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REAL VALUE ADDED AND THE MEASUREMENT OF INDUSTRIAL PRODUCTION

BY STEFANO FENOALTEA*

This article considers measures of production that allow direct comparisons across time and industries. Like value added, it is argued, real value added should measure the value of production (activity and results both, if consistently defined), but by an unchanging standard. It should thus reflect relative prices as well as quantities; the identification of "real" with "thing-like" stems from semantic confusion. While the choice of standard is arbitrary, the value-added deflator should in any case be general, not industry-specific; wage-deflation in particular is heuristically appealing. By these criteria, both orthodox "double-deflation" and own-output-price deflation yield poor measures, which do not in general outperform a simple sum of output series with value added weights; the orthodox index of activity is usually better but relatively expensive. A numerical simulation is included.

I. INTRODUCTION

Industry is activity which transforms material inputs into material outputs.[1] Physical measures of industrial production (weight, volume, etc.) are for many purposes perfectly adequate; but since they are intrinsically heterogeneous, they are ill suited to interindustry comparison and aggregation. This is precisely the breach measures of real value added are designed to fill: all industry is then measured in units of worth ("real *value* added"), as prices convert disparate physical units into a more meaningful common unit; that unit is itself kept constant ("*real* value added"), so that every unit is in fact equivalent to every other; and each stage of industrial production—each transformation of inputs into outputs—is measured separately in net terms ("real value *added*"), so that the resulting figure avoids being swollen by the contribution of previous stages of production. With all fabrication so counted only once, and in homogeneous units, direct comparison and aggregation are in principle perfectly appropriate.[2]

For all this initial straightforwardness, however, the definition of industry's real value added quickly encounters problems. Value itself is ambiguous: even

* This paper owes much to discussions with F. Gerard Adams, John C. Lambelet, Robert A. Pollak, and Stephen A. Ross. Errors, of course, are mine.

[1] I am not here concerned with such further criteria as may define "industry" to exclude production of goods incidental to the sale of services or not for market exchange; these are discussed e.g. in United Nations (1950), p. 10ff.

[2] It may be worth emphasizing that I am here concerned with "real value added" solely as a measure that allows all production to be directly and meaningfully compared or aggregated across industrial or chronological boundaries. As we shall see, "real value added" has also been used to denote measures devoted to quite different purposes; since different *desiderata* imply different criteria, the present discussion of "real value added" in the specified sense is obviously not intended to apply to "real value added" in any other sense. On the other hand, this multiplicity of meanings is clearly unfortunate; and the most fitting meaning of "real value added" seems to be the one it has here. Semantically, this use of the phrase is efficient: as noted in the text, each word in it defines one of the critical features of the desired measure. Historically, this use of the phrase follows its original meaning to the extent that it has one: the concept of "value added" was evolved, not without difficulty, precisely to allow meaningful comparison and aggregation across industries; and it was similarly recognized that intertemporal comparisons required deflating current values into what we call "real" ones (see United States Census Office, 1860; 1870, pp. 377–381; 1880, pp. x, xxiv; 1890, pp. 28–29; 1900, pp. cxxxviii–cxlii; 1910, pp. 22–26).

abstracting from intertemporal or international comparisons, it corresponds to market prices, as is well known, only on a number of highly questionable assumptions. Some of these concern the normative validity of prevailing preferences (such as may result for instance in discrimination by race or sex) or opportunities (as determined for instance by the distribution of income and wealth); and since the relevant ethical judgments are essentially arbitrary market valuations can clearly be rejected out of hand. Secondly, less fundamentally but perhaps more interestingly, market valuations are themselves unequivocal only if markets are perfect; and the empirical measurement of value and value added must often come to grips with a gap between cost and price.

In such currently orthodox treatments of the problem as United Nations (1950) or Hill (1971) this ambiguity is resolved by referring value added specifically to "the results of activity" (industrial production, net output), as distinct from "activity" (industry, input) itself. I will argue, in section II below, that in value terms this distinction is inappropriate, and results from a failure to distinguish between production *stricto sensu*, exaction (public or private taxation or subsidization), and speculation (decision-making with imperfect knowledge). In a world of perfect specialization and exchange, these diverse activities would be performed by separate entities, and each activity would be delimited by the relevant market prices; in the real world, however, the typical firm carries on two or more of these at once, and the prices that delimit each activity may need to be imputed. Once these distinctions are recognized and the corresponding imputations carried out, the formerly troublesome residual is fully allocated, the value of activity coincides with that of its results, and value added is invariant to irrelevant reallocations of activities among firms. All that is left is to decide which activities (and results) are to be included in one's definition of industry; if—as it seems natural to do—we define industry to coincide with production *stricto sensu*, value added will exclude not only materials costs but industry-specific taxes and (non-competitive) surplus, and thus correspond to the (imputed) wage and equipment-rental bill.

The measurement of value added in "real" terms will be considered in Section III. "Real" can be understood to mean either "thing-like" or "constant-worth"; while the latter meaning is here the appropriate one, the literature leans to the former. The literal interpretation of real value added as a thing in its own right proposed by Sims (1969) and Arrow (1974) is in fact irrelevant to the industrial measurement contemplated here; but even orthodox opinion takes it to mean a constant-price aggregate of physical things, so that production is in fact measured in a variety of disparate units. The first purpose of a real value added measure, however, is to reduce *all* production to the *same* unit, so that it is all directly comparable; secondarily, that unit should be empirically easy to obtain, transparent particularly in its arbitrary aspects, and of course intuitively appealing as a (constant) standard of value. On all these grounds, it would appear, the best index of real value added may be a simple deflation of current values added by the current value of common labor. This selection of human effort as the measure of all things boasts ample precedent both in the profession and beyond it; but one might complain that it fails to allow for the secularly increasing value of labor itself. This suggests the alternative of deflating current values added by the price of a fixed basket of goods; but the resulting index is rather more arbitrary than the preceding, and no less biased, albeit in the opposite direction. The goods-price-deflated index is thus not a superior substitute for the wage-deflated index, though it may be a useful adjunct to it.

Section IV considers the performance of alternative indices of real value added in the light of the above standards. The orthodox "double-deflated" quantity index of the results of activity and the corresponding quantity index of activity share the insensitivity of quantity indices to real changes in (relative) values, and the arbitrariness of constructing the component quantity series; their data costs, moreover, are relatively high. Of the two, however, the "activity" index is by far less prone to error: while it is not the best index of real value added, it is at least the quantity index that is closest to it. David (1966) instead proposed to deflate each industry's current value added by the current price of its own output: this index appears too sensitive to relative price changes, rather than not enough; and it is not invariant to subaggregation of industrial categories. Finally, a single physical series such as gross output is often used as an index of real value added: this cheapest of measures in fact often performs better than either the "double-deflated"or the David index, and with suitable disaggregation by production process will coincide with the quantity index of activity. The relative performance of these various indices is further explored, in Section V, through a simple numerical simulation.

II. Value Added: Production, Extraction, and Speculation

Value *added* is a net measure: to permit aggregation without double-counting, as noted, it must exclude what is counted elsewhere.[3] The categories that separate an industry from "elsewhere" make that measure sensitive to some changes and insensitive to others; in a world in which enterprises are not fully specialized, and market relations can readily be abandoned for bureaucratic ones, there is obvious cause to question the established practice of identifying the limits of an industry with those of particular firms.[4] If we are concerned with industry as production rather than as ownership or control, we will hardly want, for example, to register a decline in the power industry (value added producing power) and an increase in the iron industry (value added producing iron) just because an ironmaster decides to suppress the legal personality of his mill's wholly owned source of power. Such purely legal changes would be suitably ignored by a measure which did not deduct the cost of purchased power from value added in iron at all, with the disadvantage, however, that industrial power production could no longer be separately counted; and by a measure which instead recognized in-house power production as a case of vertical integration, and attributed all power production to the power industry independently of corporate organization. As with power, so too of course with any other intermediate good or service; and thus the general conclusion that industry is better measured on the basis of activities than of enterprises.

[3] This is clearly so whether the relevant input is "active" or "passive," whether the production function is somehow separable or not; compare Domar (1961), p. 726n, or Sims (1969), p. 470, and Arrow (1974), p. 4.

[4] See for instance United Nations (1950), p. 51. To be sure, the data are somewhat easier to collect if the basic administrative unit is not asked to make distinctions within itself.

To this conclusion one may object that "industrial production . . . refers not to activity as such but to the results of activity".[5] In *value* terms, however—recognizing that activities are valued by their results—such a distinction appears suspect. In point of fact, it can be argued, the foremost gap between "the value of activity" and "the value of its results"—the firm's surplus—is an illusion that stems precisely from the failure to recognize the pervasiveness of integration and the multiplicity of diverse activities carried out by the typical real-world firm. To an extent, as we shall see, the analysis carries over to the common distinction between value added "at factor cost" and "at market prices" created by public intervention; initially, however, we shall assume such intervention away, the better to focus on the significance of the firm's surplus.

Speculative Surplus

Assume, then, an economy without taxes or public subsidies. In long-run competitive equilibrium, surplus is zero, and value added can be indifferently measured as the value of activity or as the value of its results. In short-run competitive equilibrium, however, the firm's surplus is not zero: "rents resulting from dynamic disequilibrium" destroy the identity of the value of activity and of its results.[6] Such at least is the orthodox view of our textbooks; but it reflects the common assumption that the firm owns the capital goods whose services it utilizes. Clearly, such a firm is vertically integrated, in that it combines the role of industrial producer with that of speculator in its capital goods. It is the speculator, and not the producer, that earns the short-run (positive or negative) surplus: the firm engaged *solely* in production rents the equipment it uses on a spot basis, and would not reap such surplus; the firm that owns the machines (or rents them long-term) would reap the surplus even if it chose to (spot) rent them out rather than use them itself. Once the services of capital goods are valued at their current price rather than their historical price ("fixed cost"), short-run profits and losses disappear; the value of activity corresponds to the value of its results in the short run as in the long. Assuming $Q = Q(K, L, R)$, where Q, K, L, and R represent output, equipment, labor, and raw materials, and letting p, r, w, and z represent their unit values, we note that as an empirical matter value added is correctly measured by the conventional value measure of the results of activity $(pQ - zR)$, rather than the conventional value measure of activity itself $(wL + rK)$. The reason is that the difference in these conventional measures is rooted in the improper valuation of K by "historical r" rather than "current r", whereas p and z are instead the correct current values; if K is correctly valued, the two measures coincide.[7]

The preceding analysis assumes that the firm knows the demand function it faces. In fact, of course, firms often produce with uncertain knowledge of that

[5] United Nations (1950), pp. 5–6. This distinction is at the root of current concepts of value added, particularly in real terms; see for instance United Nations (1968), p. 69, or Hill (1971), p. 12ff. The point is returned to below.

[6] Kendrick (1958), p. 406.

[7] A long-term labor contract would of course be "speculation" in labor, creating exactly analogous problems of replacing the "fixed" wage that is paid out by the current value of labor (the wage that would be paid if labor were purchased on a spot basis).

demand; and when the product is sold the firm may find it has earned more or less than a normal return.[8] Such firms are also speculators, albeit in their output rather than in their input; and once again it is the speculator, and not the "pure producer" (who works only to the speculator's order) who earns that surplus. If the speculator and the producer were different (legal) persons, the value of the producer's output would be unambiguously given by the speculator's payment, whatever the latter's subsequent receipt; and (assuming current competitive pricing of both inputs and output) the value of the results of the *producer's* activity corresponds, once again, to the value of the activity itself. When producer and speculator are *not* different persons, however, the conventional value measures of activity and of its results again fail to coincide; but it is now the former, $(wL + rK)$, which correctly measures value added in production strictly defined (assuming, of course, that equipment is rented on a spot basis). The latter, $(pQ - zR)$, includes the results of speculation, in that the recorded p is the price *received*, rather than *paid*, by the speculator; if Q is correctly valued, of course, the two measures coincide.[9]

It may be pointed out that the value measure of production strictly defined—$(wL + rK)$ or $(pQ - zR)$, with the appropriate unit values as understood above—is necessarily nonnegative. The value added by the speculator's decision as to what production will take place, in contrast, is essentially unconstrained; and if the decision-maker's guesses are badly wrong, he may lose more than the value added by the producer who carries out his order. The speculator, for example, may order a machine that fails totally to perform its intended function, and is suitable only for demolition; it is thus not implausibly worth less than the materials consumed by the machine's producer. In such circumstances, the measure of the industry defined to include such speculation as well as production *stricte* is quite properly negative; but it bears repeating that such negative value added is the sum of a positive figure measuring production as such and an absolutely larger negative figure measuring speculation—and that such speculation may be performed by the producing firm or by anyone else.[10] Supervision costs being what they are, in fact, workers are not just their employer's *longa manus*, but share a measure of decision-making power. A common enough destroyer of value is thus the worker himself, who for instance critically damages a part as he installs it; but he destroys value in his (limited) capacity to decide what will be done—as his employer's agent, one might say—rather than as strict executor of decisions already taken. Risky decision-making—speculation—may thus be suffused throughout the firm, as well as concentrated at its top or altogether outside it; its value, positive or negative as it may be, is in any case conceptually distinct from the necessarily positive value of execution, i.e. of production strictly defined.

[8] I am here concerned specifically with production "on spec.", i.e. in the absence of orders. Uncertainty can also lead the firm to hold inventories; but in this case the imputation of a value to the firm's entire output (as if additions to inventory were sold from the plant to the warehouse) is already standard practice.

[9] If existing markets generate neither the correct p nor the correct r, these could not be directly calculated, but would have to be estimated. The conventional measure of the value of the results of activity $(pQ - zR)$ of course coincides with the value of the activity—strictly productive *and speculative*—that produces those results.

[10] Strictly speaking, final value should be measured by what the informed consumer would be willing to pay; the consumer disappointed by his purchase is clearly a speculator.

Non-competitive Behavior

To be sure, a gap between a good's cost (properly calculated at current values) and the price at which it is sold need not stem from uncertainty and speculation. Monopoly is an obvious alternative source of surplus; but it may be enlightening to consider first the firm that knowingly chooses to earn a *less* than normal return. The good's low market price measures its value to the buyer; but its "true" value includes the (external) benefit to the seller which justifies the decision to produce at a loss in the first place. That benefit may be purely psychic, a case of industrial Maecenatism,[11] or it may be the prospect of future profits; in any event, the phenomenon seems best assimilated yet again to the paradigm of vertical integration. Imagine that actual production is subcontracted (through competitive bidding) to an ordinary firm: the value of production is clearly given by the price paid, rather than received, by the firm that lets the contract. Once again, the value of production strictly defined is measured by the conventional measure of activity, $(wL+rK)$, assuming a proper r; the conventional measure of the results of activity, $(pQ-zR)$, understates the appropriate p by the amount of the unit subsidy.

In the presence of such departures from ordinary competitive behavior, the value of production *stricte* is properly measured at the market price plus the private subsidy. Since taxes are negative subsidies, one could argue that in strict analogy private taxes should be subtracted from the firm's income; the value of production would thus remain $(wL+rK)$, corresponding to $(pQ-zR)$ at a p exclusive of monopoly profits. The analogy might seem forced, to the extent that individual decision-makers are not normally free to levy private taxes as they are to disburse private subsidies; but the argument can be made by another route.

The basic point, once again, is that while the producer and the monopolizer are often the same legal person, they need not be; and it is of course the monopolizer who earns the monopoly profits. An industrial firm may thus earn a monopoly rent because it alone has the right to use a particular technology; it would retain that rent if it abandoned production and leased its patent rights, while the firm that paid for those rights would earn only a normal return. In the absence of artificial restraints on the right to enter into contractual agreements, indeed, monopoly profits would not accrue to producers at all, but to the true monopolizers: public officials empowered to grant patent rights, concessions, and the like—or private "enforcers" who discourage competition—would be paid the value of their service; and where public or private coercion cannot be enlisted a monopoly would be maintained only by paying off potential competitors.[12] As in the case of private subsidies, the value of production *stricte* is measured by $(wL+rK)$; the market value of the good, p, and thus $(pQ-zR)$, will reflect the value of exaction as well.

Within the industrial firm, the monopolizer may be not only the capitalist but the labor force, suitably organized. On the logic outlined above, it would then be

[11] The Lamborghini automobile concern was reputedly a case in point. Such psychic income is a form of consumption, logically value-subtracting as production is value-adding.

[12] Nineteenth-century transport history provides a rich catalog of bribes both to public officials and to potential competitors (who at times actually set up a "nuisance" company to establish their credibility); private "enforcers" are of course still with us.

appropriate to distinguish between the workers' "normal" income, which they earn as workers and is thus to be included in the value of production, and their "excess" earnings as monopolizers, which are instead to be excluded from that figure; the distinction would of course be particularly obvious if the monopoly surplus were captured not by inflating w but as a flat fee for the permission to hire at all. By the same token, labor that accepts a substandard payment for given skills and exertion should be attributed its full value, and the firm's product would be correspondingly inflated by the implicit subsidy.[13]

Public Taxes and Subsidies

Public taxes and subsidies create measurement problems largely analogous to those attributable to vertical integration in all its forms: the legal tax base is "arbitrary," i.e. determined by administrative convenience, and may be "arbitrarily" altered. A tax on output may for instance be replaced by taxes on the producer's purchases of inputs, or even by direct taxes on the inputs' incomes;[14] and such administrative changes may have no impact at all on actual production, though market prices would vary in response to shifts in the legal tax base. If our measure of production is to be unaffected by such purely administrative changes, its inclusion or exclusion of taxes must be determined *a priori*, independently of their legal base, exactly the way the production of power is to be handled independently of the legal identity of the producer. It is tempting, pursuing that analogy, to conclude that all taxes should be excluded from the measured value of production: the latter would thus correspond to $(wL + rK)$, with w and r themselves net of direct or indirect tax. The arbitrariness of the tax base would then no longer be a problem; what remains problematic, however, is the possibly arbitrary choice between public and private provision of final goods and services. Administrative convenience, for instance, may induce the members of a town to municipalize their country club, and replace the fees by equivalent taxes; and one would presumably *not* want to register a decline in industrial production as a result of a purely legal change of this particular sort.

The proper treatment of taxes thus apparently depends on a prior determination of their economic rationale: an unwelcome conclusion, surely, whose thrust may however be limited by two rather more comforting considerations. The first is that taxes which are uniform with respect to the net compensation of the primary factors of production (L and K) can of course be treated arbitrarily, since value added *relatives*—which are after all what we are interested in—will be the same whether such taxes are included or not;[15] the second is that specific departures

[13] We might imagine youths willing to work in a sports arena at a reduced wage for the pleasure of contact with the teams: their "psychic income" would be part of their wage and of the firm's product. The logic of imputing a wage to housewives' services is of course analogous.

[14] The various notions of "value added" currently accepted differ in their treatment of indirect taxes only; the questions raised by direct taxes are not broached. See for instance United Nations (1953), p. 8, United Nations (1968), p. 230ff., and Hill (1971), pp. 12–13.

[15] A flat-rate income or value-added tax is a typical example of such a uniform tax; a turnover tax would be another only in long-run institutional equilibrium, with all production vertically integrated to minimize the tax burden. In the case of a flat-rate sales tax, it would be simpler to exclude the tax from the value added of the final stage of production than to distribute it over all the stages of production. The argument that uniform taxes can be so treated assumes that public consumers' goods are consumed in proportion to income, so the mix of tax payments for consumers' goods and other payments does not vary across industries.

from uniform rates (whether differences over time or space, or, most simply, between industries) will generally be less difficult to evaluate than the tax bill as a whole. If there is evidence that an industry-specific tax is in fact returned as consumption goods specific to that industry's primary factors, that tax will be included in the value of that industry's production (which thus remains unaffected by changes between private and public financing of that consumption); normally, however, that will not be the case, and such taxes will be excluded from the value of production strictly defined on the grounds that the primary factors' value and contribution correspond to the price *net* of tax, the further increase in price being value added by the government itself.[16] An industry-specific subsidy (perhaps in the guise of a lower-than-general tax) is by the same token normally included, typically as a correction to *p* quite analogous to the case of private subsidization discussed above.

● A proper value measure of industrial production thus measures both the value of activity and the value of its results; and it is insensitive to such changes as the substitution of intrafirm relations for interfirm ones or the substitution of one tax base for another, though not of course to the possible changes in production that *may* result from them. The value measure of industrial production strictly defined will include competitive profits, since these reflect the current value of the fixed inputs; but it will exclude the profits and include the losses of risky production decisions (speculation), as it will exclude the profits of private exaction (monopoly) and include private subsidies. Differential taxes will normally be excluded, and subsidies included, whether they are "direct" or "indirect"; but uniform taxes may be included as a practical expedient.

III. Real Value Added: Criteria for Measurement

The direct comparison and aggregation of industrial production will be meaningful if the latter is uniformly measured, on a net basis, in a common unit of value. So long as all one's observations are contemporaneous, monetary magnitudes are homogeneous, and current-price value measures of industrial production are perfectly satisfactory for the purposes at hand. From one time period to another, on the other hand, monetary magnitudes are *not* meaningfully homogeneous; intertemporal comparisons or aggregation thus require that the variable value units of the current-price measures be replaced by their equivalents in units of constant, unchanging worth.[17] In the jargon, we speak of deflating (current) value added into real value added; and it is clear from the purpose of the operation that "real" has here not its literal, everyday meaning of "thing-like" but

[16] The government may then be acting "in its own right" (as in the case of so-called (de)merit wants), or as agent for some other sector which is suffering external effects; when the tax is *not* excluded, the government is the agent of the industry's own primary factors.

[17] Measurement problems are created by space as well as by time, but the two are generally considered equivalent (Arrow, 1974, p. 3). Intranational space is typically ignored; and if different nations were effectively grouped in a single market (Benelux, perhaps) there would presumably be no objection to converting own-currency values at the prevailing exchange rates the way we implicitly convert New England dollars into California dollars. Can we argue, by analogy, that if *intertemporal* markets were perfect, expectations realized, etc., we should be willing to make intertemporal comparisons directly at discounted current values? Should "real" comparisons over time discount for time as well as for inflation? Current practice is against it; but the question deserves more attention than it has so far received.

the precise technical meaning of "constant-worth." To be sure, there is between the common and the technical meanings of the word a point of contact, which marks the empirical context within which the relevant technical concept was elaborated and the common word transformed into jargon. As is well known, the distinction between (current) values and "real" values gained currency in the context of inflation: in pure or extreme inflation, changes in relative prices are negligible next to changes in absolute prices; paper money loses its value, things keep theirs, and "thing-like" corresponds to "constant-worth." Out of that context, on the other hand, "thing-like" need not correspond to "constant-worth" at all. Imagine for instance that the absolute price of every good remains constant, with the exception of one which declines to a fraction of what it was: this one thing is exactly like paper money in the case of inflation, losing value while everything else (paper money included, in the present case) retains it; it is by definition "real" in the common sense of "thing-like," but it is not "real" at all in the technical sense of "constant-worth." In the case of "real" as in the analogous one of "rent," the word's technical sense does not simply coincide with its common one; but whereas the technical understanding of "rent" is no longer corrupted by the common meaning of the term, "real" has yet to be stripped of alien connotations. The *res* in "real" casts a long shadow over the literature: the common approach is to seek not an unchanging unit of value in which to measure all industry, but the physical counterpart of value added in each industry; and the prevailing disagreements are only over its appropriate definition.

Physical Value Added

The most rigorously literal interpretation of "real" value added is that espoused by Sims (1969) and Arrow (1974). The notion of real value added makes sense, they assert, only if the production function is separable in a particular way, so that $Q = Q(K, L, R)$ can be rewritten $Q = Q(V(K, L), R)$: only in such a case can we "imagine capital and labor cooperating to produce an intermediate good, real value added (V), which in turn cooperates with materials to produce the final product[;] in other cases capital and materials may seem to be a more natural aggregate" than capital and labor.[18] From the traditional perspective of industrial measurement, however, the shape of the production function is irrelevant: "real" does not mean literally thing-like; "value added" includes capital and labor but not materials costs in order to count all production once and only once, and a different treatment of these inputs would be incongruous. In fact, Sims and Arrow are not concerned with such industrial measurement at all, but with the econometrics of production functions. "Analysis of production relations is simpler if we can restrict ourselves to looking at two inputs at a time";[19] but should one estimate a production function linking real value added to capital and labor, they ask, if such a function—$V(K, L)$—need not in fact exist? This question, however, corresponds to "Does the notion of real value added make any sense?"[20] only on a

[18] Arrow (1974), pp. 4–5; see also Sims (1969), p. 470.
[19] Sims (1969), p. 470. As an empirical matter, of course, K or L need not be a single legitimate "thing" any more than V is; the production function $Q(K, L, R)$ need not involve noticeably fewer special assumptions than $V(K, L)$ does.
[20] Sims (1969), p. 471; similarly Arrow (1974), p. 5.

literal but unusual definition of "real value added" as a thing in its own right; and it has little to do with the meaning of "real value added" in the context which coined the phrase in the first place.[21]

This tendency to consider "real" as somehow immediately thing-like is, however, shared by the literature concerned directly with the measurement of industry: "real" value added is there generally identified, if not with a thing in itself, at least with a constant-value aggregate of things. The standard measure of real value added, wholly analogous to "real" national income of which it is in fact a disaggregation,[22] is the difference between the quantity of output and the quantity of intermediate inputs, each measured at base-year prices; the appropriateness of this definition is usually considered self-evident, or nearly so.[23] David (1966) proposes instead to identify an industry's real value added with the current-price equivalent of value added in its own output, the resulting physical series being then weighted by base-year prices. These measures, which will be considered more fully below, are ordinary quantity indices, heir to the familiar "index-number problem": intertemporal real value added relatives will not in general be invariant to the choice of base year, and intratemporal relatives will not correspond to their current-price equivalents. This index-number problem is typically considered a fact of life, unwelcome perhaps but essentially unavoidable; at best, it is kept within bounds by repeatedly changing the base year and thus recalibrating the index, quite the way one might limit the error of a slow timepiece by repeatedly resetting it.[24] In fact, however, the index-number problem appears to be evidence of the fundamental inappropriateness of the usual constant-price aggregates of things as measures of real value added; and it is symptomatic that the index-number problem should vanish when relative prices remain constant, i.e. when "thing-like" happens to coincide with "constant-worth."

Within a time period, it is agreed, industries can be compared directly in current terms: the monetary unit is then a uniform measure of value, and (with the qualifications noted above) current-price values added are themselves "real" in the relevant sense. In the common perspective, this fact warrants the identification of current- and constant-price values added in the base year, and the recognition that any year can be selected as the base; but this does not quite do it justice. If within-period current-price value-added relatives are real, first of all, then changes in these relatives are also real, whether they are due to differential changes in physical output flows, in physical primary input flows per unit of output, or in the relative value of the primary inputs. Quite so: industry-specific improvements in primary factor productivity or reductions in primary factor values reduce the industry—activity and result both—just as effectively as a reduction in output does; all these influences, then, are relevant to a proper measure of real value added. If within-period current-price value-added relatives

[21] Oddly, Arrow begins his discussion of real value added by reviewing the justification of "value added" in current terms and the problem of deflating it into "an invariable standard of value," only to switch to Sims' problem with the argument that "the most natural meaning [of real value added], indeed the only one I can think of, arises from the estimation of production functions"; see Arrow (1974), pp. 3–4.

[22] See for instance Fabricant (1940), p. 26, and United Nations (1968), p. 67ff.

[23] See for instance Hill (1971), pp. 13–14 and Arrow (1974), p. 4. This definition of course also reflects the distinction between "activity" and "the results of activity" discussed above.

[24] See for instance Fabricant (1940), pp. 33–34, Sims (1969), n. 2, and Arrow (1974), p. 4.

are real, secondly, we need a further measure of "real" value added only in order to make intertemporal comparisons. Letting A and B represent industries and the subscript represent time, we have the desired real (A_1/B_1) and (A_2/B_2) by direct observation of current-price values added; what we seek are equally real intertemporal ratios such as (A_1/A_2). Given (A_1/B_1) and (A_2/B_2), of course, a single such ratio—say (A_1/A_2)—will let us calculate all other ones: for instance, $(A_1/B_2) = (A_1/A_2) \cdot (A_2/B_2)$; $(B_1/B_2) = (B_1/A_1) \cdot (A_1/B_2)$. In the same way, given the intratemporal interindustry relatives in "the base year" and the intertemporal intraindustry relatives given by our industry-specific real value added indices, we can calculate intratemporal interindustry relatives in other years: obtaining (A_2/B_2), say, from (A_1/B_1), (A_1/A_2) and (B_1/B_2). Now with the usual sort of real value added index, such calculated relatives do not in general coincide with directly observed ones; but if the observed (A_1/B_1) and (A_2/B_2) are correct real measures, then this "index-number problem" means that the calculated (A_2/B_2) is wrong, and wrong because (A_1/A_2) or (B_1/B_2) is wrong. These indices are wrong, i.e. not "real" in the proper sense of "constant-worth," precisely because they are ordinary quantity indices: "real" magnitudes being thus identified with industry-specific physical ones—current values added being thus deflated by industry-specific price indices—production is clearly measured in a variety of disparate units, and a common ("real value") measure is not achieved at all. Of course, if relative prices remain constant the index-number problem disappears: all physical units are then equivalent to each other at unchanged rates, and all industry-specific price indices are identical, so all industry is effectively measured in the same unit after all. In the context of pure inflation, "real" can be taken literally; but then, as we have seen, if that were not the case this misleading word would not here be used at all.

The Standard of Real Value

The essential objective of the desired real value added measure is to render *all* industrial production directly comparable, regardless of differences in time (or space) or technique, by expressing it in the same, unchanging unit of value. The first requisite of such a measure is thus that different industries be measured in the *same* unit; the current-price values added of different industries are accordingly to be deflated by the *same* price index. Whatever that price index may be, the mere fact of using a single, common deflator will ensure that calculated intraperiod relatives correspond to their (already "real") current-price equivalents; by the same token, each "real" time series will properly reflect all the relevant influences on relative industry size, including differential changes in the remuneration of primary factors and in their use per unit of output as well as in output itself.[25]

On the other hand, no specific deflator stands out as *the* theoretically correct one: there is no all-purpose standard of value, and no particular standard is defined by the desire to construct meaningful intertemporal comparisons of

[25] There is some recognition in the literature that "real" measures should reflect relative price as well as quantity, and thus that own-price deflation is inappropriate: in discussing capital gains on a given stock of goods, for instance, Fabricant noted the triviality of deflation by the own price of the good, and suggested deflation by "a general price index" instead; see Fabricant (1958), p. 444ff.

industrial production. In practice, then, an arbitrary choice is inevitable; but it may properly be guided by the reasonable desire to obtain a measure that is intuitively appealing, easy to grasp, and easy to construct. On these grounds, the best deflator would seem to be the current price of some particular recognizable thing (or aggregate of things): all production would then be measured in that one particular thing, considered "real"—an unchanging standard of value—by assumption; no *other* thing would be "real" in and of itself, since as relative prices changed it would be worth a varying amount of "real" units. The choice of a thing as the "real unit," to repeat, is *not* dictated by the conceptual requirement that a "real" measure be somehow a measure of or in *res*; it is warranted only as a practical expedient and so long as available alternatives appear to cost more in terms of calculation, interpretation, or communication than they are worth in terms of intuitive appeal.[26]

Of all the things that may thus be taken as the standard of value, perhaps the best is labor, in the specific sense of ordinary physical effort (whose price thus excludes the return to human capital or compensation for particularly painful working conditions). If such wage-deflation is arbitrary, its arbitrariness is wholly manifest, since the index could not be more simply conceived; and when current-price value added series are already available (and acceptable as at least approximately correct in terms of the considerations developed in Section II above), at least, its construction requires only a single additional price series which is in general readily available. Most importantly, perhaps, the choice of human effort as the measure of all things has demonstrated its wide intuitive appeal over a long (if not always entirely clear-headed) tradition that runs from the *Wealth of Nations* through the *General Theory* and beyond; and precisely because it is both straightforward and appealing it is ideally suited to communication within a broad intellectual community.[27]

The contemporary tendency, at least within the profession, is to attribute constancy of worth not to human labor but to goods. From this perspective, wage-deflation underestimates "real" growth because it neglects the growth of "real wages," and current values added should be converted into a material good rather than into labor. The difficulty, however, is that no single product is obviously preferable to any other: however appropriate deflation by corn-prices may have seemed in centuries past, here and now it would hardly command widespread assent. Most would prefer to specify an arbitrary composite good (a fixed basket of goods, conceptually equivalent to a single thing); but the costs of index construction and communication are thereby increased. A composite good can be priced only by tracing the price of all its components; where price series are not readily available, there may be a significant extra cost involved in tracing the prices of many goods rather than the wage alone. Moreover, one could hardly expect scholars working in different areas or time periods to choose the same

[26] In contrast, a complex but theoretically correct cost of living index can be constructed on the assumption that preference functions are homothetic; see for instance Pollak (1971) and Samuelson and Swamy (1974). As an empirical matter, of course, homotheticity is difficult to accept, since budget shares vary significantly with income; and this index may be considered an example of a costly construction that does not pay for itself in increased intuitive appeal.

[27] Hilton (1966), pp. 4–5, for instance, provides his readers with thirteenth-century wage rates as the best basis on which to understand modern equivalents of medieval values.

standard composite good; and since the implications of altering that standard would not be immediately apparent, the resulting indices could be shared and compared only with some difficulty.

In the case of contemporary economies amply documented by their statistical bureaus, of course, one might simply deflate current values added by some official index of prices. Among these, the most attractive would appear to be the GNP deflator: since industries would then be measured in "real" terms essentially by distributing "real GNP" among industries in proportion to their share of GNP at current prices, "real values added" would happily sum to "real GNP".[28] If "real GNP" is calculated as a Laspeyres quantity index, however, the GNP deflator is a Paasche price index; and this creates difficulties of interpretation, since we can no longer speak of the deflator as the price of a *given* composite good. Intertemporal comparisons (other than to the base year) would be among industries reduced to different physical equivalents; and it is hard to see on what grounds these physically different composites could be considered of equal worth.[29] A Laspeyres price index is not heir to this difficulty (since it keeps the same quantity weights year after year, the good assumed to be of constant worth is simply the base-year mix of final goods and services); in practice, however, GNP deflators are rarely of the Laspeyres type.[30]

On these grounds, then, deflation by the price of a (single or composite) final good seems inferior to wage deflation; and its superiority in terms of the principal criterion of intuitive appeal is certainly moot. Granting that rising living standards make labor worth more, there is nonetheless every reason to believe that increasing material abundance makes goods themselves worth less;[31] indeed, those who espouse a strict "relative-income" conception of human welfare would deny that generally shared increases in purchasing power yield any benefits at all.[32] Most probably, then, a goods-price-deflated index is to be considered biased upward with respect to the intuitively "best" index, just as a wage-deflated index is biased downward. While there is thus little reason to pursue the former *in place of* the latter (particularly in view of their relative costs), there is the usual excellent reason to pursue them both.

● As a measure of production that permits direct intertemporal and interindustrial comparisons, then, real value added is neither a thing in its own right nor a constant-price aggregate of things; it is, rather, a measure of all production in common, unchanging units of value. Such a measure can be obtained only by deflating all current-price values added by the *same* price index; while the choice among possible deflators is arbitrary, the price of common labor stands out by

[28] "Real GNP" could of course be made to coincide with the aggregate of "real values added" by deflating current GNP by the chosen deflator of current values added, be that an index of wages, corn prices, or whatever. On the other hand, the currently orthodox notion of "real GNP" is not heir to the problems—specific to *dis*aggregated measures—that plague the currently orthodox notions of "real value added"; and it does seem intellectually and bureaucratically entrenched.

[29] See for instance Phillips (1961), p. 320, and David (1962), p. 150n.

[30] See for instance Hill (1971), p. 16. In the case of international comparisons, furthermore, even Laspeyres-type GNP deflators would have different quantity weights.

[31] The *a priori* argument is made for instance in Lerner (1944), pp. 26–27; its empirical validity is confirmed by the widespread desire to smooth consumption even at the cost of postponing and reducing it (as in the case of saving for one's retirement despite negative rates of return).

[32] See for instance Easterlin (1974).

virtue of its conceptual simplicity, statistical cheapness, and wide intuitive appeal. To the extent that wage-deflation understates growth by failing to allow for increases in the "real" wage, one may wish to deflate current values added both by the wage rate and by the price of a representative basket of goods; between them, these indices straddle the measure most would consider intuitively correct.

IV. Evaluation of Alternative Indices

This proposed index of real value added defines a standard of cost and performance to which other indices may usefully be compared. From this novel perspective I shall here consider, in order, the currently orthodox measure of real value added (the "double-deflated" index) and the corresponding measure of activity; the "own-output-price deflated" index proposed by David (1966); and the simple (base-year value added weighted) "output" index that is a common *pis aller* when the data are too poor to permit more refined calculations. This general discussion will be followed by a more systematic appraisal of the comparative performance of alternative indices under particular simplifying assumptions.

Orthodox Measures of Real Value Added and of Activity

The too literal interpretation of "real" as thing-like has led, as we have seen, to the identification of "real" indices with ordinary quantity indices. "Real value added" is no exception, and its currently orthodox measure (in its usual Laspeyres form) is $(p_oQ_t - z_oR_t)$, where the subscripts o and t refer respectively to base and current periods. This index is taken to measure the real results of activity; real activity itself is measured, analogously, by $(w_oL_t + r_oK_t)$. As noted in section II, however, the current value measures of activity and of its results will coincide if they are consistently defined; and for the reasons developed in section III the corresponding "real" value measures should coincide as well. *A priori*, then, industrial production should be indifferently measurable by the real index of activity or of its results;[33] the fact that on their usual definitions these theoretically indifferent measures do not in general coincide is the premier indication that those definitions are in fact inappropriate.[34]

Even a conceptually inappropriate measure may of course be warranted by purely practical considerations; but these also argue against the orthodox quantity indices. First of all, these quantity indices will tend to cost more, not less, than the "best" measures described above. Where acceptable estimates of current values added are published as such or can be derived directly from the data, deflation by a

[33] Hill (1971), pp. 13–14, argues that real value added cannot be measured as real activity because value added includes the operating surplus for which there is no quantity unit. This argument is to be rejected on the variety of grounds noted above: because the usual competitive surplus is actually part of the value of the activity of capital equipment; because other types of surplus correspond to other types of (primary factor) activity which are (a) best excluded from a strict definition of industry and (b) conceptually no different from many other types of services (what is the quantity unit corresponding to the activity of the individual paid to be named on a "cars will be towed by x" sign, whose major purpose is deterrent?); and most of all because "real" does not refer to quantities as such at all.

[34] The difference between these measures is ordinarily used to estimate productivity change. Production functions, of course, do deal with things rather than values; and as we have seen they are best examined with a minimum of such *a priori* constraints as are implied by aggregation of any kind. See above, n. 19, and United Nations (1968), p. 66ff.

single wage or price index will be relatively straightforward; data on physical flows will be much more difficult to obtain. This is particularly true of the "activity" index, since historical information on physical capital is regularly available only in special cases (transportation and textiles, in the main); but even the "results" index faces problems, as materials input data are also rarely recorded. These are thus typically estimated from current production and net imports; but these "availabilities" neglect the at times significant changes in inventories. Secondly, these quantity indices will tend to be arbitrary: not just in the choice of base year, which is as overt and relatively harmless as the arbitrary choice of deflator for the proposed "best" measure, but in the very component series. Common deflation by an index of the price of a homogeneous good takes quality distinctions (in all other goods) in stride, since these are properly handled by relative prices; a quantity index is instead plagued by the need to reduce the number of measured goods to a manageable figure, and it does so by ignoring quality distinctions (including quality changes over time). This practically inevitable subaggregation is guided by no clear criteria; different groupings would give different results, and much arbitrariness which in theory should be manifest as the problem of weighting a multiplicity of (homogeneous) time series will instead be buried in the arbitrariness of the (heterogeneous) series themselves.[35] Once again, it is the "activity" measure which is particularly exposed to such difficulties: many industries consume and produce homogeneous goods, though many more do not; but what industry uses only a single type of skill or (especially) equipment at any point in time, let alone over years or decades?

The orthodox index of real value added is thus in general more costly and arbitrary than the "best" index proposed above, but less so than the orthodox index of activity; on the other hand, it traces real value added (properly understood) less well even than the index of activity. While the index of real value added should not be a quantity index at all, that is to say, among quantity indices real value added is better represented by the measure of activity than by the measure of its results. The "double-deflated" index is most often criticized for being able to produce negative real value added estimates.[36] in fact, as we have seen, value-*subtracting* activity is just as real as value-adding activity, and if our definition of "industry" is broad enough to include it, our measure of real value added *should* be negative whenever current-price value added is. The only paradox, as Fabricant correctly pointed out,[37] is the appearance of negative real value added estimates *in the absence* of negative current-price values added; and this oddity the "double-deflated" index is indeed heir to, because it is a quantity index with negative weights.[38] In general, of course, this is just another indication that quantities are not "real"; in particular, it is a measure of the extreme indirectness with which the "double-deflated" index traces real value added, and thus of the likelihood that it will in fact do so very poorly. The quantity index of activity, in

[35] Such buried arbitrariness is particularly pernicious in view of the widespread tendency to accept available series at face value, without troubling to examine the extent to which they really are what they purport to be.

[36] See for instance Fabricant (1940), p. 28; David (1962); and Arrow (1974), p. 4.

[37] Fabricant (1940), *loc. cit.*

[38] As is well known, $(p_oQ_t - z_oR_t) < 0$ may obtain even though $(p_oQ_o - z_oR_o) > 0$ and $(p_tQ_t - z_tR_t) > 0$ if $(z_o/p_o) > (z_t/p_t)$ or $(R_o/Q_o) < (R_t/Q_t)$; it is of course most likely to occur if value added is small in comparison to value. See most recently Hill (1971), pp. 15–19.

contrast, is altogether less indirectly related to real value added, and thus likely to perform significantly better.

As noted above, changes in real value added—the real value of activity and of its results—can usefully be attributed to changes in output, or in real value added per unit of output; the latter, in turn, will reflect changes in technique (i.e. in primary factor activity per unit of output) and in relative prices (i.e. in the real value of a unit of primary factor activity). The "double-deflated" index ($p_oQ_t - z_oR_t$) is clearly sensitive to changes in Q, but measures changes in real value added per unit of output only by changes in R/Q; and these have no necessary relation to the relevant variables at all. Assuming for simplicity a constant Q, it is easy to see that changes in R would have a generally correct influence on the index if primary inputs are being substituted for materials, with unchanging technology; or even in the presence of technical progress, if primary inputs are industry-specific and so scarce that the reduction in materials cost per unit of output simply raises their relative price. In general, of course, technology changes, and primary factors are not all (or long) in fixed supply. R, L, and K may thus decline together (given Q), with little change in real factor prices (or R and real factor prices may decline, with little change in L and K); in such cases, the "double-deflated" index registers a wholly spurious *rise* in real value added per unit. This measure is thus prone to overwhelming error; and since technical progress is industry-specific, that error will also be industry-specific, with the result that calculated real values will be totally unsuited to interindustry as well as intertemporal comparison.

The "activity" index, in contrast, measures physical activity directly, and cannot be distorted by changes in input–output ratios; it will err only in neglecting changes in real factor prices. Because it errs, the "activity" index may also rise when real value added declines: for instance, if demand is very inelastic, the expansion of capacity may so reduce its real value that on balance the real value of activity is declining even though physical activity is increasing. On balance, however, the orthodox "activity" index is clearly exposed to far fewer sources of error than the "double-deflated" index is. In contrast to goods and techniques, moreover, primary factors are far less industry-specific, particularly since crude labor costs are the largest single component of value added;[39] the errors of the "activity" index are thus not only generally smaller than those of the "double-deflated" index but far less severe in their differential impact and thus in the distortion of interindustry comparisons. At the limit, of course, we can imagine a world in which only one factor of production, homogeneous labor, transforms a variety of inputs into a variety of outputs. The "activity" index then measures all industry in the *same* physical unit, and is a perfectly correct index of real value added (obviously identical to the wage-deflated index proposed above) no matter how production functions may be shaped or changing. The "double-deflated" index would instead remain unreliable; since it is in essence an improper way of disaggregating a not unreasonable aggregate,[40] it can only be expected to behave correctly if the differences between the aggregate and the components are essentially eliminated—i.e. if the problem of industrial measurement it was designed to solve is effectively trivialized.

[39] Any tendency to "long-run equilibrium," which reduces the cost of specific capital goods to the cost of capital in general, would reinforce this point.

[40] See above, n. 28.

Own-output-price Deflated Index

David (1966) proposes to measure real value added by deflating each industry's current-price value added by the current price of its output, and aggregating the resulting physical series at base-year output prices.[41] The resulting index is in a number of respects a clear improvement over the orthodox indices just discussed; in others, it is essentially similar; and in at least one it seems clearly inferior.

Its principal advantage over the conventional indices is apparent at the level of the single industry.[42] At that level, the David index avoids the "index-number problem" altogether, since current value added is converted into a single thing (the industry's own output);[43] by this simple device, David exorcises the "unfamiliar and rather harrowing index number problem [of the 'double-deflated' index]—one which manifests itself in the appearance of negative real value added estimates".[44] More importantly, perhaps, David's straightforward deflation of current values added by a current price marks a long step in the right direction. The distinction between activity and its results is obliterated: at least in competitive equilibrium,[45] it is made clear, real value added represents *both* in real terms just as current value added does so in current ones. By the same token, the David index is sensitive to all the changes that affect current (and therefore real) value added, including changes in the unit values or productivity of labor and equipment; it is thus capable of mirroring developments that the orthodox "real" measures of activity or of its results should but cannot mirror at all. Lastly but by no means negligibly, of course, the David index can in general be obtained as (or nearly as) cheaply as the "best" index proposed above, and thus far more cheaply than either of the orthodox quantity indices.

On the other hand, the David index is not entirely free of the common misconceptions about "real" measures; and this becomes clear as soon as we consider not one industry but a variety of them. Precisely because "real value added" is given a direct physical interpretation, each industry is measured in its *own* unit—and the resulting measure is again heir to the "index-number problem." If relative goods' prices are unchanged, of course, the industry-specific price deflators are all equivalent, and even the aggregate David index avoids the "index-number problem"; but then so would the ordinary "double-deflated" index, and the two measures will in fact coincide.[46] If relative prices change,

[41] At base year prices, real values added obviously correspond to current values added.

[42] I am assuming, as David does, that the "individual industry" is unequivocal. The assumption is critical; see n. 48.

[43] As usual, there may be some question as to whether different things are in fact physically identical and thus the *same* thing. The David index thus faces the problem of changes in the quality of output the way the orthodox indices face the probiem of changes in the quality of output and/or inputs (and the "best index" the problem of changes in the chosen standard).

[44] David (1966), p. 419. Of course, the David index would also yield negative real value added estimates whenever *current* values added were negative; but as we have seen that is not a problem.

[45] David (1966), pp. 421, 425.

[46] If $(z_{t+1}/p_{t+1}) = (z_t/p_t) = (z_o/p_o)$,

$$\frac{p_oQ_{t+1} - z_oR_{t+1}}{p_oQ_t - z_oR_t} = \frac{Q_{t+1} - (z_o/p_o)R_{t+1}}{Q_t - (z_o/p_o)R_t} = \frac{Q_{t+1} - (z_{t+1}/p_{t+1})R_{t+1}}{Q_t - (z_t/p_t)R_t} = \frac{(p_{t+1}Q_{t+1} - z_{t+1}R_{t+1})/p_{t+1}}{(p_tQ_t - z_tR_t)/p_t}.$$

If industry-specific David and "double-deflated" indices thus coincide, so of course do any aggregates of these that are similarly weighted by base-year values added.

however, the David index (like the orthodox indices) will measure industrial production in units that are inherently different, and of variable real value: the index, though properly sensitive to all changes in current values added, is also improperly sensitive to changes in relative goods' prices. Like the orthodox indices, then, the David index is in general distorted, and capable of moving in the wrong direction altogether. If technical progress saves L and K, for instance, the David index will register the decline in (real) value added per unit of output; but at the same time it will tend to understate it, since the (relative) decline in p that accompanies such technical progress means that that industry's value added is now measured in *smaller* real units. If zR/Q drops faster than $(wL+rK)/Q$, indeed, so will p—and (given Q) the David index registers a *rise* in real value added instead of the decline that is actually taking place.[47] If changes in materials costs instead *offset* increases in the productivity of the primary factors, and thus tend to maintain the real value of a unit of output, the David index will perform well. The relative performance of the David index and of the orthodox quantity measures of activity or of its results will thus depend on the particular empirical constellation of changes in prices and quantities; since industry-specific prices and materials costs often move together in response to changes in supply and demand as well as in materials-saving innovation, however, there is at least a general presumption that the David index, like the "double-deflated" index, will on average perform less well than the orthodox quantity index of activity.

By yet another standard, however, the David index appears quite inferior to all the alternatives considered so far. One of the primary purposes of defining (real) value added as a *net* measure is to make it insensitive to subaggregation: for example, we want to obtain the *same* measure of the steel-from-iron ore industry whether we obtain it directly or as the sum of the steel-from-pig-iron and pig-iron-from-iron ore industries. Current-price value added, the "best" measure of real value added proposed above, and the orthodox quantity indices of activity and of its results all retain this highly desirable feature; the David index does not. If the steel-from-iron ore industry is measured directly, the current value added of the aggregate steel-from-ore industry is deflated by the price of steel; if it is measured as the sum of its two components, part of that total current value added is deflated by the price of steel, and part by the price of pig—and unless relative (pig/steel) prices remain constant this partial difference in the deflator will alter the resulting aggregate estimate.[48]

Value-added-weighted Output Index

A widely used index of an industry's real value added is simply that industry's output.[49] For purposes of interindustry comparison or aggregation, these output

[47] I am assuming, for simplicity, that the standard of value is the wage unit—or, equivalently, that over the economy as a whole technical progress is negligible.

[48] In practice, then, the David index even for a single industry is not the unique, base-invariant time series it at first appears to be: since any ordinary industry can be vertically disaggregated into almost as many successive steps as one chooses to contemplate, the David formula leads to a whole family of indices: one for the industry considered as a unit, and another for each possible level of disaggregation and (in consequence) base year as well.

[49] At times, of course, output is itself estimated by dividing an input series (typically the principal raw material) by a constant input–output ratio; in such cases the index of real value added is obviously the input. The essential point, for present purposes, is that the industry is represented by a single physical series, be it Q, K, L, or R; I will deal explicitly only with the most common practical case, in which the index is Q, since the extension to other cases is perfectly straightforward.

series are weighted by base-year values added per unit of output: the result is thus a *net* output index, very different in substance from a *gross* output index obtained by using gross value (price) weights. In form, however, the result is an orthodox quantity index, and it duly shares the "index number problem"—which once again reminds us that this index will properly measure all production in the *same* "real" unit only when the implicit assumption that real values added per unit of output remain constant turns out to be correct. Out of the trivial context of pure inflation, of course, the validity of this assumption will be a matter of luck; at best, one can hope, the sources of distortion will largely cancel each other out (as they may when capital is substituted for labor, say, or when increased fabrication from lower-quality materials or for higher-quality output accompanies technical progress).

In the literature, this "output" index has generally been used to measure real value added where the absence of series on raw materials precluded the calculation of the desired "double-deflated" index.[50] While certainly a *pis aller* justified by its minimal data needs, this "output" index should not, however, be considered an inferior substitute for the "double-deflated" index—not only because the latter is not the theoretically correct index of real value added at all, but because in comparison to a correct index the "double-deflated" index is not likely to outperform the "output" index at all. If (R/Q) remains constant, for instance, these two indices obviously coincide;[51] and since as we have seen the constancy of (R/Q) is perfectly compatible with changes in real value added per unit, these indices can clearly coincide in a wrong measure as in a right one. If (R/Q) varies, these indices will differ, as the "double-deflated" index alone then registers a change in real value added per unit of output; but as we have seen that change need not even be in the right direction, so there is no general presumption that the "double-deflated" index is superior to the simple "output" index even where these measures do in fact differ.

The "output" index may be similarly compared to the David index. These indices will coincide if the ratio of value added to value remains constant,[52] since in the David formula the change in current-price value added is then exactly matched by the change in the current price of the (output) unit in which value added is measured; and as we have seen the resulting measure may well fail to register an actual change in real value added per unit. If the ratio of value added to value varies, these indices will not coincide; but as we have seen the change in real value added per unit of output registered by the David index need not even be of

[50] In a similar vein, Hill (1971) is devoted largely to the question of properly weighting indices of output and of input in order to achieve the best estimate of the true "double-deflated" measure when these component series are subject to error.

[51] If $(R_{t+1}/Q_{t+1}) = (R_t/Q_t)$,

$$\frac{p_o Q_{t+1} - z_o R_{t+1}}{p_o Q_t - z_o R_t} = \frac{Q_{t+1}(p_o - z_o(R_{t+1}/Q_{t+1}))}{Q_t(p_o - z_o(R_t/Q_t))} = \frac{Q_{t+1}}{Q_t}.$$

[52] If $\dfrac{(p_{t+1}Q_{t+1} - z_{t+1}R_{t+1})}{p_{t+1}Q_{t+1}} = \dfrac{(p_t Q_t - z_t R_t)}{p_t Q_t}$,

$$\frac{(p_{t+1}Q_{t+1} - z_{t+1}R_{t+1})/p_{t+1}}{(p_t Q_t - z_t R_t)/p_t} = \frac{Q_{t+1}((p_{t+1}Q_{t+1} - z_{t+1}R_{t+1})/p_{t+1}Q_{t+1})}{Q_t((p_t Q_t - z_t R_t)/p_t Q_t)} = \frac{Q_{t+1}}{Q_t}.$$

the right sign, so the "output" index (which assumes no such change) can again be more accurate than its more elaborate counterpart. There is thus no general reason to replace an "output" index by its David counterpart—or of course vice versa, since the information which yields a David index will as readily yield the "best" index described above.

Unlike the "double-deflated" index but like the David index, finally, the "output" index is not insensitive to subaggregation: unless the ratio of pig iron output to steel output is constant, for instance, the estimated real value added index for the steel-from-ore industry will not be the same whether it is calculated directly as a single series (steel output times the value added in producing a unit of steel from ore) or indirectly as the sum of two series (steel output times the value added in producing a unit of steel from pig, plus pig output times the value added in producing a unit of pig from ore). But whereas in the case of the David index the data base is presumed complete and the choice of vertical disaggregation is therefore arbitrary, the "output" index is in essence a way of stretching an incomplete data base. It is thus in the logic of the "output" index to incorporate new series as they become available: if the ferrous metals industry is represented by a single (steel) series, it is presumably because no distinct pig iron series can be obtained; if it can, it is natural to introduce it into the index (suitably reducing the weight of the steel series), since it conveys extra information (on changes in the ratio of output to input, inventories, or international trade) that will in general improve the estimate. Unlike the David index, the "output" index is sensitive to—i.e. can be improved by—*horizontal* disaggregation by production process. A single "industry" can be differentiated into a number of more homogeneous sub-industries on the basis of technical information alone, thus increasing the likelihood that within each disaggregated component real value added per unit did indeed remain reasonably constant. At the limit, each "industry" would be identified with a *single* well-defined production process, using given amounts of (particular) primary input services per unit of output; and upon aggregation at base year values the resulting index would obviously coincide with the corresponding orthodox index of activity $(w_oL_t + r_oK_t)$—which as we have seen is probably the best measure of real value added one can obtain by constant-price aggregation of physical series, and in the absence of full information on current values added.

● In sum, if acceptable series on current values added are available, one should simply deflate all of these by the price of a single, intuitively appealing unit of constant worth. The deflation of each industry's value added by the price of its own output would give clearly inferior results, since different industries would be measured by different standards; and the result would hinge on the inevitably arbitrary degree of (vertical) disaggregation one chooses to impose. In fact, neither this own-price deflation nor the orthodox "double-deflated" index of the results of activity appears superior to the simple addition of gross output series with value added weights, which requires a good deal less information. Among the various quantity indices, in fact, the least incorrect index of real value added appears to be the orthodox index of activity; and one notes that the simple (value-added-weighted) output index approaches that activity index as (with the addition of information on techniques) it is suitably disaggregated by production process.

V. A Numerical Simulation

A more systematic appraisal of the comparative performance of these various indices can be obtained by means of numerical simulation. In order to focus on the interaction between the impetus to change and the shape of the production function, I will assume that labor is the only primary factor of production, and that technical change in the economy at large is negligible. In such circumstances, the "best" measure of real value added is unequivocal, and the orthodox index of activity will always give correct results. The comparisons will thus bear on the relative performance of the orthodox "double-deflated" index, the David index, and the simple "output" index.

I assume two industries in competitive equilibrium. In period 1, each produces 100 units of output from 100 units of labor and 100 units of raw material; unit monetary values are 2.0 for output, 1.0 for labor, and 1.0 for raw material. In each industry, output is obtained from the inputs by a constant-elasticity-of-substitution production function[53] of the form

$$Q_i = g_i[a_i L_i^{-b} + (1-a_i)R_i^{-b}]^{-1/b}.$$

The initial values assume $g_i = 1$ and $a_i = 0.5$, so that function reduces to

$$Q_i = [0.5L_i^{-b} + 0.5R_i^{-b}]^{-1/b}.$$

The parameter b may vary from -1 (in which case the elasticity of substitution $s = 1/(1+b)$ is infinite and $Q_i = [0.5L_i + 0.5R_i]$) through 0 (in which case the elasticity of substitution is unity and $Q_i = [L_i^{0.5} R_i^{0.5}]$) to $+\infty$ (in which case the elasticity of substitution is zero and $Q_i = \min[L_i, R_i]$). Assume, further, that demand is unit elastic (so that $p_i Q_i$ is constant, whatever Q_i); that raw materials are in perfectly inelastic supply to the first industry ($R_1 = 100$ whatever z_1) and in perfectly elastic supply to the second industry ($z_2 = 1.0$ whatever R_2); and that labor is in perfectly elastic supply to both industries ($w_1 = w_2 = 1.0$, whatever L_1 or L_2).

Table 1 records the new competitive-equilibrium input and output values and quantities in both industries consequent upon a variety of specified changes in the original conditions (all the others being held constant): these include four cases of supply increase (through three varieties of technical progress, respectively neutral, labor-augmenting, and raw materials-augmenting; and through an increase in the supply of raw materials) and one case of demand increase. In each case, the elasticity of substitution is allowed to vary over its full range. One may argue that (at this level of simplification) an elasticity of substitution greater than 1 ($-1 \leq b < 0$) is not empirically interesting, as it implies that the output can be produced by raw materials alone, without any expenditure of labor and thus without any value added, in which case the "output" is the untransformed input and the industry, *stricto sensu*, does not exist at all. The corresponding magnitudes are included all the same, for the sake of illustration; it should be noted, however, that the figures for L_2 and R_2 with $b = -1$ in sections (a), (b), and (f) represent only one of the possible equilibria (as the two factors of production are indistinguishable by either productivity or conditions of supply).

[53] See Arrow, Chenery, Minhas and Solow (1961).

TABLE 1

EQUILIBRIUM INPUT AND OUTPUT QUANTITIES AND VALUES FOR TWO HYPOTHETICAL INDUSTRIES

b	s	L_1	R_1	Q_1	w_1	z_1	p_1	L_2	R_2	Q_2	w_2	z_2	p_2
(a) Initial values:													
$b \geq -1$	$s \geq 0$	100	100	100	1.0	1.0	2.0	100	100	100	1.0	1.0	2.0
(b) Terminal values, following technical change doubling the efficiency of L_i and R_i:													
$b \geq -1$	$s \geq 0$	100	100	200	1.0	1.0	1.0	100	100	200	1.0	1.0	1.0
(c) Terminal values, following technical change doubling the efficiency of L_i:													
−1	∞	150	100	200	1.0	0.50	1.00	200	0	200	1.0	1.0	1.00
−1/2	2	122	100	164	1.0	0.78	1.22	133	67	150	1.0	1.0	1.33
0	1	100	100	141	1.0	1.00	1.41	100	100	141	1.0	1.0	1.41
1/2	2/3	86	100	129	1.0	1.13	1.55	88	112	139	1.0	1.0	1.44
1	1/2	78	100	122	1.0	1.22	1.64	83	117	137	1.0	1.0	1.46
2	1/3	69	100	114	1.0	1.31	1.75	77	123	136	1.0	1.0	1.47
∞	0	50	100	100	1.0	1.50	2.00	67	133	133	1.0	1.0	1.50
(d) Terminal values, following technical change doubling the efficiency of R_i:													
−1	∞	0	100	100	1.0	2.00	2.00	0	200	200	1.0	1.0	1.00
−1/2	2	76	100	125	1.0	1.24	1.60	67	133	150	1.0	1.0	1.33
0	1	100	100	141	1.0	1.00	1.41	100	100	141	1.0	1.0	1.41
1/2	2/3	114	100	148	1.0	0.86	1.35	112	88	139	1.0	1.0	1.44
1	1/2	124	100	153	1.0	0.76	1.31	117	83	137	1.0	1.0	1.46
2	1/3	136	100	159	1.0	0.64	1.25	123	77	136	1.0	1.0	1.47
∞	0	200	100	200	1.0	0.00	1.00	133	67	133	1.0	1.0	1.50
(e) Terminal values, following a doubling in the availability of R_i (to $R_1 = 200$ for all z_1, and $z_2 = 0.5$ for all R_2):													
−1	∞	0	200	100	1.0	1.00	2.00	0	400	200	1.0	0.50	1.00
−1/2	2	76	200	125	1.0	0.62	1.60	67	267	150	1.0	0.50	1.33
0	1	100	200	141	1.0	0.50	1.41	100	200	141	1.0	0.50	1.41
1/2	2/3	114	200	148	1.0	0.43	1.35	112	177	139	1.0	0.50	1.44
1	1/2	124	200	153	1.0	0.38	1.31	117	166	137	1.0	0.50	1.46
2	1/3	136	200	159	1.0	0.32	1.25	123	155	136	1.0	0.50	1.47
∞	0	200	200	200	1.0	0.00	1.00	133	133	133	1.0	0.50	1.50
(f) Terminal values, following a quadruplication of sales:													
−1	∞	700	100	400	1.0	1.00	2.00	400	400	400	1.0	1.0	2.0
−1/2	2	563	100	284	1.0	2.37	2.81	400	400	400	1.0	1.0	2.0
0	1	400	100	200	1.0	4.00	4.00	400	400	400	1.0	1.0	2.0
1/2	2/3	295	100	160	1.0	5.05	5.01	400	400	400	1.0	1.0	2.0
1	1/2	237	100	141	1.0	5.63	5.69	400	400	400	1.0	1.0	2.0
2	1/3	183	100	124	1.0	6.17	6.44	400	400	400	1.0	1.0	2.0
∞	0	100	100	100	1.0	7.00	8.00	400	400	400	1.0	1.0	2.0

Table 2 indicates the various indices of real value added for the terminal situation (initial real value added being set equal to 100) as calculated from the prices and quantities reported in Table 1.[54] The most straightforward comparison is between the "output" index i^3 and the David index i^4. These coincide whenever $b = 0$ ($s = 1$), as (VA/V) is then constant; and they may coincide for all b, whether

[54] While a "raw materials" index (R) analogous to the "output" index (Q) is not included in Table 2, its value can be obtained directly from Table 1.

TABLE 2

Indices of Real Value Added, with Initial Real Value Added = 100, as Calculated from Table 1

Industry:		1					2					1+2						
Index:		$i^1=i^2$	i^3	i^4	i^5		$i^1=i^2$	i^3	i^4	i^5		$i^1=i^2$	i^3		i^4		i^5	
Weights:		1, 2	1, 2	1, 2	1	2	1, 2	1, 2	1, 2	1	2	1, 2	1	2	1	2	1	2
b	s																	
(a) Initial values:																		
$b \geq -1$	$s \geq 0$	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
(b) Terminal values, following technical change doubling the efficiency of L_i and R_i:																		
$b \geq -1$	$s \geq 0$	100	200	200	300	∞	100	200	200	300	∞	100	200	200	200	200	300	∞
(c) Terminal values, following technical change doubling the efficiency of L_i:																		
−1	∞	150	200	300	300	300	200	200	400	400	∞	175	200	200	350	350	350	700
−1/2	2	122	164	200	228	277	133	150	200	233	400	128	157	156	200	200	230	330
0	1	100	141	141	183	241	100	141	141	183	241	100	141	141	141	141	183	241
1/2	2/3	86	129	111	158	205	88	139	123	166	199	87	134	134	117	117	162	202
1	1/2	78	122	95	144	186	83	138	114	158	182	80	130	130	105	104	151	184
2	1/3	69	114	79	129	157	77	136	105	149	164	73	125	125	92	91	139	161
∞	0	50	100	50	100	100	67	133	89	133	133	58	117	117	70	67	117	117
(d) Terminal values, following technical change doubling the efficiency of R_i:																		
−1	∞	0	100	0	100	?	0	200	0	200	?	0	150	?	0	?	150	?
−1/2	2	76	125	96	150	210	67	150	100	167	200	72	138	136	98	98	158	205
0	1	100	141	141	183	241	100	141	141	183	241	100	141	141	141	141	183	241
1/2	2/3	114	148	169	196	233	112	139	155	189	252	113	143	143	162	161	193	242
1	1/2	124	153	189	206	227	117	137	161	192	258	120	145	145	175	174	199	241
2	1/3	136	159	218	219	221	123	136	167	195	261	130	148	147	192	190	207	238
∞	0	200	200	400	300	200	133	133	178	200	267	167	167	167	289	267	250	222
(e) Terminal values, following a doubling in the availability of R_i (as in Table 1):																		
−1	∞	0	100	0	0	0	0	200	0	0	0	0	150	?	0	?	0	0
−1/2	2	76	125	96	50	78	67	150	100	33	80	72	138	136	98	98	42	79
0	1	100	141	141	83	110	100	141	141	83	110	100	141	141	141	141	83	110
1/2	2/3	114	148	169	96	124	112	139	155	100	118	113	143	143	162	161	98	121
1	1/2	124	153	189	106	133	117	137	161	109	122	120	145	145	175	174	107	128
2	1/3	136	159	218	119	146	123	136	167	117	126	130	148	147	192	190	118	136
∞	0	200	200	400	200	200	133	133	178	133	133	167	167	167	289	267	167	167
(f) Terminal values, following a quadruplication of sales:																		
−1	∞	700	400	700	700	700	400	400	400	400	400	550	400	400	550	550	550	550
−1/2	2	563	284	401	469	1285	400	400	400	400	400	482	342	323	400	401	434	670
0	1	400	200	200	300	∞	400	400	400	400	400	400	300	267	300	267	350	800
1/2	2/3	295	160	118	219	∞	400	400	400	400	400	347	280	244	259	198	310	726
1	1/2	237	141	83	181	4069	400	400	400	400	400	319	270	237	242	166	291	602
2	1/3	183	124	57	148	669	400	400	400	400	400	292	262	235	228	138	274	458
∞	0	100	100	25	100	100	400	400	400	400	400	250	250	250	213	100	250	250

Key: i^1: "true" index of real value added (VA_t/w_t)
i^2: quantity index of activity ($w_oL_t + r_oK_t$)
i^3: "output" index ($(VA_o/Q_o)Q_t$)
i^4: David index ($p_o(VA_t/p_t)$)
i^5: "double-deflated" index ($p_oQ_t - z_oR_t$)

in error (as in the event of neutral technical change, illustrated by case (b)), or not (as in the case of growth without change in relative quantities or values, illustrated by the response of industry 2 to the demand shift of case (f)). If i^3 and i^4 do not coincide for all b, then $i^3 > i^4$ to one side of their equality, and $i^3 < i^4$ to the other side; and as both indices err in the same direction throughout the range of s, one index clearly outperforms the other throughout the empirically more interesting part of that range ($s \leq 1$).[55] It is apparent, from cases (c) through (f), that i^4 measures the change in real value added better than i^3 (for $s < 1$) only in the case of labor-augmenting technical progress (case (c)); with growth induced by demand shifts, or by shifts in the supply or productivity of raw materials (cases (d) through (f)), the "output" index errs less than the David index. More: in all these cases, i^3 actually coincides with $i^1 = i^2$ if labor and raw materials are not substitutable ($s = 0$); and while in the case of labor-augmenting technical change i^4 similarly does best with $s = 0$, it need not even then be free of error (thus industry 2 in Table 2).

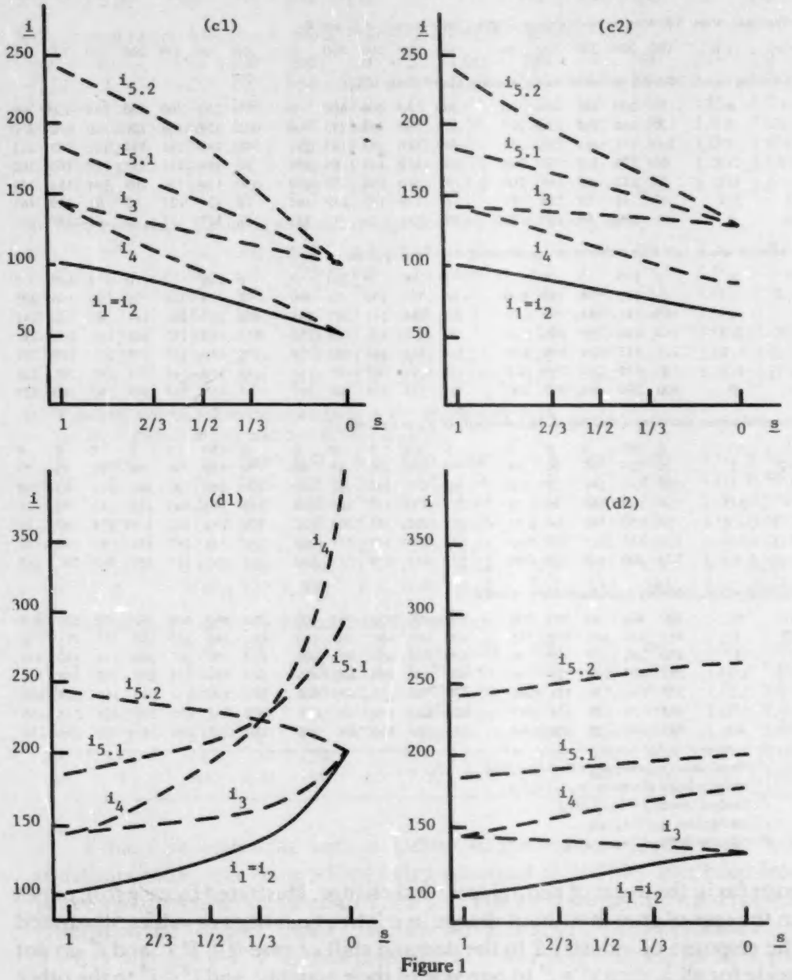
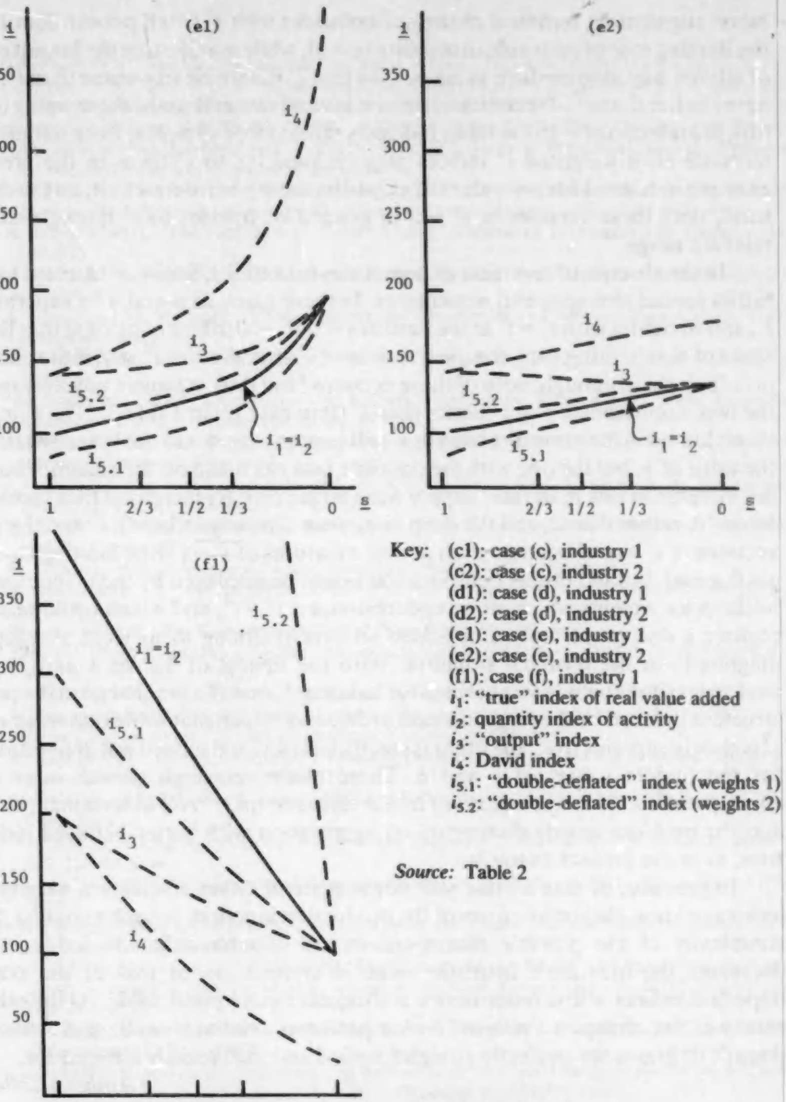


Figure 1

[55] See also Figure 1. These indices' relative performance is of course reversed in the less interesting part of the range of s ($s > 1$).



Source: Table 2

Figure 1 (continued)

The comparative performance of the "double-deflated" index i^5 is not so simply assessed. Only in the case of equiproportionate growth—when no measurement problems in fact exist—does i^5 perform as well as any other index (thus industry 2, case (f)); only in the case of neutral technical change will i^5 be ever the worst of all measures (indeed, as in the present example, case (b), the index based on advanced-technology prices need not even be defined). In case (c), with labor-augmenting technical change, i^5 coincides with i^3 (itself poorer than i^4) at the limiting case of zero-substitutability ($s = 0$), while manifesting the largest error of all over any intermediate value of s; so that i^5 is here clearly worse than i^4, and never better than i^3. If technical change is instead raw materials-augmenting (case (d)), the relation of i^5 to the other indices is rather more complex. Both initial- and terminal-price-weighted i^5 indices may be superior to i^4 (thus in the present example industry 1 for low values of s), and both may be inferior to it; on the other hand, both these versions of i^5 will in general be inferior to i^3 throughout the relevant range.[56]

In the absence of technical change (cases (e) and (f)), finally, i^5 displays to the full its special strengths and weaknesses. In these cases, as noted, i^3 is superior to i^4, and coincides with $i^1 = i^2$ at the limit of $s = 0$; i^5 is also free of error at that limit. Short of that limiting case, the two versions of i^5 straddle $i^1 = i^2$, and if the change in (z/p) is small enough, both of these versions (and thus, *a fortiori*, any average of the two) approximate $i^1 = i^2$ better than i^3 (thus case (e) in Table 2). The i^5 index that is biased in the same direction as i^3 will remain superior to the latter, whatever the value of s; but the one with the opposite bias can make no such claim. Thus, if the increase in raw materials' supply were larger (say R_1 increased by a factor of 10 or 20, rather than 2, and the drop in z_2 were similarly inflated), i^3 and the two versions of i^5 would become even poorer measures of $i^1 = i^2$ (the limiting case of $s = 0$ aside); but the fastest deterioration would be displayed by the i^5 index with initial price weights which alone underestimates $i^1 = i^2$, and which could readily register a decline of real value added so overwhelming as to yield a negative magnitude in the terminal situation. With the figures of Tables 1 and 2, the analogous situation is in fact verified for industry 1, case (f): the change in the price structure induced by the large increase in demand is such that while one version of i^5 is clearly superior to i^3, the other is clearly inferior, and indeed not defined at all for the middle values of s and b. These alarmingly high growth rates are consequent upon low (or negative) initial values of $(pQ - zR)$ at terminal prices; and the problem largely disappears on aggregation with better-behaved industries, as in the present example.

In practice, of course, one will not in general either calculate a variety of indices or know the specific form of the production function. In such a context, the complexity of the possible distortions in the "double-deflated" index itself decreases the measure's heuristic value in comparison to that of the other imperfect indices whose misbehavior is altogether more predictable. At the other extreme, the cheapest "output" index performs relatively well; and when it doesn't its biases are perfectly straightforward and thus readily allowed for.

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[56] In the present example, the i^5 index with terminal weights for industry 1 coincides with $i^1 = i^2 = i^3$ for $s = 0$; but the reason is the anomaly of a terminal $z = 0$, which gives raw materials a zero weight and thus obviously reduces the "double-deflated" index to a simple "output" index. If the augmentation of raw materials were limited to 50 percent, for instance, with $s = 0$, $i^1 = i^2 = i^3 = 150$, $i^4 = 226$, and $i^5 = 200$ (initial weights) or 181 (terminal weights). Note that with $s = 1$, $i^1 = i^2 = 100$, $i^3 = i^4 = 122$, and $i^5 = 144$ and 159 with initial and terminal weights respectively: in contrast to the present example (100 percent augmentation), the terminal-price-weighted i^5 is then monotonically increasing, rather than decreasing, as s declines from 1 to 0.

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