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USE OF THE LINEAR QUADRATIC APPROACH TO STUDY THE DYNAMIC POLICY RESPONSES OF A NONLINEAR MODEL OF THE FRENCH ECONOMY

BY B. A. OUDET*

In this paper a study of the dynamic policy responses of a model of the French economy is presented. Advantage is taken of the linear behavior of the model around a reference trajectory. The linear approximation of the implicit state variable representation is determined by stepwise regressions. The examination of the ten control variables has little influence in modifying the autonomous dynamics of the model. The hypothesis is verified by simulations on the nonlinear model of feedback controls computed on the linear approximation.

I. INTRODUCTION

The current level of inaccuracy of macroeconomic models[1] and the limited knowledge by the policymaker of which objective function to use are obstacles to the use of optimal control in the choice of economic policies. Optimal control can, however, be of immediate interest as a tool for studying the dynamic behavior of our macroeconomic models. The value of the control approach for this purpose is not in its accuracy but rather in its ease of application and in the wealth of information it provides.

In this paper I propose a control system that achieves this informational objective. It takes advantage of the fact that some macroeconomic models are in reality slightly nonlinear. The approach consists of computing the control rules based on a linear approximation in the state variables, which rules are in turn applied to the nonlinear model. The method is described briefly in section II. An application in section III to the STAR[2] model, a yearly nonlinear model of the French Ministry of Finance, illustrates the information on its dynamic policy responses that can be gained from this approach.

II. THE CONTROL SYSTEM

The proposed approach for the control of the nonlinear econometric model includes the following steps: First establish a list of the state (X), output (Y) and control variables (U). Second, generate from the nonlinear model a reference trajectory given projected input variables. Third, identify an openloop, linear, state variable model that best predicts deviations from the reference trajectory. Fourth, apply to the nonlinear model the feed-back control

(1)
$$U_t = U_t^* + \Delta U_t$$
$$\Delta U_t = -L(t) \cdot \Delta X(t) + G(t)$$

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[1] See Haitovsky, Treyz, and Su [4] for a critical appraisal of two well known U.S. models.

[2] Schéma Théorique d'Accumulation et de Répartition [2].

where U_t^* is the control vector on the reference trajectory, ΔU_t is a perturbation added to U_t^* to create the total current control U_t, $\Delta U(t)$ is the deviation of the state variables from the reference trajectory; and $L(t)$ and $G(t)$ are the matrix and vector respectively that minimizes, given the linear constraint, the criteria

$$J = \sum_{t=1}^{T-1} (\Delta Y_t - \Delta Y_t^D)^T Q (\Delta Y_t - \Delta Y_t^D)$$
$$+ \sum_{t=0}^{T-1} (\Delta U_t - \Delta U_t^D)^T R (\Delta U_t - \Delta U_t^D) + (\Delta Y_t - \Delta Y_t^D)^T F (\Delta Y_t - \Delta Y_t^D) \tag{2}$$

In Eq. (2), ΔY_t^D and ΔU_t^D are the vectors of desired output and control deviations from the reference trajectory respectively and Q, R, and F weighting matrices.

Our approach differs from the Linear Quadratic Gaussian approach proposed by Athans [1] or Kendrick [5] for the stochastic control of nonlinear models. The purpose of ΔU_t is not to bring the model *back* to its *optimal* trajectory in case of perturbations but rather to *deviate* from the reference trajectory in some desired *optimal fashion*. The control system is thus a tool for generating evolution scenarios in the direction specified by the choice of Q, R, and F in the criteria. Its outputs are *controlled simulations* which are believed to be an improvement over the traditional trial and error simulations presently used in the study of the dynamic behavior of large macroeconomic models.

The second innovation of our control approach is the estimation of the transition matrix A and the control matrix B using a least squares method.[3] Each line of the combined matrices $[A(t), B(t)]$ is determined successively by stepwise regression. The observations of the independent variables are deviations in the state and control variables at t; the observations of the dependent variables are the resulting deviations of the state variables at $t+1$. Stepwise regression has the advantage of ranking the state and control variables[4] according to their effectiveness in changing values of the state variables over the next period.

It is a powerful tool for clearing the A and B matrices of unimportant coefficients and, more important, isolating these coefficients crucial to the dynamics of the system.

III. Application to the Star Model

STAR is a yearly model of 77 equations. The state vector contains, as in Kendrick [5], the one period lagged endogenous variables and one additional state variable for each endogenous variable of lag greater than one for a total of 32 states variables.[5] There are two output variables, the rate of inflation, YQ, and

[3] See Cooper and Fischer [3] for the estimation of input-output relationships of the St. Louis model using a similar method. In their study, least squares determines the order of input and output lags.

[4] To obtain a valid ranking, we imposed an orthogonality constraint (zero covariances) on the independent variables. The observations on the independent variables were generated using pseudo-random binary sequences.

[5] 51 of the 77 endogenous variables do not appear with lags in the model; moreover they are not considered as objective variables. They are thus not included in the list of state and control variables.

trade balance, *BACO*, and ten control variables which are the changes in:

VNP = personal liabilities[6]

XHA = rate of growth of employment in government

CA = government consumption

KA = government investment

WKM = personal capital expenditure[6]

SCSA = government wages including contributions to social security

PSA = government transfer payment to individuals

IIE = indirect business tax

IDE = business income tax

IM = personal income tax.

The reference trajectory was computed for a five year period (1972–76). The validity domain of the linearized model turned out to be suprisingly large about the reference trajectory. On this basis it was decided to estimate a linear, time-invariant model around the 1971 point.

Among the data provided by the standard stepwise program are the percentages of variation of the dependent variable caused by a change in the independent variables. Equating to zero the coefficients of the variables that explain less than 5 percent of the total variation greatly simplifies the *A* and *B* matrices. An examination of the non-zero coefficients and their associated percentages offers insight into the dynamics of the model. For example it permits us to locate four control variables, ΔXHA, $\Delta SCSA$, ΔPSA, ΔIM, that have little influence in modifying the autonomous dynamics of STAR.

In the *B* matrix each of the four corresponding columns has at most three non-zero coefficients. The examination of the *A* matrix shows that the states modified by the controls do not affect the states crucial to the dynamics of the model.

The controlled simulations of Tables I and II permits one to verify the small action of the four control variables. In Table I we pursue a trajectory with trade equilibrium (as opposed to the reference trajectory) with no costs attached to the use of the controls. The criteria are specified as follows:

$$\Delta y_1^D(t) = \Delta YQ^D(t) = 0 \qquad t = 1, \ldots 5$$

$$\Delta y_2^D(1) = \Delta BACO^D(1) = 15{,}174 \qquad \Delta BACO^D(2) = 13{,}644$$
$$\Delta BACO^D(3) = 17{,}305 \qquad \Delta BACO^D(4) = 21{,}325$$
$$\Delta BACO^D(5) = 23{,}513$$

[6] These variables represent government action on personal credit and personal capital expenditure.

(the trade deficit of the reference trajectory)

$$\Delta u_i^D(t) \qquad i=1,\dots 10$$
$$t=1,\dots 5$$
$$q_{ij}=f_{ij}=0 \qquad i\neq j$$
$$q_{11}=f_{11}=0$$
$$q_{22}=f_{22}=100$$
$$r_{ij}=0 \qquad i\neq j \quad i=1,\dots 10$$
$$j=1,\dots 10$$
$$r_{ii}=0{,}5$$

TABLE I

TRAJECTORY WITH TRADE EQUILIBRIUM

	States and Output*				
	1972	1973	1974	1975	1976
ΔAUT	−4,326 (−4,416)	5,020 (4,714)	4,240 (3,953)	1,943 (1,864)	3,338 (3,683)
ΔVKENA	−5,208 (−5,271)	−3,577 (−3,685)	−458 (−635)	61 (−359)	3,375 (3,249)
ΔRDSTAR	12,680 (11,932)	21,910 (21,614)	22,494 (22,990)	28,190 (28,395)	25,648 (24,454)
ΔYQ	3.7 (3.5)	2.1 (2.5)	0.8 (1.2)	0.6 0.9	−0.8 (−1.5)
ΔXQNA	−2.4 (−2.4)	0.2 (0.2)	−0.4 (−0.5)	−0.2 (−0.3)	0.2 (0.4)
ΔWCM	−2,251 (−3,160)	12,515 (14,582)	21,509 (23,311)	30,637 (32,473)	31,649 (32,615)
BACO	−249 (−72)	−182 (−47)	−341 (−41)	−1,080 (−38)	−1,694 (−28)

* *ΔYQ, ΔXQNA, ΔXHA* are in percent, the others are in millions of francs as computed by the nonlinear model. Enclosed in parenthesis are the results obtained by simulation of the control rules on the linear model. All variables with the exception of *BACO* are deviations from the reference trajectory.

We provide in the tables the value of the output controls, and the following states:

AUT = after tax business profits
VKENA = industry investment
RDSTAR = disposable personal income
XQNA = G.N.P. rate of growth
WCM = personal consumption expenditures
(All variables are in current francs except *XQNA* and *VKENA*.)

In Table II we pursue the same trajectory with a height cost (10,000) attached to the use of the four control variables, while the weights of the other are kept unchanged.

TABLE I

TRAJECTORY WITH TRADE EQUILIBRIUM (continued)

Controls					
	1972	1973	1974	1975	1976
ΔVNP	−9,350	−6,083	−3,639	−2,029	−2,170
ΔXHA	2.03	1.01	0.3	0.0	0.0
ΔWCA	2,374	1,870	1,107	417	−2,422
ΔWKA	1,843	1,479	818	230	−2,534
ΔWKM	7,675	5,754	3,337	1,681	−293
ΔSCSA	1,796	996	504	154	50
ΔPSA	1,845	1,128	675	352	−25
ΔIIE	2,167	563	2,114	3,471	3,512
ΔIDE	9,073	5,570	5,621	6,002	4,318
ΔIM	−2,663	−1,663	−1,011	−541	−20

TABLE II

TRAJECTORY WITH TRADE EQUILIBRIUM WITHOUT THE USE OF THE CONTROLS XHA, SCSA, PSA, IM

States and Output*					
	1972	1973	1974	1975	1976
ΔAUT	−4,246	5,111	5,340	3,443	5,330
ΔVKENA	−4,749	−2,945	893	2,102	6,081
ΔRDSTAR	5,548	17,835	19,978	26,868	25,410
ΔYQ	3.6	2.4	0.9	0.7	0.8
ΔXQNA	−2.4	0.2	−0.4	−0.2	0.2
ΔWCM	−3,379	12,798	22,099	32,200	33,918
BACO	−247	−156	−241	−1,028	−1,783

Controls					
	1972	1973	1974	1975	1976
ΔVNP	−9,414	−5,992	−3,836	−2,226	−2,381
ΔXHA	0.0	0.0	0.0	0.0	0.0
ΔWCA	2,480	2,068	1,109	320	−2,656
ΔWKA	1,940	1,671	816	129	−2,777
ΔWKM	7,631	5,506	3,585	1,710	−320
ΔSCSA	0	0	0	0	0
ΔPSA	0	0	0	0	0
ΔIIE	2,106	861	2,168	3,796	3,847
ΔIDE	9,200	5,888	5,859	6,296	4,727
ΔIM	0	0	0	0	0

* See footnote Table I.

The elimination of the trade deficit is obtained in each case: in Table I for example in 1972 the trade deficit goes from 15,174 to 249 million francs. The values of the outputs, states and the six controls match up closely in the two tables. The suppression of the action of the four controls thus has little effect on the trajectory.

Comparison between the simulation results from the linear and the nonlinear models shows that errors due to the linear approximation are small, at least over the first three years of the simulation, which correspond to the range of application of STAR. The proposed control system performs well, at least on the present model. The simulation is obtained at reasonable computer expense (one simulation run used ≃ 1.5 minutes of IBM 360/67 CPU time) and it provides important information about the dynamic behavior of the model.

IV. Conclusions

Optimal control is viewed in this paper as a tool for understanding the dynamics of large nonlinear models. For this purpose, control systems do not have to be sophisticated: most important is their ease of application. Such a control system is proposed: its outputs are controlled simulations applied to STAR which are believed to be improvements over present trial and error simulations.

The use of the proposed control is, not limited to the study of the model dynamics. It can save computer time if applied as a preliminary step to nonlinear programming for determining the appropriate values of weighting matrices.

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