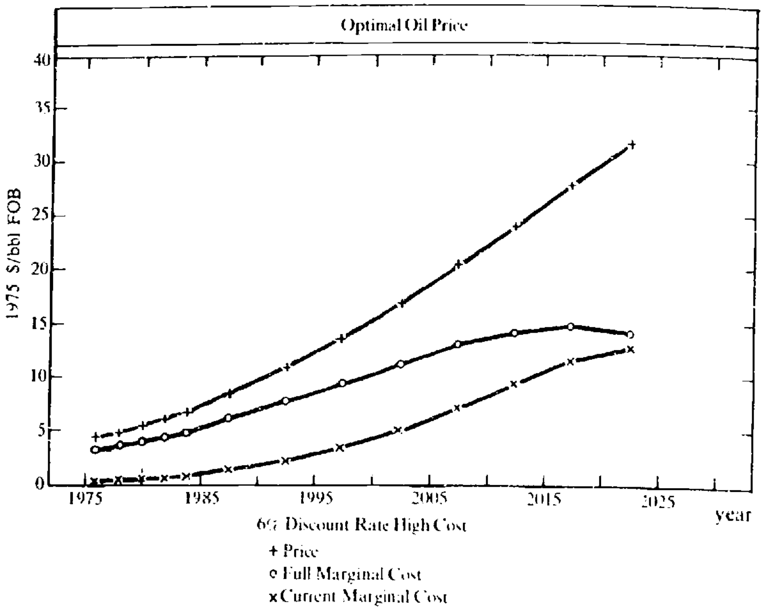

Figure V-1 Base Case Run E

This eventual decline of the user cost occurs because the number of future barrels of oil whose extraction costs are affected by current production decreases as the ultimate exhaustion date is neared. Thus, the current cost eventually comes to represent a larger and larger share of the marginal cost, reaching 100 percent on the very last barrel depleted.

The optimal initial scarcity rent can be seen in column 5 to be only $1.18. This rent rises at the interest rate of 6% over the whole period during which OPEC produces oil. By the last period, the royalty has grown to $17.67 per barrel. The optimal price path, broken into its marginal cost and royalty components is illustrated graphically in Figure V-1.

OPEC supply rates (which include indigenous use) in column 6, can be seen to be rather high by historical standards. They begin at almost 33 MMBD and rise to 45 MMBD by 1985. The highest rate ever reached is 56 MMBD in the 2006–2010 period.

Demands and supplies from the various regions in the rest of the world can be observed in the second and third blocks of the table. They are fairly self-explanatory. The synthetics supply results (column 5 of the third block of the table), however, deserve special comment. Synthetic liquids can be seen to play virtually no role in the world liquid fuels market prior to the year 2000. Supply does not exceed 1 MMBD until after that year. However, in subsequent years the results represent a rapid buildup of the industry and reflect high price elasticity of supply, so that as oil reserves in OPEC and the rest of the world decline, synthetic liquids from coal come to dominate the liquid fuels market. The percentages of total world demand for liquid fuels supplied by synthetic liquids over the latter half of the problem are given in the following table:

Time Period	Percent of liquid fuel demand supplied by synthetics
1996 2000	less than 1
2001 2005	3
2006 2010	9
2011 2015	23
2016 2020	38
2021 2025	64

Note that OPEC and non-OPEC oil suppliers each continue to supply about 18% of world demand in the period 2021 to 2025. However, OPEC has essentially exhausted its resources by the end of this period and supplies nothing in the next five-year period (not reported in table V-1). Non-OPEC suppliers (who possess some higher cost resources) do continue to supply a minor part of world demand for about ten more years beyond 2025.

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