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Appendix A

THE UTILITY MAXIMIZATION APPROACH

We assume that the individual consumes n commodities during the period t. Of these n goods, m, where $1 \leqq m < n$, are devoted to present consumption and the remaining $n-m$ to future consumption. The quantity consumed by the individual of the ith commodity in period t is c_i, and the price of the good is p_i; if $i > m$, p_i represents the expected future price of the good discounted for time.

The individual's scale of preferences for these goods is represented by a utility function:

$$u = \phi(c_1, \cdots, c_n) \ . \tag{A.1}$$

The manner in which he distributes a given income, y, among these goods is determined by maximizing (A.1) subject to the budgetary constraint:

$$\sum_1^m p_i c_i + \sum_{m+1}^n p_i c_i = y \ . \tag{A.2}$$

The first sum may be called consumption expenditures, c, and the second savings, s, both in current dollars.

The solution of this problem leads to the $n-1$ equilibrium conditions:

$$\frac{\partial\phi/\partial c_i}{\partial\phi/\partial c_1} = p_i/p_1 \qquad (i = 2, \cdots, n) \ . \tag{A.3}$$

The last equation of the system is obtained from (A.2), which can be written as

$$\sum_1^n (p_i/p_1)\, c_i = y/p_1 \ . \tag{A.4}$$

The solution of these n equations yields functions of the form

$$c_i = c_i(y^*, p_2/p_1, \cdots, p_n/p_1) \tag{A.5}$$

where

$$y^* = y/p_1 \ .$$

If we define $c^* = c/p_1$ and $s^* = s/p_1$ then (A.5) becomes

$$c^* = g(y^*, p_2/p_1, \cdots, p_n/p_1) \tag{A.6}$$

and correspondingly

$$s^* = y^* - g(y^*, p_2/p_1, \cdots, p_n/p_1) = h(y^*, p_2/p_1, \cdots, p_n/p_1) \ . \tag{A.7}$$

In other words, the individual divides a given income between consumption and savings in such a way as to maximize his utility. Although the content of the utility function and of the budgetary restraint is a matter of debate, the foregoing presentation brings out two facts. One is that the individual is concerned with real quantities and not with the money value of these quantities. This is so because money of itself has no utility until related to the goods that can be bought for it. Hence, the maximization of utility can be accomplished only with respect to real goods, not to money values.

Nevertheless, the prices of the various goods are also highly relevant, as is evident from (A.7), for they determine the distribution of the given income among the commodities. The same thing would be true if instead of using p_1, as a deflator, one used a linear combination of prices

$$P = \sum_i w_i p_i$$

called a price index, w_i being constant weights, because a unique correspondence exists between $(y/p_1, p_2/p_1, \cdots, p_n/p_1)$ and $(y/P, p_1/P, \cdots, p_n/P)$.

The price ratio variables would lose their relevance only if they remained constant, in which case (A.6) and (A.7) would reduce to

$$\text{(A.8)} \qquad c^* = g(y^*) \text{ and } s^* = h\,(y^*)\,.$$

In the further discussion we shall assume that changes in the relative prices are small enough to admit (A.8) as a reasonable approximation.

The variables entering into (A.8), or (A.6) and (A.7), depend clearly on the content of (A.1) and (A.2). In the case of (A.1), the individual's utility function, the variables other than current income that are admissible from a micro-economic viewpoint, i.e., those variables whose inclusion can be supported on the basis of micro-economic considerations alone, are so numerous that there is little to be gained in discussing them at this stage. In Chapter I the implications to the aggregate function of including some of the most prominent of these variables in the individual function have been explored, and the value of these variables has been determined empirically in Chapter II.

The problem about the budgetary constraint is whether to include liquid assets and borrowing as well as income; that is, whether instead of (A.2) we should not use

$$\text{(A.9)} \qquad \sum_1^n p_i c_i = y + a + b$$

where a is the liquid assets available to the individual at the beginning of period t, and b is the potential borrowing of the individual during this period.

From a logical point of view, the inclusion of a and b in the budgetary constraint is surely required. Nevertheless, they could be omitted from empirical work with aggregate consumption functions if either of the following two statements could be proved:

1) The amount of consumption expenditures financed by *a* or *b* is negligible relative to expenditures out of current income.
2) Consumption expenditures financed by *a* and *b* remain approximately constant over time, as does the proportion of total funds available from this source (which is the same as saying that the proportion of total consumption expenditures financed by *a* and *b* should remain constant).

Little work has been done on this problem, a situation no doubt partly attributable to the unavailability, until recently, of statistics on liquid asset holdings by consumers. To omit these variables altogether with little or no explanation, as was done in most aggregate consumption function studies, is to assume implicitly the validity of one of the above propositions. Whether or not one or the other of them is valid is a matter that can only be settled by empirical research.