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Volume Author/Editor: Frank A. Hanna, Joseph A. Pechman, and Sidney M. Lerner

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Chapter Author: Frank A. Hanna, Joseph A. Pechman, Sidney M. Lerner

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Appendix B

Methods Used to Determine Effects of Changes in the Absolute and Relative Distributions of Total Income upon Its Composition

The computations to estimate the effects of changes in the absolute and relative distributions of aggregate income on its composition were based on the size distribution of the incomes of persons filing returns in 1936. In general, the first step was to transform the 1936 distribution of aggregate income in one of two ways: (a) the total income of each person filing a return was increased either 25 or 50 percent; (b) the aggregate income of all filers was held constant, but its relative distribution was so altered as to move the Lorenz curve either 25 or 50 percent toward the line of equal distribution. The new aggregates for each income group were distributed among the various receipts according to the percentage composition of income for each income group, as shown in Table 1. Finally, the new amounts of each receipt, at each income group, were added and a new composition of income for all groups computed on the basis of these aggregates.[1]

1 Calculating the Basic Frequency Distribution

The size distribution of aggregate income is by $1,000 intervals up to the $10,000 level; by $5,000 intervals $10,000-25,000; and

[1] The methods used in this Appendix were developed in *Consumer Expenditures in the United States,* pp. 164-95.

the three top classes are $25,000-50,000, $50,000-100,000, and $100,000 and over (App. Table 3). Before shifting the proper number of individuals in the manner described above, the inter-

APPENDIX TABLE 3

The Actual and Basic Distribution of Income, 1936

ACTUAL DISTRIBUTION			BASIC DISTRIBUTION		
Income Group	No. of Returns	Income (dollars)	Income Group	No. of Returns	Income (dollars)
			$1- 199	8,672	867,200
			200- 399	13,770	4,131,000
$1- 999	128,699	86,302,000	400- 599	18,869	9,434,500
			600- 799	33,803	23,662,100
			800- 999	53,585	48,226,500
			1,000- 1,199	50,393	55,432,300
			1,200- 1,399	46,770	60,801,000
1,000- 1,999	206,610	298,239,150	1,400- 1,599	41,832	62,748,000
			1,600- 1,799	36,613	62,242,100
			1,800- 1,999	31,002	58,903,800
			2,000- 2,199	23,235	48,793,500
			2,200- 2,399	15,537	35,735,100
2,000- 2,999	66,151	156,430,090	2,400- 2,599	12,204	30,510,000
			2,600- 2,799	8,712	23,522,400
			2,800- 2,999	6,463	18,742,700
			3,000- 3,249	7,340	22,937,500
			3,250- 3,499	4,100	13,837,500
3,000- 3,999	17,848	60,636,230	3,500- 3,749	3,600	13,050,000
			3,750- 3,999	2,808	10,881,000
4,000- 4,999	7,280	32,253,480	4,000- 4,499	4,092	17,391,000
			4,500- 4,999	3,188	15,143,000
5,000- 5,999	4,020	21,824,360	5,000- 5,499	2,252	11,823,000
			5,500- 5,999	1,768	10,166,000
6,000- 7,999	4,137	28,370,780	6,000- 6,999	2,492	16,198,000
			7,000- 7,999	1,645	12,337,500
8,000- 9,999	2,031	18,035,560	8,000- 8,999	1,155	9,817,500
			9,000- 9,999	876	8,322,000
10,000-14,999	2,281	27,403,230	10,000-12,499	1,524	17,145,000
			12,500-14,999	757	10,408,750
15,000-19,999	891	15,254,220	15,000-17,499	538	8,742,500
			17,500-19,999	353	6,618,750
20,000-24,999	421	9,390,790	20,000-22,499	237	5,036,250
			22,500-24,999	184	4,370,000
			25,000-29,999	311	8,552,500
25,000-49,999	692	23,386,820	30,000-39,999	210	7,350,000
			40,000-49,999	171	7,695,000
			50,000-74,999	159	9,937,500
50,000-99,999	206	13,929,380	75,000-99,999	47	4,112,500
100,000 & over	74	16,569,080	100,000 & over	74	16,569,080
Total	441,341	808,025,170		441,341	812,194,030

vals in the actual distribution were divided into narrower intervals to minimize the error made when it was later necessary to assume that the frequencies were evenly distributed in each inter-

val. Two methods of interpolation were used: one for incomes below $3,000, the other, for incomes above $3,000.

Interpolation below $3,000

The interpolations below the $3,000 group were based on the distribution of individual filers by $100 economic income brackets.[1] The absolute distribution by economic income below $3,000 was converted to a percentage distribution and the percentages cumulated. The cumulated frequency distribution based on the economic income classification below $3,000 is practically the same as the cumulated frequency distribution based on the total income classification. The interpolation was made by assuming that the differences between the total income and the economic income cumulative distributions, at the known points ($1,000 and

	CUMULATED FREQUENCY DISTRIBUTION BY	
LESS THAN	Economic Income	Total Income
	(percentages)	
$1,000	32.82	32.06
2,000	84.48	83.52
3,000	100.00	100.00

$2,000), were due to a cumulative error distributed evenly within each interval; i.e., the error allocated to each $100 interval within each $1,000 group was one-tenth of the total error in the $1,000 group.[2] After the cumulative frequency distribution by $100 total income groups had been calculated in this manner, the percentages were decumulated, and the frequency in each $100 group was calculated by applying the decumulated percentages to the total number of individuals with less than $3,000 total income.

To calculate the incomes for each group, the $100 intervals were combined into $200 intervals, and the number of returns in each $200 group was multiplied by the midpoint of the interval. The differences between the estimated and actual amounts were 0.6 percent or less in the three $1,000 groups below $3,000.

1 *Wisconsin Individual Income Tax Statistics, 1936,* I, A8. For a definition of economic income, see Part I.

2 For example, at the $1,000 level, the difference was −0.76%: (32.06% − 32.82%). The error allocated to each interval was, therefore, −0.076%. The cumulative error at the $100 level was −0.076%; at the $200 level 2 (−0.076%), at the $300 level 3 (−0.076%), and so on.

Interpolation above $3,000

To interpolate above the $3,000 level, a graphic method was used. The cumulated number of returns was plotted in ogive form and a smooth curve run through the known points. Interpolated points were then read off the ogive at the desired intervals, and decumulated to give the number of returns in each new group. The number of returns multiplied by the midpoint of each interval gave the income for each group. The differences between the estimated and actual amounts were less than 0.9 percent in all intervals above $3,000.

The final aggregate income in the basic distribution is slightly over $812 million; the actual amount, over $808 million, a difference of only five-tenths of one percent.

2 Transforming the Basic Frequency Distribution

Proportionate Increase in Aggregate Income

When aggregate income is increased 25 percent and the increase is distributed proportionately, each individual income is increased 25 percent. Let X be an income in 1936: To calculate what income in 1936 will be $500 after a 25 percent increase, we use the equation: $1.25X = \$500$; $X = \$400$. The limits of the new intervals expressed in 1936 incomes were fixed in this way. For example, the first five intervals in the distribution after a proportionate increase of 25 percent in aggregate income were:

Interval limits after 25% increase in aggregate income	1936 interval limits before 25% increase in aggregate income
$0- 200	$0-160
200- 400	160-320
400- 600	320-480
600- 800	480-640
800-1,000	640-800

In other words, individuals who formerly received $0-160 will be in the $0-200 group after their incomes are increased 25 percent; individuals who formerly received $160-200 will be in the $200-400 group, etc.

To find the number of individuals in each group in the new distribution, it was assumed that the individuals were distributed

evenly in each group of the basic frequency distribution. For example, the individuals who were in the $600-800 group after total income was increased 25 percent had incomes of $480-640 in 1936. The number of individuals in the $480-600 group in the basic frequency distribution is, on the assumption of an even distribution within the group, 60 percent of the total in the $400-600 group. Similarly, the number of individuals in the $600-640 group is 20 percent of the total in the $600-800 group. The number in the $480-640 group is computed from data in the basic frequency distribution.

BASIC FREQUENCY DISTRIBUTION (App. Table 3)		RESULT OF 25% INCREASE IN AGGREGATE INCOME	
Interval	Number	Interval	Number
$400-600	18,869	$480-600	11,321 (60% of 18,869)
600-800	33,803	600-640	6,761 (20% of 33,803)
Total new $600-800 group			18,082

To find the aggregate income in each group in the new distribution, the frequencies in each subinterval found above (that is, $480-600, $600-640, etc.) were multiplied by the midpoints of the subintervals, and the amounts in these subintervals combined into the 30 groups shown in the basic frequency distribution. This step gave the 1936 income in each new group. Since each particular income is increased 25 percent, the 1936 incomes in the new intervals were multiplied by 1.25. The aggregate income in the new $600-800 group was computed as follows:

INTERVAL	NUMBER (1)	MIDPOINT (2)	AGGREGATE 1936 INCOME[a] (3)	RESULT OF 25% INCREASE IN AGGREGATE INCOME[b] (4)
$480-600	11,321	$540	$6,113,340	$7,641,675
600-640	6,761	620	4,191,820	5,239,775
Total new $600-800 group	18,082		10,305,160	12,881,450

[a]Col. (1) x col. (2). [b]Col. (3) x 1.25.

The income for each group in the new distribution except that above $100,000 was calculated in this manner. Since all incomes above $100,000 would remain above that level after the 25 percent increase, the aggregate income for the group was increased 25 percent directly and was inserted into the new distribution.

Moving the Lorenz Curve toward the Line of Equal Distribution

"Given a frequency distribution and the Lorenz curve corresponding to it, the new frequency distribution obtained by moving every family some proportion of the way from its income in the given distribution toward the mean income has, as its Lorenz curve, the curve obtained by moving the given Lorenz curve the same proportion of the way vertically toward the line of equal distribution." [3] Thus, in order to shift the Lorenz curve 25 percent closer to the line of equal distribution, assuming the same total income, each individual income is shifted 25 percent toward the mean income. Let X be an income in 1936; to find what a 1936 income would have to be to become an income of $500 after it is shifted 25 percent toward the mean, the following equation is used: $X + .25(\$1{,}840 - X) = \$500$ ($1,840 is the mean income in 1936). Solving, $X = \$53.33$. The group limits, expressed in 1936 income, of the new distribution were computed in the manner described above, using this equation. The number of returns in each group of the new distribution was computed by assuming once again that the individuals in each group are distributed evenly within it.

The income of each group in the new distribution is the sum of the new incomes. An individual who had received an income of X in 1936 has an income of $X + .25(\$1{,}840 - X)$ after the shift. Therefore, the new aggregate of income for any group of N individuals is:

$$\Sigma X + \Sigma .25(\$1{,}840 - X) \text{ or } .75\Sigma X + N(\$460).$$

For example, the individuals in the $600-800 group after the shift had previously had incomes of $187-453.[4] The number of individuals in the $187-453 group in the basic frequency distribution is computed as follows:

BASIC FREQUENCY DISTRIBUTION (App. Table 3)		RESULT OF 25% SHIFT TOWARD LINE OF EQUAL DISTRIBUTION	
Interval	Number	Interval	Number
$0-200	8,672	$187-200	564 (6.5% of 8,672)
200-400	13,770	200-400	13,770 (100% of 13,770)
400-600	18,869	400-453	5,000 (26.5% of 18,869)
Total new $600-800 group			19,334

[3] *Consumer Expenditures in the United States,* p. 188.

[4] For the lower limit: $X + .25 (\$1{,}840 - X) = \$600$; $X = \$187$.
For the upper limit: $X + .25 (\$1{,}840 - X) = \$800$; $X = \$453$.

The income in the new $600-800 group is as follows:

Interval	Number (1)	Midpoint (2)	Aggregate 1936 Income[a] (3)	Result of 25% Shift toward Line of Equal Distribution[b] (4)
$187-200	564	$193.50	$109,134	$341,290
200-400	13,770	300.00	4,131,000	9,432,450
400-453	5,000	426.50	2,132,500	3,899,375
Total new $600-800 group	19,334		6,372,634	13,673,115

[a]Col. (1) x col. (2).

[b]Obtained from the formula: $.75\Sigma X + N(\$460)$, where N is the number in col. (1) and ΣX is the amount of income in col. (3).

These procedures were followed to calculate the income for each group in the new distribution, except that above $100,000. The income of each individual in this group was taken from unpublished tables of the Wisconsin Income Tax Study, and the new income computed directly.