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Chapter Title: An Integrated View of Tests of Rationality, Market Efficiency, and the Short-Run Neutrality of Aggregate Demand Policy

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3 An Integrated View of Tests of Rationality, Market Efficiency, and the Short-Run Neutrality of Aggregate Demand Policy

3.1 Introduction

This chapter highlights the common elements in procedures for testing (1) rationality of forecasts in either market or survey data, (2) capital market efficiency, (3) the short-run neutrality of aggregate demand policy, and (4) Granger (1969) causality in macroeconometric models. It answers the following questions: How do the test statistics from these procedures relate to one another, and can they be used for inference under quite general conditions?

We will begin with the simplest case and then treat increasingly complex cases. The simplest case, discussed in Section 3.2, involves cross-equation tests of rationality when some measure of expectations is available. To make inferences about expectations in the absence of directly observable expectations, some model of market behavior is needed. This case is discussed in Section 3.3. Section 3.4 discusses cross-equation tests of short-run neutrality of aggregate demand policy. A final section summarizes the results.

3.2 Test of Rationality

As in Chapter 2, let ϕ_{t-1} denote the set of information available at the end of period $t-1$, and let $E(\ldots|\phi_{t-1})$ denote the objective expectation conditional on ϕ_{t-1}. Suppose that X_t is generated by the following linear model:

$$X_t = Z_{1,t-1}\alpha_1 + Z_{2,t-1}\alpha_2 + u_t, \tag{1}$$

This chapter is based on joint work with Andrew Abel (Abel and Mishkin 1983).

where

$Z_{1,t-1}$ and $Z_{2,t-1}$ = vectors of variables known at time $t - 1$,
α_1, α_2 = coefficients,
u_t = error term which is assumed to have the property that $E(u_t|\phi_{t-1}) = 0$.

The distinction between $Z_{1,t-1}$ and $Z_{2,t-1}$ is that $Z_{2,t-1}$ includes variables relevant for forecasting X_t but ignored by the econometrician in conducting tests of rationality. Of course $Z_{2,t-1}$ could be empty. It is clear from (1) that the objective expectation of X_t, conditional on ϕ_{t-1}, is

(2) $$E(X_t|\phi_{t-1}) = Z_{1,t-1}\alpha_1 + Z_{2,t-1}\alpha_2.$$

Now consider a one-period-ahead forecast X_t^e, which is some observable measure of an expectation of X_t made at time $t - 1$. Rationality of expectations requires that the forecast X_t^e must equal the objective expectation of X_t conditional on ϕ_{t-1}: that is, $X_t^e = E(X_t|\phi_{t-1})$. Thus in the following equation,

(3) $$X_t^e = Z_{1,t-1}\alpha_1^* + Z_{2,t-1}\alpha_2^* + v_t,$$

rationality implies that $\alpha_1 = \alpha_1^*$, $\alpha_2 = \alpha_2^*$ and v_t is identically zero. However, in dealing with actual data on expectations, the following weaker definition of rationality is used which allows for a nonzero observation error v_t:

(4) $$E(X_t - X_t^e|\phi_{t-1}) = 0.$$

This definition still requires that $\alpha_1 = \alpha_1^*$ and $\alpha_2 = \alpha_2^*$, yet it allows the observation error v_t to be nonzero with the restriction that $E(v_t|\phi_{t-1}) = 0$. If v_t is identically zero, then X_t^e is a minimum-variance unbiased forecast of X_t. Replacing the restriction that v_t be identically zero with the restriction that $E(v_t|\phi_{t-1}) = 0$ will remove the minimum variance property of X_t^e but not the unbiasedness conditional on ϕ_{t-1}.

Observe that (4) implies that the forecast error is uncorrelated with information in ϕ_{t-1}. This implication of rational expectations is the basis for the following test procedure. The null hypothesis of rationality is tested by testing the coefficient $\omega = 0$ in the regression equation

(5) $$X_t - X_t^e = Z_{1,t-1}\omega + \eta_t,$$

when η_t = error term where $E(\eta_t|\phi_{t-1})$ is assumed to equal zero. This is the most common test of rationality used, for example, to study forward rate forecasts in the foreign exchange market (see Levich 1979).

The effect of ignoring relevant information in this test becomes clear when equation (3) is subtracted from (1) to obtain the following equation for the forecast error:

(6) $$X_t - X_t^e = Z_{1,t-1}(\alpha_1 - \alpha_1^*) + Z_{2,t-1}(\alpha_2 - \alpha_2^*) + u_t - v_t.$$

Recall that rationality implies that $\alpha_1 - \alpha_1^* = 0$, $\alpha_2 - \alpha_2^* = 0$, and $E(u_t - v_t|\phi_{t-1}) = 0$. Therefore, under the hypothesis of rationality, the coefficient $\hat{\omega}$ estimated from the OLS regression of $X_t - X_t^e$ on $Z_{1,t-1}$ in (5) will be a consistent estimate of $\alpha_1 - \alpha_1^*$ and should not be significantly different from zero. This follows directly from the orthogonality of Z_{t-1} and η_t $[\eta_t = Z_{2,t-1}(\alpha_2 - \alpha_2^*) + u_t - v_t]$. Note that under rationality $\hat{\omega}$ is a consistent estimate of $\alpha_1 - \alpha_1^*$ even if Z_2, which is the set of relevant variables excluded from the regression, is not empty. Thus leaving out relevant variables from the OLS regression (5) will not affect the rationality implication that $\hat{\omega}$ should not differ significantly from zero.

Another way of stating the point is that the test described here is a test of rationality no matter what available past information is included in Z_1 (or no matter what information is excluded from the regression equation). That is, plim $\hat{\omega}$ can differ from zero only if there is a violation of rationality. However, it is possible that plim $\hat{\omega}$ could equal zero even in the presence of irrationality. For example, suppose that $\alpha_1 = \alpha_1^*$, $E(u_t - v_t|\phi_{t-1}) = 0$ and Z_2 is orthogonal to Z_1, yet there is irrationality because $\alpha_2 \neq \alpha_2^*$. In this case, plim $\hat{\omega} = 0$. Therefore, a failure to reject the null hypothesis, even asymptotically, does not rule out irrationality because, in this case, the probability of Type II error does not go to zero as the sample size goes to infinity.

Studies that test for the rationality of survey forecasts (Pesando 1975; Carlson 1977; Mullineaux 1978; Friedman 1980) use the following alternative procedure. Consider the following least-squares regression equations:

$$X_t = Z_{1,t-1}\gamma + u_{1t}, \tag{7}$$

$$X_t^e = Z_{1,t-1}\gamma^* + u_{2t}, \tag{8}$$

where

γ, γ^* = coefficients
u_{1t}, u_{2t} = error terms where $E(u_{1t}|\phi_{t-1})$ and $E(u_{2t}|\phi_{t-1})$ are assumed to equal zero.

As is pointed out in Modigliani and Shiller (1973), rationality of expectations requires that plim $\hat{\gamma}$ = plim $\hat{\gamma}^*$. This implication of rationality becomes clear if we suppose that Z_2, the set of variables excluded from the regressions in (7) and (8), is empty; that is, the regressions in (7) and (8) contain all information in ϕ_{t-1} relevant for forecasting X_t. In this case, $\hat{\gamma}$ and $\hat{\gamma}^*$ are each consistent estimates of α_1 under the null hypothesis of rationality, and they should not differ significantly. One way to test $\gamma = \gamma^*$ is to stack (7) and (8) into a single regression and perform a Chow (1960) test for the equality of coefficients (see Pesando 1975). However, if the variance of residuals in (7) differs, as is likely, from the variance of residuals in (8), a correction must be made for this heteroscedasticity (see Mullineaux 1978). Note that testing the cross-equation restriction $\gamma = \gamma^*$ is equivalent to testing $\omega = 0$, in (5), since

$$\hat{\omega} = (Z_1'Z_1)^{-1}Z_1'(X - X^e) = (Z_1'Z_1)^{-1}X - (Z_1'Z_1)^{-1}Z_1'X^e = \hat{\gamma} - \hat{\gamma}^*, \tag{9}$$

where X_1 and X^e are $n \times 1$ vectors with X_t and X_t^e, respectively, in row t. Similarly Z_1 is a matrix of n rows with $Z_{1,t-1}$ in row t.

Now suppose that Z_2 is not empty, so that relevant variables are excluded from (7) and (8). In this case, the estimates $\hat{\gamma}$ and $\hat{\gamma}^*$ generally will not be consistent estimates of α_1 and α_1^*, respectively, even if expectations are rational. However, rationality of expectations still implies that plim $\hat{\gamma}$ = plim $\hat{\gamma}^*$ because, as shown above, $\hat{\gamma} - \hat{\gamma}^*$ is numerically equal to $\hat{\omega}$ and plim $\hat{\omega} = 0$. Another way to understand this finding is to calculate the plims of $\hat{\gamma}$ and $\hat{\gamma}^*$. They are

$$\text{plim } \hat{\gamma} = \alpha_1 + (Z_1'Z_1)^{-1}Z_1'Z_2\alpha_2, \tag{10}$$

$$\text{plim } \hat{\gamma}^* = \alpha_1^* + (Z_1'Z_1)^{-1}Z_1'Z_2\alpha_2^*. \tag{11}$$

Rationality implies that $\alpha_1 = \alpha_1^*$, $\alpha_2 = \alpha_2^*$, and hence plim $\hat{\gamma}$ = plim $\hat{\gamma}^*$. As is obvious from (10) and (11), the equality of plim $\hat{\gamma}$ and plim $\hat{\gamma}^*$ reflects the equal asymptotic bias in the two estimates.

This section has analyzed tests of rationality in the presence of some observable measure of expectations. The general conclusion is that a rejection of $\gamma = \gamma^*$ or, equivalently, of $\omega = 0$, is a rejection of rational expectations regardless of the completeness of the information set specified by Z_1. The two alternative procedures discussed here are thus tests of rationality under quite general conditions.

In the absence of direct observations of expectations, we must infer information on expectations from observed market behavior. The next section discusses the use of security price data to test for the rationality of expectations.

3.3 Test of Rationality and Market Efficiency

The most common tests of rationality (efficiency) in capital markets focus on the condition derived in the previous chapter:

$$E(y_t - \tilde{y}_t | \phi_{t-1}) = 0, \tag{12}$$

where y_t is a one-period return for a security and $\tilde{y}_t$ is the expected return generated from a model of market equilibrium. Equation (12) above implies that $y_t - \tilde{y}_t^*$ should be uncorrelated with any past information in ϕ_{t-1}. It is the basis for a common test of market efficiency (see Fama 1976*a*) in which the null hypothesis that $\alpha = 0$ is tested in the regression equation below:

(13) $$y_t = \tilde{y}_t + Z_{t-1}\alpha + \mu_t$$

where

Z_{t-1} = variables contained in ϕ_{t-1},
α = coefficients,
μ_t = error term where $E(\mu_t|\phi_{t-1})$ is assumed to equal zero.

A test of the null hypothesis that $\alpha = 0$ is a test of the joint hypothesis of market efficiency (rationality) and the model of market equilibrium, no matter what past information is included in Z.

The "efficient-markets model" of the previous chapter that satisfies (12) is:

(14) $$y_t = \tilde{y}_t + (X_t - X_t^e)\beta + \epsilon_t,$$

where

ϵ_t = a scalar disturbance with the property $E(\epsilon_t|\phi_{t-1}) = 0$—thus ϵ is serially uncorrelated and uncorrelated with X_t^e,
X_t = the k-element row vector containing variables relevant to the pricing of the security at time t,
X_t^e = the k-element row vector of one-period-ahead rational forecasts of X_t, that is, $X_t^e = E(X_t|\phi_{t-1})$,
β = $k \times 1$ vector of coefficients.

As in Chapter 2, the linear forecasting equation for the k variables in X_t is

(15) $$X_t = Z_{t-1}\gamma + u_t,$$

where

$\gamma = l \times k$ matrix of coefficients.
u_t = k-element row vector of disturbances where $E(u_t|\phi_{t-1})$ is assumed to equal zero.

When we apply rational expectations, (14) becomes

(16) $$y_t = \tilde{y}_t + (X_t - Z_{t-1}\gamma^*)\beta + \epsilon_t,$$

where $\gamma = \gamma^*$.

The system of (15) and (16) can be estimated with the methodology outlined in the previous chapter. The cross-equation constraints implied by market efficiency (rationality), $\gamma = \gamma^*$, can be tested with a likelihood ratio test and are analogous to the rationality constraints for the regressions (7) and (8). Although expectations are not directly observable, we can test their rationality by maintaining the equilibrium model of $\tilde{y}$ and the condition that only contemporaneous unanticipated movements in X_t are correlated with $y_t - \tilde{y}_t$. Any rejection of the constraint $\gamma = \gamma^*$ could indicate a failure either of the rationality of expectations about X_t or of the maintained equilibrium model. This interpretation of such a test is discussed in the previous chapter.

Two questions arise about the econometric properties of this procedure. First, does it provide a test of market efficiency (rationality) under the maintained model of $\tilde{y}_t$ even if Z_{t-1} excludes variables relevant to forecasting X_t? Second, what is the relation of this test to the common test for market efficiency using equation (13)? These questions are related; the following theorem provides answers.

3.3.1 Theorem

Consider the system of equations

(a)
$$X_t = Z_{t-1}\gamma + u_t,$$
$$y_t = \tilde{y}_t^* + (X_t - Z_{t-1}\gamma^*)\beta + \epsilon_t,$$

where X_t is a k-element row vector, Z_{t-1} is an l-element row vector, y_t and $\tilde{y}_t$ are scalars, γ and γ^* are $l \times k$ parameter matrices, β is a $k \times 1$ parameter vector, u_t is a k-element row vector, and ϵ_t is a scalar. Consider also the equation

(b)
$$y_t = \tilde{y}_t + Z_{t-1}\alpha + \mu_t,$$

where α is an $l \times 1$ parameter vector. The quasi-likelihood ratio test of the null hypothesis $\gamma = \gamma^*$ in (a) is asymptotically equivalent to a quasi-F test of the null hypothesis $\alpha = 0$ in (b). (The quasi-likelihood ratio and quasi–F tests are constructed as if the disturbances, u_t, ϵ_t, and μ_t are i.i.d. normal.)

Outline of Proof

(See Abel and Mishkin [1980] for a more detailed and formal proof.) The key insight in the proof of this theorem is to observe that the system (a) can be rewritten as

(17)
$$X_t = Z_{t-1}\gamma + u_t,$$
$$y_t = \tilde{y}_t + (X_t - Z_{t-1}\gamma)\beta + Z_{t-1}\theta + \epsilon_t,$$

where $\theta = (\gamma - \gamma^*)\beta$. The null hypothesis $\gamma = \gamma^*$ will be true only if $\theta = 0$, and this constraint can be tested using the nonlinear least-squares procedures described in the previous chapter. The constraint that γ is the same in both equations in (17) is not binding, so we can estimate the parameters in (17) by OLS on each equation. Specifically, the estimate $\hat{\gamma}$ is obtained by OLS on the first equation. Treating $\tilde{y}_t$ as known, $\hat{\beta}$ and $\hat{\theta}$ are obtained from an OLS regression of $y_t - \tilde{y}_t$ on $X_t - Z_{t-1}\hat{\gamma}$ and Z_{t-1}. Since the residuals from the first equation in (17), $X_t - Z_{t-1}\hat{\gamma}$, are orthogonal to Z_{t-1} by construction, the estimate of θ will not be affected if $X_t - Z_{t-1}\hat{\gamma}$ is omitted from the list of regressors when OLS is applied to the second equation in (17). Thus the estimate of θ is numerically identical to, and has the same distribution as, the OLS estimate of α in (b). Although the test statistic associated with the null hypothesis $\alpha = 0$ may differ in small samples from the test statistic associated with the null hypothesis $\theta = 0$, these test statistics will be asymptotically equal.

3.3.2 Remarks

The theorem is valid regardless of the properties of the error terms u and ϵ. If they are not i.i.d., the two test procedures will be asymptotically equivalent, but neither will yield test statistics with the assumed asymptotic distributions. If the contemporaneous correlation of u and ϵ is zero, the OLS regression of y on $\hat{u}$ ($\hat{u} = X - Z\hat{\gamma}$) and Z will provide consistent estimates of both β and θ. If the contemporaneous correlation of u and ϵ is unknown, then β is unidentified. Nevertheless, in this case the OLS estimate of θ is still consistent and the theorem continues to apply. Since β is, in general, unidentified, there is an alternative demonstration of this theorem. The maximized value of the likelihood function is not affected by an arbitrary choice of β. Therefore, set β equal to zero, and observe that we now have a seemingly unrelated system (Zellner 1962) in which the right-hand-side variables are identical in each equation. The estimates of γ and θ thus can be obtained from OLS equation by equation.

Observe that the second equation in (17) contains a model of market equilibrium. The proof outlined above treats $\tilde{y}_t$ as known. If it is unknown and assumed to be a linear function of past variables W_{t-1}, then W_{t-1} must also be included as explanatory variables in the time series model for X_t. The orthogonality of the residuals in the equations for X_t with the other right-hand-side variables in the second equation of (17) is thus preserved, and the proof of the theorem may proceed as above. This becomes clear in the proof of the corollary in Section 3.4. Of course, if the coefficients of W_{t-1} in the model of market equilibrium are estimated, then we cannot test the rationality restriction that $y_t - \tilde{y}_t$ is uncorrelated with W_{t-1}. The question of the testability of such restrictions has been discussed in Appendix 2.1.

Observe also that $\theta = (\gamma - \gamma^*)\beta$ is an $l \times 1$ vector. Thus the test of $\theta = 0$ (or, equivalently, $\alpha = 0$) is a test of only l constraints. However, there are $l \times k$ constraints in $\gamma = \gamma^*$. Therefore, all these constraints are testable only if $k = 1$. Even when $k > 1$, imposing the constraint $\gamma = \gamma^*$ places only l binding restrictions on the system in (a). For example, consider the case in which $l = k = 2$. The system of equations can be written as

$$
\begin{aligned}
(18) \qquad X_{1t} &= \gamma_{11}Z_{1,t-1} + \gamma_{21}Z_{2,t-1} + u_{1t},\\
X_{2t} &= \gamma_{12}Z_{1,t-1} + \gamma_{22}Z_{2,t-1} + u_{2t},\\
y_t &= \beta_1 X_{1t} + \beta_2 X_{2t} - (\gamma_{11}^*\beta_1 + \gamma_{12}^*\beta_2)Z_{1,t-1}\\
&\quad - (\gamma_{21}^*\beta_1 + \gamma_{22}^*\beta_2)Z_{2,t-1} + \epsilon_t .
\end{aligned}
$$

The four parameters γ_{ij} can be estimated from the first two equations. If $\mathrm{Cov}(\epsilon_t, u_{it})$ is known to be zero, we can estimate β_1, β_2, $(\gamma_{11}^*\beta_1 + \gamma_{12}^*\beta_2)$, and $(\gamma_{21}^*\beta_1 + \gamma_{22}^*\beta_2)$ from the third equation. Since we cannot estimate the four elements γ_{ij}^*, separately, we cannot separately test the four restrictions $\gamma_{ij} = \gamma_{ij}^*$. However, we can test $l = 2$ linear combinations of the rationality restrictions.

$$(\gamma_{i1} - \gamma_{i1}^*)\beta_1 + (\gamma_{i2} - \gamma_{i2}^*)\beta_2 = 0 \qquad \text{for } i = 1 \text{ and } 2. \tag{19}$$

If we do not know the covariances of ϵ_t and u_{it}, then β_1 and β_2 are not identified. However, we can still test whether the two linear combinations above are equal to zero. To see this, rewrite the third equation as

$$\begin{aligned} y_t = {} & [(\gamma_{11} - \gamma_{11}^*)\beta_1 + (\gamma_{12} - \gamma_{12}^*)\beta_2]Z_{1,t-1} + [(\gamma_{21} - \gamma_{21}^*)\beta_1 \\ & + (\gamma_{22} - \gamma_{22}^*)\beta_2]\, Z_{2,t-1} + \beta_1 u_{1t} + \beta_2 u_{2t} + \epsilon_t . \end{aligned} \tag{20}$$

Observe that the coefficients of $Z_{1,t-1}$ and $Z_{2,t-1}$ in the rewritten equation are the testable linear combinations of rationality restrictions.

3.3.3 Implications

The most interesting implication of the above theorem is similar to the finding in Section 3.2: a rejection of the cross-equation restriction $\gamma = \gamma^*$ is a rejection of market efficiency or, equivalently, rationality (maintaining the model of market equilibrium) whether or not the information set in Z_1 is complete. This is demonstrated by noting that the test of $\gamma = \gamma^*$ is asymptotically equivalent to the test of $\alpha = 0$, which is clearly a test of the efficient-markets condition (12), regardless of what past information is included in Z. However, if the model generating X_t is not correctly specified, then in general there is an errors-in-variables bias that leads to inconsistent estimates of β and γ. Nonetheless, any asymptotic bias in $\hat{\gamma}$ will be identical to that in $\hat{\gamma}^*$.

The theorem implies further that rationality (or market efficiency) does not rule out significant correlations of $y_t - \tilde{y}_t$ with current variables. Therefore, if information not available at time $t - 1$ is included in the Z_{t-1} vector—as in earlier work mentioned in Chapter 2—then neither procedure provides a test of rationality.

3.4 Tests of the Short-Run Neutrality of Aggregate Demand Policy

Sargent (1976*a*) discusses tests of a classical equilibrium macroeconometric model with a Lucas (1973) supply function of the form

$$y_t = \tilde{y}_t + (X_t - X_t^e)\beta + \epsilon_t , \tag{21}$$

where

y_t = a scalar representing output or unemployment at time t,
$\tilde{y}_t$ = the equilibrium (natural rate) level of output or unemployment at time t,
X_t = a k-element vector of aggregate demand variables, such as the price level or the money supply at time t,
ϵ_t = scalar disturbance term with the property $E(\epsilon_t | \phi_{t-1}) = 0$.

This equation has the neutrality property that only unanticipated changes in X_t have an effect on $y_t - \tilde{y}_t$. Note that it is one form of the MRE equation discussed in the preceding chapter and has the same form as the efficient-markets model (14). As before, we must specify how $\tilde{y}_t$, the equilibrium level of output or unemployment, is calculated in order to give the supply function empirical content. A particular specification often used with the Lucas supply function is

$$\tilde{y}_t = \sum_{i=1}^{L} \lambda_i y_{t-i}. \tag{22}$$

Suppose that X_t is generated by the forecasting model

$$X_t = Z_{t-1}\gamma + \sum_{i=1}^{L'} \psi_i y_{t-i} + u_t, \tag{23}$$

where

Z_{t-1} = an l-element row vector of predetermined variables other than lagged y_t,
γ = an $l \times k$ matrix of coefficients,
ψ_i = a k-element row vector of coefficients.

Note that (23) has the same form as the forecasting model (15) in the preceding section, except that in (23) we distinguish between lagged values of y_t and other predetermined variables. We assume for the moment that $E(u_t | \phi_{t-1}) = 0$ and combine (21)–(23) to obtain the system

$$\begin{aligned} X_t &= Z_{t-1}\gamma + \sum_{i=1}^{L'} \psi_i y_{t-i} + u_t, \\ y_t &= \left(X_t - Z_{t-1}\gamma^* - \sum_{i=1}^{L'} \psi_i^* y_{t-i}\right)\beta + \sum_{i=1}^{L} \lambda_i y_{t-i} + \epsilon_t \end{aligned} \tag{24}$$

with the cross-equation rationality constraints $\gamma = \gamma^*$ and $\psi_i = \psi_i^*$, $i = 1, \ldots, L'$. Any rejection of these constraints could indicate a violation of the null hypothesis of rationality, or of the maintained hypothesis of the equilibrium model.

Sargent (1976*a*) uses Granger (1969) causality tests to test the joint hypothesis of rationality of expectations and the equilibrium model described in (21) in (22) above, which embodies the neutrality of anticipated policy. Substituting (22) into (21) and taking expectations conditional on ϕ_{t-1}, we have

(25) $$E(y_t|\phi_{t-1}) = \sum_{i=1}^{L} \lambda_i y_{t-i} = E(y_t|y_{t-1}, y_{t-2}, \ldots, y_{t-L}).$$

In other words, the optimal linear forecast for y_t does not benefit from the use of other information besides past y's. Hence, the equilibrium model in Sargent (1976*a*) requires that any past information, Z_{t-1}, fails to Granger-cause y_t. Specifically, if OLS is used to estimate the parameters ν_i and α in the regression equation,

(26) $$y_t = \sum_{i=1}^{L'} \nu_i y_{t-i} + Z_{t-1}\alpha + \mu_t,$$

where $L' \geq L$, the estimate of α should not differ significantly from zero.

The relationship between tests of the cross-equation constraints in (24) and the Granger-causality test in (26) is made clear by the following corollary.

Corollary

If $L' \geq L$, then a quasi-likelihood ratio test of the null hypothesis $\gamma = \gamma^*$ in (24) is asymptotically equivalent to a quasi-F test of the null hypothesis that $\alpha = 0$ in (26).

Outline of Proof

As in the proof of the theorem, the unconstrained system (24) can be rewritten as

(27) $$X_t = Z_{t-1}\gamma + \sum_{i=1}^{L'} \psi_i y_{t-i} + u_t,$$

$$y_t = \left(X_t - Z_{t-1}\gamma - \sum_{i=1}^{L'} \psi_i y_{t-i}\right)\beta + \sum_{i=1}^{L} \lambda_i y_{t-i}$$
$$+ Z_{t-1}\theta_o + \sum_{i=1}^{L'} \theta_i y_{t-i} + \epsilon_t,$$

where $\theta_o = (\gamma - \gamma^*)\beta$ and $\theta_i = (\psi_i - \psi_i^*)$ for $i = 1, \ldots, L'$, and it can be estimated by OLS on each equation. Note that since θ_i and λ_i are both coefficients of y_{t-i} in (27), the separate parameters θ_i and λ_i are not identified for $i \leq L$. Hence, the constraints $\psi_i = \psi_i^*$ for $i \leq L$ are not testable. In order to test the testable cross-equation restrictions, the system (27) can be estimated by OLS on each equation, as explained in the proof of the theorem in Section 3.3. Since the estimated residuals from the first equation will be orthogonal to Z_{t-1} and y_{t-i} for $i = 1, \ldots, L'$, the deletion of this residual vector from the second equation will not affect the OLS estimates of the coefficients on Z_{t-1} and y_{t-i}. Hence, as in the previous proof, the least-squares estimates of α and θ_o will be numerically identical, and the test statistics associated with the null hypotheses $\alpha = 0$ and $\theta_o = 0$ will be asymptotically equal.

Remarks

Obviously, OLS cannot be applied directly to the second equation of (27) as it is written since for $i \leq L$, y_{t-i} appears twice on the right-hand side because we must estimate the parameters of the $\tilde{y}_t$ model. OLS can be used after this equation has been rewritten to eliminate the perfect collinearity of right-hand-side variables. Thus we cannot obtain testable restrictions on ψ_i^* for $i = 1, \ldots, L$. However, the constraints $\theta_i = 0$ and hence $\psi_i = \psi_i^*$ for $i = L + 1, \ldots, L'$ are testable with the identifying restriction that the lag length L in (22) is shorter than the lag length L' in (23). This seems a rather strong assumption to impose on the basis of a priori knowledge, and one should be cautious in interpreting results based on estimates of θ_i in this case.

Implications

It is important to consider the effects of specifying the list of variables included in Z_{t-1} incorrectly. Irrelevant predetermined variables in Z_{t-1} will not lead to inconsistent parameter estimates but will, in general, reduce the power of tests. On the other hand, excluding relevant variables from Z_{t-1} will lead to a breakdown of the assumption that $E(u_t|\phi_{t-1}) = 0$, and will lead to inconsistent estimates of γ. Even in this case, however, any rejection of the constraint $\gamma = \gamma^*$ in (24) indicates a failure of rationality, or of the equilibrium model which embodies neutrality, since a rejection of this constraint indicates that Z Granger-causes y. As demonstrated above, this implication holds regardless of the information included in Z.

The procedure outlined therefore provides a test of the joint hypothesis of rationality and the equilibrium model, even if relevant predetermined variables are omitted from Z_{t-1}. This result can be used to show that Lucas's (1972) conjecture that tests of neutrality cannot be conducted when there is a change in policy regime is not always correct. If there are two policy regimes in the sample period 1 to T with the break occurring at T_1, then there is a separate forecasting equation for each regime: for example,

$$X_t = Z_{t-1}\gamma_1 + u_{1t} \qquad \text{for } t = 1 \text{ to } T_1 - 1, \tag{28}$$
$$X_t = Z_{t-1}\gamma_2 + u_{2t} \qquad \text{for } t = T_1 \text{ to } T.$$

Using dummy variables, we can write one forecasting equation for both regimes:

$$X_t = Z_{t-1}\gamma_1 + Z_{t-1}^*\xi + u_t \qquad \text{for } t = 1 \text{ to } T, \tag{29}$$

where

$$Z^*_{t-1} = \begin{cases} 0 & \text{for } t = 1 \text{ to } T_1 - 1 \\ Z_{t-1} & \text{for } t = T_1 \text{ to } T \end{cases}$$

$$\xi = \gamma_2 - \gamma_1$$

$$u_t = \begin{cases} u_{1t} & \text{for } t = 1 \text{ to } T_1 - 1 \\ u_{2t} & \text{for } t = T_1 \text{ to } T \end{cases}$$

Neglecting to take account of a change in policy regime is, therefore, equivalent to omitting the relevant set of variables Z^*_{t-1} from the forecasting equation. But as we have seen, even if Z_{t-1} excludes this relevant information because its variables are chosen without considering the change in policy regime, a test of the cross-equation restriction $\gamma = \gamma^*$ continues to be a test of the joint hypothesis enbodying neutrality. An important caveat, however, needs to be mentioned. The change in policy regime could alter the population variances of the error terms in both the forecasting equation and the output or unemployment equations. Unless attention is devoted to correcting potential heteroscedasticity that can arise as a result, the test statistics may lead to misleading inference.

McCallum (1979*a*) and Nelson (1979) emphasize the point raised by Sargent (1973, 1976*b*) that the Granger-causality tests are tests of the neutrality of anticipated policy only if (1) lagged values of $X_t - X^e_t$ do not enter the supply function (21), or (ii) the disturbance ϵ_t in (21) is serially uncorrelated. That is, if either of these two conditions does not hold, then it is possible for Z to Granger-cause y even though anticipated policy is neutral.

The analysis in the present chapter demonstrates these points also. The corollary above breaks down if there are lagged surprises in (21) and hence in (24). Although the contemporaneous residual from the first equation in (27) is, by construction, orthogonal to Z_{t-1} and y_{t-i}, the lagged residuals are not. Thus, the test of $\gamma = \gamma^*$ will no longer be equivalent to a Granger-causality test. Granger-causality will no longer be a test of the joint hypothesis of rationality and the model of equilibrium output.

Now consider the case in which only contemporaneous innovations in X_t appear in (21) and (24), but ϵ_t is serially correlated, implying that μ_t is serially correlated. Here, the corollary holds and the Granger-causality test is asymptotically equivalent to the test of $\gamma = \gamma^*$. However, since the right-hand sides of both (24) and (26) include lagged dependent variables, the estimates of α and θ_o will no longer be consistent. Test statistics from both procedures are invalid in this case. To obtain valid test statistics for the joint hypothesis, we correct the supply function (21) for serial correlation by quasi-differencing and generate specification with a serially uncorrelated error. The resulting specification will contain lagged as well as current $X_t - X_t^e$. We are then dealing with the case above where the Granger-causality test is no longer a test of the joint hypothesis.

3.5 Summary and Conclusions

The framework in this chapter ties together a range of issues in testing rationality, financial market efficiency, and the short-run neutrality of aggregate demand policy. Two main themes stand out in this integrated framework:

1. The cross-equation tests of rationality, market efficiency, and short-run neutrality discussed here are asymptotically equivalent to more common single-equation regression tests.
2. The exact specification of the relevant information set used in rational forecasts is not necessary for the cross-equation tests of rationality, market efficiency, and short-run neutrality to have desirable asymptotic properties.