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A NOTE ON INTERSECTORAL SHIFTS AND AGGREGATE PRODUCTIVITY CHANGE

BY JACK BEEBE*

INTRODUCTION

This note is an elaboration of an earlier paper, "Intersectoral Shifts and Aggregate Productivity Change," by Grossman and Fuchs [1]. The note investigates certain aspects of their analysis; the conclusions presented here support those of Grossman and Fuchs (G&F). In particular, this note: (a) derives equations for aggregate output per manhour under two-sector growth in a simplified and more straightforward fashion; (b) discusses and interprets these equations in light of G&F's (equivalent) equations for aggregate output per manhour; (c) explores the conditions under which intersectoral (labor) shifts have significant effects on aggregate output per manhour; and (d) suggests very briefly sectoral breakdowns other than "goods and services" to which their methodology is applicable.

The work presented here began independently of G&F's study, but the refinements have benefited greatly from the detailed treatment provided in their paper. An earlier paper by Mark [2] suggested the problem to this author. Mark's paper focused on measuring productivity in government. But it also raised the question as to how a different measure of productivity in government (attributable to more refined measures of government real output) would affect aggregate output per manhour in the total economy. Mark weighted annual gains in sector outputs per manhour by sector labor shares to obtain the annual gain in the aggregate index. The weighting scheme raised serious questions about the proper weights—questions that are answered by G&F and this note. This note closes with a very brief mention of applications such as this one.

AN ALTERNATIVE DERIVATION OF THE AGGREGATE OUTPUT PER MANHOUR EQUATIONS

This section will derive the equations for aggregate output per manhour in a different fashion than done by G&F. The intended result is both a more straightforward derivation and hopefully, equations that are more easily interpreted. The derivations use a notation identical to that of G&F:

XG_t = goods output in constant dollars in year t
XS_t = service output in constant dollars
$X_t = XG_t + XS_t$ = total output in constant dollars
$x_t = XG_t/X_t$ = goods sector's share of constant dollar output

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HG_t = manhours employed in the goods sector
HS_t = manhours employed in the service sector
$H_t = HG_t + HS_t$ = total manhours employed
$h_t = HG_t/H_t$ = goods sector's share of manhours employed
$AG_t = XG_t/HG_t$ = output per manhour in the goods sector in constant dollars
$AS_t = XS_t/HS_t$ = output per manhour in the service sector in constant dollars
A_t = aggregate output per manhour in constant dollars
rg = constant annual growth rate of AG_t
rs = constant annual growth rate of AS_t
$k_t = AS_t/AG_t$ = output per manhour in the service sector relative to the goods sector
$Z_{t,i} = Z_t/Z_i$ = growth factor of Z (any variable) in year t relative to year i, where $t > i$.

In order to derive weights for the calculation of aggregate output per manhour, consider the following simple formulation of dual sector growth:

$$\begin{aligned} &\text{Goods Sector: } XG_i(HG_{t,i})(AG_{t,i}) = XG_t \\ &\text{Service Sector: } \underline{XS_i(HS_{t,i})(AS_{t,i}) = XS_t} \\ &\text{Aggregate: } X_i(H_{t,i})(A_{t,i}) = X_t. \end{aligned} \tag{1}$$

The above formulation says that for each sector and for the aggregate, real product in period t increases over real product in period i by the product of the growth factor in manhours and in output per manhour. Solving the bottom line of (1) for $A_{t,i}$ and substituting values for the first two lines,

$$\begin{aligned} A_{t,i} &= \frac{X_t}{X_i H_{t,i}} = \frac{XG_t + XS_t}{X_i H_{t,i}} = \frac{XG_i HG_{t,i} AG_{t,i} + XS_i HS_{t,i} AS_{t,i}}{X_i H_{t,i}}, \\ \text{or } A_{t,i} &= x_i(HG_{t,i}/H_{t,i})AG_{t,i} + (1 - x_i)(HS_{t,i}/H_{t,i})AS_{t,i}. \end{aligned} \tag{2}$$

Since,

$$h_i(HG_{t,i}/H_{t,i}) = h_t, \tag{3}$$

equation (2) can be written as

$$A_{t,i} = x_i[h_t/h_i]AG_{t,i} + (1 - x_i)[(1 - h_t)/(1 - h_i)]AS_{t,i}. \tag{2'}$$

Equations (2) and (2′) give two ways in which to express the general formula for aggregate output per manhour. Note that in equation (2) the weights for each sector are the real product shares in period i and the relative growths of labor inputs over $t - i$ periods (that is, the growth factor of labor in that sector relative to the aggregate growth factor of labor). In the equivalent equation (2′), the weights are the real product shares in period i and the ratios of the sectors' shares of labor in period t relative to period i. Equations (2) and (2′) are equivalent to G&F's

more elaborate equation (2).[1] It may be argued that these equations (2) and (2′) do not decompose output per manhour into the "rate, level, and interactions" effects. Actually, equations (2) and (2′) break output per manhour into two components: a "rate" effect and a "level plus interaction" effect. (Instead of the effects' being additive, they are multiplicative, which is more manageable in the opinion of this author.) The note will return to why combining the "level and interaction" effects is perhaps more meaningful.

Consider two important special cases of equations (2) and (2′), both of which are developed by G&F.

Case # 1: Constant Labor Shares

If sector labor shares remain constant over time, then sector manhours must grow at identical rates, and conversely. In this case, equations (2) and (2′) reduce to

$$A_{t,i} = x_i AG_{t,i} + (1 - x_i)AS_{t,i}. \tag{4}$$

If one assumes constant labor shares over time, then real product shares in period i become the weights in the calculation of aggregate output per manhour, and conversely.

Case # 2: Constant Real Product Shares

For real product shares to remain constant over time, real products in both sectors must grow at identical rates. A necessary and sufficient condition is that the combination of manhour increase and output per manhour increase be identical in both sectors, or

$$HG_{t,i}AG_{t,i} = HS_{t,i}AS_{t,i} = H_{t,i}A_{t,i}. \tag{5}$$

Solving for $A_{t,i}$,

$$A_{t,i} = (HG_{t,i}/H_{t,i})AG_{t,i} = (HS_{t,i}/H_{t,i})AS_{t,i}, \tag{6}$$

or

$$A_{t,i} = [h_t/h_i]AG_{t,i} = [(1 - h_t)/(1 - h_i)]AS_{t,i}. \tag{6′}$$

Equations (6) and (6′) say that under constant real product shares, aggregate output per manhour can be found simply by weighting output per manhour in either sector by its relative growth factor of labor input or by the ratio of its

[1] G&F's equation for aggregate output per manhour contains an additional variable, k_t. However, there is a fixed relation among x_t, h_t, and k_t, so that k_t is determined given x_t and h_t. A short derivation of this relation for any year t is:

$$k_t \equiv \frac{AS_t}{AG_t} = \frac{XS_t/HS_t}{XG_t/HG_t}.$$

Dividing all terms by X_t to obtain shares

$$k_t = \frac{(XS_t/X_t)/(HS_t/X_t)}{(XG_t/X_t)/(HG_t/X_t)} = \frac{(1 - x_t)/(1 - h_t)}{x_t/h_t} = \frac{(1 - x_t)/x_t}{(1 - h_t)/h_t}.$$

This equation says that output per manhour in the service sector relative to the goods sector, k_t, is the service sector's share of output relative to the goods sector's divided by the service sector's share of manhours relative to the goods sector's. The relation among x_t, h_t, and k_t is also derived by G&F (p. 234) and is used throughout their analysis and this note.

labor share in period t to that in period i. (Note that real products do not enter this calculation.) What happens if equation (6) is substituted into the general result in equation (2)? Substituting equation (6) into equation (2),

$$A_{t,i} = x_i A_{t,i} + (1 - x_i) A_{t,i} = A_{t,i}.$$

The real product share weights have no effect on the calculated $A_{t,i}$. Under the assumption of constant real product shares, any weights which sum to one could be used in place of x_i and $(1 - x_i)$ in equation (2). In order to simplify equation (2) under the assumption of constant real product shares, then, substitute h_i and $(1 - h_i)$ for x_i and $(1 - x_i)$.

$$A_{t,i} = h_i (HG_{t,i}/H_{t,i}) AG_{t,i} + (1 - h_i)(HS_{t,i}/H_{t,i}) AS_{t,i}. \tag{7}$$

Using equation (3) to simplify equation (7),

$$A_{t,i} = h_t AG_{t,i} + (1 - h_t) AS_{t,i}. \tag{8}$$

Under the assumption of constant real product shares, the weights become labor shares in the final period. Conversely, if one uses final period labor shares as weights, then the assumption of constant real product shares is implied. (Note that equation (6) can also be used.)

The above results are no different from those of G&F, but the derivations are perhaps shorter and more easily understood. In the next section, the interpretation of the "rate" and "level plus interaction" effects in equation (2) will be somewhat different from theirs.

The Effect of Changing Employment Shares on Aggregate Output per Manhour

This section will (a) interpret equation (2) by decomposing the change in aggregate output per manhour into two "effects" and comparing these to G&F's three "effects"; (b) discuss some implied assumptions resulting from G&F's choice of simulation parameters for their secular simulations; and (c) comment very briefly on output per manhour differentials between some sectors other than goods and services.

Equations (2) and (2′) in this note (or equation (2) in G&F) give aggregate output per manhour for the general case—that is, allowing for shifts in employment and real output shares over time. But suppose employment shares remain constant over time. What then would aggregate output per manhour be? Aggregate output per manhour in this case can be found by using equation (4). (This equals G&F's "rate" effect.) What is the difference between aggregate output per manhour in this case first assuming constant employment shares and then allowing for changing employment shares? It is the difference between that found using equations (2) and (4). (This equals G&F's "level" plus "interaction" effects.)

The similarity of the method used in this note to G&F's approach can be summarized as follows: (a) Aggregate output per manhour assuming constant employment shares over time—equation (4)—is equivalent to G&F's "rate" effect; and (b) the difference between aggregate output per manhour allowing for shifting employment shares—equation (2)—and that calculated assuming no change in employment shares—equation (4)—is equivalent to G&F's "level" plus "interaction" effects.[2]

Combining the "level" and "interaction" effects simplifies the analysis, but one is entitled to ask the cost of this simplification in terms of lost generality. The cost is negligible unless one wants to examine specifically the "pure level" effect. This case is really only interesting when $rs = rg$, that is, when the rates of productivity increase are identical in the two sectors, or in the special case where $rs = rg = 0$. But G&F do not consider this case (except to exemplify the "pure level" effect), and this author feels also that it is not terribly interesting. Furthermore, it can be analyzed using equations (2) or (2′) in this note, although the importance of k_i in this case is implicit in the values of x_i and h_i.

Let us now turn to G&F's choice of parameter values for their secular simulations. Consider G&F's examples of the "level" effect (the top of page 237 of their paper). The analysis here will combine their "level" and "interaction" effects.

TABLE 1

INDEX OF AGGREGATE OUTPUT PER MANHOUR

	Explicit Assumptions $rg = 2\%$, $k_1 = 2.00$, 1st decade	
Effects on Aggregate Index	$x_1 = 0.4$	$x_1 = 0.6$
Labor shares variant[a]	104.8	132.2
Labor shares constant[b]	115.0	117.3
Difference[c]	−10.2	14.9

[a] Calculated using equation (2) in this note or equation (2) in G&F.

[b] Calculated using equation (4) in this note or the first term of equation (2) in G&F.

[c] Calculated by subtracting row 2 from row 1 in this table or using the last two terms of equation (2) in G&F.

Why is it that the "differences" in Table 1 are so large and in opposite directions? The answer becomes obvious when, from the "explicit" assumptions in Table 1, one derives "implicit" assumptions for these cases.[3] The picture is completed in Table 2.

TABLE 2

CALCULATED IMPLICIT ASSUMPTIONS

	$rg = 2\%$, $k_1 = 2.00$, 1st decade	
Implicitly Assumed	$x_1 = 0.4$	$x_1 = 0.6$
k_{11}	1.813	1.813
x_{11}	0.585	0.415
h_1	0.571	0.750
h_{11}	0.719	0.562

[2] Note that the same analysis can be performed for (a) aggregate output per manhour assuming constant real output shares over time, equations (6) or (8); and (b) the difference between aggregate output per manhour allowing for shifting real output shares—equation (2)—and that calculated assuming no change in output shares—equations (6) or (8).

[3] Footnote 1 of this note gives the relation among x, h, and k, and G&F give other formulae on p. 234.

Examining the "implicit" assumptions in Table 2, two explanations stand out: (a) there are very large differences in the levels of output per manhour in both cases ($k_1 = 2.00$ and $k_{11} = 1.81$); and (b) there are extremely rapid shifts in employment (and real output) shares in both cases. Furthermore, these shifts are in opposite directions. In the first case where $x_1 = 0.4$, the real output share of the goods sector (whose productivity level is only half that in the service sector) goes from 40 percent to almost 59 percent and the labor share rises from 57 percent to 75 percent over a period of only 10 years. In the second case where $x_1 = 0.6$, the same sort of shift occurs except that the shift in output and employment is from the high (level) productivity sector to the low (level) productivity sector. It's no wonder that the "level" effects are large and in opposite directions.

G&F use these cases as illustrations and, of course, dismiss secular shifts of this consequence as unrealistic. In fact, their cyclical shifts are not even this great on an annual basis. However, their simulations leave the reader with the impression that the secular "level" or "level plus interaction" effects can be larger than reality suggests. This impression is caused by the wide (extreme) range of choices for k and the specifications of x over time relative to the rather conservative ranges of rs and rg. For example, consider their specification of x_t, given x_1 and x_{51}:

$$x_t = \hat{x} + b/t.$$

This specification of x_t has an important property which is not discussed. For $x_1 = 0.4$ or $x_1 = 0.6$, almost the entire (large) change in real output shares occurs in the first decade. and this forces rapid labor shifts in the first decade. (Table 2 of this note shows that 18.5 of the 20 percentage point shift in output shares for the five decades occurs in the first decade.) Beyond the first decade, one can safely assume constant real output shares.

Upon careful examination of G&F's Tables 4 and 5, one finds two general situations which lead to large "level" or "level plus interaction" effects. The first occurs in the first decade when $x_1 = 0.4$ or $x_1 = 0.6$ and k_1 differs significantly from 1. This case was discussed above, and it was found that there were extremely rapid real output and labor shifts between sectors with significantly different levels of output per manhour. The second case occurs in the fifth decade (where $rg = 3$ percent). Since real output shares are essentially constant for all simulations in the fifth decade, changing output shares are not the cause. When the "level plus interaction" effect is significant in the fifth decade, one finds relative output per manhour levels substantially different between the two sectors and labor shifting rapidly. For example, take the case in the fifth decade where $x_1 = 0.4$, $rg = 3$ percent, and $k_1 = 0.80$. The "level plus interaction" component is -6.7 (the largest value for the fifth decade). For this case, $k_{41} = 0.37$, $k_{51} = 0.30$, $h_{41} = 0.35$, and $h_{51} = 0.31$. In other words, the "level plus interaction" effect is relatively large because labor is shifting into the service sector which during this decade has a level of output per manhour only *one-third* that in the goods sector. (Note that this shift is necessary in order that real output shares remain constant.)

What does one conclude from all this? First, G&F's conclusion that, except possibly for agriculture, intersectoral shifts have not had a major impact on aggregate productivity change (p. 238) is strengthened when one examines the assumptions underlying cases where the "level plus interaction" effect is large. Second, it may be simpler to combine G&F's "level" and "interaction" effects without loss of generality, and use more readily interpreted equations such as (2) and (4) in this note.

In closing, G&F's equations (or the equations in this note) and their simulation methodology are very helpful in understanding the effect of intersectoral shifts on aggregate output per manhour. The methodology should be applied to differentials between other sectors such as agriculture and nonagriculture, or government and private. The simulation parameter values used by G&F for goods and services are not directly applicable to these other sectoral breakdowns. For example, the values of x for these cases are nowhere near 0.5 and they shift substantially over time (at least for the postwar span); the rate differentials are also greater for these cases, particularly for agriculture. The methodology is also applicable to Mark's study [2] of alternative productivities in government mentioned earlier, and to studies such as Denison's recent paper [3] in which he considers several sectors. The equations and simulation method can be extended to more than two sectors, although the algebra will undoubtedly become tedious.

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