





























$$(A.17) \quad \gamma_0(j) = \gamma_0(j+1) - \frac{1}{2} H'_{0,u}(j) \mathcal{H}_{0,uu}^{-1}(j) H_{0,u}(j) \\ j = N-1, \dots, k+1; \quad \gamma_0(N) = 0.$$

Then

$$(A.18) \quad g_0(k+1) = \gamma_0(k+1) + \frac{1}{2} \sum_{j=k+1}^{N-1} \text{tr} [\mathbf{K}_0(j+1) \mathbf{Q}(j) + \mathcal{A}_{0,xx}(j) \Sigma_0(j|j)]$$

This completes the proof of (2.20). If the sequence of nominal controls is optimal for the deterministic problem, then the Hamiltonian (A.3) achieves its minimum and  $H_{0,u} = \mathbf{0}$  (unless one has a constrained optimization and the minimum occurs at the boundary). In this case  $\gamma_0(j) = 0$  for all  $j$ .

## REFERENCES

- [A1] M. Aoki. *Optimization of Stochastic Systems*. Academic Press. 1967.
- [A2] K. J. Åström, *Introduction to Stochastic Control Theory*. Academic Press. 1970.
- [A3] A. B. Abel. "A Comparison of Three Control Algorithms as Applied to the Monetarist Fiscalist Debate." *Annals of Economic and Social Measurement*. Vol. 4. No. 2. pp. 239-252. 1975.
- [B1] R. Bellman. *Adaptive Control Processes: A Guided Tour*. Princeton University Press. 1961.
- [B2] Y. Bar-Shalom and R. Sivan. "On the Optimal Control of Discrete-Time Linear Systems with Random Parameters." *IEEE Trans. Automatic Control*. Vol. AC-14, pp. 3-8, February 1969.
- [B3] Y. Bar-Shalom and E. Tse. "Dual Effect, Certainty Equivalence and Separation in Stochastic Control." *IEEE Trans. Automatic Control*. Vol. AC-19, pp. 494-500, Oct. 1974.
- [B4] Y. Bar-Shalom and E. Tse. "Concepts and Methods in Stochastic Control" in *Control and Dynamic Systems: Advances in Theory and Applications*. C. T. Leondes, ed., Academic Press, 1975.
- [C1] R. E. Curry, "A New Algorithm for Suboptimal Stochastic Control." *IEEE Trans. Auto. Control*. Vol. AC-14, pp. 533-536, Oct. 1969.
- [C2] G. Chow. *Analysis and Control of Dynamic Economic Systems*, Wiley. 1975.
- [C3] G. Chow. "On the Control of Nonlinear Systems with Unknown Parameters." *Econometric Research Program Report No. 175*. Princeton University, March 1975.
- [D1] S. E. Dreyfus. *Dynamic Programming and the Calculus of Variations*. Academic Press. 1965.
- [D2] J. G. Deshpande, T. N. Upadhyay and D. G. Lainiotis. "Adaptive Control of Linear Stochastic Systems." *Automatica*. Vol. 9, pp. 107-115. 1972.
- [F1] A. A. Feldbaum. *Optimal Control Systems*. Academic Press. 1965.
- [J1] O. L. R. Jacobs and J. W. Patchell. "Caution and Probing in Stochastic Control." *International Journal of Control*. Vol. 15, pp. 189-199. 1972.
- [M1] L. Meier, R. E. Larson, and A. J. Tether. "Dynamic Programming for Stochastic Control of Discrete Systems." *IEEE Trans. Auto. Control*. Vol. AC-16, pp. 767-775, Dec. 1971 (Special Issue on Linear Quadratic Gaussian Problem).
- [M2] E. C. MacRae. "Linear Decision with Experimentation." *Annals of Economic and Social Measurement*, Vol. 1, pp. 437-447. 1972.
- [N1] A. L. Norman. "First Order Dual Control." *Annals of Economic and Social Measurement*. 1976.
- [R1] H. Raiffa and R. Schlaifer. *Applied Statistical Decision Theory*. M.I.T. Press. 1972.
- [R2] G. C. Rausser and J. W. Freebairn. "Approximate Adaptive Control Solutions to U.S. Beef Trade Policy." *Annals of Economic and Social Measurement*. Vol. 3, No. 1, pp. 177-203. 1974.
- [S1] G. N. Saridis. *Self-Organizing Control Stochastic Systems*. M. Dekker. 1976.
- [S2] J. Spcyer, J. Deyst, and D. Jacobson. "Optimization of Stochastic Linear Systems with Additive Measurement and Process Noise Using Exponential Performance Criteria." *IEEE Trans. Auto Control*. Vol. AC-19, pp. 358-366, August 1974.
- [S3] H. Simon. "Dynamic Programming under Uncertainty with a Quadratic Function." *Econometrica*, Vol. 24, pp. 74-81. 1956.
- [T1] E. Tse, Y. Bar-Shalom, and L. Meier. "Wide-Sense Adaptive Dual Control of Stochastic Nonlinear Systems." *IEEE Trans. Automatic Control*. Vol. AC-18, pp. 98-108, April 1973.
- [T2] E. Tse, and Y. Bar-Shalom. "An Actively Adaptive Control for Discrete-Time Systems with Random Parameters." *IEEE Trans. Auto. Control*. Vol. AC-18, pp. 109-117, April 1973.
- [T3] E. Tse and M. Athans. "Adaptive Stochastic Control for a Class of Linear Systems." *IEEE Trans. Automatic Control*. Vol. AC-17, pp. 38-51, February 1972.

- [T4] E. Tse and Y. Bar-Shalom, "Adaptive Dual Control for Stochastic Nonlinear Systems with Free End-Time," *IEEE Transactions Auto. Control*, Vol. AC-20, pp. 600-605, October 1975.
- [T5] H. Theil, "A Note on Certainty Equivalence in Dynamic Planning," *Econometrica*, Vol. 25, pp. 346-349, 1957.