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A METHOD FOR STOCHASTIC CONTROL OF NONLINEAR ECONOMETRIC MODELS AND AN APPLICATION: ABSTRACT

BY J. PHILLIP COOPER AND STANLEY FISCHER

In this paper, we describe a method for controlling stochastic, nonlinear, econometric models and illustrate the method by studying monetary and fiscal policy in the St. Louis econometric model. The method consists essentially of estimating the means and variances of the dynamic responses of the nonlinear model to changes in instrument variables, in order to produce a linear representation of the model with random coefficients, and then using this linear, stochastic version in optimization. The major virtue of the method is that it is as easily applied to large as to small models.

In more detail, the method has five steps. First, choose nominal paths for the control variables in the problem. Second, carry out a series of stochastic simulations of the model being optimized: in these simulations, stochastic elements of the model are generated by a pseudorandom number generator *and* stochastic disturbances are generated around the nominal paths of the control variables. Third, calculate linear regressions in which the simulated target variables of the model are the dependent variables and the instrument variables are among the independent variables of the regressions. Fourth, compute optimal feedback control rules for the instrument variables using the linear regressions calculated at the third step to represent the model. Fifth, evaluate these feedback control rules in stochastic simulations of the original model.

The nominal paths chosen for the instruments in our work were simple constant growth rate paths: in principle, these nominal paths could also be obtained as the solution to a deterministic, nonlinear control problem. In the second step, the underlying model is treated as stochastic: the stochastic disturbances of the instrument variables around the nominal paths have the same variance as historical experience. As a result of the second step, we have a large set of data representing the response of the model to changes in policy. The third step summarizes this information in linear, random coefficients regressions, consisting of parsimonious, autoregressive-moving average models representing the target variables as functions of current and lagged instruments and lagged target variables. A quadratic loss function is used at the fourth step to obtain optimal feedback control rules for the linearized version of the model represented by the linear regressions. The fifth step is undertaken as a check on the operation of the rules in the original model.

In our work with the St. Louis model, we took the rates of inflation and unemployment to be target variables, and the growth rates of money and high employment federal expenditures to be instruments. The control rules produced were applied in twenty replications of a stochastic simulation of the original model in which both coefficients and additive errors were treated as random. The control rules produced losses significantly below those obtained under constant growth rate rules for the instruments and historical experience: changing the relative

weights on arguments in the loss function produced the expected changes in the results of policy. For example, reducing the cost of deviations of unemployment from its desired level reduced the average deviation of inflation from its target level. Comparison of the optimal rule obtained in this paper with the best heuristic rule found in Cooper and Fischer (1973) showed that losses due to deviations of the rates of inflation and unemployment from their target values obtained here were similar to those in our previous paper but with less variation in the instruments. It is also shown that while fiscal policy contributes little added stabilization when an active monetary policy is used, fiscal policy does stabilize the model if the money supply grows at a constant rate.

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Reference

Cooper, J. Phillip and Stanley Fischer, "Monetary and Fiscal Policy in the Fully Stochastic St. Louis Model," in *Journal of Money, Credit and Banking*, (February, 1974).

Annals of Economic and Social Measurement, 3/1, 1974

STUDIES OF ECONOMIC PROBLEMS

OPTIMAL POLICIES FOR MONETARY CONTROL*

BY ROBERT S. PINDYCK AND STEVEN M. ROBERTS

This paper will present some optimization experiments using a linearized version of the Federal Reserve Board's monthly money market model, which was designed primarily to study the impact of policy instruments on monetary and financial targets. Using linear-quadratic optimal control, we calculated optimal policies for a single instrument, unborrowed reserves, with the objective of forcing monetary aggregates and interest rates to follow desired paths. There is a conflict between the choice of policy target, i.e., there is a trade-off between the control of monetary aggregates and the control of interest rates. By calculating a set of optimal policies using different objectives, that trade-off can be demonstrated. The optimal strategies are also calculated using closed-loop control so as to correct for random disturbances. It is shown how the existence of random disturbances modifies the target trade-offs between monetary aggregates and interest rates, and requires greater flexibility in the movements of the control variable.

1. INTRODUCTION

Recent applications of optimal control theory to economic stabilization policy problems have usually involved calculating time paths for one or more "global" policy variables so as to minimize some macroeconomic cost functional.[1] The aim of these exercises has been to indicate how policy objectives relating to GNP, employment, prices, and the balance of payments might best be attained. The policy variables which can be manipulated might include tax rates, the level of government expenditures, and the money stock. Tax rates and the level of government expenditures are subject to rather direct control. However, the money stock cannot be controlled directly by the Federal Reserve: the Fed can however, manipulate other variables which in turn affect the money stock.[2]

The ultimate concern of monetary policy-makers is with the real economy and how policy involving monetary (e.g., the money stock) and financial (e.g., interest rates) variables can best be used to attain the desired levels of GNP, employment, prices, and the balance of payments. The inability to directly control these policy "instruments" has resulted in a two-stage optimization process in which these instruments are in fact "intermediate" targets and the true policy instruments are those variables over which the Fed has direct control, e.g., required reserve ratios, the discount rate, ceilings on interest payments on bank liabilities, and the use of open market operations to affect either unborrowed reserves or the

* This paper does not necessarily reflect the views of the Board of Governors of the Federal Reserve System or its staff. We wish to express our appreciation to Franco Modigliani, James Pierce, William Poole, Thomas Thomson, and Peter Tinsley for their helpful comments. We would like to thank Walter Davis and Lucy McCurdy for their programming assistance and Nancy Wilson for her expert typing. Revised July 1973.

[1] See, for example, recent work by Chow [6], [7], Friedman [10], Livesey [14], Pindyck [16], [17], and Sengupta [18].

[2] During the past few years, there has been a controversy over the ability of the Federal Reserve to control monetary aggregates. For a discussion of some of the issues, see Pierce and Thomson [15], Davis [8], and Andersen [1].