Volume Title: Annals of Economic and Social Measurement, Volume 5, number 2

Volume Author/Editor: Sanford V. Berg, editor

Volume Publisher: NBER

Volume URL: http://www.nber.org/books/aesm76-2

Publication Date: April 1976

Chapter Title: Neighboring Stochastic Control of an Econometric Model

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Chapter URL: http://www.nber.org/chapters/c10441

Chapter pages in book: (p. 211 - 221)

Annals of Economic and Social Measurement, 5/2, 1976

NEIGHBORING STOCHASTIC CONTROL OF AN ECONOMETRIC MODEL*

BY PETER WALSH AND J. B. CRUZ, JR.

In this paper, an econometric model with parameter uncertainty considered by Kendrick and Majors, which extends the deterministic linear econometric model used by Pindyck, is modified further to account for additive errors in the structural equations and additive observation errors in variables. The use of a linearized neighboring optimal stochastic control with a Kalman filter is investigated. Simulation results indicate that when a Kalman filter is used to improve the estimates of the state variables and parameters, deviations from the desired paths tend to be attenuated.

1. INTRODUCTION

In recent years there have been many applications of modern control theory to economic stabilization and planning, such as the work done by Chow [4, 5], Friedman [6], Livesey [10], Pindyck [15, 16], and Sengupta [18]. Pindyck [15] used a 28 state variable linearized model, assumed to be deterministic, and a quadratic tracking approximation for the criterion function. The validity of a linearized model has been discussed by Pindyck [15, 16] and the economic justification of the quadratic criterion approximation has been discussed by Theil [19]. An approach to the problem which takes parameter variations into account has been done by Kendrick and Majors [8] utilizing Pindyck's model for the case of stochastic state variable coefficients.

In this paper, we allow for further uncertainties in the form of additive noise in the system equations. This is a common method of representing uncertainties in economic estimation [3]. Furthermore, we will allow for error in the measurement of the state and parameter values and for uncertainty in the initial conditions.

We will utilize a Kalman filter to improve the estimates of the state and parameter values. Kalman filter techniques have had a wide variety of successful applications in aerospace systems [7, 9], yet relatively few in economics [1, 14]. We will show that by replacing the actual measurements by the improved estimates in Kendrick's control rule that the resultant state and control paths will deviate less from the desired paths, thereby reducing the criterion function. We will also discuss a reformulation of Kendrick's quadratic penalty in the criterion function which will better reflect how closely the resultant solution comes to Pindyck's original policy formulation.

2. THE AUGMENTED SYSTEM AND THE THE FORMULATION OF THE CRITERION FUNCTION

In this section we will briefly review the model used [15], Kendrick's method of dealing with the parameter variations [8] and present our results on the formulation of the criterion function. Pindyck's model consists of ten linear

* This work was supported by the National Science Foundation under Grant ENG 74-20091.

structural equations in which the variables appear with multiple lags. The system is rewritten as twenty–eight first order equations in the form

$$x_{t+1} = A_0 x_{t+1} + A_1 x_t + B_1 u_t + C_1 z_t \tag{1}$$

where x is the state vector (28×1), u is the control vector (3×1), and z is the vector of exogenous variables (2×1). Equation (1) can be rewritten as

$$(I - A_0)x_{t+1} = A_1 x_t + B_1 u_t + C_1 z_t. \tag{2}$$

It is desired to put the system equations into the form

$$x_{t+1} = Ax_t + Bu_t + Cz_t. \tag{3}$$

Define α_t to be a vector of the stochastic parameters. Kendrick considered the problem of mapping the statistics of those elements of α contained in the A_0 matrix into the A matrix of (3). This was done by translating those components of α from (1) into (3) via a first order Taylor series approximation of (2).

Defining

$$f_1(x_{t+1}, \alpha_{t+1}) = (I - A_0)x_{t+1}.$$

Equation (2) is approximated by

$$G_t x_{t+1} + e_t \alpha_{t+1} = A_1 x_t + B_1 u_t + q_t, \tag{4}$$

where

$$q_t = C_1 z_t - f_1(x^*_{t+1}, \bar{\alpha}) + G_t x^*_{t+1} + e_t \bar{\alpha}, \tag{5}$$

$$G_t = \frac{\partial f_1}{\partial x_{t+1}}, \qquad e_t = \frac{\partial f_1}{\partial \alpha},$$

with the derivatives evaluated at $(x^*_{t+1}, \bar{\alpha})$, x^* is the solution of the certainty equivalence problem, and $\bar{\alpha}$ is the mean value of α.

Kendrick assumed that α follows a first order process of the form[1]

$$\alpha_{t+1} = \alpha_t + \hat{\theta}_t \tag{6}$$

where $\hat{\theta}_t$ is a vector of uncorrelated additive noise, with zero mean and a covariance matrix Σ. Combining (4) and (6) into an augmented system and solving yields

$$y_{t+1} = \tilde{A} y_t + \tilde{B} u_t + \tilde{q}_t + \tilde{\theta}_t \tag{7}$$

where

$$\tilde{A}_t = H_t^{-1} S, \qquad \tilde{q}_t = H_t^{-1} w_t, \qquad \tilde{B}_t = H_t^{-1} V, \qquad \tilde{\theta}_t = H_t^{-1}\theta_t$$

with

$$H_t = \left[\begin{array}{c|c} G_t & e_t \\ \hline 0 & I \end{array}\right] \qquad S = \left[\begin{array}{c|c} A_1 & 0 \\ \hline 0 & I \end{array}\right] \qquad y_t = \left[\begin{array}{c} x_t \\ \hline \alpha_t \end{array}\right]$$

$$V = \left[\begin{array}{c} B_1 \\ \hline 0 \end{array}\right] \qquad w_t\left[\begin{array}{c} q_t \\ \hline 0 \end{array}\right] \qquad \theta_t = \left[\begin{array}{c} 0 \\ \hline \hat{\theta}_t \end{array}\right]$$

[1] A more general expression is to premultiply α_t by ρ. In our case ρ equals the identity matrix.

Kendrick used a criterion function of the same form as that used by Pindyck [15]. However, he chose to track the solution to the certainty equivalence problem (x^*, u^*) which is found by solving the optimal control problem with the elements of α_t set equal to their mean values. Kendrick's criterion function is

$$J_1 = E\left\{\sum_{t=1}^{T} [(y_t - y_t^*)'\tilde{Q}(y_t - y_t^*) + (u_t - u_t^*)'\tilde{R}(u_t - u_t^*)]\right\} \tag{8}$$

where

$$y^* = \left[\begin{array}{c} x^* \\ \hline \bar{\alpha} \end{array}\right], \quad \tilde{Q} = \left[\begin{array}{c|c} Q & 0 \\ \hline 0 & 0 \end{array}\right], \quad \tilde{R} = R$$

where Q and R are those used in Pindyck's criterion function. The pair (x^*, u^*) are the solution of Pindyck's problem which used $(\hat{x}, \hat{u})$ as the desired paths.

It is appropriate to linearize about x^* since in the limit, as the noise goes to zero, the state and control paths would go to (x^*, u^*). Moreover, in the presence of small stochastic disturbances the linearization is valid and we have a neighboring optimal control. However, it seems more reasonable to try to track $\hat{x}$ rather than x^* since the parameter variations may occur in a manner more favorable to tracking $\hat{x}$ yielding better results. The advantage of these variations would not be utilized if one were tracking x^*. Thus, our formulation of the criterion function is

$$J_2 = E\left\{\sum_{t-1}^{T} [(y_t - \hat{y}_t)'\tilde{Q}_t(y_t - \hat{y}_t) + (u_t - \hat{u}_t)'\tilde{R}(u_t - \hat{u}_t)]\right\} \tag{9}$$

where

$$\hat{y} = \left[\begin{array}{c} \hat{x} \\ \bar{\alpha} \end{array}\right].$$

The two performance indices would generally yield different optimal controls. The control rule used by Kendrick is based on a linearized model. This approximation results in a suboptimal control. Due to the approximation, the control rule results in the same solution whether one uses (8) or (9).

The control rule is of the form,[2]

$$u_t = D_t y_t + h_t. \tag{10}$$

Notice that u_t is explicitly a function of the parameter values via the augmented state vector, y_t.[3]

When the Riccati equations are solved using the a priori parameter values, $\bar{\alpha}$, the resultant values of the matrix D_t and the open loop vector h_t will be the same whether one tracks $(\hat{x}, \hat{u})$ or (x^*, u^*). The controls will, therefore, be the same.

In order to obtain a more optimal solution, thereby gaining further advantage by tracking $(\hat{x}, \hat{u})$, it would be necessary to reformulate the optimization problem at each quarter, resolving the Riccati equations from the terminal time to the present time, utilizing our most recent parameter estimates.

[2] For derivation, see Pindyck [15].
[3] See Kendrick [8] for adaptation of the control rule to the augmented system.

Since the added computation involved is prohibitive, we have simply solved Riccati equations, and therefore D_t and h_t, with the a priori parameter values, $\bar{\alpha}$.

It should be recognized that although D_t and h_t are only dependent on $\bar{\alpha}$, the control, u_t, is still explicitly dependent on the most recent parameter estimate, α_t, via the augmented vector y_t.

If, indeed, the pair $(\hat{x}, \hat{u})$ are the desired target paths based on economic considerations, then using $(\hat{x}, \hat{u})$ in the criterion function, (9) is a more natural expression of deviations from the original policy objectives.

3. The Estimation Problem

In this section we will present the application of Kalman filtering techniques to improve state estimation and in turn to improve the effectiveness of the control law utilizing the estimates.[4] We will discuss the details of applying the filter to a macroeconomic planning problem and the resultant improvement in the effectiveness of the control. The improvement can be seen by comparing the criterion function values and by analyzing the state and control paths of Monte Carlo runs performed with and without the filter. The system model is

$$y_{t+1} = \tilde{A}y_t + \tilde{B}u_t + \tilde{q}_t + \tilde{\theta}_t \tag{11}$$

which is identical to (7) except that general system noise, w_t, is included

$$\tilde{\theta}_t = H_t^{-1}\begin{bmatrix} w_t \\ \hat{\theta}_t \end{bmatrix}$$

where $\tilde{\theta}_t$ is the vector of parameter variations as before. The vector, w_t, has zero mean and covariance matrix, $\bar{Q}_t$, which is assumed to be diagonal. So the covariance matrix of the vector $[w_t \vdots \hat{\theta}_t]'$ is

$$\begin{bmatrix} \bar{Q}_t & 0 \\ 0 & \Sigma \end{bmatrix}. \tag{12}$$

For the initial conditions we have

$$E[y(0)] = \hat{y}_0, \qquad E[(y(0)-\hat{y}_0)(y(0)-\hat{y}_0)'] = P_0$$

where $E[\cdot]$ denotes expected value.

The measurement noise is assumed additive, with z_t the measurement

$$z_t = y_t + v_t \tag{13}$$

where the noise v_t has zero mean and a covariance matrix $\bar{R}_t$, also assumed diagonal (i.e. measurement errors are uncorrelated with each other). For our experiments, the covariance matrices $\bar{R}_t$ and $\bar{Q}_t$ are assumed constant.

The measurement and system errors are also assumed uncorrelated

$$E[w_t v_\tau'] = 0 \qquad \forall t, \tau.$$

[4] For a standard derivation see for example Gelb [7], Meditch [11], or Rhodes [17].

In our notation, $\hat{y}_t(+)$ is the expected value of y_t given the measurements through time t, i.e. $\hat{y}_t(+)=\hat{y}(t|t)=E(y_t|\{z_\tau, \tau=0, 1, \ldots, t\})$. Similarly, $\hat{y}(-)=\hat{y}(t|t-1)=E(y_t|z_\tau, \tau=0, 1, \ldots, t-1)$. The caret ($\hat{\ }$) will denote expected value. The state estimate extrapolation is

$$\hat{y}_{t+1}(-)=\tilde{A}_t\hat{y}_t(+)+\tilde{B}_t u_t+\tilde{q}_t. \tag{14}$$

The error covariance propagation is

$$P_t(-)=\tilde{A}_{t-1}P_{t-1}(+)\tilde{A}'_{t-1}+H_{t-1}^{-1}\tilde{Q}_{t-1}(H_{t-1}^{-1})'. \tag{15}$$

The state and covariance updates are

$$\hat{y}_t(+)=\hat{y}_t(-)+K_t[z_t-\hat{y}_t(-)] \tag{16}$$

$$P_t(+)=[I-K_t]P_t(-) \tag{17}$$

where the Kalman gain is

$$K_t=P_t(-)[P_t(-)+\tilde{R}_t]^{-1}. \tag{18}$$

These equations are for measurements in the form of (13). More general equations can be derived for the case when the measurements are of the form

$$z_t=C_t y_t+v_t.$$

The controls, u_t, are the decisions made during quarter t. The information, z_t, for the same quarter is not yet available. Therefore, when employing the filter, the state extrapolation (14) must be used in the control rule

$$u_t=D_t\hat{y}_t(-)+h_t.$$

If the filter is not used the state must be extrapolated from the previous measurement by

$$\hat{y}_t(-)=\tilde{A}_{t-1}z_{t-1}+\tilde{B}_{t-1}u_{t-1}+\tilde{q}_{t-1}. \tag{19}$$

This puts complete confidence in the measurements, equivalent in our case to setting the Kalman gain equal to the identity matrix.

The filter equations (14–18) are for a linear system and linear measurements. The actual system is not in fact linear when we allow for parameter variations since some of the elements of the system matrices are state variables of this augmented system. For this case, the system can be written in a general nonlinear form

$$y_{t+1}=f(y_t, u_t, z_t)+\tilde{\theta}_t.$$

The problem introduced by the nonlinearities is in calculating the expected value of $f(y_t, u_t, z_t)$ for which exact knowledge of the probability distribution functions of y_t are required. It is therefore desirable to make a reasonable approximation such as utilizing some form of an extended Kalman filter.[5]

[5] For a derivation, see Chapter 6 of Gelb [7].

One applicable form of an extended Kalman filter is based on the approximation of $f(y_t, u_t, z_t)$ by a linearization about $\hat{y}_t$, the most recent estimate of y.

$$f(y_t, u_t, z_t) \approx f(\hat{y}_t, u_t, z_t) + \frac{\partial f}{\partial y}(y_t - \hat{y}_t)$$

where $\partial f/\partial y$ is evaluated at $\hat{y}_t$. The resultant filter equations are identical to (14–18) except that $\tilde{A}_{t-1}$ is replaced by $\partial f/\partial y$ evaluated at $\hat{y}_{t-1}$ in the error covariance propagation equation (15) and the state estimate extrapolation (14) is determined by the nonlinear system model, i.e. the matrices in (14) are evaluated with the most recent estimates of α_t. For our simulations, however, we have simply applied the Kalman filter (14–18) to the model as linearized in (7).

The system and measurement noise covariance matrices, $\bar{Q}_t$ and $\bar{R}_t$, must be chosen by the economist in accordance with his confidence in the system model and the estimation (measurement) process. For example, if the model is considered to be inaccurate, such as a linearization neglecting important nonlinearities, and if the measurement process is assumed to be fairly accurate, it would be appropriate to choose a relatively large system covariance matrix, $\bar{Q}_t$, and a relatively small measurement covariance matrix, $\bar{R}_t$. The large $\bar{Q}_{t-1}$, and therefore $P_t(-)$, and the small $\bar{R}_t$ cause the Kalman gain (18) to be large. That is, the updating of the state (16) depends heavily on the information in the latest measurement. Conversely, if the model is considered to be accurate and the measurements are inaccurate, the update will tend to ignore the current measurement and rely on the model.

The first ten equations are corrupted with system noise, corresponding to the errors of the original ten structural equations. The augmenting equations describing the parameter behavior are, of course, also corrupted with noise. The remaining equations, however, which propagate the delayed values of the endogenous variables, are not corrupted.

The initial measurement of a state, say at time t, is corrupted but it is not recorrupted at a future time when it appears as a delayed state. The measurements of the parameter values are also corrupted at each time interval.

By employing a Kalman filter, we have a method of accurately quantifying our confidence in both our model and in the latest measurements. Inherent in the filtering process is the effect of weighting differently the value of the information contained in the measurements of the various states. Thus, for example, by proper choice of the covariance matrices we can be very selective, utilizing the measurement of one state while ignoring another.

In some applications, one might have perfect measurements of the endogenous variables, where only the parameter values are uncertain. In this special case, the order of the system, which now only describes the parameter behavior (6), is reduced to the number of parameters being estimated. The same parameter measurement equation could be used and the filter algorithm applied. However, we could take much better advantage of the measurements of the endogenous variables. Since the system equations give us a relationship between the endogenous variables and the parameters, these equations can be reformulated as the parameter "measurement" equations, that is, the system equations can be rewritten as a measurement equation

$$x_{t+1} = C_t\alpha_t + \theta_t,$$

where x_{t+1} is the vector of the directly measured endogenous variables, α_t is the vector of parameters, C_t is a matrix of present and past endogenous variables, and θ_t is the actual system's noise which now takes the role of the measurement error.

Thus, the often interesting case in which only parameter values are uncertain can be readily treated by a Kalman filter.

4. Estimation of the Covariance Matrices

It is our intention to demonstrate the possible advantages of applying Kalman filtering techniques to the estimation of state and parameter values in a macroeconomic planning problem. The results, in practice, are dependent on one's ability to accurately estimate the values of the covariance matrices which characterize the uncertainties.

A realistic approach to characterization of the system noise is to estimate the standard deviations of the system noise by the standard errors of the estimation process. However, for the purpose of our simulations, we chose to set the standard deviations of the system disturbances equal to a fraction of the corresponding initial conditions. To choose the scaling factor, we adjusted it until reasonable state and control paths were obtained. A realistic level was found when the standard deviations were set at one percent of the initial condition values.

In simulations, our covariance estimate and the standard error would both yield consistent results in that equivalent improvements in performance would be achieved. In practice, of course, the standard errors from the estimation process would be more appropriate estimates of the standard deviations of the system noise.

The parameter covariance estimates are known from the estimation of the consumption equation. In our experiment, as in Kendrick's, three parameters in Pindyck's consumption function are chosen to be stochastic, the coefficient of current disposable income, the coefficient of current disposable income lagged one period, and the constant term. The covariance matrix chosen to drive the parameter noise was proportional to the covariance matrix Σ_p for the parameters from the estimation of the consumption function.

$$\Sigma_p = \begin{bmatrix} 0.001833 & -0.001345 & 0.04254 \\ -0.001345 & 0.003295 & 0.05318 \\ 0.042540 & 0.053180 & 6.55000 \end{bmatrix}$$

where the mean values of the parameters are 0.415, −0.0282, and 5.299. When the covariance matrix was set at about one or two percent of Σ_p, realistic state and control paths were obtained.

For the purpose of simulations, we have assumed that the standard deviations of the measurement noise are proportional to the corresponding initial conditions. This choice at least takes into account the effects of the units of measurement on the covariance levels. Again, setting the standard deviations equal to about one percent of the initial conditions gave the most realistic results.

In the economic literature, Vishwakarma [20] has determined estimates of the measurement error covariance for a model of the Netherland's economy but has offered no justification nor insights into the procedure for obtaining the estimates.

Chow [2] has discussed the applicability of Kalman filter algorithms but, again, has not considered the procedural aspects of obtaining estimates of the measurement error covariance matrices.

In practice, the accurate estimation of the statistics of the measurement error is a formidable problem. Empirically, one could base the estimates of the statistics on the historical patterns of revisions in preliminary data estimates.

To develop a more accurate estimation procedure one might consider the specific structure of the measurement process of each variable and then, in combination with data on past revisions of preliminary data estimates, the covariance levels could be estimated.

The problem can be approached more readily from within the framework of the Kalman filter. As discussed by Mehra [13], one can test the optimality of the filter based on the innovation property of the filter. That is, from the estimable statistics of the innovation sequence $\{z_t - \hat{y}_t(-)\}$, one can test whether the covariance matrices $\bar{Q}$ and $\bar{R}$ are accurate. If the covariance matrices are not accurate, then the autocorrelation function of the innovation process can be used to obtain asymptotically unbiased and consistent estimates of the covariance matrices $\bar{Q}$ and $\bar{R}$. Further discussion on identification can be found in Mehra [12].

A thorough treatment of the problem is beyond the scope of this paper. We have demonstrated the applicability of Kalman filter algorithms assuming that, in practice, a procedure is utilized for making sufficiently accurate estimates of the measurement error covariance matrices.

5. Simulation Results

We performed experiments for the period from the first quarter of 1957 through the first quarter of 1962. For all runs the use of the Kalman filter reduced the value of the criterion function, typically by 10 to 20 percent. Some representative values are shown below.

Normalized Criterion Function Values	
with filter	without filter
1.002	1.190
7.609	8.873
3.584	3.826
7.656	7.706
5.832	6.479

Examples of some of the paths followed are shown in Figures 1 through 4. In all of the runs we notice a general tendency of the state and control paths to be more oscillatory when the filter is not employed. Without the filter, decisions are made based solely on the measurements. These decisions will be inaccurate and, in general, will have a detrimental effect. Corrections must be made during the following period, but again based solely on the measurements, thus the oscillations. This effect can be seen in Figure 1 which shows the short term interest rate. Both the short and long term interest rates were very sensitive to this effect. Using the filter not only suppresses the variations but also maintains the states closer to the desired levels. The disturbances of this particular run caused a general increase in consumption, Figure 2, and in the price level, Figure 4, above the desired values but the employment of the filter mitigated this rise. The disturbances were favorable toward the money supply objectives, Figure 3, and by using the filter, better advantage was taken of this shift. A particular control or state may often deviate more from its desired path when the filter is in use; however, the overall performance, reflected by the criterion function, is consistently improved.

For all figures, the horizontal axis is time in quarters beginning with the first quarter of 1957.

Line #1. With Kalman filter
#2. Without Kalman filter.
#3. Desired path ($\hat{x}$).
#4. Pindyck's certainty equivalence solution (x^*).

The interest rate is in percent.
The consumption and the change in the money supply in billions of dollars.
The price level equals 100 in 1958.

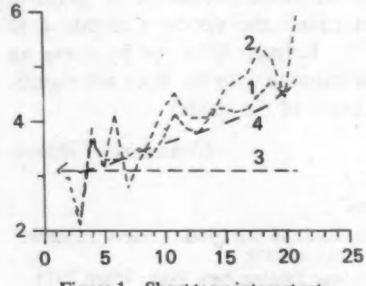

Figure 1 Short term interest rate

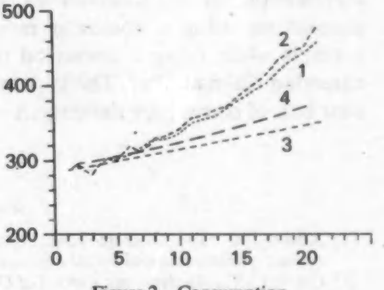

Figure 2 Consumption

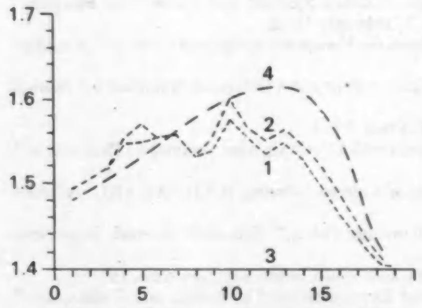

Figure 3 Change in the money supply

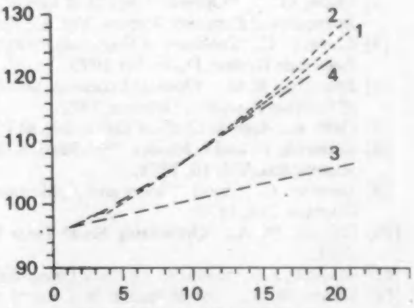

Figure 4 Price level

6. Conclusions

Our results have shown that when applying Kendrick's technique for dealing with parameter variations, the penalization about the original desired path leads to the same neighboring optimal solution as that obtained by penalizing about the certainty equivalence solution, when the state equations are linearized using the a priori parameter estimates, $\bar{\alpha}$. We have argued that conceptually the penalization about the original desired path will make the criterion function a more meaningful expression of the deviations from the original policy objectives. In general the two stochastic control problems will yield different optimal solutions.

In order to deal with imperfect measurements of the states and parameters, a Kalman filter can be used to obtain more accurate estimates. The Kalman filter algorithm does not replace conventional econometric estimation, but rather it supplements it, giving improved estimates. Due to its recursive nature, the filter's dimension does not increase with increased sample size. By using the improved estimates in the control rule, we have shown that a significant improvement in the performance of the system results. This is demonstrated by the state and control paths tracking the desired paths more closely. This, of course, results in a decrease of the criterion function value.

Improper choice of covariance matrices and inaccurate model formulation can result in a degradation of the performance. If the measurements are sufficiently accurate, modeling errors can be partially compensated for by appropriate choice of the covariance matrices. An approach to study how well the filter can compensate for the inaccuracies of a linearized model would be to perform simulations using a nonlinear model to determine the system's response to controls while using a linearized model in the Kalman filter, or by using an extended Kalman filter. The improvements obtainable with the filter are significant but, of course, are dependent on the accuracy of the model.

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