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# 4 The Rate of Obsolescence of Patents, Research Gestation Lags, and the Private Rate of Return to Research Resources

Ariel Pakes and Mark Schankerman

The recent interest of economists in knowledge-producing activities has two main strands. The first is an attempt to explain the growth in the measured productivity of traditional factors of production by incorporating research resources in production function and social accounting frameworks (for a review see Griliches 1973, 1980). The second derives from two fundamental characteristics of knowledge as an economic commodity, its low or zero cost of reproduction and the difficulty of excluding others from its use. These features give knowledge the character of a public good and suggest that the structure of market incentives may not elicit the socially desirable level (or pattern) of research and development expenditures. In particular, it has been argued that market incentives may create either underinvestment or overinvestment in knowledge-producing activities (see Arrow 1962; Dasgupta and Stiglitz 1980). To investigate this possibility, economists have applied the techniques of productivity analysis and estimated the private (and social) rate of return to research from production functions incorporating research resources as a factor of production. The estimates of the *private* return for the late 1950s and early 1960s fall in the range of 30–45 percent.[1] Despite these

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1. For example, see Griliches (1980) and Mansfield (1968). Note that both these studies are based on firm or microdata. More aggregative data bases are not directly relevant to estimates of the private rate of return to research.

high estimated private rates of return, the share of industrial resources allocated to research expenditures did not increase over the succeeding decade.[2] This suggests a paradox: Why has research effort not been receiving more attention from industrial firms if the private rate of return to research is so attractive?

Two important parameters in these calculations of the private return to research are the rate of decay of the private revenues accruing to the industrially produced knowledge and the mean lag between the deployment of research resources and the beginning of that stream of revenues. These parameters, of course, are necessary ingredients in *any* study involving a measurement of the stock of privately marketable knowledge. The rate of decay in the returns of research has not previously been estimated. In this paper we present a method of explicitly estimating that parameter. We also use information provided by others to calculate the approximate mean R & D gestation lags. Since previous research has not included the latter and seems to have seriously understated the rate of decay of appropriable revenues in calculations of the private rate of return to research expenditures, we then use our estimates to improve on previous results on this rate of return.

Of course, all previous work in this area has been forced to make some assumption, either implicit or explicit, about the value of the decay rate. The problem arises because it has been assumed to be similar to the rate of decay in the *physical* productivity of traditional capital goods. The fact that the rate of deterioration of traditional capital and the rate of decay in appropriable revenues from knowledge arise from two different sets of circumstances seems to have been ignored.[3]

The employment of research resources by a private firm produces new knowledge, with some gestation lag. The new knowledge or innovation may be a cost-reducing process, a product, or some combination of the two. The knowledge-producing firm earns a return either through net revenues from the sale of its own output embodying the new knowledge, or by license and nonmonetary returns collected from other firms which lease the innovation. Since the private rate of return to research depends on the present value of the revenues accruing to the sale of the knowledge produced, the conceptually appropriate rate of depreciation is the rate at which the appropriable revenues decline for the innovating firm. However, as Boulding (1966) noted, knowledge, unlike traditional capital,

2. The share of net sales of manufacturing firms devoted to total R & D (publicly and company funded) actually declined from 0.046 in 1963 to 0.029 in 1974, or by about 40 percent, though the share devoted to company-funded R & D remained constant at 0.019. See National Science Foundation, (1976, table B-36).

3. The commonly assumed rate of decay of the knowledge produced by firms is between 0.04 and 0.07 (Mansfield 1968). Griliches (1980), noting some of the conceptual distinctions between the rates of decay in traditional capital and in research, assumes an upper bound of 0.10 for the latter.

does not obey the laws of (physical) conservation. The rate of decay in the revenues accruing to the producer of the innovation derives not from any decay in the productivity of knowledge but rather from two related points regarding its market *valuation*, namely, that it is difficult to maintain the ability to appropriate the benefits from knowledge and that new innovations are developed which partly or entirely displace the original innovation. Indeed, the very use of the knew knowledge in any productive way will tend to spread and reveal it to other economic agents, as will the mobility of scientific personnel. One might expect then that the rate of decay of appropriable revenues would be quite high, and certainly considerably greater than the rate of deterioration in the physical productivity of traditional capital.[4]

In section 4.1 we examine two independent pieces of evidence bearing on the rate of decay of appropriable revenues. The information from various sources on the mean lag between R & D expenditures and the beginning of the associated revenue stream is summarized in section 4.2. In section 4.3 we attempt to get a rough idea of how seriously the existing estimates of the private rate of return to research overstate the true private rate of return. Brief concluding remarks follow.

## 4.1 The Rate of Decay of Appropriable Revenues

The first piece of evidence on the rate of decay of appropriable revenues (hereafter, the rate of decay) is based on data presented in Federico (1958). Federico provides observations on the percentage of patents of various ages which were renewed by payment of mandatory annual renewal fees during 1930–39 in the United Kingdom, Germany, France, the Netherlands, and Switzerland. A theoretical model of patent renewal will lead directly to a procedure for estimating the rate of decay from these data.

Consider a patented innovation whose annual renewal requires payment of a stipulated fee. Letting $r(t)$ and $c(t)$ denote the appropriable revenues and the renewal fee in year $t$, the discounted value of net revenues accruing to the innovation over its life span, $V(T)$, is

$$V(T) = \int_0^T [r(t) - c(t)]e^{-it}dt, \tag{1}$$

where $i$ is the discount rate and $T$ is the expiration date of the patent.

4. The models used in this paper do not assume that the rate of decay in appropriable revenues is exogenous to the firm's decision-making process. In a dynamic context, a firm processing an innovation has to choose between increasing present revenues and inducing entry, and charging smaller royalties to forestall entry. This choice is the basis of Gaskins's (1971) dynamic limit pricing analysis of situations involving temporary monopoly power. The Gaskins model can be used to show that the optimal revenue stream will decline over time and that the rate of decline will depend on certain appropriability parameters.

Differentiating (1) with respect to $T$, the optimum expiration date, $T^*$, is written implicitly as

(2) $$r(T^*) = c(T^*),$$

provided that $r'(t) < c'(t)$ for all $t$. Equivalently, the condition for renewal of the patent in year $t$ is that the annual revenue at least covers the cost of the renewal fee

(3) $$r(t) \geq c(t).$$

Let the annual renewal fee grow at rate $g$, and the appropriable revenues decline at rate $\delta$. Then condition (3) can be written as

(4) $$r(0) \geq c(0)e^{(g+\delta)t}.$$

Allowing for differences in the initial appropriable revenues among patents, and letting $f(r)$ represent the density function of the distribution of their values, the percentage of patents renewed in each year, $P(t)$, is[5]

(5) $$P(t) = \int_{C(t)}^{\infty} f(r)dr,$$

where $C(t) = c_0 e^{(g+\delta)t}$. It follows that

(6a) $$P'(t) = -C'(t)f(C),$$

and

(6b) $$P''(t) = -f(C)C''(t)\left[1 + C\frac{f'(C)}{f(C)}\right],$$

where the primes denote derivatives. That is, as long as $(g + \delta) > 0$, the percentage of patents renewed will decline with their age. The curvature of $P(t)$, however, will depend on the distribution of the values of the innovations patented.[6] For example, if $f(r)$ is lognormal, $P(t)$ will have one point of inflection, being concave before it and convex thereafter (see curve 1, fig. 4.1). Alternatively, Scherer (1965) cites evidence presented in Sanders, Rossman, and Harris (1958) which indicates that the value of patents tends to follow a Pareto-Levy distribution. If $f(r)$ is Pareto-Levy, $P(t)$ will be a strictly decreasing convex function of patent age, as shown by figure 4.1, curve 2. Figure 4.2 presents the actual time paths of $P(t)$

5. We are implicitly assuming that the rate of decay of appropriable revenues, $\delta$, does not differ among patented innovations. This assumption allows us to compare directly our estimate of $\delta$ to those assumed in previous empirical work (since the same implicit assumption is prevalent in that work) and to consider the economic implications of the different values of $\delta$ (see section 4.3). The model could be generalized to allow for differences in decay rates and, if sufficient data were available, one could estimate the parameters of the joint distribution of the values of the initial revenues and the decay rates.

6. Since the value of patents (eq. [1]) is a monotonic transformation of the adjusted initial revenues, we shall use the two terms interchangeably.

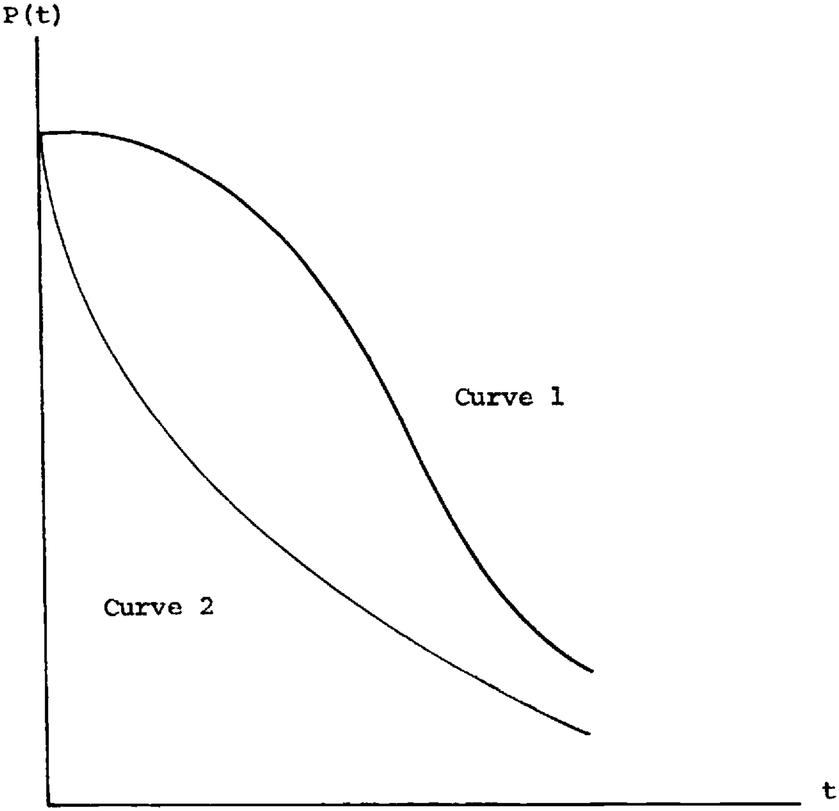


**Fig. 4.1**

from Federico (1958). Four of the five curves tend to support Sanders et al.'s data and are consistent with an underlying Pareto-Levy distribution of patent values. The time path for Germany, however, indicates that the underlying distribution for that country has at least one mode. Since it is futile to estimate both the parameters of the underlying lognormal (for example) and the decay rate in appropriable revenues from only eighteen observations available on Germany, we shall disregard the German data in the remainder of this empirical work.[7]

We now use this simplified model to obtain rough estimates of the decay rate of appropriable revenues $\delta$. Consistent with the evidence in figure 4.2, the relative density function of initial revenue is taken to be of the Pareto-Levy type:

(7) $$f(r) = \beta r_m^{\beta} r^{-(\beta+1)}, \ r_m > 0, \beta > 0,$$

7. The United Kingdom patent system requires no renewal payments until the fifth year. Hence, the underlying distribution of patent values may have an inflection point, but it cannot be ascertained from the data.

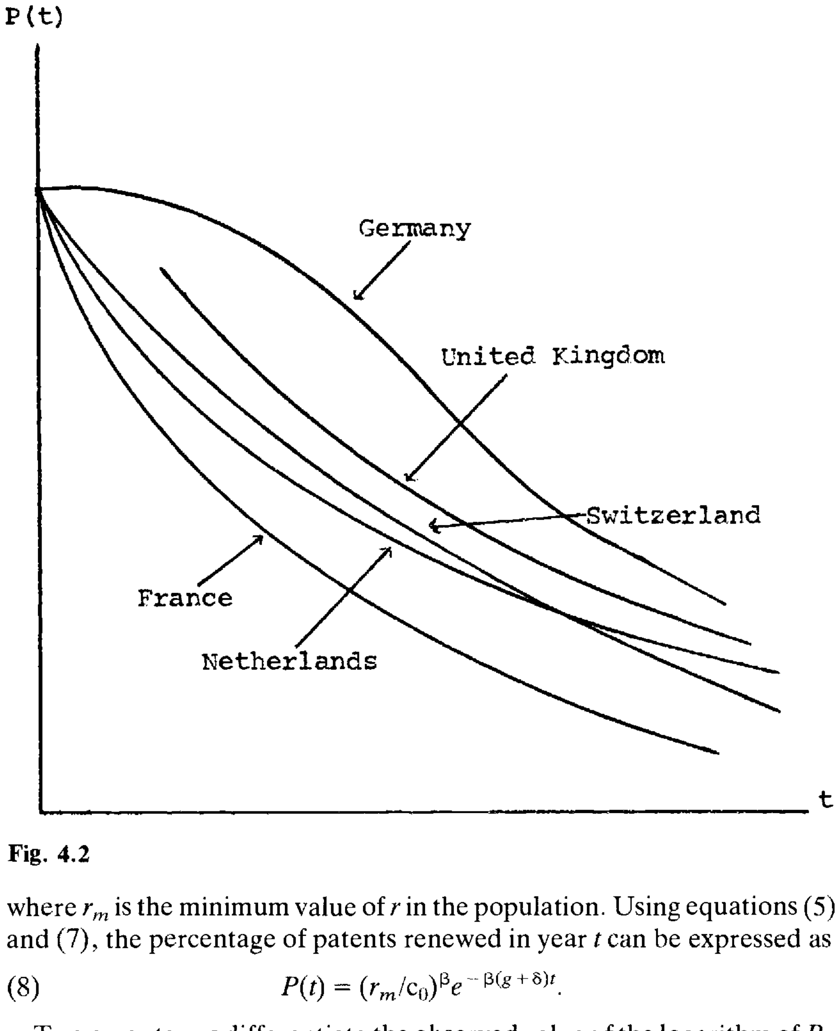


**Fig. 4.2**

where $r_m$ is the minimum value of $r$ in the population. Using equations (5) and (7), the percentage of patents renewed in year $t$ can be expressed as

(8) $$P(t) = (r_m/c_0)^{\beta} e^{-\beta(g+\delta)t}.$$

Two error terms differentiate the observed value of the logarithm of $P_t$, $\log P_t^m$, from the value predicted by equation (8).[8] The first, $\nu_1$, is a sampling or measurement error, while the second, $\nu_2$, is a structural error in the model. Assuming that $P_t^m$ is derived from a binomial sampling process around the actual value, that is, $P_t^m \sim b(P_t, N)$, it follows that

$$\sigma^2_{\nu_1} = V(\log P_t^m) \approx \frac{1 - P_t}{P_t N},$$

where $N$ is the (unobserved) number of patents sampled. The structural

8. Since $P_t$ is bounded by zero and unity, the composite error cannot be independently and identically distributed. The analysis which follows corresponds closely to the treatment of similar problems in logit regressions. See Berkson (1953) and Amemiya and Nold (1975).

error, $v_2$, will be assumed to be an independent, identically distributed normal deviate with variance $\sigma^2$.

Letting $j$ index a country, for sufficiently large $N_j$ the logarithmic transform of $P_{tj}^m$ can be written as

$$\text{(9)} \qquad \log P_{tj}^m = \alpha_{0j} + \alpha_{1j}t_j + \mu_{tj},$$

where $\mu_{tj} \sim N[0, \sigma^2 + (1 - P_{tj})/P_{tj}N_j]$, $\alpha_{1j} = -\beta_j(g_j + \delta_j)$, and $\alpha_{0j} = \beta_j \log (r_m/c_0)_j$. The estimating question (9) embodies the basic prediction of the model, namely a negative relationship between $\alpha_{1j}$ and $g_j$, where the slope coefficient is the parameter of the underlying distribution of patent values.

Consistent estimates of $\alpha_{0j}$, $\alpha_{1j}$ and their standard errors can be derived from the following two-stage procedure. First, estimate (9) by ordinary least squares. Next, define $e^2$ as the squared residuals from (9) and regress

$$\text{(10)} \qquad e_{tj}^2 = \sigma^2 + \frac{1}{N_j}\frac{1 - P_{tj}^m}{p_{tj}^m}.$$

Letting $F$ be the fitted value from (10), use $F^{-1/2}$ to weight and perform weighted least squares on (9).

If our model is correct, and if $\beta$ and $\delta$ do not vary between countries, then

$$\text{(11)} \qquad \alpha_{1j} = -\beta\delta - \beta g_j.$$

Since $g_j$, the rate of growth of renewal fees, is available from Federico's data,[9] (11) can be tested by using $F^{-1/2}$ to weight and by performing weighted least squares on the equation

$$\text{(12)} \qquad \log P_{tj}^m = \alpha_{0j} - \beta\delta t_j - \beta(g_j t_j) + \mu_{tj},$$

where all symbols are as defined above.

If (11) is the true specification, then minus twice the logarithm of the likelihood ratio from (12) and the weighted least-squares version of (9) will distribute asymptotically as a $\chi_2^2$ deviate. Moreover, equation (12) will provide estimates of both the rate of decay of appropriable revenues ($\delta$) and the underlying distribution of patent values ($\beta$).

Table 4.1 summarizes the empirical results. The observed value of the $\chi^2$ test statistic is 5.4, while the 5 percent critical value is 5.99. Though a little high, the test statistic does indicate acceptance of the hypothesis in (11). The estimates of $1/N_j$ and $\sigma^2$ are all positive thereby lending support to the weighting procedure described above.

Turning to the parameters of interest, the point estimate of $\beta$ is 0.57

9. These growth rates were calculated from a semilog linear regression of costs against time for each country. The growth rates (and their standard errors) for the Netherlands, the United Kingdom, France, and Switzerland were 0.085, (0.002), 0.129 (0.015), 0.089 (0.006), and 0.143 (0.008), respectively.

**Table 4.1 Estimates from the Patent Renewal Model**[a,b]

| Common Parameters (57 observations) | | Country-Specific Parameters | | |
|---|---|---|---|---|
| | | | $\alpha_{0j}$ | $1/N_j$ |
| $\beta\delta$ | 0.14 | France | 0.04 | 0.00014 |
| | (0.01) | | (0.02) | |
| $\beta$ | 0.57 | United Kingdom | 0.55 | 0.00016 |
| | (0.07) | | (0.02) | |
| $\delta$ | | Netherlands | 0.09 | 0.00040 |
| | | | (0.02) | |
| Point estimate[c] | 0.25 | Switzerland | 0.32 | 0.00032 |
| Confidence interval[d] | 0.18–0.36 | | (0.02) | |
| $\sigma^2$ | 0.0002 | | | |
| $R^2$ | 0.996 | | | |

[a]The data on patent renewal are taken from Federico (1958). These data cover the percentage of patents of different ages in force during 1930–39 which were renewed by payment of a mandatory annual renewal fee. For example, if a patent was granted in 1925, it would appear in the data as five years old in 1930, six years old in 1931, and so on. Therefore, the percentage of patents renewed after five years is based on the total number of patents issued five years earlier in a particular country.

[b]Standard errors are in parentheses.

[c]$\hat{\delta} = \widehat{\beta\delta}/\hat{\beta}$.

[d]The confidence interval corresponds to the 95 percent Fieller bounds on $\delta$.

with a standard error of 0.07. One can check this estimate against an independent source of information: as mentioned earlier, Sanders, Rossman, and Harris (1958) provide evidence on the distribution of the value of patents in the United States. Fitting a Pareto-Levy distribution to these data, we obtain a point estimate for $\beta$ of 0.63 with a standard error of 0.06. That is, the estimate of $\beta$ from Sanders et al.'s data is very close to that obtained using our model and Federico's data.[10]

Our primary interest is in $\delta$, the (average) decay rate in appropriable revenues. The point estimate of $\delta$ is 0.25, while a 95 percent confidence interval places the true value of $\delta$ between 0.18 and 0.36.[11] An estimated $\delta$

10. These data correspond to appropriable revenues minus costs associated with the patent, but since cost data were not available we were forced to use net value data. A cumulative Pareto-Levy distribution was fitted to the five positive net value observations on expired patents which therefore have observable net values. The $R^2$ from this regression was 0.97. Note that Pareto-Levy distributions with $\beta < 1$ do not have either a finite mean or variance and hence do not behave according to the law of large numbers. Therefore, if the distribution of patent values approximates the distribution of project values, diversification into many independent projects will not reduce risk. This point was originally made by Nordhaus (1969). Of course, if the returns to different projects are negatively correlated, diversification may still reduce risk.

11. Note that since the estimate of $\delta$ is obtained as the ratio of two coefficients, its confidence interval (obtained by using Fieller bounds) is not symmetric around its point estimate. Two remarks on the robustness of these results are also in order. First, the assumption that the revenue stream can be described by an exponential rate of decay can be

of 0.25, though consistent with theoretical arguments concerning the unique characteristics of knowledge as an economic commodity, implies that earlier researchers have assumed values of $\delta$ which are far too small. In particular, the *lower* bound of the 95 percent confidence interval for $\delta$ is nearly twice the *maximum* value of the rate of decay of private returns used in previous research.

Of course, our estimate of $\delta$ may reflect some sample selection bias. The rate of decay of patented innovations may differ from that of all innovations. The direction of the bias is indeterminate since it depends on the correlation between the patent selection process and the rates of obsolescence in the universe of all innovations. However, the estimates of $\delta$ may be biased downward for two reasons: First, the fact that patents create property rights in the embodied knowledge may result in a lower rate of obsolescence for those patentable innovations. Second, given a patentable innovation, it is easy to show that the innovator will actually take out a patent only if patenting lowers the rate of decay. As we show presently, however, evidence of a completely different nature suggests that whatever bias exists is negligible.

The second source of evidence on the magnitude of the decay rate of appropriable revenues is derived from data presented in Wagner (1968) on the life span of applied research and development expenditures. Survey data on applied research and development were collected from about thirty-five firms with long R & D experience in thirty-three product fields, using the product field description employed by the National Science Foundation (NSF) in its annual industry reports. Included in the survey was a question on the life span of R & D defined as the period after which the product of the R & D was "virtually obsolete."

This definition does not correspond directly to the decline in the appropriable revenues accruing to research and development. However, a rough correspondence can be established by assuming that R & D is virtually obsolete when the appropriable revenues reach some small fraction of the initial value, and then by experimenting with different fractions to examine the sensitivity of the implied decay rate to the assumption. Table 4.2 presents the average life span of R & D for durable and nondurable product field categories, product- and process-oriented R & D, and the implied decay rates based on various reasonable definitions of virtual obsolescence. While the implied decay rates do vary with

viewed as a first-order (logarithmic) approximation to a more general stream of revenue. We also experimented with a second-order approximation, namely, $r(t) = r(0)\exp(At + Bt^2)$. The estimates of $B$ and its standard error were both zero to two decimal places and the rest of the results were almost identical to those reported here. Apparently market-induced obsolescence is well approximated by an exponential pattern. Griliches (1963) reaches the same conclusion with respect to the obsolescence component of the deterioration in the value of traditional capital goods. Second, the results from the unweighted version of (12) yielded a point estimate of $\delta = 0.22$, with Fieller bounds of 0.16 to 0.33, and an estimate of $\beta = 0.62$.

Table 4.2 Estimates of δ from Average Life Span of R & D[a,b]

| | Ratio of Revenue in Year $T$ to Initial Revenue ("virtual obsolescence") | | |
|---|---|---|---|
| | 0.15 | 0.10 | 0.05 |
| Durable goods R & D | | | |
| Product ($T$ = 9) | 0.21 | 0.26 | 0.33 |
| Process ($T$ = 11) | 0.17 | 0.21 | 0.27 |
| Nondurable goods R & D | | | |
| Product ($T$ = 9) | 0.21 | 0.26 | 0.33 |
| Process ($T$ = 8) | 0.24 | 0.29 | 0.38 |

[a]Taken from Wagner (1968, p. 196, table 5), which refers to "applied research and development" (AR & D). These life span figures (denoted by $T$ in parentheses) are averages of survey responses, weighted by 1965 product-field expenditures and by frequencies of the response distribution.

[b]Calculated as $\delta = -(\log x)/T$, where $x$ is the assumed ratio of revenue in year $T$ to initial revenue accruing to the R & D.

the definition of virtual obsolescence, the rate of values is nearly identical to the Fieller bounds on δ in table 4.1.[12]

The responses of firms to Wagner's question can also be used to check the reasonableness of the rates of obsolescence commonly assumed in the literature. If in fact $\delta = .05$ (.10), that would imply (using $T = 9$ from table 4.2) that firms consider the product of their R & D virtually obsolete even though the annual revenue flow is still 64 (41) percent of its initial value. This seems highly implausible and casts additional doubt on the conventionally assumed values of δ.

## 4.2 Mean R & D Lags

Two independent sources of information are used to estimate the mean R & D lags, defined as the average time between the outlay of an R & D dollar and the beginning of the associated revenue stream. This lag consists of a mean lag between project inception and completion (the gestation lag), and the time from project completion to commercial application (the application lag).

Rapoport (1971) presents detailed data on the distribution of costs and time for forty-nine commercialized innovations and the total innovation time for a subset of sixteen of them in three product groups—chemicals, machinery, and electronics. The innovation process is decomposed into

12. The only other estimate of the decay rate is produced knowledge of which we are aware is reported in a footnote in Griliches (1980). A regression of productivity growth against R & D flow and stock intensity variables in his microdata set yielded an estimated δ of 0.31. Griliches points out the discrepancy between this result and the rest of his analysis but offers no reconciliation.

**Table 4.3** **Estimates of the Mean R & D Lag (years)**

| | R & D Gestation Lag | Application Lag | Total Lag |
|---|---|---|---|
| Rapoport | | | |
| Chemicals | 1.48 | 0.24 | 1.72 |
| Machinery | 2.09 | 0.31 | 2.40 |
| Electronics | 0.82 | 0.35 | 1.17 |
| Wagner | | | |
| Durables | 1.15 | 1.47 | 2.62 |
| Nondurables | 1.14 | 1.03 | 2.17 |

Source: Calculated from data contained in Rapoport (1971) and Wagner (1968).

five stages: applied research, specification, prototype or pilot plant, tooling and manufacturing facilities, and manufacturing and marketing start-up. Since the expenditures on manufacturing and marketing start-up are not included in the NSF definition of R & D expenditures, the time involved in that stage is treated here as the application lag. The remaining data are used to calculate the gestation lag. The first part of table 4.3 summarizes the R & D gestation and application lags for the three product groups.[13]

Additional information on the average R & D and application lags is provided in Wagner (1968). Survey data on process- and product-oriented R & D were gathered from about thirty-six firms with long R & D experience in a variety of durable and nondurable goods industries. Included was information on the duration of applied research and development, project duration for projects successfully completed in 1966, the distribution of R & D expenditures for successfully completed projects classified by project duration, the percentage of total funds accounted for by projects abandoned before completion together with the time of abandonment, and the interval between the completion of R & D and commercial application of the innovations. These data are used to calculate both an application lag and a mean gestation lag which, unlike those based on Rapoport's data, take into account expenditures on both technically successful and unsuccessful projects. The results are given in the second part of table 4.3.

The gestation lags based on Wagner's data are broadly similar to those

13. The details of the calculations are omitted here for the sake of brevity but are available on request. However, the limitations of these estimates should be noted. First, all the projects analyzed by Rapoport resulted in significant innovations, and as Scherer (1965) and Mansfield (1968) have noted, mean lags tend to be longer for more significant technical advances. Second, we have not taken into account the time overlap between stages, which, according to Rapoport, is considerable. Both of these factors would tend to cause upward biases in our estimates of the mean lag, $\theta$. On the other hand, the R & D costs of technically unsuccessful projects should be taken into account, which would tend to raise the estimates of $\theta$.

derived from Rapoport, but the application lags are considerably longer,[14] causing some discrepancy between the two sets of results. Mansfield (1968), using data gathered from extensive personal interviews with R & D project evaluation staff, concluded that the mean application lag was about 0.53 years. Substitution of this number for Wagner's would bring the two sets of results closer together and put the total lag at about 1.75 years. For present purposes, however, a range of values between 1.2 and 2.5 years is good enough.[15]

### 4.3 Implications for Measuring the Private Rate of Return to Investment in Research

The preceding sections of this paper provide estimates of the decay rate and the mean R & D lag whose values are substantially higher than those assumed in previous research. These estimates are now used to get a rough indication of the implications for production function estimates of the private rate of return to research expenditures.

Let $Q_R$ denote the increment in value added (or sales) generated by a unit increase in research resources $\theta$ years earlier. Then the equation for the private (internal) rate of return to investment in research is

$$e^{\theta r} = \int_0^\infty Q_R e^{-(r+\delta)\tau} d\tau \,, \tag{13}$$

where $\theta$ and $\delta$ were defined earlier, and $r$ is the private rate of return or the implicit discount rate that would make investment in research marginally profitable. Integrating (13) yields the following nonlinear equation for $r$:

$$e^{\theta r}(r+\delta) - Q_R = 0 \,. \tag{14}$$

In the special case where $\theta = 0$, this reduces to $r = Q_R - \delta$, corresponding to the equation used by previous researchers.

Given estimates of $Q_R$, $\delta$, and $\theta$, we can compute the private rate of return from equation (14). Two points should be noted. First, since research expenditures are usually included in the measures of traditional capital and labor expenditures in the production functions used to estimate $Q_R$, the private rates of return to research reported in the literature

14. Since Wagner does not precisely define the "end of AR & D" or the "application of innovations," some caution should be exercised in interpretating the application lags. Wagner does indicate that the longer application interval in durables reflects in large part the defense-space-atomic-energy-oriented fields, so that the application lag for other industries is probably closer to the nondurable estimate. On the other hand, Rapoport's "manufacturing and marketing start-up" stage may understate the actual application lag.

15. The maximum of this range is considerably shorter than the midpoint of the interval between project inception and marketing, reflecting the fact that the distribution of research expenditures on projects is considerably skewed to the left.

represent excess returns above and beyond the normal remuneration to traditional factors (see Griliches 1973). To avoid this problem, we base the calculations on estimates of $Q_R$ corrected for this double-counting (Schankerman 1981). Second, these estimates of $Q_R$ are calculated by multiplying the estimated sales (value added) elasticity of the stock of knowledge times the ratio of sales to the stock of knowledge. The stock of knowledge is taken as the undepreciated sum of research expenditures over the period of observation. For the calculations in (14) to be consistent, however, the stock of knowledge must be calculated according to declining balance depreciation. We therefore calculate the depreciated sum of research expenditures with a decay rate of $\delta$ and then use this stock of knowledge to convert the estimated sales elasticity into a value of $Q_R$.[16] This has been done for three values of $\delta$, corresponding to the point estimate and Fieller bounds obtained earlier (0.18, 0.25, and 0.36), and for three different values of $Q_R$ (0.30, 0.35, and 0.40), corresponding to the pooled (across industries) point estimate plus or minus one standard deviation computed from Schankerman (1981). The results are presented in table 4.4 for $\theta = 2$.

Turning to the results, it is apparent that the net private rates of return to investment in research are greatly reduced by our adjustments. The net private rate of return varies between .075 and .174. If the normal (net) rate of return to traditional capital is about 0.08 (Griliches 1980), this implies risk premiums for investment in research of between zero and about 9 percent. In view of the abnormal riskiness associated with research expenditures, these risk premiums appear modest.

In short, table 4.4 suggests that the private rates of return to investment in research and traditional capital are roughly equated at the margin. Another way of checking this possibility is to ask: What is the decay rate of appropriable revenues implied by the assumption that firms equate, at the margin, the private rates of return to investment in research and traditional capital? With a mean R & D lag of $\theta$, the return to a dollar of research is $(r + \delta + 1)(1 + r)^{-\theta}$, while for traditional capital, with depreciation rate $\delta_c$, it is $(r + \delta_c + 1)$. Using $\delta_c = 0.06$ and $r = 0.08$ from

16. We thank Zvi Griliches for pointing out this problem and suggesting a solution. The data for the calculations are taken from the National Science Foundation (1976), table B-1. Three additional points should be noted. First, Schankerman's estimates of $Q_R$ are based on the large microdata set used by Griliches (1980). Griliches's (uncorrected) pooled estimate of $Q_R$ was about 0.30, while his estimates for research intensive and nonintensive industries were 0.40 and 0.20, respectively. Mansfield's (uncorrected) estimates, averaged over the ten firms he used, range from about 0.20 to 0.30, depending on the specific assumptions made. Second, if private returns to knowledge do in fact decay, there is an error in the measured stock of knowledge used as an independent variable in the regressions to estimate $Q_R$. However, it can be shown that the ratio of the variance in measurement error to the variance in the true stock is very small (less than 0.0026), so the sales elasticity of the stock of knowledge can be taken directly from Schankerman's regressions. Finally, the private rates of return are much less sensitive to $\theta$ than to $\delta$.

Table 4.4 Estimates of the Net Private Rate of Return to Research ($\theta$ = 2)

| | $Q_r$ | | |
|---|---|---|---|
| $\delta$ | .30 | .35 | .40 |
| .18 | .090 | .118 | .144 |
| .25 | .111 | .144 | .174 |
| .36 | .075 | .107 | .139 |

Griliches (1980), the value of $\delta$ which equates these two terms is $\delta = 0.25$ if $\theta = 2.0$. This value is identical to the point estimate of $\delta$ in table 4.1.

## 4.4 Concluding Remarks

In this paper we stress the conceptual distinction between the rates of decay in the physical productivity of traditional capital goods and that of the appropriable revenues which accrue to knowledge-producing activities. An estimate of the private rate of obsolescence of knowledge is necessary in any study which requires constructing a stock of privately marketable knowledge. We estimate this parameter from a simple patent renewal model and find the estimate comparable to evidence provided by firms on the life span of the output of their R & D activities. The empirical results indicate that the rate of obsolescence is considerably greater than the rates typically assumed in the literature. The estimated decay rates, together with mean R & D gestation lags, are used to calculate the net private rate of return to investment in research. Our results suggest that the *private* rate of return to research expenditures, at least in the early 1960s, was *not* unreasonably high. It is important to emphasize, however, that to draw conclusions regarding the divergence between the private and social rates of return to knowledge-producing activities, information on the social rate of return must be added to the information contained in this paper.[17] Nonetheless, if our calculations of the private rate of return are even approximately correct, they do suggest a partial resolution to the paradox presented in the introduction to this paper: Why did private firms not increase the share of their resources devoted to R & D if their previous research efforts were so highly profitable? Part of the answer may be that research was not as privately profitable as has been thought.

17. In this connection, the social rate of decay may well be smaller than the rate of decay of appropriable revenues. See Hirshleifer's (1971) distinction between real and distributive effects in the production of knowledge.

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