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SOME "RAS" EXPERIMENTS WITH THE MEXICAN INPUT–OUTPUT MODEL

BY PEDRO URIBE*

This paper deals with the results obtained from a RAS adjusted series of matrices in the forecasting of intermediate demand given the final demand; moreover, the consequences of coefficient change in intermediate demand are analyzed.

1. INTRODUCTION

The purpose of this paper is to comment on some empirical results obtained from the application of the RAS method of Stone *et al.* to the Mexican input–output model. Not having much evidence to rely on for testing RAS-adjusted matrices against reality, we have tried to analyze the consistency of results and the empirical plausibility of conclusions derived from our set of matrices. Another possibility, still to be explored, is to use other adjustment methods, such as the linear programming approach of Matuszewski *et al.* (1964).

Section 2 discusses RAS from a new angle, proposing an interpretation of RAS coefficients in terms of prices, which will be tested in Section 7. Section 3 describes the various steps used in the estimation of the series of matrices. Section 4 describes a short test run for the prediction of future marginals from a known matrix and constant RAS multipliers. One concludes that, although errors grow linearly with time, a few sectors account for most of them, so that the extrapolation may be safe in the short run, if one keeps track of some key sectors.

Section 5 deals with the prediction of intermediate demand, given final demand and a coefficient matrix. The age of the coefficient matrix turns out to be crucial, final-demand blow-up being better than input–output when the age of the matrix exceeds 10 years. Section 6 studies the effect of coefficient changes on intermediate demand. It turns out that, on the one hand, coefficient changes—minimized on the average by RAS—are an extremely important determinant of changes in intermediate demand in the Mexican economy. On the other hand, results are plausible in the sense that they indicate a substitution process occurring in the economy, where "traditional" inputs (agricultural goods, minerals) are being heavily replaced by chemicals; a large proportion of the growth of the so-called "modern" sector is accounted for by coefficient changes.

Section 7 assumes coefficients are generated by a Cobb–Douglas production function, through profit maximization, given exogenous prices. It seems that price movements have been so small that no significant changes are predicted by the

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Cobb–Douglas model; hence coefficient changes are in the main the result of ex-ante technical change. Computed ex-ante change from the Cobb–Douglas model leads to practically the same results as those observed in Section 6, with a high empirical plausibility.

Finally, Section 8 hints at a production-function-free model of technological change, the empirical testing of which is still in the research stage.

2. On RAS and its Interpretations

Although the RAS method for updating input–output tables is well known, it is convenient to mention briefly some of its main characteristics that will be relevant in the sequel. Let A and B be two non-negative matrices of the same order, with none of the rows or columns of either A or B consisting entirely of zeros. Call B a RAS transform of A, given strictly positive diagonal matrices $\hat{r}$ and $\hat{s}$,[1] so that $B = \hat{r}A\hat{s}$. Clearly, the relation "B *is a RAS transform of* A" is an equivalence relation.

The RAS transform of the input–output coefficient matrix in t, $A(t)$, was proposed originally by Leontief as a model for the coefficient matrix $A(t+1)$. Stone and his group at Cambridge, England (see Stone *et al.*, 1963) proposed a way to find one RAS transform, the one for which $\hat{r}A(t)\hat{s}$ will add up to the marginals for $(t+1)$. RAS transforms have been extensively studied by M. Bacharach, a former member of Stone's group (see Bacharach, 1970). He shows that, if A, B are scaled so that $\Sigma_{ij}a_{ij} = \Sigma_{ij}b_{ij} = 1$, then the quantity:

$$I = \Sigma_{ij}b_{ij} \log \frac{b_{ij}}{a_{ij}} \tag{2.1}$$

is minimized under $\Sigma_i b_{ij} = v_j$, $\Sigma_j b_{ij} = \mu_i$, say, when B is a RAS transform of A. Equation (2.1) may be interpreted as the information gain from a posterior bivariate distribution (b_{ij}), given a prior distribution (a_{ij}). It was first proposed by the present author, de Leeuw, and Theil (1966).

If the log in (2.1) is expanded, the dominant term is chi-square; hence RAS is approximately a chi-square minimizer; chi-square criteria have been proposed to adjust frequency matrices to known marginals by Deming and Stephan (1940) and Friedlander (1961).

None of these "minimizer" interpretations have an economic character. Stone calls r_i a "substitution effect" and s_j a "fabrication effect." It is also possible to see RAS in the light of the gravitational models of W. Isard. I propose here another strictly economic interpretation of RAS.

If one considers input–output coefficients as the outcome of profit maximization, given a production function and a set of exogenous input and factor prices (as do for example, Morishima and Murata, 1972), one sees that physical coefficients are given by

$$a_{ij} = p_j\alpha_{ij}/p_i \tag{2.2}$$

[1] We adopt the convention of writing $\hat{x}$ for the diagonal matrix obtained from vector x. All vectors will be columns; accents will denote transposition.

in the Cobb–Douglas, and by

$$a_{ij} = (p_j \theta_{ij}/p_i)^{\sigma_j} \tag{2.3}$$

in the CES, where α_{ij} is the exponent for input (or factor i) in the Cobb–Douglas production function of sector j, and θ_{ij} is the distribution parameter and σ_j the elasticity of substitution in the CES production function of sector j. The p_i are prices.

Two-stage production functions, such as Uzawa's generalized CES (1962), may be treated in a similar way. Consider first the CES mixture of Cobb–Douglas components:

$$X_j = \Sigma_g \left(\theta_{gj} \prod_{i \in S_g} x_{ij}^{-\alpha_{ij}\rho_{gj}}\right)^{-1/\rho_{gj}} \tag{2.3'}$$

Profit maximization leads to

$$a_{ij} = \frac{p_j \alpha_{ij}}{p_j} \theta_{gj} A_{gj}^{-}\left(\frac{1 - \sigma_{gi}}{\sigma_{gj}}\right)$$

Notice that the first multiplicative component, $p_j \alpha_{ij}/p_j$, is a Cobb–Douglas input coefficient. If A_{gj} is consistent with CES profit maximization, then $A_{gj} = (\theta_{gj} p_j / P_g)^{\sigma_{gj}}$ and hence

$$a_{ij} = (P_g \alpha_{ij}/p_i)(p_j \theta_{gj}/P_g)^{\sigma_{gj}}, \tag{2.4}$$

the product of a within-Sg Cobb–Douglas coefficient and the between-Sg CES coefficients. Pg is the aggregate price $P_g = \sum_{i \in S_g}(X_{ij}/X_{gj})p_i$, X_{gj} the g-th Cobb–Douglas component of the CES (2.3).

Similarly, a Cobb–Douglas mixture of CES components,

$$X_j = \prod_g \left\{ \sum_{i \in S_g} (\theta_{ij} x_{ij}^{-\rho_{gj}})^{-1/\rho_{gj}} \right\}^{\alpha_{gj}}$$

leads in a straightforward manner to:

$$a_{ij} = (\theta_{ij} P_g/p_i)^{\sigma_{gj}} (p_j \alpha_{gj}/P_g) \tag{2.5}$$

According to (2.2), price changes without a change in the production function lead to

$$a_{ij}(t+1) = \{p_i(t)/p_i(t+1)\} a_{ij}(t) \{p_j(t+1)/p_j(t)\},$$

that is, $r_i = p_i(t)/p_i(t+1)$, $s_j = p_j(t+1)/p_j(t)$. These values I call the Cobb–Douglas-RAS coefficients, r_i^{CD}, s_j^{CD}. Clearly, one sees that (2.3′) implies CES–RAS coefficients:

$$r_i^{CES} = (r_i^{CD})^{\sigma_j}$$

$$s_j^{CES} = (s_j^{CD})^{\sigma_j}$$

Of course, r_i is independent of j in the Cobb–Douglas case, and/or sectors having the same elasticity of substitution. Coefficients defined in (2.4) will change according to

$$a_{ij}(t+1) = r_i^{CD} r_g^{CES} a_{ij}(t) s_g^{CES} s_j^{CD} \qquad (i \in Sg)$$

Those defined in (2.5) will follow:

$$a_{ij}(t+1) = r_i^{\text{CES}} r_g^{\text{CD}} a_{ij}(t) s_g^{\text{CES}} s_j^{\text{CD}}$$

Some empirical results on this will be reported in Section 7.

3. The Estimation Procedure of Mexico's Input–Output Matrices

The only full input–output table for Mexico was compiled in 1960 by the Bank of Mexico.[2] This will be referred to as "the original 1960 matrix." Other sources of information are the National Accounts,[3] an unpublished table on import composition,[4] referred to as "Importación" and the joint study of ECLA and Nacional Financiera,[5] designated as ECLA–NAFINSA. The Mexican input–output system includes 46 intermediate sectors, listed in Table 4, Section 6, and four primary inputs (imports, labor, capital, and indirect taxes minus subsidies). Final demand is divided into consumption, exports and gross capital formation (plus or minus changes in inventories). The National Accounts give the 50 row totals at current and constant 1960 prices to 1967, save row 47 (imports), which has been deflated with index numbers from ECLA–NAFINSA. Row totals 1 to 46 for 1968/72 at current and constant 1960 prices are also given in the National Accounts; row total 47 is deflated using unpublished ECLA figures. Row totals 48 to 50 are estimated as follows: indirect taxes are a constant proportion of GDP (4.84 percent). Disposable income is distributed between labor and capital using

$$\text{(3.1)}\qquad \underset{(0.167)}{w_t = -0.011356} + \underset{(0.931)}{0.623038 w_{t-1}} + \underset{(1.030)}{0.389621 w_{t-2}} + \underset{(0.125)}{0.00638 w_{t-3}}$$

($R^2 = 0.999364$), where w_t is the share of labor in year t.

Column 47 sums up to row 47 ($\sum_{i=1}^{46} a_{ij}$). Columns 1 to 46 are reported in the National Accounts, both in current and 1960 prices; exports (column total 48) are reported only at current prices and deflated with ECLA-NAFINSA index members up to 1969 and ECLA unpublished figures for 1970–1972. Column total 49 (gross capital formation) is reported at current prices and deflated with the implicit GDP deflator; consumption is estimated as a residual from GDP (+imports–exports–gross capital formation). All this covers the period 1950–1972.

Matrix estimates are the two-stage RAS equivalent of the original 1960 matrix, as follows: for the purpose of RAS estimation, intermediate matrices include imports, and are called the augmented intermediate matrix (AIM): reduced

[2] *Banco de México, Cuadro de Insumo-Producto para* 1960, *undated.*

[3] *Cuentas Nacionales y Acervos de la Capital, undated. Estadísticas de la Ofna. de Cuentas de Producción y Precios* 1973. Includes a Statistical Appendix (Apéndice Estadístico No. 1) published in October, 1973.

[4] "Importación de Mercancías." Ofna. de Cuentas del Exterior. October 26, 1973.

[5] ECLA and Nacional Financiera. *La Política Industrial en el Desarrollo Económico de México.* Mexico City, Nacional Financiera, 1971.

primary input matrices (RPM) exclude imports. Then, all matrices being given at constant 1960 prices:

1. For $t = 1960$, the stage I-AIM is RAS-equivalent to the original 1960 matrix, under National Accounts row and column totals.
2. For $t = 1961$ to $t = 1972$, the stage I-AIM for t is RAS-equivalent to range I-AIM for $t - 1$, under row and column totals from the National Accounts.
3. For $t = 1950$ to $t = 1959$, the stage I-AIM is RAS-equivalent to stage I-AIM for $t + 1$, under row and column totals from the National Accounts.
4. Stage I-RPM is given in the National Accounts, for 1950 to 1967.
5. Stage I-RPM for 1968–1972 is RAS-equivalent, for each t, to stage I-RPM for $t - 1$, under (2.1) and the resulting distribution of GDP.
6. Stage II row 47 is defined as follows: for intermediate sectors, the element $(47, j)$ is row total 47 × the share of intermediate goods in total imports (given in "Importación") times the share of sector j in stage I row 47. This covers $j = 1$ to 46. Imports of consumption goods (47, 47) and capital goods (47, 49) are defined in a similar way.
7. Stage II column 48 incorporates information from ECLA–NAFINSA on the exports of 16 groups of goods; they are disaggregated from the share of each sector in the exports of its group, according to the stage I matrix.
8. The resulting stage II sub-matrix of the AIM is RAS-equivalent to the corresponding sub-matrix of the stage I-AIM, under National Accounts (minus the value of the elements in row 47 or column 48 defined above).
9. This covers 1950 to 1969. Stage II-AIM for t is RAS-equivalent to stage II-AIM for $t - 1$, with t from 1970 to 1972.
10. Matrices from 1973 to 1975 are all RAS-equivalent to the $(t - 1)$ matrix, with RAS coefficients defined as the average stage I values for 1960–1969.

4. A Test for the Quality of RAS Forecasts

Only very restricted testing can be done on the quality of a RAS-adjusted matrix; the only conclusive one implies having a "real" matrix for the forecast period. One can test the extrapolation of a matrix to periods where the marginals are not known: let $\bar{r}_i$ and $\bar{s}_j$ be the last-known values of r_i and s_j, say those carrying $A(t)$ into $A(t + 1)$. One may try $A(t + 2) = \bar{r}^2 A(t)\bar{s}^2$, or, in general, $A(t + h) = \bar{r}^h A(t)\bar{s}^h$.

A simple test was carried out along these lines, with the 1973, 1974, and 1975 matrices defined as above, using the 1972 matrix and the average 1960–1969 r and s values. Intermediate flow matrices for 1970, 1971, and 1972 were predicted, using the 1969 matrix and the average 1960–1969 r and s. Then marginals were compared with their true values from the National Accounts.

It seems that, on the one hand, row and column totals were forecast with more or less the same degree of success, although column errors are a bit larger (see Table 1). Root-mean square errors (RMS),

$$\left\{ \frac{1}{46} \sum_{i=1}^{46} \left(\frac{x_i^P - x_i^A}{x_i^A} \right)^2 \right\}^{1/2}$$

where x_i^P is the predicted and x_i^A the actual value of x_i, were computed for both column and row totals for the three years. They tend to increase heavily with time: gross output, for example, was predicted with a RMS error of 8.35 percent at one year's distance, 15.80 percent for two years, and 23.5 percent for three years.

TABLE 1
ROOT MEAN SQUARE ERRORS FOR RAS—PREDICTED MARGINALS
(in percents)

	1970	1971	1972
Gross Output			
Total	8.3538	15.8044	23.4878
Without 5 sectors	5.1805	10.2817	12.1362
Intermediate Inputs			
Total	11.7454	17.9810	26.3472
Without 5 sectors	9.7860	12.7193	15.2139

On the other hand, five sectors seem to account for a large fraction of forecasting errors; in the five cases errors mean overestimation. They are shown in Table 2, together with their forecasting errors $(x_i^A - x_i^A)/x_i^A$.

TABLE 2
FORECASTING ERRORS FOR SOME CRUCIAL SECTORS (percents)

Sector \ Period	Gross Output 1970	Gross Output 1971	Gross Output 1972	Intermediate Inputs 1970	Intermediate Inputs 1971	Intermediate Inputs 1972
Mining, non-metals	19.1964	32.2034	67.8571	16.0494	25.8824	54.3210
Basic chemicals	16.4265	36.4362	69.9495	22.4771	47.1522	88.7550
Fertilizers and pesticides	31.6384	51.0000	78.8546	32.8244	54.7973	86.3095
Other chemicals	17.8451	24.8408	32.1637	20.5882	28.7037	39.5652
Electric machinery	12.3306	39.6601	50.8750	12.4668	38.9503	50.9804

The period running from 1970 to 1972 is a difficult one for the Mexican economy. 1970 witnessed a change of Administration, an event which is traditionally thought to have significant consequences. 1971 was a semi-recession year, when the rate of growth of GNP was halved, and 1972 is considered as the initial year of a mild inflationary period—an experience not known in the Mexican economy for a long time. One may see that so-called "traditional" sectors continue to grow and were underestimated by RAS; "dynamic" sectors were generally overestimated. This is an interesting subject for further research.

Pending an extension of the test examined above (for another 20 years), one may conclude that short-run RAS extrapolation, possibly aided by some exogenous hypotheses (such as rate of growth of GDP, imports, exports, etc.) may not be too bad, provided one keeps track of some crucial sectors that account for a large fraction of forecasting errors.

5. Forecasting Intermediate Demand

We are concerned in this section with the predictive power of the intermediate demand predictor:

$$z^P_{ih} = [(I - A_t)^{-1} - I]f_{t+h} \tag{5.1}$$

Tilanus (1966), Tilanus and Rey (1963), Rey and Tilanus (1964), and Theil (1966) have studied this field extensively for the Dutch economy; it can also be extended to primary inputs (see Tilanus and Harkema 1962).

Our experiment is rather artificial, of course; all matrices A_t are estimated and the estimation process includes f_t. It may be worth while to point out that errors of (5.1) are unlikely to be overestimated, at least on the average, since matrix estimates minimize[7] change according to information theory.

The Dutch studies enter into many interesting details which we leave aside; we do not look for the statistical structure of prediction errors, but for a measure of the efficiency of an "aged" coefficient matrix to predict intermediate demand, given a perfect forecast of final demand.

We are interested in comparing the performance of (5.1) as against a simpler predictor, starting also from perfect prediction of final demand, but without knowledge of input coefficients:

$$z^B_{ith} = z_{it}\left(\frac{f_{i,t+h}}{f_{it}}\right). \tag{5.2}$$

Predictor (3.2) has been called "final demand blow-up" predictor. We call (3.1) the 'input–output' predictor.

We are interested in the behavior of the root mean-square errors:

$$e^P_{ih} = \frac{1}{T-h}\left[\sum_{t=1}^{T-h}\left(\frac{z^P_{ith} - z_{i,t+h}}{z_{i,t+h}}\right)^2\right]^{1/2}$$

$$e^B_{ih} = \frac{1}{T-h}\left[\sum_{t=1}^{T-h}\left(\frac{z^B_{ith} - Z_{i,t+h}}{Z_{i,t+h}}\right)^2\right]^{1/2}$$

where T is the length of the period under analysis (here $T = 26$).

The performance of the input–output predictor worsens with the length of the prediction period, as should be expected, and there is a great sector variation in the behavior of the e^P_{ih}. For example, the mining sectors reach errors higher than 10 percent in one and two years, "other textiles" (non soft-fiber) and construction, in 2, fertilizers, in 3, basic chemicals, in 6, synthetic materials, in 5, and forestry, in 6, while others—like the food industry, communications, trade, rubber,

[7] Not quite, since the estimation process is not straightforward RAS, but only close to it.

printing and editorial, and petroleum and petrochemicals—do not reach this level in the period under analysis (25-year horizon). "Critical" periods (in the sense of less than 10 percent prediction errors) are large for the services (other than transportation).

This is not the pattern of the blow-up predictor: after a critical point, 6 years on the average, errors start to decline to an average of 18 percent for 20 years and rise again afterwards (see Table 3).[8] Thus, the short-run performance of the input–output predictor seems better than the blow-up; the situation reverses for the long run (after 4 years on the average). Table 3 shows the values of

$$\bar{e}_h^P = \frac{1}{46} \sum_{i=1}^{46} e_{ih}^P$$

and

$$\bar{e}_h^B = \frac{1}{46} \sum_{i=1}^{46} e_{ih}^B$$

These values are highly dominated by the errors in Sectors 5, 6, and 14, so that, side by side, Table 3 contains

$$\bar{e}_h'^P = \frac{1}{43} \sum_{i \neq 5,6,14} e_{ih}^P \qquad \bar{e}_h'^B = \frac{1}{43} \sum_{i \neq 5,6,14} e_{ih}^B$$

If $\bar{e}_h'^P$ and $\bar{e}_h'^B$ are used, blow-up prediction performs better than input–output prediction after 8 years (again, the average over the sectors' critical levels of 10 percent average error are: 4 years for input–output and 2 years for blow-up if all sectors are taken into account; 11 years for input–output and 12 for blow-up if the dominant sectors are excluded).

6. Sources of Change in the Demand for Intermediate Inputs: Coefficient Change

Consider the familiar input–output equation for intermediate demand:

$$z = x - f = \{(I - A)^{-1} - I\}f = A(I - A)^{-1}f = Cf \tag{6.1}$$

Take Index z, f, and A with subscript t, lag one period, and subtract to obtain:

$$\begin{aligned} z_t - z_{t-1} &= C_t f_t - C_{t-1} f_{t-1} = C_t(f_t - f_{t-1}) + (C_t - C_{t-1})f_{t-1} \\ &= C_{t-1}(f_t - f_{t-1}) + (C_t - C_{t-1})f_t \end{aligned} \tag{6.2}$$

In what follows I will take the simple mean of the two last right-hand members of (6.2) and define $C^* = (C_t + C_{t-1})/2$, $f^* = (f_t + f_{t-1})/2$, $\Delta C = C_t - C_{t-1}$, $\Delta f = f_t - f_{t-1}$, so that:

$$\Delta z = z_t - z_{t-1} = C^* \Delta f + (\Delta C) f^* \tag{6.3}$$

The first term on the right is the effect of changes in final demand holding C constant at C^*, upon the change in intermediate demand. The second term is the effect of coefficient change, holding final demand constant at f^*. A word of caution

[8] This is not the behavior found in the Dutch studies: see Theil (1966), p. 186.

TABLE 3

AVERAGE ROOT MEAN—SQUARE PREDICTION ERRORS FOR INPUT–OUTPUT AND FINAL DEMAND BLOW-UP PREDICTORS OF INTERMEDIATE DEMAND, BY LENGTH OF HORIZON

	Input–Output		Final Demand Blow-Up	
Years ahead	A	B*	A	B*
1	0.020369	0.016423	0.076105	0.018927
2	0.041807	0.027270	0.257118	0.031225
3	0.076551	0.042152	0.488365	0.053787
4	0.115902	0.056720	0.694574	0.075978
5	0.154525	0.061742	0.898923	0.096592
6	0.191774	0.062417	1.015625	0.081417
7	0.197753	0.070831	0.624096	0.071478
8	0.204174	0.079390	0.319685	0.072247
9	0.237075	0.087272	0.308324	0.072575
10	0.262386	0.097593	0.258630	0.069537
11	0.298328	0.109508	0.233216	0.082903
12	0.325883	0.121670	0.238642	0.101889
13	0.371613	0.134517	0.283794	0.117014
14	0.449625	0.146087	0.311779	0.096908
15	0.510355	0.158047	0.307058	0.094017
16	0.481798	0.172252	0.218269	0.100588
17	0.468006	0.182266	0.188489	0.092035
18	0.465440	0.198109	0.157746	0.081595
19	0.527681	0.207107	0.146353	0.066511
20	0.639423	0.223229	0.182646	0.110780
21	0.805617	0.247337	0.235497	0.153827
22	1.044452	0.277170	0.290216	0.189102
23	1.317865	0.326539	0.366483	0.240061
24	1.359723	0.393608	0.384313	0.297411
25	1.295315	0.413919	0.209831	0.155952

* Excluding sectors 5, 6, and 14.

is in order. This is *not* technological change, as contended by H. Simon (1951). Input–output coefficients are the result of both an ex-ante technology (for example, as expressed by a neoclassical production function) and relative input and factor prices. This subject will be explored in Sections 7 and 8.

Table 4 shows the average value of the components of (6.3) relative to intermediate demand change: $(\Delta C)f^*/\Delta z$ and the sign of $C^*\Delta f/\Delta z$, expressed in percentages, over the ten periods 1963–1964 to 1972–1973 for the 46 intermediate sectors.

Signs of $C^*\Delta_f/\Delta_z$ are shown, although it is not necessary, for in all cases with positive $(\Delta C)f^*/\Delta_z$ the percentage value is below 100. If one looks at the number of negative signs of $(\Delta C)f^*$ one finds that most "traditional" sectors have a large number of them: agriculture and forestry have 9, other textiles (rough fibers, mainly sisal), tannery and leather goods, wood and cork, real estate, and, surprisingly, construction, have 8. Another surprise is the 7 negative signs in the petroleum sector.

On the other hand, the "modern" sectors (to which petroleum should belong) show little or no trace of negative $(\Delta C)f^*$. In general, leaving aside very large

TABLE 4

COMPONENTS OF CHANGES IN THE DEMAND FOR INTERMEDIATE INPUTS (percentages with respect to Δz) 1963–1974

		Coefficient change		Final demand change	
Sector	Negative Δz	Average 1963–1974	No. of Negative Signs of Component	No. of Negative Signs of Component	Sign of Average 1963–1974
1. Agriculture	5	369.5	9	0	–
2. Livestock	0	−39.4	7	0	+
3. Forestry	2	−239.5	9	0	+
4. Fishing	4	152.3	6	0	–
5. Mining, metals	5	−1757.9	6	0	+
6. Mining, non-metals	4	663.1	6	0	–
7. Petroleum and first stage petrochemicals	0	−15.4	7	0	+
8. Meat and dairy products	0	0.3	3	0	+
9. Wheat and corn products	2	33.3	3	1	+
10. Other foodstuffs	2	52.5	5	0	+
11. Beverages	1	−0.02	4	1	+
12. Tobacco	1	30.5	5	0	+
13. Textiles, soft fibers	0	20.2	2	0	+
14. Other textiles	8	149.0	8	0	–
15. Clothing and footwear	1	46.6	3	0	+
16. Wooden and cork products	2	−3023.3	8	0	+
17. Paper and pulp	1	30.3	5	0	+
18. Printing, editorial	2	212.1	5	0	–
19. Tannery and leather goods	2	1.2	8	0	+
20. Rubber industry	0	−12.5	5	0	+
21. Basic chemicals	0	−11.5	4	0	+
22. Synthetic materials	0	51.8	0	0	+
23. Fertilizers and pesticides	2	111.1	3	0	–
24. Soaps and detergents	0	−4.0	3	0	+
25. Pharmaceuticals	0	−30.6	3	0	+
26. Cosmetics	0	37.9	0	0	+
27. Other chemicals	1	80.2	2	0	+
28. Processing of non-metals	0	6.2	4	0	+
29. Basic metallurgical industry	1	177.5	6	0	–
30. Metal-mechanical	1	22.4	4	0	+
31. Non-electric machinery	1	−65.2	4	0	+
32. Electric machinery	2	50.9	4	0	+
33. Transport equipment	3	28.1	6	0	+
34. Automotive	3	233.4	5	0	–
35. Other manufactures	0	−54.4	6	0	+
36. Construction	3	−4.3	8	0	+
37. Electricity	0	34.2	0	0	+
38. Films and recreation	2	770.2	6	0	–
39. Transportation	0	−112.7	7	0	+
40. Communications	0	20.2	1	0	+
41. Trade margins	0	6.4	4	0	+
42. Real estate	0	−70.6	8	0	+
43. Hotels and restaurants	0	−0.03	4	0	+
44. Banking, finance and insurance	0	54.2	5	0	+
45. Other services	0	−49.3	7	0	+
46. Government services	0	−34.3	4	0	+

TABLE 5
1973 GROSS OUTPUT: ACTUAL AND UNDER CONSTANT 1963 INPUT–OUTPUT MATRIX
(millions of 1960 pesos)

Sector	Gross Output			
	1963 (1)	1973 (2) Actual	1973 (3) under 1963 coeff.	Ratio (2/3)
1. Agriculture	21,762	26,304	31,646	0.831195
2. Livestock	12,805	22,020	22,312	0.986913
3. Forestry	1,056	1,388	1,792	0.774554
4. Fishing	676	902	1,089	0.828283
5. Mining, metals	3,088	3,354	5,770	0.581282
6. Mining, non-metals	1,444	2,390	3,480	0.686782
7. Petroleum and first stage petrochemicals	11,668	24,893	25,109	0.991398
8. Meat and dairy products	4,077	7,807	7,728	1.010223
9. Wheat and corn products	10,107	15,021	14,657	1.037115
10. Other foodstuffs	14,400	26,942	27,467	0.980886
11. Beverages	4,968	9,649	9,620	1.003015
12. Tobacco	1,405	2,301	2,302	0.999566
13. Textiles, soft fibers	5,350	15,592	14,634	1.065464
14. Other textiles	1,556	1,338	2,334	0.573265
15. Clothing and footwear	5,982	16,202	16,422	0.986604
16. Wooden and cork products	1,983	2,008	2,611	0.769054
17. Paper and pulp	2,935	6,518	6,421	1.015107
18. Printing, editorial	1,903	4,020	3,986	1.008530
19. Tannery and leather goods	1,253	2,159	2,743	0.787094
20. Rubber industry	1,417	3,308	3,393	0.974948
21. Basic chemicals	1,480	4,449	4,054	1.097435
22. Synthetic materials	959	6,665	3,795	1.756258
23. Fertilizers and pesticides	1,035	2,536	2,015	1.258560
24. Soaps and detergents	1,155	2,682	2,617	1.024838
25. Pharmaceuticals	2,394	5,666	5,525	1.025593
26. Cosmetics	910	2,943	2,746	1.071743
27. Other chemicals	1,539	3,753	3,614	1.038463
28. Processing of non-metals	2,910	8,419	8,084	1.041350
29. Basic metallurgical industry	6,048	15,122	15,620	0.968118
30. Metal mechanical	3,200	7,604	7,801	0.974799
31. Non-electric machinery	1,149	4,975	4,243	1.172524
32. Electric machinery	3,072	9,075	8,777	1.033894
33. Transport equipment	1,301	2,806	2,949	0.951502
34. Automotive	3,774	15,598	15,191	1.026799
35. Other manufactures	1,778	3,881	3,761	1.031907
36. Construction	16,921	38,814	39,846	0.9741102
37. Electricity	3,005	9,266	7,735	1.197986
38. Films and recreation	2,332	3,682	3,672	1.002942
39. Transportation	7,993	14,660	14,877	0.985380
40. Communications	1,091	3,023	2,511	1.204206
41. Trade margins	63,274	126,645	123,337	1.026825
42. Real estate	14,932	23,836	24,617	0.968290
43. Hotels and restaurants	5,257	11,374	11,231	1.012697
44. Banking, finance and insurance	3,589	8,126	7,822	1.038800
45. Other services	9,518	15,989	15,615	1.023945
46. Government services	13,235	28,725	27,796	1.033423

figures (due to very small Δ_z), Table 4 shows that coefficient change is a non-trivial determinant of the changes in intermediate demand.

Another way of looking at this is the following: hold the coefficient matrix constant at $t = t_0$, and look for the effects of only final demand changes up to $t = T$. Table 5 shows the result of doing this for $t_0 = 1963$ and $T = 1973$; that is, hold the matrix for 1963 constant and look for the value of z (or X) in 1973, letting final demand vary from 1964 onwards as observed. Table 5 shows 1963 gross output (the starting point), observed 1973 gross output, and the values of gross output in 1973 if the intermediate coefficient matrix were held constant at its 1963 value. The ratio of actual to hypothetical 1973 values is the proportional gain (or loss if less than 1) in gross output by each sector due to coefficient change during the decade. We find thus that agriculture loses gross output at an annual rate of 1.7 percent $\{(1 - 0.831195)/10\}$, forestry, of 2.25 percent per year, mining of metals, 4.2 percent, mining of non-metals, 3.13 percent, rough-fiber textiles, 4.3 percent, while basic chemicals gain at a rate of almost 1 percent per year, synthetic materials, at 7.6 percent, fertilizers and pesticides, 2.6 percent, etc. This is fairly consistent with what can be seen in Table 4. One sees that the surprising 7 negative signs of the petroleum sector have little consequence: a gross output loss rate of less than one percent. Its considerable growth is therefore due to final demand.

It can be concluded that, as an effect of prices or as an effect of technological change, there is a clear substitution of "traditional" inputs (agricultural stuffs, minerals) for synthetic inputs. Exploration into the causes (prices and technology) will be pursued in the next section.

7. Technological Change in a Cobb–Douglas World

In this section we assume a Cobb–Douglas production function generating both input and factor coefficients. Of course, all is reducible to a production function with factors as arguments, but we have seen that there are important changes in intermediate coefficients and must try to derive an explanation. I am going to be concerned only with input coefficients. Using the results in Section 2, we get the basic equation

$$a_{ij}(t + 1) = \mathrm{r}_i^{\mathrm{CD}} a_{ij}(t) s_j^{\mathrm{CD}} \tag{7.1}$$

where $\mathrm{r}_i^{\mathrm{CD}} = p_i(t)/p_i(t + 1)$ and $s_j^{\mathrm{CD}} = p_j(t + 1)/p_j(t)$. We call (7.1) the Cobb–Douglas RAS model. Our purpose in this section is to compare it with the "ordinary" RAS model, i.e., $a_{ij}(t + 1)$ obtained from Stone's algorithm, subject to National Accounts marginals. Again, some way of synthesizing a large amount of information (46 × 46 coefficients × 25 years) will be needed. Let $a_{ij}(t + 1)$ be obtained from ordinary RAS. If the Cobb–Douglas model is sustained, the discrepancy with (7.1) is explained as a change in α_{ij}; we know that α_{ij} is the money coefficient $p_i a_{ij}/p_j$; hence we may estimate:

$$\begin{aligned} \frac{\alpha_{ij}(t + 1)}{\alpha_{ij}(t)} &= \frac{p_i(t + 1)a_{ij}(t + 1)p_j(t)}{p_j(t + 1)a_{ij}(t)p_i(t)} \\ &= \mathrm{r}_i s_j / \mathrm{r}_i^{\mathrm{CD}} s_j^{\mathrm{CD}} \end{aligned} \tag{7.2}$$

where r_i and s_j are ordinary RAS coefficients. Table 6 shows yearly averages from 1963 to 1972 of average values of $a_{ij}(t + 1)/a_{ij}(t)$ along rows (output coefficients) and columns (input coefficients). Table 6 also shows the mean values of the predicted change under Cobb–Douglas assumptions. From there, an approximation to (7.2) is obtained. Indeed, the predicted Cobb–Douglas change is $r_i^{CD}s_j^{CD}$, so that (7.2) is the ratio of ordinary RAS to Cobb–Douglas RAS. I approximate the mean (7.2) by the ratio of the mean ordinary RAS to the mean Cobb–Douglas RAS changes.

Changes in the rows are related to increases in the intermediate demand of input i; row averages (7.2) will show the average annual increase ratio (if greater than 1) in the use of input i, *independent of price changes.* The Cobb–Douglas predicted change will show that part of the increase in the use of i which is due to a lower relative price; the final (multiplicative) result is the RAS-predicted change.

Changes along the column may denote several things. If less than one, average (7.2) will show that the technology of sector j (*ex-ante production function*) consumes less intermediate inputs. With a great deal of wishful thinking, one may interpret this component as the complement of an annual productivity increase rate, but I am not much willing to venture such a debatable interpretation. The Cobb–Douglas prediction will show the expected increase (or decrease) in physical input due to price changes; if less than one, it shows that the price of output has risen above the price of inputs, and vice versa. The final result is the RAS-predicted change, showing "actual" increase or decrease as the product of changes due to prices and changes due to "technology."

On examining Table 6 one concludes that, even if most changes are small, there is a striking consistency with the results of Section 6. We find that predicted Cobb–Douglas changes are very small; "traditional" sectors show a considerable rate of ex-ante change (agriculture, 6.5 percent per year, mining of metals, 11.4, mining of non-metals, 5.8) leading to a decrease in average *output* coefficients. Petroleum is predicted to decrease on the base of prices (Cobb–Douglas forecast), but grow on account of ex-ante change. "Modern" sectors show the same pattern as in Section 6: synthetic materials with ex-ante average growth of 16 percent per year (decrease if only prices are taken), fertilizers, 7 percent, electricity, almost 8 percent, etc.

Input coefficients would increase as a result of prices in all sectors (thus probably leading to smaller added values); their increase occurs also in observation; in the "traditional" sectors, there is an ex-ante increase, while those identified as "modern" show decreases. Probably this could reinforce the "productivity" conjecture raised above.

One may also think that the presence of what I have been handling as ex-ante changes may be interpreted as evidence that the elasticity of substitution is not equal to one. This is a matter for further research.

8. A Production-Function-Free Model of Technological Change

Let x_j be gross output of sector j, x_{ij}, the flow of input i into sector j. We know that money coefficients $p_i x_{ij}/p_j x_j$ are nothing but the exponents of a

TABLE 6

ORDINARY RAS AND COBB–DOUGLAS RAS COEFFICIENT CHANGES; APPROXIMATE EX-ANTE CHANGES
(TIME AVERAGES OF ROW AND COLUMN AVERAGE PROPORTIONAL CHANGES)

	Output Coefficients			Input Coefficients		
	RAS	Cobb–Douglas	ex-ante (approx.)	RAS	Cobb–Douglas	ex-ante (approx.)
1. Agriculture	0.931723	0.995898	0.935561	1.101504	1.064676	1.034591
2. Livestock	0.991286	0.993453	0.997819	1.115847	1.064936	1.047807
3. Forestry	0.959341	0.992927	0.966174	1.062003	1.069831	0.992683
4. Fishing	0.944377	1.027358	0.919229	1.085964	1.028348	1.056028
5. Mining, metals	0.915238	1.032182	0.886789	1.135180	1.027369	1.104939
6. Mining, non-metals	0.951225	1.008807	0.942921	1.076140	1.055968	1.019103
7. Petroleum and first stage petrochemicals	1.008422	0.979757	1.029257	1.052706	1.085403	0.969876
8. Meat and dairy products	1.001062	1.004097	0.996977	1.069771	1.058522	1.010627
9. Wheat and corn products	0.984064	0.985222	0.998824	1.098506	1.075298	1.021583
10. Other foodstuffs	0.975993	1.000650	0.975359	1.105121	1.059708	1.042854
11. Beverages	1.005922	1.016248	0.989839	1.072753	1.037651	1.033828
12. Tobacco	0.995057	0.992558	1.002518	1.077406	1.068084	1.008728
13. Textiles, soft fibers	1.032729	0.994724	1.038207	1.061889	1.067799	0.994465
14. Other textiles	0.921024	1.003371	0.917930	1.107976	1.059589	1.045666
15. Clothing and footwear	1.033424	1.022873	1.010315	1.060727	1.037537	1.022351
16. Wooden and cork products	0.978263	0.983850	0.994321	1.084358	1.078522	1.005411
17. Paper and pulp	1.005950	0.996640	1.009341	1.065985	1.063188	1.002631
18. Printing, editorial	1.003600	0.994312	1.009341	1.062540	1.067623	0.995239
19. Tannery and leather goods	0.975897	1.020077	0.956690	1.066214	1.037287	1.027887
20. Rubber industry	1.008652	0.971425	1.038322	1.042108	1.094192	0.952400
21. Basic chemicals	1.014258	0.981641	1.033227	1.068200	1.082579	0.986718

22. Synthetic materials	1.101757	0.949761	1.160036	1.042744	1.119669	0.931297
23. Fertilizers and pesticides	1.029184	0.960659	1.071331	1.062801	1.104836	0.961954
24. Soaps and detergents	1.026202	-0.988907	1.037713	1.066132	1.075100	0.991658
25. Pharmaceuticals	1.018786	0.985221	1.034068	1.048184	1.078394	0.971986
26. Cosmetics	1.055907	0.984600	1.072422	1.058008	1.079489	0.980101
27. Other chemicals	1.008585	0.977082	1.032242	1.080572	1.086084	0.994925
28. Processing of non-metals	1.018246	1.000810	1.017422	1.062930	1.062106	1.000776
29. Basic metallurgical industry	1.003134	0.985746	1.017639	1.075779	1.077620	0.998292
30. Metal mechanical	1.008001	0.999303	1.008704	1.066057	1.062848	1.003019
31. Non-electric machinery	1.066220	0.999318	1.066948	1.071935	1.062396	1.008979
32. Electric machinery	1.033827	0.992305	1.041844	1.047592	1.073063	0.976263
33. Transport equipment	0.994109	1.034779	0.960692	1.068490	1.027840	1.039549
34. Automotive	1.058586	0.984474	1.075280	1.053750	1.081188	0.974677
35. Other manufactures	1.029775	1.003403	1.026283	1.067475	1.051852	1.014853
36. Construction	0.973637	1.012945	0.961194	1.062212	1.048222	1.013346
37. Electricity	1.045044	0.970323	1.077006	1.028329	1.098521	0.936103
38. Films and recreation	1.006413	1.025257	0.981620	1.061248	1.038058	1.022340
39. Transportation	1.007250	0.983410	1.024242	1.058311	1.080776	0.979214
40. Communications	1.035518	0.999801	1.037241	1.051876	1.062189	0.990291
41. Trade margins	1.013760	1.003511	1.010214	1.054345	1.059972	0.994691
42. Real estate	0.988303	1.012572	0.976032	1.063765	1.049778	1.013324
43. Hotels and restaurants	1.021269	1.015265	1.005913	1.078929	1.047135	1.030363
44. Banking, finance and insurance	1.009613	1.020183	0.989639	1.089968	1.040966	1.047074
45. Other services	0.992274	1.043743	0.950688	1.059866	1.018779	1.040330
46. Government services	1.002181	1.012957	0.989362	1.030892	1.051019	0.980850

Cobb–Douglas production function, if this is the picture of technology ex-ante. If technology ex-ante obeys a CES, physical coefficients will be given by:

$$a_{ij} = x_{ij}/x_j = \left(\frac{p_i}{p_j\theta_{ij}}\right)^{-\sigma_j}$$

where

$$x_j = (\Sigma_i\theta_{ij}x_{ij}^{-\rho_j})^{-1/\rho_j} \quad \text{and} \quad \rho_j = (1-\sigma_j)/\sigma_j$$

Suppose elasticities of substitution σ_{ikj}, of input i for input k in sector j are not necessarily equal, as in the CES, but at least locally constant:

$$\sigma_{ikj} = -\Delta\log(x_{ij}/x_{kj})/\Delta\log(p_i/p_k) \tag{8.1}$$

Let, for fixed j, δ be the vector of elements $\Delta\log p_i$, ξ_j be the vector of elements $\Delta\log x_{ij}$, and S_j the matrix of elements σ_{ikj}. Let, in general, $\hat{v}$ be the diagonal matrix constructed from vector v and ι (iota) be a vector of units. Then (8.1) may be written as

$$S_j\hat{\delta} - \hat{\delta}S_j = \xi_j\iota' + \iota\xi_j' \tag{8.2}$$

which allows in principle the determination of ξ_j. If one approximates the right-hand side to the left-hand side of (5.2) by least squares, under $\iota'\xi_j = 0$, one gets[9]

$$\xi_j = \frac{1}{m}(S_j\delta - \hat{\delta}S_j\iota) \tag{8.3}$$

where m is the number of inputs. Re-writing (5.3) one sees that:

$$\begin{aligned}\Delta\log x_{ij} &= \frac{1}{m}(\Sigma_k\sigma_{ikj}\Delta\log p_k - \Delta\log p_i\Sigma_k\sigma_{ikj})\\ &= \mu_{ij}\{\Sigma_k\xi_{ikj}\Delta\log p_k - \Delta\log p_i\} \qquad (8.4)\end{aligned}$$

say. The first term within braces in the last expression in (8.4) is a weighted average of log-changes in prices, with weights proportional to the *ex-post* elasticities of substitution (8.1); it may be seen as the log-change of a price index:

$$\Delta\log\pi_{ij} = \Sigma_k\xi_{ijk}\Delta\log p_k \tag{8.6}$$

Now, μ_{ij} can be evaluated as follows:

$$\begin{aligned}\mu_{ij} &= \frac{1}{m}\Sigma_k\sigma_{ikj} = \frac{1}{m}S_j\iota\\ &= \frac{1}{m}\hat{\delta}^{-1}(S_j\hat{\delta} - \xi_j\iota' + \iota\xi_j')\iota\end{aligned}$$

using (5.2). This equals

$$\frac{1}{m}S_j\iota = \frac{1}{m}\hat{\delta}^{-1}S_j\delta - \hat{\delta}^{-1}\xi_j$$

[9] Minimize tr $(S_j\hat{\delta} - \hat{\delta}S_j - \xi_j\iota' + \iota\xi_j')'(S_j\hat{\delta} - \hat{\delta}S_j - \xi_j\iota' + \iota\xi_j')$ under $\iota\xi_j = 0$.

since $\iota'\xi_j = 0$. The first term above is

$$\frac{1}{m}\Sigma_k \sigma_{ikj}(\Delta \log p_k / \Delta \log p_i) = \mu_{ij} \Delta \log \pi_{ij} / \Delta \log p_i$$

The second is $\Delta \log x_{ij} / \Delta \log p_i$. Hence:

$$\mu_{ij}(1 - \Delta \log \pi_{ij} / \Delta \log p_i) = \Delta \log x_{ij} / \Delta \log p_i$$

μ_{ij} is a modified direct price elasticity for x_{ij} if $\Delta \log \pi_{ij}/\Delta \log p_i = 0$ (as in any partial equilibrium model) μ_{ij} is the direct price elasticity of x_{ij}.

All this suggests a Barten–Theil type of demand equations for x_{ij}, say

$$\Delta \log x_{ij} = \mu_{ij} \Delta \log (p_i / \pi_{ij})$$

$$\mu_{ij} \Sigma_k C_{ikj} \Delta \log p_k \tag{8.7}$$

Ex-ante technological change will be measured by changes in the parameters of equations (8.7). This would require a very large sample indeed; it has been suggested to use equations of the type

$$\Delta \log x_{ij} = C_{i0j} + \Sigma_k C_{ikj} \Delta \log p_k \tag{8.8}$$

where C_{i0j} is a rate of change of x_{ij} in time, and would be taken as caused by technological changes, and C_{ikj} is proportional to an average or long-run elasticity of substitution. Empirical research on equations (8.7) and (8.8) is being conducted presently.

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