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THE THEORY OF HOUSEHOLD BEHAVIOR: SOME FOUNDATIONS

BY KELVIN LANCASTER*

This paper is concerned with examining the common practice of considering the household to act as if it were a single individual. It concludes that aggregate household behavior will diverge from the behavior of the typical individual in two important respects, but that the degree of this divergence depends on well-defined variables—the number of goods and characteristics in the consumption technology relative to the size of the household, and the extent of joint consumption within the household. For appropriate values of these, the degree of divergence may be very small or zero.

For some years now, it has been common to refer to the basic decision-making entity with respect to consumption as the "household" by those primarily concerned with data collection and analysis and those working mainly with macroeconomic models, and as the "individual" by those working in microeconomic theory and welfare economics. Although one-person households do exist, they are the exception rather than the rule, and the individual and the household cannot be taken to be identical.

In the total absence of trade between the micro-welfare and macro-empirical branches of the profession, it might not matter that the consumption units were different in the different contexts. But there is trade—perhaps less than there ought to be—and this is where the danger lies. It is not uncommon to take analysis that has been devised to provide a reasonable model of the single individual and then apply that analysis to the household, as if it were the same thing. The most surprising offender is Arrow and Hahn (1971) where, in a book designed to meet the highest standards of analytical rigor, the basic decision-maker in consumption is called the "household"—and then has ascribed to it a set of properties that are appropriate only for the single individual.

If it could be shown that households did, indeed, behave like the individuals of microtheory, then there would be no problem, but we know that this can be taken to be evidently true, if at all, only in a household run in a dictatorial fashion by a single decision-maker. If the household does behave like an individual in any other circumstances, we must be able to prove this and be able clearly to state those circumstances.

The purpose of this paper is to concentrate on the fact that the typical household consists of more than one person and to investigate the extent to which it is (a) entirely or (b) approximately legitimate to ascribe to that household those properties traditionally ascribed to the single consumer.

I. THE INDIVIDUAL AND THE HOUSEHOLD

Since a household is composed of individuals, we must either construct a theory of the household which is based on and derived from the theory of behavior

*This is a revised version of the paper delivered at the conference, reworked to strengthen the substantive results and stripped of some interesting but unessential material.

of individuals, suitably modified to take account of their association within the household, or ignore those individuals altogether and construct a theory of the household which is *sui generis* and not based on individuals. The theory of the consumption function is of this latter kind, not derivable from standard microtheory, as are *ad hoc* models such as the stock adjustment model of Houthakker and Taylor (1970).

We shall be entirely concerned here with models of the first kind, based on the theory of individual behavior and using the results of microtheory. Remarkably little has been done in this area, although Samuelson (1956) tackled the problem directly in what is probably the fullest discussion in the economics literature of the relation of household decisions to individual preferences. Becker (1965), Muth (1966) and others since have considered the problems associated with production (implicit and explicit) within the household and with time allocation within the household, but assumed away any problems associated with the household's decision function. There is, of course, an extensive literature on both aggregation and the construction of social welfare functions, two problems directly relevant to the theory of the household, but with the emphasis placed on large, rather than small, groups. The marketing literature contains much discussion of intrahousehold decision processes, primarily from the point of view of trying to influence sales by manipulating these,[1] and there has been considerable recent work in the empirical investigation of who makes what decisions within the household.[2] Finally there is decision theory, especially the work on teams by Marshak and Radner (1972), which has relevance to the household decision process. For the basic problem with which we are concerned here, however, we cannot draw on any of this literature except that on aggregation and social welfare functions (including the Samuelson article), since we shall confine ourselves to the pure demand properties of households under conditions that do not involve production, time or uncertainty.

The Individual

The individual, who will appear both as the typical member of the household and as a reference with whose typical behavior we shall compare the behavior of the household, is the standard individual consumer of microtheory. He (she) is assumed to have complete and well-ordered preferences, full information, and to optimize perfectly subject to a budget constraint with exogenous prices and income (or endowments). We shall consider consumption behaviour in two contexts:

(1) *Traditional*, in which every good fits into the preference system in a way which is unique to itself, and

(2) *The characteristics model*, in which goods possess characteristics which are typically obtainable also from other goods or their combinations.[3]

[1] For example, in considering a household with some differences in tastes (the wife likes opera, the husband golf), a well-known text in this area goes on to conclude, "in this case it may be desirable for a manufacturer of golf equipment to attempt to convince the wife that the husband's need for recreation is, in fact, legitimate" (Engel, Kollat and Blackwell, *Consumer Behavior*, Holt Rinehart and Winston, 1968, pp. 330–1.)

[2] See, for example, Ferber and Nicosia (1973).

[3] See Lancaster (1966) or (1971).

In using the characteristics model, we shall confine ourselves to the case in which the act of consumption (that is, of extracting characteristics from goods) is linearly combinable, in the sense that the characteristics obtained from any collection of goods is the sum of the characteristics contained in the specified quantities of the individual goods.

When investigating whether the household can be considered to behave like a single individual, we shall concentrate on the following criteria which are either directly observable or can be derived from observable behavior:

(1) *The individual is efficient.* In the traditional context, this is either trivial or unobservable (how could we know whether a consumer was equating marginal rates of substitution to price ratios or not?), but is nontrivial and observable in the characteristics model. In the latter model, a consumer who is efficient will, when faced with choices among collections of M goods possessing between them only R different characteristics ($R < M$), need to consume no more than R different goods.

(2) *Substitution predominates in demand.* This basic property, a consequence of choice under a budget constraint with preferences having the generally assumed properties, can be expressed in a variety of different ways. The two we shall be particularly interested in are the alternatives:

(a) The Slutsky matrix is symmetric and negative semi-definite

(b) The strong and weak axioms of revealed preference are satisfied.

(The substantive contents of both (a) and (b) are equivalent.)

A household can be considered to act *as if* it was a single individual only if the aggregate household consumption vectors (obtained by summing the individual consumption vectors over all members of the household), the aggregate household income, and the price vector for goods, are related in such a way as to satisfy (1) and (2) above.

The Household

The household is composed of individuals, but it is clearly more than that. We shall consider the household to possess three leading properties

(1) The household is a collection of individuals

(2) It is a *small* collection of individuals

(3) It is a *closely-knit* collection of individuals.

Insofar as the household is a collection of individuals, a theory of the household can draw on general theoretical results for groups, such as aggregation properties and the properties of social decision rules. Since it is a small collection, we must reject group properties which depend on large numbers and search for properties which depend on smallness of numbers. Since the household is a closely-knit group, we can accept some things—like interpersonal utility comparisons—that we would not over random aggregates, and must be prepared to emphasize others, like joint consumption, that would be peripheral phenomena for large groups.

We shall first investigate the extent to which smallness as such enables us to reach different conclusions about the household as an aggregate than we would reach for a large group, then go on to investigate the effect of close-knitness on the household decision function and on joint consumption phenomena.

II. Households as Small Aggregates

It is well known[4] that, if individual consumption vectors x^i chosen subject to prices p and incomes y^i are aggregated into a group consumption vector X $(= \sum x^i)$, then X, Y $(= \sum y^i)$, p do not necessarily bear the same kind of relationship to each other as to the equivalent quantities for the individuals. In particular, the aggregate quantities need not satisfy the weak axiom of revealed preference.

The best known example is the 2×2 case given by Hicks (1956), where two persons facing choices among two goods each choose collections in two different price-income situations which are consistent with all standard assumptions about individual behavior, but in which the aggregate vectors have the properties $pX' < pX, p'X < p'X'$, contravening the revealed preference axiom.

Since the 2×2 example is particularly clearcut, it was presumed for many years that the aggregate properties of small groups were, if anything, more divergent from those of the individual than the aggregate properties of large groups. The Hicks example requires that the income consumption curves of the individuals be so related that the consumer choosing the lowest ratio of the first to the second good in the initial situation shows a much greater increase in the consumption of the first good, relative to the second, as income rises, than does the other consumer. Thus it has been argued that this kind of effect will wash out over a large group—an argument which can be found spelled out in detail in Pearce (1964).

It is only recently that it has been realized that it is not the number of consumers, as such, that is relevant, but the number of consumers relative to the number of goods. The failure of the aggregate demand in the Hicks case to possess any well-defined substitution property is now seen to depend, not on the actual number of consumers, but on the fact that the numer of individuals is the same as the number of goods.

The Number of Goods and the Size of the Group

The particular developments in theory that turn out to be especially relevant to the household as a small group were set in motion by Sonnenschein (1972, 1973), who asked, and gave the first answer to, the following question; If the aggregate excess demand function is continuous and satisfies Walras' Law (essentially an accounting identity), but is otherwise arbitrary, could this excess demand function have been obtained as the aggregation of individual excess demand functions each of which possesses all the properties derived from traditional preference maximizing behavior? Sonnenschein's affirmative answer was sharpened by Mantel (1974) and given definitive shape by Debreu (1974) who showed that the affirmative answer required that the number of individuals be at least as great as the number of goods. Further clarifications have been made in a paper by McFadden, Mantel, Mas-Collel and Richter (1974?), and mopping up still continues.

[4] See, for example, Samuelson (1948).

If we invert the reasoning of the Sonnenschein–Mantel–Debreu result, it implies that, in a world of N goods, there can exist N consumers with acceptable individual behavior whose aggregate behavior does not necessarily exhibit the traditional substitution properties. This explains the Hicks example, but shows that it is a special case because it assumes only two goods, rather than because the group comprises only two people.

Since a single individual exhibits all the standard demand behavior but an aggregate of N individuals (assuming N goods) may show none, the interesting question—and the crucial one for the theory of the household—is what happens when there are at least two, but less than N, individuals? This question has been answered by Diewert (1974?) whose analysis we shall follow.

Consider first an individual consumer. The Slutsky equations for this individual can be written in the matrix form

$$V = K + bx^T$$

where V is the matrix of uncompensated price partials, K is the Slutsky matrix, b is the column vector of income partials and x the column vector of initially chosen quantities. The demand properties of the individual are usually summarized by noting that K is symmetric and negative semi-definite. For our purposes here, it is more useful to consider the properties of V, the uncompensated matrix.

Let us choose a set of linearly independent vectors A^k, each of which is orthogonal to x. There will be $n - 1$ such vectors, which we assemble into the $n \times (n - 1)$ matrix A, where n is the number of goods. From the Slutsky equation we then obtain

$$\begin{aligned} A^TVA &= A^TKA + A^Tbx^TA. \\ &= A^TKA \end{aligned}$$

since $x^TA = 0$.

Now the properties of symmetry and negative semi-definiteness of K are left unchanged by the transformation $A^T \ldots A$, so that the matrix A^TVA possesses these properties. Thus, although the $n \times n$ uncompensated matrix V is not itself necessarily either symmetric or negative semi-definite, there always exists a transformation $A^T \ldots A$ such that the $(n - 1) \times (n - 1)$ matrix A^TVA is symmetric negative semi-definite.

If we think of the compensated demand function as exhibiting "full" concavity properties (since we can associate with it a negative semi-definite matrix of order n), the uncompensated demand function can be considered to have one degree less concavity, since the negative semi-definite property is associated with a matrix of order $n - 1$. Alternatively we can note that symmetry imposes $\frac{1}{2}n(n - 1)$ restrictions on the compensated matrix K, but only $\frac{1}{2}(n - 1)(n - 2)$ restrictions on the uncompensated matrix V.

Now consider the aggregation of m consumers, each choosing independently with his own preferences and budget (but all facing the same prices). Denoting aggregates by bars and values for the individuals by superscript $s(s = 1, \ldots, m)$, we have[5]

$$\bar{V} = \sum_s V^s$$
$$= \sum_s K^s + \sum_s b^s(x^s)^T.$$

Choose a set of linearly independent vectors $\bar{A}^k$, each orthogonal to all the vectors x^s. In general the vectors x^s will be linearly independent, and thus we can certainly find $n - m$ vectors $\bar{A}^k$ but not, in general, more than that. Assembling the vectors into $n \times (n - m)$ matrix $\bar{A}$, we then obtain

$$\bar{A}^T\bar{V}\bar{A} = \sum_s \bar{A}^T K^s \bar{A} + \sum_s \bar{A}^T b^s (x^s)^T \bar{A}$$
$$= \sum_s \bar{A}^T K^s \bar{A}$$

since $(x^s)^T\bar{A} = 0$, all s.

By the same reasoning as used in the individual case, the matrix $\bar{A}^T\bar{V}\bar{A}$ is symmetric negative semi-definite, but of order $n - m$. Thus aggregate demand exhibits "less" concavity than individual demand, the divergence increasing as the number of individuals increases. The number of implied symmetry restrictions is $\frac{1}{2}(n - m)(n - m - 1)$, a number which declines as m increases. If $m \geq n$ no matrix $\bar{A}$ can be found (unless there is linear dependence among the vectors x^s) and aggregate demand does not necessarily exhibit any concavity or symmetry properties at all, the Sonnenschein–Mantel–Debreu result. On the other hand if n is large and m small (2 for many households), the properties of $\bar{V}$ do not diverge greatly from those for the individual. Thus we can state the following:

Result 1

Even if the individuals in the household receive their own budgets and make totally independent choices, the behavior of the household will be "close", in a clearly defined sense, to that of an individual consumer, provided the number of goods is large relative to the number of members of the household.

This is a statement that could not have been made on any firm basis even two or three years ago.

The Characteristics Model

In the characteristics model, the individual has preferences over characteristics which are taken to have the same properties as the traditional preferences over

[5] Note that $\bar{V}$ is the exact aggregate analog of V^s, but ΣK^s is not the aggregate analog of the Slutsky matrix. The latter would require a different decomposition of $\bar{V}$ from that given in the text, namely

$$\bar{V} = S + \bar{b}(\bar{x})^T$$

where $\bar{b}_i = (\sum x_i)/(\sum y)$ and $\bar{x}_i = \sum x_i$, and S is then the true analog of the Slutsky matrix for the individual. Since the terms $\bar{b}(\bar{x})^T$ and $\sum_s b^s(x^s)^T$ are quite different, so are S and $\sum_s K^s$. Note also that we will have a different S for every different rule for distributing the aggregate income.

goods. The characteristics are obtained, in the case we shall use in this paper, from goods in such a way that the vector of characteristics is a linear transformation of the vector of goods of the form $z = Bx$, where z is the characteristics vector.

If B is a square nonsingular matrix, there is a unique inverse transformation $x = B^{-1}z$ from characteristics into goods. In this case we can aggregate the behavior of the members of the households over characteristics to obtain an aggregate characteristics vector $\bar{z}$. Since there will be a unique vector $q = pB^{-1}$ of implicit characteristics prices, and since we have assumed behavior over characteristics to fit the traditional pattern of behavior over goods, the matrix C of uncompensated partials of characteristics with respect to their implicit prices will have the same properties as ascribed to V in the traditional case. The matrix of price partials of goods in this case is equal to BCB^{-1}, which has the same symmetry and negative semi-definite properties as C and thus as the traditional V. The characteristics model gives identical results with respect to the demand properties of goods as the traditional model so long as the matrix B is square and nonsingular, requiring that the number of goods and the number of characteristics be equal.

But if the number of goods exceeds the number of characteristics, the matrix B is of order $r \times n$ (where r is the number of distinct characteristics). From standard optimizing theory, the individual will attain his optimum subject to a linear budget constraint on goods by consuming only r goods. The choice of those r goods will depend on the consumer's optimal characteristics vector and thus will, in general, vary from consumer to consumer. The choice of those r goods will then give a basis in B from which the inverse relationships will be determined. In particular, if B^s is the $r \times r$ basis chosen from B by the sth consumer, we will have $x^s = (B^s)^{-1}z^s$ and $q^s = p(B^s)^{-1}$. The latter relationship implies that the implicit prices on characteristics differ between consumers, ruling out all standard aggregating procedures for characteristics over individuals.

Solution of the individual's optimization problem over characteristics gives, of course, a unique solution in terms of goods—typically with the quantities of $n - r$ of the goods being zero. We cannot use the Slutsky analysis on the demand for goods because the solution is a corner solution, but the demand for goods satisfies the axioms of revealed preference.[6] Aggregation over consumers whose individual behavior satisfies the strong axiom of revealed preference has been shown, in McFadden Mantel Mas-Collel and Richter (1974) to lead to results similar to the Sonnenschein–Mantel–Debreu conclusions. Although we do not have an analysis of the case in which $m < n$ which shows a continuous relationship between the size of the group and the degree of divergence from the individual pattern, as we do when we can use the Slutsky equations, it can be conjectured that some relationship of this kind does exist.

The most important, and most observable, effects of the characteristics model on household behavior lie in efficiency considerations. An individual consumer will not need to consume more goods than there are characteristics but, if $r < n$, different individuals may, in general, choose different collections of r goods. Thus a household, each of whose members is efficient in consuming only r

[6] See Lancaster (1971), p. 58–9.

goods, may consume more than r goods in the aggregate and thus will appear to be consuming inefficiently. Indeed, the aggregate characteristics vector may be such that it could be more efficiently attained by consuming a set of goods different from those chosen by any member of the household. Figure 1 shows an example with 6 goods and 2 characteristics where the household consumes 4 goods (G_1, G_2 by one member, G_5, G_6 by the other) when, if the household had really been a single individual, the efficient choice would have been the two goods G_3 and G_4.

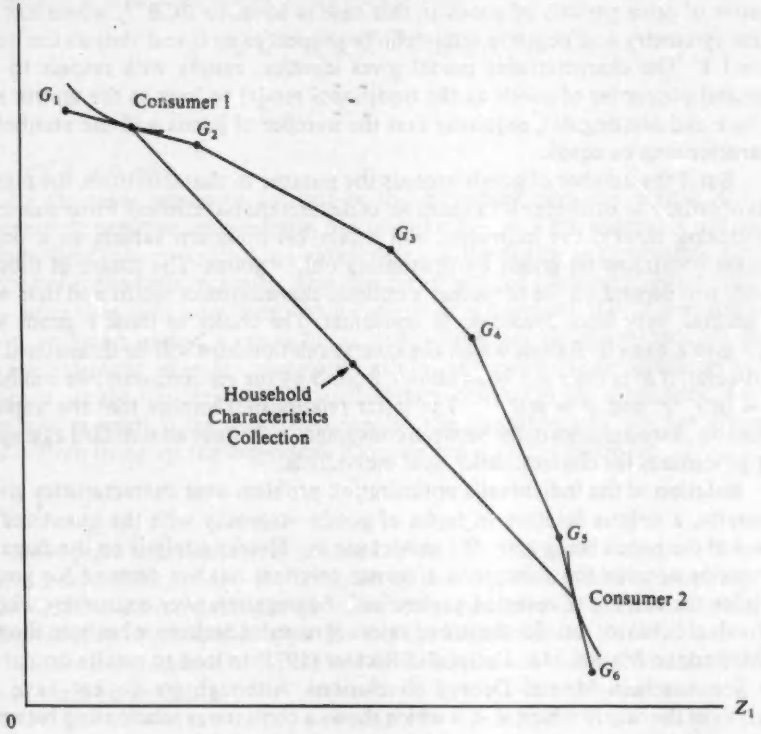

Figure 1.

It is obvious that, in a general way, the number of goods that the household might be observed to consume in excess of the number that would be consumed by a single individual will increase as the number of goods increases relative to the number of characteristics, and as the number of persons in the household increases. We can state the following;

Result 2

If the individuals in the household receive their own budgets and make totally independent choices, and if there are more goods than distinct characteristics, the behavior of the household may be apparently inefficient, in the sense that is purchases more goods than there are characteristics. The potential divergence between the number of goods purchased by the household and the number that would be purchased by an individual will be less, the smaller the household and the smaller the number of goods relative to characteristics.

Conclusions on Households as Simple Aggregates

If a household is a simple aggregate of individuals each of whom makes his own choices subject to his own preferences and his own budget (and in the absence of joint and externality effects), then the household cannot, in general, be considered to behave as if it was a single individual. Its observable behavior will differ from that of the single individual in that demand may show a lesser degree of symmetry and concavity and consumption may appear to be inefficient as evidenced by the purchase of more goods than there are distinct characteristics. In both these respects, however, the household's behavior will be "close" to that of an individual if the size of the household is small and the number of characteristics and goods is large. Simple examples in two goods and two individuals vastly overstate the degree of divergence between household and individual behavior, as compared with the more realistic case of few individuals and many goods.

III. Household Decision Functions

We now turn from the model of the household as a mere collection of independent individuals to consider models in which decisions are made by the household as a unit, but in which the individuals in the household still possess their own preferences.

If there is to be a single household decision function which reflects and is based upon the preferences of the individual members of the households, then the Arrow impossibility theorem applies just as it does to larger groups. There can be no rule that will generate, from the preference orderings of the members alone, a household preference ordering that is Paretian, has unrestricted domain, satisfies the condition of independence from irrelevant alternatives, and is non-dictatorial, unless the preferences of the individuals are related in some particular way. To have a household behaving as a single decision-making unit, one or more of the Arrow conditions must be dropped, we must work with a household preference ordering over a restricted domain, we must assume that the preferences of household members are always related in such a way as to always lead to unambiguous household preferences, or we must be willing to have the household decision function based on more information than is contained in individual preference orderings alone.

The traditional approach of simply regarding the household as a single person can be considered to have rested upon one of two implicit assumptions, that the household decision function is dictatorial and reflects the preferences of its "head", or that the members of the household have identical preferences and unanimity is found on all choices.

Our purpose is, of course, to go beyond this kind of simplification. For the household, we can break out of the Arrow prison by using the "close-knitness" property that is not applicable in the case of the broader social welfare function. This property makes it reasonable that the household can make decisions on the basis of more information about the effect on its members than is contained in preference orderings alone. In particular, we can contemplate a household decision function which takes account of degrees of preferences and of relative weights to be given to the preferences of different members.

Samuelson Households

The household decision function discussed at some length by Samuelson (1956) has the form $U[u^1(x^1), u^2(x^2), \ldots, u^m(x^m)]$ where $u^i(x^i)$ is the utility function of the i-th household member derived from his own consumption vector x^i. There are no externalities, interdependencies or joint effects. U is an increasing function of the u^i's (so the household is Paretian) and the concavity properties of U on the u^i's and of the u^i's on the x^i's are such as to make U a strictly quasi-concave function of the ultimate arguments u^i_r. A sufficient, but not necessary, condition for this is that the u^i's are strictly concave and U is a strictly quasi-concave function of the u^i's. Concavity, not merely quasi-concavity, is appropriate for the u^i's since the household function is assigning cardinal measures to the utilities of all its members.

With a household utility function of this form, the household optimum for given household income Y can be achieved by dividing income among members of the household in such a way as to equalize weighted marginal utilities of income. That is, we must have:

$$U_i \frac{\partial u^i}{\partial y} = U_j \frac{\partial u^j}{\partial y}$$

for all i, j. The individual members then optimize on their personal budgets.

Since U is strictly quasi-concave on the individual goods quantities u^i_r, and since all household members face the same prices, the goods aggregates $X_r = \sum_i x^i_r$ satisfy the properties of Hicksian composite goods and thus U is strictly quasi-concave over the household goods vector X. The concavity properties of household demand are identical to those of an individual consumer.

There is, however, one respect in which the behavior of the household may differ from that of the individual. This will be when there are more goods than characteristics.

Characteristics and Paretian Households

Consider the Samuelson household in the context of the characteristics model, in which we shall assume the consumption technology is such as to have more goods than there are distinct characteristics. The overall structure of the optimizing process is the same as in the traditional model, each member of the household maximizing his own utility function subject to his own budget constraint (on goods), the budgets being allocated in accord with the Samuelson rule.

The kinds of choices that the individuals will make will be the same as in the aggregate model and the reasoning given in Part II which led to Result 2 will be applicable here. In particular, if members of the household differ sufficiently in their individual preferences, different members may obtain their optimal collections of the same set of characteristics by consuming different bundles of goods. Thus even with a unified household decision rule like that of the Samuelson model, we can still have a situation like that depicted in Figure 1, with apparent inefficiency and with the household purchasing more goods than there are characteristics.

The above argument is not applicable only to households of the Samuelson type, but applies for any kind of household decision function which is Paretian (that is, in which U is an increasing function of the u^i's). The optimum for any such household must be such that the utility level attained by any member of the household has been attained with the least possible expenditure on goods from the household budget. For members with sufficiently different preferences, this minimum expenditure criterion will imply different goods in the optimum bundles of different members, giving us once again the situation leading to Result 2 and depicted in Figure 1.

We can summarize our findings in this Part as follows:

Result 3

In the Samuelson model, aggregate household demand will exhibit the same concavity properties as for the individual, whatever the relationship between the number of goods and the number of characteristics.[7]

Result 4

If there are more goods than distinct characteristics, neither the Sanuelson household nor any other household with a Paretian decision function need necessarily be observed to behave as if it were a single individual, since the household may purchase more goods than there are distinct characteristics, contravening the efficiency condition for the individual.

We should note that Result 4 may be applicable even in a *dictatorial* household. The dictator can get the other members of the household to any utility level he has chosen for them most cheaply by giving each the appropriate optimal goods bundle, leaving the maximum residue for his own use. This will lead to the same kind of results as in the Paretian household.

Conclusions on Households with Centralized Decisions

In the absence of externalities, interdependencies, or joint consumption effects, even a centralized household decision function is not sufficient to guarantee that the household behaves like a single individual in all respects. In particular, although the household may possess the demand substitutability properties of the individual, it may not possess the efficiency properties.

[7] In the characteristics analysis, u^i is taken to be strictly quasi-concave on characteristics but when mapped into a function of goods it becomes quasi-concave only, due to the prevalence of zero goods quantities. Thus we must drop the "strictly" from the specification of all the concavity properties of the demand for goods, for the individual as well as the household.

IV. Joint Consumption and Related Matters

Since a household consists of a small number of individuals with close associations, we can expect every kind of joint and externality effect, including interdependence of utilities, to play a far more significant role than in the case of a large aggregate. A considerable share of typical household activities (meals, recreation, simple occupancy of the home, for example) involve joint consumption in some sense or major consumption externalities. The taxonomy, alone, of all the possible effects would be a considerable task, while a full exploration of territory which has but a few signposts at present would be well beyond the scope of a single paper.

We shall not take up at all the question of interdependent preferences,[8] but confine our investigation to external interdependence, through joint consumption or otherwise, in which the utilities of other household members do not appear as direct arguments in the utility function of any one. The only direct interrelation between utilities is confined to the formulation of the household decision function.

Joint Consumption and Externalities

There is an externality effect within the household whenever the utility (preferences) of one member are affected by the quantities consumed of any good by any other member. We shall concentrate on the fullest kind of positive externality, joint consumption, in which the consumption of any quantity of the joint good (or characteristic) by any member of the household has the same effect on other members of the household as if they had directly consumed it themselves. (If someone turns on the radio, everyone hears it). Thus the total quantity consumed within the household appears in everyone's utility function, making the good or characteristic a kind of household public good or public characteristic. It will be convenient to refer to these characteristics or goods as "public" within the context of the household, although they are private goods from the point of view of society as a whole and are purchased by the household through the market.

The property of being private or public can be taken to reside in the individual characteristic (a food may have a private flavor but a public odor). A true household public good is then one with characteristics which are all public, a private good with characteristics which are all private, and a mixed good with characteristics of both kinds.

If all goods available to the household are either public or private in the above sense, the analysis is precisely the same as if the household was a mini-economy facing a linear transformation curve (the budget line) and the ordinary theory of public goods can be applied. Although the household is small, its size is essentially fixed and phenomena such as "crowding"[9] are not important. Provided the household has a proper decision function, an optimal solution can be reached by central purchase and allocation, by lump-sum contributions towards purchase of the public goods from members' budgets or even (in a highly sophisticated household!) by a Lindahl solution with the public good sold to different members at different prices.

[8] One could develop the relevant analysis along the lines suggested in Winter (1969).
[9] See Buchanan (1965) or Ellickson (1973).

In the traditional case, where there is a one to one relationship between a characteristic and a good, the structural properties of household demand are not changed by the existence of joint consumption within the household, provided the household has a mechanism for attaining an optimum.[11] In particular, if the household decision function has the Samuelson form $U[u^1(x^1, V), \ldots, u^m(x^m, V)]$, with U a strictly quasi-concave function of all the ultimate arguments x^i_j, V_k (V is the vector of public goods), then U is a strictly quasi-concave function of the aggregate household goods quantities X_j, V_k, for the same reasons as in the all-private case. Thus the concavity properties of household demand are not affected by the introduction of the public goods.

More Goods than Characteristics

If there are more goods than characteristics, the extent to which the household behavior conforms to the efficiency conditions for the individual depends on the separate relationships between the number of public goods and public characteristics and between the number of private goods and characteristics, in the assumed absence of mixed goods. It is obvious that, since each member's utility from the public characteristics depends on the household totals, those totals should be obtained in the least cost way in order to achieve optimality. Thus the number of public goods will not exceed the number of public characteristics, whatever the size of the household. On the other hand, the relationship between the number of private goods purchased by the household and the number of private characteristics may exceed the number that would be purchased by the single individual, for the same reasons as in Parts II and III.

Mixed Goods

In the overall economy, many of what are regarded as pure public goods undoubtedly have private aspects (defense is not a pure public good to someone living next to an airbase), but these private aspects are scattered and of variable impact in most cases. Thus it is a reasonable first approximation to consider a division of goods into public goods and private goods over the economy as a whole, and only when we consider smaller segments of the economy (especially localities) do we need to consider goods as having a mix of public and private characteristics.

For a unit as small as the household, however, many or most goods that have public characteristics (in the special household sense) will also have private characteristics. The dwelling itself will have a mixture of public characteristics (the areas of joint use) and private (in individual bedrooms). Thus we can regard the mixed good as typical within the household, not an unusual special case.

Since our concern in this paper is with the extent to which the household can be treated as if it were a single individual, we shall not give any descriptive analysis of the mixed goods case but proceed immediately to consider the effect of mixed goods on the concavity and efficiency properties of household demand.

[10] Some interesting possibilities arise in the absence of household cooperation, including game-type behavior based on mutual "free ride" considerations.

It is obvious that concavity properties, which are not affected by the presence of pure public goods, will not be affected by the presence of mixed goods. Any effects will be confined to the efficiency criteria. Since pure public goods give efficiency properties identical with those for the individual, while pure private goods lead to a divergence between the properties of the household and the individual, we can expect that mixed goods tend to lower the divergence as compared with pure private goods. Confirmation of our expectations commences with the following theorem.

Theorem

If there are two characteristics, one public (within the household) and one private, a household decision function, and an array of n mixed goods, the household will purchase only two of the goods, however large may be the number of goods and the size of the household.

Denote by z_1^s the amount of the private characteristic received by the s-th member of the household, and by z_2 the total household quantity of the public characteristic. Let a_{1j}, a_{2j} be the quantities of the two characteristics contained in unit quantity of the j-th good, and let x_j^s denote the quantity of the j-th good consumed by the s-th individual. The household optimum is then the solution of the problem

$$\max U[u^1(z_1^1, z_2), u^2(z_1^2, z_2), \ldots]$$

subject to

$$z_1^s - \sum_j a_{1j} x_j^s = 0 \qquad s = 1, \ldots, m$$

$$z_2 - \sum_j a_{2j}\left(\sum_s x_j^s\right) = 0$$

$$\sum_j \sum_s x_j^s = I$$

where goods units have been chosen so that prices can be taken as unity, and I is the household income. In addition we have the mn nonnegativity restrictions $x_j^s \geq 0$, all s, j.

The dual problem involves the $m + 2$ dual variables $w_1^s, s = 1, \ldots, m$ (the shadow price of the private characteristic to each individual), w_2 (the common shadow price of the public characteristic), and v (the shadow marginal valuation of household income). There is a dual constraint for each x_j^s of the form[11]

$$a_{1j} w_1^s + a_{2j} w_2 \leq v \qquad s = 1, \ldots, m; \qquad j = 1, \ldots, n$$

such that $x_j^s = 0$ unless the corresponding constraint is satisfied as an equation.

In general, the optimum will involve the consumption of exactly two goods by any individual, and these will be goods which are adjacent along the efficiency

[11] These constraints correspond to taking the derivatives of the Lagrangean with respect to the x_j^s's. The value assigned to v can be taken to be essentially arbitrary, but v must be positive.

frontier (see Figure 1). Suppose the optimum for the first individual is consumption of goods $j, j+1$ (the goods are taken to be numbered successively along the frontier), then the following must be true:

$$\begin{aligned} a_{1j}w_1^1 + a_{2j}w_2 &= v \\ a_{1j+1}w_1^1 + a_{2j+1}w_2 &= v \\ a_{1k}w_1^1 + a_{2k}w_2 &< v \qquad k \neq j, \quad j+1. \end{aligned}$$

The two equations determine w_1^1 and w_2, the inequalities must hold if j, $j+1$ are truly the optimal pair of goods. Now consider any other consumer for whom the shadow price on the private characteristic is w_1^s. The value of w_2 must be the same as for the first consumer, and the inequalities to be satisfied have the same coefficients a_{1j}, a_{2j} as for the first consumer. Now if we had $w_1^s > w_1^1$, the left hand sides of the relationships corresponding to the two equations above would be greater than v, contravening the dual constraint. If we had $w_1^s < w_1^1$, then none of the inequalities would be satisfied as an equation and the s'th individual would receive no goods. Thus we must have $w_1^s = w_1^1$, so that the same inequalities are satisfied as equations for the s-th individual as for the first, and his consumption will consist of the same two goods as for the first individual, proving the theorem. The difference between this result and that for the non-joint case, in which every individual may consume a different pair of goods (if there are a sufficient number of goods relative to the number of characteristics) arises from the common shadow price w_2 which would be replaced by individual shadow prices w_2^s in the absence of joint consumption.

Note that the proof of the theorem is based on efficiency conditions alone and is independent of the specific properties of U, u^s, so it holds for any degree of dispersion among the private preferences of household members, provided they can agree on a household decision function which leads to an proper optimum.

If we extend the theorem to cover a situation in which there are r_1 private characteristics and r_2 public characteristics, we can expect any one consumer to consume not more than $r_1 + r_2$ different goods. The appropriate dual relations will give us $r_1 + r_2$ equations in r_1 private shadow prices and r_2 common shadow prices. Now suppose that there are two consumers whose optimal choice of goods differs in r' goods. The total number of dual equations to be satisfied is thus $r_1 + r_2 + r'$, while the number of dual variables is $2r_1 + r_2$. Thus the optimal choices for the two individuals cannot differ in more goods than there are private characteristics.

Consider any individual consuming $r_1 + r_2$ goods and thus whose shadow prices satisfy $r_1 + r_2$ dual equations. The public characteristic shadow prices are common and thus exogenous to the individual. Thus the individual's equation system in his private shadow prices consists of $r_1 + r_2$ equations in r_1 unknowns, so that r_2 of the equations are linearly dependent on the remainder. This means that any other individual who consumes at least r_1 of the same goods, and thus whose private shadow prices satisfy r_1 of the equations, will have the remaining equations also satisfied and thus will consume all $r_1 + r_2$ of the same goods as the first individual.

But we have shown that the two goods collections cannot differ by more than r_1 goods, and thus have a minimum of r_2 goods in common. If $r_2 \geq r_1$, therefore, the individuals will consume identical sets of goods (in types of goods, not necessarily in their proportions).

We can bring together the basic theorem and its extension into a result that also serves to summarize the effect of joint consumption on observed household behavior:

Result 5

Joint consumption effects do not change the concavity properties of household demand, as compared with the situation in their absence, but have important repercussions on the extent to which the household satisfies the efficiency criteria of the single individual. In particular, if goods possess some characteristics that are consumed jointly (public characteristics) and others which effect only the individual directly consuming the good (private characteristics), the effect of the joint consumption is to reduce the divergence between the number of goods that the household would be observed to purchase and the number that would be purchased by an efficient single individual. If the number of public characteristics is at least as great as the number of private characteristics, the household will purchase the same number of goods as would a single individual.

V. General Conclusions

We can summarize the overall results of the paper as follows:

(1) There is no general warrant for considering the household to behave as if it were a single individual. The observable behavior of the household may differ from the typical behavior of the individual in two respects

(a) the concavity properties of its demand function may differ from that of the individual

(b) the efficiency properties (observed as numbers of goods purchased relative to the number of distinct characteristics) may not conform to those of the individual.

(2) If the household is an aggregate of independent consumers, the concavity properties of its demand function will be weaker than those for the individual, but will come closer to the individual properties as the size of the household decreases. The efficiency properties will diverge from those of the individual, with the maximum extent of this divergence declining as the size of the household decreases.

(3) The existence of a well-behaved household decision function of a Samuelson or similar kind will remove all the divergence in concavity properties between the household and the individual, but will *not* remove the divergence in efficiency properties.

(4) The existence of joint consumption effects within the household will not affect the concavity properties of demand but will reduce the divergence in efficiency properties between the household and the individual.

The household that *will* behave as if it were a single individual is either dictatorial or has a well-behaved decision function and joint effects in consumption covering at least half the characteristics relevant to its members.

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