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Chapter I

REVIEW OF PAST STUDIES

Introduction

The relationship between aggregate consumption and national income aroused only occasional interest before the advent of Keynes' *General Theory*. Though several studies had been made of the relationship at a given time between family income and family consumption, not much attention appears to have been given to the temporal relations of total consumption, national income, and related variables.

In his *General Theory,* Keynes emphasized the dependence of consumption on income and developed the concepts of the marginal and average propensities to consume. He expounded the notion of a consumption function and suggested that the marginal and average propensities to consume have certain properties — for example, that a given increase in real income produces a smaller increase in real consumption. To test the validity of these concepts, researchers began to undertake empirical investigation of the nature of the consumption function and to estimate the marginal and average propensities at different stages of business fluctuations. Most of these studies have been statistical inquiries, proceeding from a preconceived theory to the construction of a consumption function and then to numerical estimates of the parameters of the function. Eighteen examples of these statistical efforts to establish the characteristics of the consumption function have been selected for examination here.

The objective of this study is to determine some of the factors that increase the predictive accuracy of an aggregate consumption function. This is accomplished in two steps:

1) A summary of the main statistical studies of the aggregate consumption function (Chapter I). No summary or critical evaluation of these studies has hitherto appeared, nor has any attempt been made to reconcile the results of the studies on the basis of their many differences in method and approach.

2) A recomputation of most of these functions with a standard set of data (Chapter II). Alternative forms of each function have been computed to isolate specific factors and to permit an evaluation of their effect on predictive accuracy.

This study places some emphasis on the micro-economic point of view, that is, on what particular aggregate consumption functions imply about individual behavior. This approach assumes that the consumption of different individuals can be related to certain measurable factors common to all individuals, and that these individual functions can then be "aggregated." Theoretically, at least, one might begin by developing a micro-economic function that explains the variations in an individual's consumption and then arrive at a relation for the economy as a whole by some operation called "aggregation." The micro-economic function itself possesses a certain rationale, which can be brought out through the utility maximization approach. For those who may be interested, an outline of this approach is given in Appendix A.

In this study of aggregate consumption functions it is useful to remember that essentially such a function is an attempt to estimate the relation between aggregate consumption and aggregate income in much the same manner as a demand schedule expresses the estimated relationship between the quantity demanded of a particular good and its price. To derive either schedule, it is necessary to make use of other relevant variables, whose omission might lead to distorted estimates. The specification of these other variables may be considered to be the *ceteris paribus* condition. The *ceteris paribus* of an aggregate consumption function consists of the specification of the one or more variables entering into the function other than current income.

The *ceteris paribus* of a consumption function clearly depends on the purpose of the particular study, on exactly what the consumption function is designed to predict. In this respect, the period over which each single measurement of income or consumption is made is of particular importance, for the variables entering into a consumption function predicting year-to-year changes in consumption are not likely to be the same as those entering into a function predicting quarterly changes. In particular, the *ceteris paribus* of consumption functions seeking to predict secular shifts in consumption ought to differ from the *ceteris paribus* of functions measuring intracyclical changes. Actually, considerable variation exists among the conditions specified by different studies having the same objective, as is shown in a later section. If the notion of the aggregate consumption function is found to be of use, however, further research may lead to more general agreement on the nature of the *ceteris paribus* condition in a consumption function for a particular purpose.

It would seem, therefore, that any evaluation of past statistical studies of the aggregate consumption function must start by an examination of the *ceteris paribus* conditions used to achieve a particular end. When the various theories of the aggregate consumption function are reduced to a common denominator in this fashion, the relationships between them can be brought out and some opinion may be rendered as to the desirability of each.

Notation

A list of the main symbols used in the study will prove useful at the outset. It might be noted that throughout this study consumption in time t means expenditures on consumption goods made during a given period and is not necessarily related to the amount of goods physically consumed or used up during that period.

A = Pareto's α, taken without regard to sign

B = a measure of income inequality defined as: (see pp. 18-20 below)

$$\frac{\text{Cumulative median income} - \text{Ordinary median income}}{\text{Cumulative median income}}$$

C = aggregate consumption expenditures in billions of current dollars

E = aggregate enterprise saving in billions of dollars

M = liquid assets in billions of dollars

N = population

P = the Bureau of Labor Statistics index of consumer prices

P_f = index of farm prices

P_w = index of wage rates

R = index of retail sales

S = aggregate savings in billions of dollars

T = time in years

U = random disturbance

V = aggregate personal taxes and social security contributions in billions of current dollars

Y = aggregate disposable income in billions of current dollars $= C + S$

Y' = aggregate national income in billions of current dollars

Y'' = aggregate income payments in billions of current dollars

Y_e = aggregate "higher" income (i.e., dividends, interest, rents, nonfarm entrepreneurial withdrawals, and corporate officers' salaries) in billions of current dollars

Y_f = aggregate farmers' income in billions of current dollars

Y_g = aggregate speculative income (capital gains on securities) in billions of current dollars

Y_l = aggregate labor income in billions of current dollars

Y_0 = highest aggregate disposable income ever attained in any previous year, in billions of current dollars

$X_{-\lambda}$ = the value of X, λ years previous. The absence of time subscript indicates reference to the current period

$\Delta X = X - X_{-1}$

c = consumption expenditures of an individual

s = savings of an individual

y = income of an individual
$\alpha, \beta, \gamma, \ldots$ = unknown parameters
u = random disturbance

The basic classification of consumption functions

Nearly one hundred aggregate consumption functions have been fitted to data for the United States during the past fifteen years. To list all of these functions in the present summary would be pointless — such a list already exists[1] and most of the functions have the same form, having been developed to test alternative hypotheses preparatory to determining "the" function.[2] In addition, several studies employ identical functions, units of measurement, and method of fit, and cover almost the same period.[3] To summarize each of these studies would also be of little value as a review of the past progress on the subject. Instead we include in the present summary only those studies which appear to have made some original contribution either to the form or to the interpretation of the aggregate consumption function, and from these studies the function(s) incorporating the original contribution have been selected for review.

In this manner, eighteen studies have been chosen, seventeen dealing with consumption fluctuations in the United States and one with consumption in Germany. The German consumption function has been included because it possesses some distinctive characteristics not present in any of the consumption functions for the United States. The principal consumption functions of the eighteen studies are shown in Table 1, segregated according to whether the economic variables are deflated or not, and then further classified by the variables contained in the *ceteris paribus* assumption, that is, by the variables used other than national income.

Variables entering into the consumption function

An examination of the functions in Table 1 indicates that, with but two exceptions (1.20 and 1.22), the various forms differ according to the treatment given three factors: prices, population, and income.[4] Our discussion

[1] G. H. Orcutt and A. D. Roy, "A Bibliography of the Consumption Function" (University of Cambridge, 1949), mimeographed release.

[2] For example, W. S. Woytinsky derives seven different consumption or savings functions in his study in the 1946 *Review of Economic Statistics,* but recommends only one as most desirable.

[3] E.g., R. B. Bangs, "The Changing Relation of Consumer Income and Expenditure," *Survey of Current Business,* Vol. 22 (April 1942), pp. 8-12, and J. L. Mosak, "Forecasting Postwar Demand: III," *Econometrica,* Vol. 13 (1945), pp. 25-53; Office of Business Economics, "Income, Consumption, and Savings," *Survey of Current Business,* Vol. 26 (May 1946), pp. 5-7, and J. Steindl, "Post-war Employment in the U. S. A.," *Bulletin of the Oxford University Institute of Statistics,* Vol. 6 (1944), pp. 193-202.

[4] Actually, time constitutes yet a fourth variable. However, as we shall see later, for all practical purposes time is synonymous with population during the period covered by the functions, and it is therefore considered in conjunction with the population factor.

of the functions therefore centers on the different ways in which each of these variables is taken into account, and on the rationale behind these various treatments. The general procedure is to construct a theoretical framework for each factor explaining its relevance, from the point of view of economic behavior, to a consumption function. The means by which account is taken of the factor are then assessed with reference to this framework.

The three factors to be considered on the following pages by no means exhaust the field of inquiry. Two other promising variables are corporate savings and liquid assets, used by Samuelson and by Klein (1.20), respectively. These, with other possible variables such as consumer borrowing, are omitted from the discussion, however, because of practical limitations on the scope of the study. To cover a small area in detail would seem to be of more value in this subject than to cover a large area superficially.

Price effects

Two questions may be raised in considering the place of a price variable in an aggregate consumption function. First, is a price variable relevant? Second, if so, how should the price variable be included in the consumption function?

The answer to the first question is generally affirmative, as is noted in Appendix A where the utility maximization approach is applied to derive the micro-economic consumption function. However, even if this approach is not recognized, cogent reasons exist for allowing for price changes, as Samuelson stated:[5]

> Because of changes in prices, changes in money income and consumption are not the same thing as changes in real income and consumption. From economic theory and from observation, we should not expect to find an invariant relationship between money consumption and money income, regardless of the real levels which these represent. A doubling of *all* prices simultaneously would presumably leave each individual in the same position as previously; we should expect, therefore, no change in real quantities, abstracting from the dynamical effects of *changing* prices. Unless a correction were made for price changes, it would appear that two different observations on the consumption function were available, and that the marginal propensity to consume were equal to the average propensity to consume. Thus, if previously money consumption equaled national income (investment being zero), and suddenly all prices doubled evenly, presumably money consumption would double as income doubled. This might be erroneously interpreted to indicate a marginal propensity to consume of unity, when in fact only one observation of the true *real* consumption function had been made, and no basis exists for inferring the magnitude of the marginal propensity to consume.

In view of the fact that the need for price adjustment was clearly recognized in Staehle's pioneer study[6] as well as by Keynes,[7] it is surprising to

[5] P. A. Samuelson, *op. cit.*, p. 252.

[6] Hans Staehle, *op. cit.*, p. 139.

[7] J. M. Keynes, *op. cit.*, Ch. 8.

Table 1

CLASSIFICATION OF AGGREGATE CONSUMPTION FUNCTIONS

AUTHOR	YEAR PUBLISHED	PERIOD COVERED	VARIABLES OTHER THAN LINEAR INCOME TERM		FUNCTION	R^2 ADJUSTED FOR SAMPLE SIZE	REMARKS
			A *Variables in current price aggregates*				
Mosak	1945	1929-40		(1.1)	$C = 8.62 + .803Y$	.989	
Paradiso	1945	1923-40	T	(1.2)	$C = 5.50 + .828Y + .04(T - 1935)$	.994	
Mack	1948	1929-40	ΔY	(1.3)	$C = 7.02 + .8615Y - .0587\Delta Y$	.995	
		1929-40	$\Delta Y, (Y_{-1} - Y_{-3})/2$	(1.4)	$C = 4.95 + .8929Y - .0524\Delta Y - .0776(Y_{-1} - Y_{-3})/2$	.996	
Stone	1942	1929-41	T, Y^2	(1.5)	$C = -2.42 + 1.005Y - .0029Y^2 - .6235(T - 1935)$		C at factor cost
Ezekiel	1942	1920-39	$\Delta Y', T, T^2$	(1.6)	$C = 10.26 + .71Y' - .09\Delta Y' + .92T - .37T^2$	.986	
Polak*	1939	1919-32	$T, \Delta P_f$	(1.7)	$C = .95Y_l + .70Y_e + .35Y_g + .05\Delta P_f + Y_f + .27T$	.983	
			B *Variables in deflated price aggregates*				
Bennion	1946	1923-30		(1.8a)	$\frac{C}{P} = -2.0 + .93\frac{Y}{P}$	.958	$P_{1941} = 100$
		1931, '36-37, '39-40		(1.8b)	$\frac{C}{P} = 1.9 + .88\frac{Y}{P}$	.997	$P_{1941} = 100$
		1932-35, 1938		(1.8c)	$\frac{C}{P} = 4.9 + .85\frac{Y}{P}$	.997	$P_{1941} = 100$
Woytinsky	1946	1923-30, '35-40		(1.9)	$\frac{C}{P} = -1.3 + .925\frac{Y}{P}$		$P_{1941} = 100$
		1923-30, '35-40	T	(1.10)	$\frac{C}{P} = 9.8 + .75\frac{Y}{P} + .21(T - 1930)$		$P_{1941} = 100$
Klein	1947	1922-41	T	(1.11)	$\frac{C}{P} = 11.87 + .73\frac{Y}{P} + .04(T - 1931)$		Implicit price index, 1934 price = 100; function fitted by reduced form
Klein	1950	1921-41	T, YT	(1.12)	$\frac{C}{P} = 9.7 + .77\frac{Y}{P} - .01\frac{Y}{P}(T - 1931) + .76(T - 1931)$	.977	Implicit price index 1934 price = 100

Bassie	1946	1910-16, '23-40	N	(1.13)	$\frac{C}{P} = -12.6 + .77\frac{Y}{P} + .17N$		$P_{1935\text{-}39} = 100$
			C *Variables in deflated price per capita units*				
Haavelmo	1947	1922-41		(1.14a)	$\frac{C}{NP} = 113.1 + .672\frac{Y}{NP}$		$P_{1935\text{-}39} = 100$; functions fitted by reduced forms
		1929-41		(1.14b)	$\frac{C}{NP} = 95.05 + .712\frac{Y}{NP}$		
Smithies	1945	1923-40	T	(1.15)	$\frac{C}{NP} = 76.58 + .76\frac{Y}{NP} + 1.15(T - 1922)$	.933	$P_{1929} = 100$
Friend	1946	1923-40	T	(1.16)	$\frac{C}{NP} = 67.6 + .775\frac{Y}{NP} + .83(T - 1930)$		$P_{1941} = 100$
		1923-30, '35-40	T	(1.17)	$\frac{C}{NP} = 92.4 + .728\frac{Y}{NP} + .90(T - 1930)$		$P_{1941} = 100$
		1923-40	$T, \Delta Y$	(1.18)	$\frac{C}{NP} = 61.2 + .788\frac{Y}{NP} - .04\Delta\frac{Y}{NP} + .84(T - 1930)$		$P_{1941} = 100$
		1923-30, '35-40	$T, \Delta Y$	(1.19)	$\frac{C}{NP} = 75.3 + .764\frac{Y}{NP} - .092\Delta\frac{Y}{NP} + .90(T - 1930)$		$P_{1941} = 100$
Klein	1947	1922-41	Y_{-1}, M_{-1}	(1.20)	$\frac{C}{NP} = 79.04 + .58\frac{Y}{NP} + .13\left(\frac{Y}{NP}\right)_{-1} + .06\left(\frac{M}{NP}\right)_{-1}$		$P_{1935\text{-}39} = 100$; function fitted by reduced form
Modigliani	1949	1921-40	Y_o	(1.21)	$\frac{C}{NP} = 2 + .773\frac{Y}{NP} + .125\left(\frac{Y}{NP}\right)_o$	.982	$P_{1929} = 100$
Samuelson	1941	1921-35	E	(1.22)	$\frac{C}{NP} = 7.5 + .81\frac{Y''}{NP} + .23\frac{E}{NP}$	.942	$P_{1929} = 100$
			D *Consumption or savings expressed as ratio to income*				
Duesenberry	1948	1923-40	Y/Y_o	(1.23)	$\frac{S}{Y} = -.066 + .165\frac{Y/NP}{(Y/NP)_o}$	.798	$P_{1941} = 100$
Staehle	1937	1928-34	B	(1.24)	$\frac{R}{Y_l} = 197.4 - .33\frac{Y_l}{P_w} - 2.31B$	.713	Function relates to Germany; based on quarterly data; R/Y_l and Y_l/P_w are indices, 1928 = 100

(For notes see next page.)

* Polak actually used ($C - Y_f$) as his dependent variable and then transferred Y_f to the right-hand side, for the reason that the estimates of Y_f were derived in such a manner that they represent "more nearly farmers' withdrawals for consumption than farmers' incomes including their business savings. Hence it has been assumed that these amounts are equal to farmers' consumption; in all calculations, they have been subtracted from the consumption series. . . ." (*op. cit.*, p. 3).

Sources

Bassie, V. L., "Consumers' Expenditures in War and Transition," *Review of Economic Statistics*, Vol. 28 (1946), pp. 117-30.
Bennion, E. G., "The Consumption Function: Cyclically Variable?" *Review of Economic Statistics*, Vol. 28 (1946), pp. 219-24.
Duesenberry, James, "Income-Consumption Relations and Their Implications," *Income, Employment, and Public Policy: Essays in Honor of Alvin H. Hansen* (W. W. Norton, 1948), pp. 54-81.
Ezekiel, Mordecai, "Statistical Investigations of Saving, Consumption, and Investment: I," *American Economic Review*, Vol. 32 (1942), pp. 22-49.
Friend, Irwin, "Relationship between Consumers' Expenditures, Savings, and Disposable Income," *Review of Economic Statistics*, Vol. 28 (1946), pp. 208-15.
Haavelmo, Trygve, "Methods of Measuring the Marginal Propensity to Consume," *Journal of the American Statistical Association*, Vol. 42 (1947), pp. 105-22.
Klein, L. R., *Economic Fluctuations in the United States, 1921-1941* (John Wiley and Sons, 1950), pp. 88 f., 112, and "The Use of Econometric Models as a Guide to Economic Policy," *Econometrica*, Vol. 15 (1947), pp. 111-51.
Mack, R. P., "The Direction of Change in Income and the Consumption Function," *Review of Economics and Statistics*, Vol. 30 (1948), pp. 239-58.
Modigliani, Franco, "Fluctuations in the Saving-Income Ratio: A Problem in Economic Forecasting," *Studies in Income and Wealth, Volume Eleven* (NBER, 1949), pp. 371-443.
Mosak, J. L., "Forecasting Postwar Demand: III," *Econometrica*, Vol. 13 (1945), pp. 25-53.
Paradiso, L. J., in *National Budgets for Full Employment*, National Planning Association, 1945.
Polak, J. J., "Fluctuations in United States Consumption, 1919-1932," *Review of Economic Statistics*, Vol. 21 (1939), pp. 1-12.
Samuelson, P. A., Appendix to Ch. XI in Hansen, A. H., *Fiscal Policy and Business Cycles* (W. W. Norton, 1941), pp. 250-60.
Smithies, Arthur, "Forecasting Postwar Demand: I," *Econometrica*, Vol. 13 (1945), pp. 1-14.
Staehle, Hans, "Short-Period Variations in the Distribution of Incomes," *Review of Economic Statistics*, Vol. 19 (1937), pp. 133-43.
Stone, Richard, "National Income in the U. K. and the U. S. A.," *Review of Economic Studies*, Vol. X (1942-43), pp. 1-27.
Woytinsky, W. S., "Relationship Between Consumers' Expenditures, Savings, and Disposable Income," *Review of Economic Statistics*, Vol. 28 (1946), pp. 1-12.

In addition to differences in the numerical estimates of parameters of any two functions caused by using current dollars or deflated dollars, or by differences in period covered, method of fit, form of function, and variables included, comparison is further complicated by the fact that different sets of aggregate consumption-income data for the United States were used in these studies. One set of data was used as the basis for consumption function studies until 1942 (and perhaps even in Paradiso's study in 1945). These data were then revised by the Department of Commerce, and the revised figures were used in the consumption function studies published from 1945 through 1948, with the exception of Ruth Mack's. The adoption of new concepts of aggregate income and consumption led to a complete overhaul of the Department of Commerce estimates in 1947 (National Income Supplement to *Survey of Current Business*, July 1947), resulting mainly in increased estimates of consumption expenditures relative to disposable income. Only Ruth Mack's work, of the studies listed in this table, is based on these revised figures.

note that six of the studies in Table 1 did not take price effects into consideration.[8] None of the six gave any reason for the omission. Woytinsky's study, appearing in 1946, only then focused attention on the value of adjusting aggregates for price changes, and it was in this that his main contribution lay. His reasons were the same as Samuelson's.

Where price adjustment is made in a consumption function, various forms can be used. The relationship between them is perhaps best illustrated by considering the different aggregate functions that can be developed by use of the two simplest forms incorporating price in a microeconomic function (assuming that some such function exists). These forms are:

$$\frac{c_i}{P} = \alpha + \beta \frac{y_i}{P} + u_i \tag{1.25}$$

and

$$c_i = \alpha + \beta y_i + \gamma P + u_i . \tag{1.26}$$

The first yields the aggregate function

$$\frac{C}{P} = \alpha N + \beta \frac{Y}{P} + U \tag{1.27}$$

or

$$\frac{C}{NP} = \alpha + \beta \frac{Y}{NP} + \frac{U}{N}. \tag{1.27a}$$

In nonstochastic terms — omitting the variable, U — both forms are equivalent, but the best linear empirical estimates of α and β would be obtained with existing methods by using (1.27), because that is the only form where a nonobservable stochastic variable (U), not correlated with the observable variables, enters additively. However, (1.27a) is the form that has most generally been used.[9]

The aggregate function resulting from (1.26) is

$$C = \alpha N + \beta Y + \gamma NP + U \tag{1.28}$$

or

$$\frac{C}{N} = \alpha + \beta \frac{Y}{N} + \gamma P + \frac{U}{N} \cdot \tag{1.28a}$$

Neither of these forms seems to have been used in the past. Of the two,

[8] J. J. Polak used a cost-of-living index variable in fitting some of the functions that he finally rejected. His use of an index of farm prices (P_f) was intended to reflect only speculative gains on commodities. *Op. cit.*, p. 2.

[9] E.g., (1.14), (1.21), (1.22).

(1.28) is clearly better for estimating the values of the parameters. Polak used a modification of (1.28a), omitting any adjustment for population, and the empirical results were rejected by him as not meeting his criteria for a good consumption function.

Time trends and adjustment for population changes

To what extent is adjustment for population necessary in an aggregate consumption function? The answer is that, except for certain trivial cases,[10] some such adjustment would seem to constitute a basic criterion of a valid aggregate function. Adjustment is obviously necessary when aggregate functions have been developed from functions describing individual behavior, as illustrated in the preceding section by functions (1.25)-(1.27). With (1.25) as the basic premise, adjustment for population could be made in the manner shown by either (1.27) or (1.27a); for estimation purposes (1.27) would again be preferable. Most studies using one of these forms relied on (1.27a), however, and only one of the past consumption functions, (1.13), is modeled after (1.27).

Whether or not some hypothesis as to individual behavior is explicitly postulated, an aggregate function nevertheless requires adjustment for population because it is intended to represent the behavior of an aggregation of individuals. And from the standpoint of policy formation, it is clearly of considerable importance whether a shift in aggregate consumption is attributable to population change or to change in some other variable. Despite this, twelve of the twenty-four functions listed in Table 1 make no adjustment for population.[11] Insofar as population changes have influenced consumption during the periods studied — and Bassie's study shows such an influence to have existed — these twelve functions are open to criticism on theoretical grounds for neglecting a relevant variable.[12] Considered on practical grounds, some of the functions show a rectification of the omission by insertion of a time trend, which is effective because there was an approximately linear relationship between time and population during the periods studied.[13] For such periods, use of a time variable is practically equivalent to adjustment for population. If $N = \gamma_0 + \gamma_1 T$ then (1.27) becomes

$$(1.29) \qquad \frac{C}{P} = \alpha\gamma_0 + \alpha\gamma_1 T + \beta\frac{Y}{P} + U,$$

[10] E.g., $\alpha = 0$ in (1.27).

[11] In Staehle's study, such an adjustment would seem unnecessary in view of the short period involved — seven years.

[12] The authors of these studies might protest on the ground that their α is equivalent to N in (1.27). But then α cannot be a parameter since it incorporates the variable, N, and its value therefore cannot remain constant over a period of time (omitting the case when N is constant).

[13] Thus the coefficient of determination between the two variables for 1923-41 is .912.

which is almost indistinguishable from (1.27) if N and T are highly correlated.

In fact, however, use of a time trend to adjust for population changes was apparently made only inadvertently. For purposes of prediction this is not reliable, for the linear relationship between time and population that prevailed during the period of observation may not continue in the future. Principally the time trend was inserted as a means of allowing for shifts in the consumption habits and tastes of the population; for example, at any given income level (money) expenditures may tend to rise over time because of a general change in cultural patterns brought on in part by technological progress. In addition, it may have been used as a catch-all variable to adjust for changes in the composition of the population that might affect the micro-economic relation: for example, changes in the distribution of the population by age, urban or rural, status, or income.

The manner in which a time trend is included in the consumption function depends upon the assumptions made about (1.25). The more general hypothesis, usually implicit, has been that the intercept of the individual function with respect to income depends on time. This means that, assuming linearity, $\alpha = \alpha_0 + \alpha_1 T$ in (1.25), which leads to an aggregate function of the type

$$\frac{C}{NP} = \alpha_0 + \alpha_1 T + \beta \frac{Y}{NP}. \tag{1.30}$$

This type of assumption is implicit in such functions as (1.2), (1.10), and (1.19) in Table 1. Ezekiel's function (1.6) assumes that α is affected by a nonlinear time trend, but the coefficient of T^2 was shown to be non-significant at the usual, .05, level of significance.[14]

Another possible assumption is that the individual's marginal propensity to consume — β in (1.25) — is altered through time for much the same reasons as were noted in the case of α. Then we would have in (1.25), again assuming linearity, $\beta = \beta_0 + \beta_1 T$, which would lead to the aggregate function

$$\frac{C}{NP} = \alpha + \beta_0 \frac{Y}{NP} + \beta_1 \frac{YT}{NP}. \tag{1.31}$$

Or, if both assumptions were made,

$$\frac{C}{NP} = \alpha_0 + \alpha_1 T + \beta_0 \frac{Y}{NP} + \beta_1 \frac{YT}{NP}. \tag{1.32}$$

This is the rationale for Klein's function (1.12). That there may be some basis for such a function is indicated by the fact that β_1 proved statistically significant in (1.12).

[14] Irwin Friend, "Ezekiel's Analysis of Saving, Consumption, and Investment," *American Economic Review*, Vol. 32 (1942), pp. 829-35.

The wisdom of inserting a time trend in a consumption function may be questioned in view of the danger of an unforeseen shift in the rate of change of the variables that time is supposed to approximate. Thus the rate of change in population during the 1940's, about 1,900,000 per year, was more than twice the rate of change in population during the 1930's. Hence extrapolation into the 1940's of a function like (1.30) fitted to data for the 1930's would tend to yield underestimates of consumption, other things equal.

Wherever possible, it is desirable to seek the underlying cause rather than to assume that the future rate of change in a particular variable will be the same as in the past. Changes in economic relationships can never be simply the result of the passage of time. The element of time is the medium through which economic changes take place. For example, the degree to which vegetables become cooked depends on the amount of time the pot with the vegetables remains on the fire, but time is not the cause of the vegetables being cooked. In the sense that "time" is always present it is a contributory factor, but the true cause is the heat from the fire, and were it not for this heat the vegetables would not cook no matter how much time were to elapse.

To attribute changes in economic relationships to time carries one or both of two implications. One is an admission of failure to answer the basic question posed by the observed change: *why* did the change occur when it did? If we cannot supply an answer to this question, the utility of economic relationships for prediction or forecasting purposes is seriously impaired. For how would we know when, or under what conditions, another shift might occur?[15] The other implication is that the causal variables are so numerous, or so difficult to measure, that it is not practicable to include all of them. Only on this basis would the use of a time trend in an economic relationship seem justifiable, and it would be warranted only after some attempt had been made to include the true causal variables behind the time variable.

Income effect

There appears to be well-nigh universal agreement on the importance of income as a major determining factor of consumption. Where question does arise, and where consumption studies differ greatly, is over the manner in which income should be inserted into the aggregate function. At least three distinct facets of this question deserve consideration:

[15] Actually, the insertion of a time variable in an economic relationship can be useful for checking the validity of the relationship, *but in a way opposite to that in which time coefficients have been interpreted in the past.* A statistically significant value for the coefficient of the time variable would indicate an incomplete relationship, in the sense that one or more relevant variables had been omitted. Of course, a statistically nonsignificant time coefficient does not of itself validate a relationship.

1) Granted that average income, or total income, is of major importance, how should it be inserted in the function?
2) Should some measure of income distribution also be included?
3) Can cyclical variations in the consumption-income relationship be distinguished from secular variations?

Selection of income variable

Should all income be combined into one variable, or should separate variables be used for each major type of income? If income is lumped into one variable, what definition of income should be used? No definitive answers can be offered at this time, but it will be useful to explore the implications of the possible alternatives.

The rationale underlying the use of several income variables, one (or more) for each type of income, is essentially as follows: (a) The consumption function varies by "type" of income;[16] and (b) the distribution of income by type varies over time sufficiently to affect aggregate consumption expenditures when (a) is taken into consideration. In this event, estimates of aggregate consumption will be appreciably[17] more accurate when income is segregated by type than when one aggregate income variable is used in the aggregate consumption function.

A linear aggregative function separating income by type might have the following form:

$$\text{(1.33)} \quad \frac{C}{NP} = \alpha + \beta_1 \frac{Y_1}{NP} + \beta_2 \frac{Y_2}{NP} + \cdots + \beta_m \frac{Y_m}{NP}$$

where Y_1, Y_2, etc., represent different types of incomes, as illustrated in footnote 16 above.

This is the manner in which Polak (1.7) arranged his income variables. It is evident that the use of one over-all income variable in a consumption function, as in (1.27a), is a special case of (1.33), namely when $\beta_1 = \beta_2 = \ldots = \beta_m$. Hence the desirability of (1.33) as against (1.27a) can be tested by determining whether the estimated values of $\beta_1, \beta_2, \ldots, \beta_m$ could have been drawn from a population in which these parameters are all equal. To disprove the null hypothesis, however, is, though a necessary, not a sufficient condition for preferring (1.33), because the validity of assumption (b) underlying (1.33) is left in question. In effect, condition (a) asserts that the β_i in (1.33) are not equal, and condition (b) asserts that the Y_i do not remain constant fractions of Y over time, i.e., $Y_i \neq k_i Y$. If (a) does not hold, (1.27a) is preferable to (1.33) regardless

[16] Type of income may be defined according to nature of compensation (wages and salaries, rents, etc.), or industrial origin, or on any other basis.

[17] The meaning of "appreciably" depends on the purpose of the particular study.

of the validity of (b). If (a) does hold, (1.33) is preferable if (b) also holds. But if (a) holds and (b) does not, (1.33) reduces to

$$\frac{C}{NP}=\alpha+\beta_1\frac{k_1Y}{NP}+\beta_2\frac{k_2Y}{NP}+\ldots+\beta_m\frac{k_mY}{NP}=\alpha+\sum_i\beta_i\frac{k_iY}{NP}.$$

In this case, $\sum_i\beta_ik_i=\beta$, say, could be estimated directly by (1.27a) unless there is some special interest in the values of the β_i.

If all income is lumped into one variable, what should this variable include? Most studies have used disposable personal income, which is essentially total income payments to individuals less personal taxes and employee social security contributions, on the ground that only this amount is available for consumption expenditures. The assumption seems justifiable under the present pay-as-you-go plan (combined with the Declaration of Estimated Tax for the higher income brackets), but was this also true for the prewar period when taxes were not collected currently? In the latter case, one would have to assume either that income recipients estimated their tax in advance and then put aside the requisite sum and no longer considered it in planning their consumption expenditures, or that disposable income in year t is defined as total income in year t, less personal tax payments made in that year (mostly the income tax on income earned in year t–1). The latter assumption seems the more plausible of the two in justifying the use of a disposable income variable.

It may, however, be argued that consumers in the prewar period based their expenditures partly on their disposable income and partly on the income that was taxed away. In other words, the aggregate function would perhaps be of the form

$$\frac{C}{NP}=\alpha+\beta\frac{Y}{NP}+\gamma\frac{V}{NP} \tag{1.34}$$

where V = tax liability and social security contribution, and $Y+V$ = total income received by all individuals.

If this hypothesis were wholly false, γ would equal zero. Unfortunately, the only two studies to use a national income or total income payments variable, Ezekiel (1.6) and Samuelson (1.22), did not separate Y from V; that is, they may be said to have assumed $\beta=\gamma$; so that a test of this theory from the past studies is not possible.[18]

In principle, the difficulty could be resolved if we could use the known tax schedule as a relation between individual income and tax, neglecting the number of dependents, etc. Because of tax exemption limits and the progressivity of tax, this would mean replacing the right-hand side of

[18] Stone used still another concept, expressing consumption expenditures at factor cost as a function of disposable income. This procedure diminishes the consumption variable to some extent, insofar as certain types of consumption taxes are removed from consideration, but it makes no allowance for variations in the relative prices of consumption goods. Stone does not discuss his reasons for expressing consumption at factor cost.

(1.34) by a nonlinear expression involving not only the mean income but also some other parameters of the income distribution.

Relevance of income distribution

From a theoretical point of view, it is reasonable to insert some income distribution measure in an aggregate consumption function because the aggregate function is based on individual behavior. In general, moments of income higher than the first may enter into the aggregate function. This is not true for the micro-economic function considered so far, (1.25), because it contains the simplifying assumption that the parameters of any given function are the same for all individuals, and because it contains income terms no higher than the first degree. If these assumptions were valid, higher moments of the income distribution would be unnecessary in the aggregate function. If, on the other hand, the parameters of the micro-economic relation are not the same for all individuals, or if income terms of the second degree or higher enter into this relation, an aggregate consumption function linear in income can be justified only by showing that these moments either are constant or exert negligible effect on consumption.

The assumption stated above, that $\alpha_i = \alpha$, $\beta_i = \beta$, etc., is therefore basic to the support of an aggregate function such as (1.1). It will be instructive to investigate the implications arising from the possible invalidity of this assumption. Suppose that the micro-economic relation involved parameters α_i, β_i depending on the individual i thus:

(1.35) $$c_i = \alpha_i + \beta_i y_i.$$

To aggregate (1.35) some assumption must be made about the nature of its parameters. A reasonable first approximation is to assume that they depend solely on income, so that by (1.35) we have

(1.36) $$c_{y_j} = \alpha_{y_j} + \beta_{y_j} y_j$$

where

$$\alpha_{y_j} = \sum_{i=1}^{n_{y_j}} \alpha_{iy_j} \quad \text{and} \quad \beta_{y_j} = \sum_{i=1}^{n_{y_j}} \beta_{iy_j}$$

n_{y_j} being the number of individuals in the income bracket having average income, y_j.

Suppose the parameters of (1.36) have the following form:

(1.37) $$\alpha_{y_j} = \alpha_0 + \alpha_1 y_j$$

$$\beta_{y_j} = \beta_0 + \beta_1 y_j .$$

In that case (1.36) becomes:

(1.38) $$c_{y_j} = \alpha_0 + (\alpha_1 + \beta_0) y_j + \beta_1 y_j^2.$$

Even in this simplified case, it is clear that moments of the income distribution higher than the first enter into the aggregate consumption function. More realistic assumptions as to the nature of the parameters of (1.35) would probably substantiate this result and involve other variables as well, such as family size and liquid assets. In that event, the joint distribution of income and these other variables must be taken into consideration.

Hence, unless the assumption that the parameters of a micro-economic consumption function are the same for all individuals is valid, the theoretical formulation of even such a simple aggregate consumption function as one derived from (1.36) must allow for the effects of income distribution not included in that of the first moment. In practice this would not be necessary if the effect of higher moments of the income distribution on aggregate consumption could be shown empirically to be negligible. Yet their negligibility has apparently never been proved, and hence every one of the functions in Table 1 except (1.24) is questionable because it does not allow for income distribution effects. This failure is the more striking because the only students who have treated the question empirically, Staehle and Polak, were almost the earliest to attempt a construction of the aggregate consumption function,[19] and both found some statistic of dispersion of the income distribution to be highly relevant (though Polak, ironically enough, rejected his functions containing an income distribution variable under the mistaken belief that its coefficient had the wrong sign: see p. 21). A review of their findings therefore seems desirable.

Staehle's study was probably the first statistical investigation of the aggregate propensity to consume to be published in this country. Staehle pointed out that the income distribution of a particular population should be taken into account in measuring its propensity to consume, since the total demand (or consumption) curve is essentially the aggregation of the individual consumption curves weighted by income. Though he did not actually go through the process of mathematical aggregation, his function (1.24) fits neatly into the preceding discussion. Starting with the micro-economic relation (1.25), Staehle's function can be derived by assuming that the parameters of this relation are the same for all individuals, i.e.,

$$(1.39) \qquad \begin{aligned} \alpha_{iy} &= \alpha_0 y_i \\ \beta_{iy} &= \beta_0 y_i + \beta_1 B \,. \end{aligned}$$

Except, perhaps, for the omission of constant terms, (1.39) is not un-

[19] Jan Tinbergen's earlier estimate in *Business Cycles in the United States of America, 1919-1932* (Vol. II of *Statistical Testing of Business-Cycle Theories,* Geneva, League of Nations, 1939), pp. 35-49, should also be mentioned in this respect. It is not considered in detail because his consumption function is essentially a modification of Polak's with additional equations to explain farmers' consumption and residential construction.

realistic. B is the measure of income inequality, which is not necessarily related to Y. It is defined thus:

$$B = \frac{\text{Cumulative median income} - \text{Ordinary median income}}{\text{Cumulative median income}}.$$

The cumulative median income "is obtained by putting all incomes in the order of increasing size, and cumulating them from the first (i.e., the lowest) income upwards to the point where the cumulated sum just reaches half the total amount of all incomes. The size of the last income which must just be added to the cumulated sum in order to reach this amount is the cumulative median income."[20]

B may vary from zero to one, inclusive. If all incomes are equal, B would be zero; B would be one if at least half (but not all) the incomes are zero. Thus one might say that the inequality of the income distribution, the degree to which income is concentrated in the upper part of the income scale, increases with B.[21]

In his empirical analysis Staehle raised the question whether the effect of changes in income inequality on aggregate consumption is negligible and can therefore be neglected, or whether the opposite is true. He attempts to show that, at least in the short run, income inequality is not constant, and that it has a substantial effect on the propensity to consume.

This is accomplished by a multiple regression analysis of the aggregate average propensity to consume of German wage earners on B and on an index of their labor income deflated by a wage index, (1.24). The propensity to consume is measured by the ratio of an index of retail sales to the index of labor income. Quarterly data are used, covering the period

[20] Staehle, *op. cit.*, p. 136, fn. 7.

[21] If the income distribution can be approximated by Pareto's formula, B can be expressed in terms of Pareto's α, the absolute value of which we call A. This is possible because the ordinary median income, y_m say, is the solution of the following equation:

$$\int_{y_m}^{\infty} f(y)\,dy = \frac{N}{2}$$

when incomes are ranked in ascending order, whereas the cumulative median income, y_c say, is derived from

$$\int_{y_c}^{\infty} yf(y)\,dy = \frac{Y}{2} = \frac{1}{2}\int yf(y)\,dy,$$

where the last integration is over the whole range of values of y.

The Pareto law $f(y) = \frac{k}{y^{A+1}}$, k = constant, can be substituted into the above equations, the equations solved for y_m and y_c, and the solutions substituted into the formula for B. The result is:

$$B = 1 - 2^{\frac{1}{A(1-A)}}.$$

1928-34, taken from German insurance statistics relating to all wage earners. Adjustment for seasonal variation was made by means of four-quarter moving averages.

Staehle shows that if changes in B are neglected, almost no relation exists between the propensity to consume and labor income; the determination coefficient[22] is 10 per cent. But when B is introduced into the regression equation, the value of the determination coefficient rises to 73 per cent, with the income variable explaining 37 per cent of the variance in the propensity to consume and the B variable explaining the remaining 36 per cent. As the value of B increases, i.e., as income inequality increases, the regression equation shows that the proportion of income spent declines. Hence, Staehle remarks,

> . . . the degree of inequality in the distribution of labor income is about as important as the amount of labor income in terms of wage units in explaining fluctuations in the proportion of retail sales to labor income. Furthermore, no explanation at all would have been obtained, if labor income (in wage units) had been taken as the only influencing factor. In conclusion, therefore, it may be said that it is indispensable to take into account the variations in the distribution of incomes in constructing the "propensity to consume."

Staehle then brings out the distinctive effect of the income distribution variable by showing that, for the period under consideration, the fluctuations of B were not correlated with those of either industrial production or wage rates in Germany. Although the data used refer to only a part of the German economy, Staehle says: "If measurement should reveal a considerable variability of the essential characteristics of the income-distribution among wage earners, then it is very likely that a measurement covering all income-recipients would show the total distribution to be at least as variable." This statement is, however, debatable, especially so in the light of the findings of Mendershausen (p. 21) to the effect that changes in inequality at different segments of the income distribution may not be in the same direction.

Polak included a measure of income distribution among other variables in attempting to explain the annual fluctuations of United States consumption between 1919 and 1932. The measure used was Pareto's α, the absolute value of which we call A. This measure, α, is defined as the slope of the line fitted to the points obtained when the number of income recipients with incomes greater than y is plotted against y on double-logarithmic paper.[23] The slope is always negative, hence $A = -\alpha$. The smaller A is, the more unequal the distribution of incomes.

Polak derived a number of regression equations, some including A and others excluding it. Although the inclusion of A increased the coefficient

[22] Staehle, *op. cit.*, pp. 141-2.

[23] Staehle had considered using this measure in his study but had rejected it because of the very small changes in A over time. It is not valid, in any case, to fit cumulative distributions by least squares, because of the dependence of successive items.

of determination substantially in every case, Polak nevertheless rejected all the functions containing A because the negative sign of the coefficient of A yielded by all of these functions apparently contradicted one of his basic a priori conditions, namely that consumption should increase as the distribution of incomes becomes less unequal; i.e., as A rises. Polak attributed this negative sign of the coefficient of A in part to the negative correlation between A and high income and speculative income (capital gains). And since he believed that high income and speculative income should be included in his consumption function, he resolved the dilemma by discarding A.

The true explanation of the negative sign of the coefficient of A is probably the one advanced by Mendershausen.[24] In his analysis of intertemporal changes in the income distribution, Mendershausen discovered a negative correlation between changes in inequality within the entire income distribution and changes in inequality within the higher income groups. Polak's values for A were based on income tax statistics relating to recipients of $5,000 net incomes and above; that is, to the higher income groups. Hence, as Mendershausen points out, the negative sign of the coefficient of A did not imply that changes in inequality in the income distribution as a whole were inversely related to consumption. If this is the true explanation, Polak's results clearly do not mean that income distribution should be left out of account in a consumption function.

The only other empirical evaluation of the influence of income distribution on consumption is contained in a postwar forecasting study by J. L. Mosak.[25] In deriving a relationship between total consumption and disposable income, he advances the theory that income distribution (and prices) exert their main effect in determining the distribution of expenditures among commodities rather than in regulating the absolute amount of consumption. In support of this point on the matter of income distribution, Mosak offers the following:

> Multiple regressions, using the ratio of salaries and wages to total income payments as a rough index of income size distribution, were examined for the three major classes of consumer expenditure, namely, durable goods, nondurable goods, and services. Although the separate effect for each class of expenditure appeared important, the combined effects on aggregate consumer expenditure substantially canceled out.[26]

This question, however, as well as the influence of prices on consumption, needs to be explored more fully, as Mosak recognized. In particular, since the various possible measures of income inequality do not show the same variations over time, the choice of an appropriate measure is impor-

[24] *Changes in Income Distribution during the Great Depression* (NBER, 1946).

[25] *Op. cit.*

[26] *Ibid.*, p. 33.

tant. For example, Simon Kuznets' recent study[27] shows that the shares in total income of upper income groups below the top 1 per cent of the population arrayed by income move inversely to business cycles, but that the share of the top 1 per cent has moved positively in some cycles and inversely in others. Hence the cyclical movements in income shares depend on the specific level observed in the array. Somewhat similar results were obtained by Horst Mendershausen in his study.

Cyclical considerations and lag variables

Most of the studies summarized in Table 1 make no attempt to distinguish interannual cyclical fluctuations from secular trends in aggregate consumption functions, perhaps partly because the problem was not recognized and partly because no workable solution could be found. Where such attempts have been made, they have taken one of two forms.

One approach has been to make some adjustment for "atypical" years in deriving estimates of the parameters of an aggregate consumption function. Thus Woytinsky eliminated the years 1931-34 from the period he studied, as being unduly depressed and not representative of the more normal interannual fluctuations. On the other hand, Bennion classified each year in the period 1923-40 as prosperous, semidepressed, or depressed and then fitted linear regressions of C/P on Y/P to each of the three sets of data. In effect, Bennion's (and Woytinsky's) argument is that we do not have one consumption function for a period of years, but several such functions, the appropriate one for a particular year depending upon the category into which that year is classified. But this presents a somewhat dubious situation since the criterion of classification is itself dependent upon some measure of cyclical fluctuations. Bennion, for example, used the percentage-of-full-employment criterion, which we may call X. In terms of micro-economic behavior, Bennion's argument then reduces to the contention that the individual's consumption is

$$\frac{c_i}{P} = \alpha + \beta \frac{y_i}{P} \tag{1.40}$$

where

$$\alpha = \phi(X) \quad \text{and/or} \quad \beta = \psi(X) \; . \tag{1.41}$$

But if so, then why not simply substitute (1.41) into (1.40) and use as the base for the aggregate consumption function a relation such as

$$\frac{c_i}{P} = \phi(X) + \psi(X) \frac{y_i}{P}. \tag{1.42}$$

[27] "Shares of Upper Income Groups in Income and Savings," *Occasional Paper 35* (NBER, 1950), pp. 39-44.

If X is some function of Y/P, the aggregate form of (1.42) may be simply a nonlinear relation in Y/P. But if X depends on variables (generally) other than Y, the aggregation process might lead to a heretofore unsuspected consumption function. In any event, such a procedure would dispense with the more or less arbitrary methods of classification used by Bennion and Woytinsky, which forced continuous variables into discrete classes whose limits are necessarily a matter of judgment.

A second and more recent approach has been to combine both cyclical and secular influences into one aggregate function by inserting a variable that remains approximately constant over a particular cycle but may vary as between cycles. This approach is employed by Modigliani (1.21) and Duesenberry (1.23), the variable used by both being the highest previous income: Y_0. The reasoning behind the use of Y_0, which is treated in some detail by Duesenberry,[28] is that people plan their expenditures not only on the basis of current income but also with reference to their highest preceding income. Under our social system, it is explained, people are constantly striving toward a higher standard of living. Their actions are very sensitive to the neighbors' behavior, and they seek to maintain at least the highest standard of living attained in the past. Hence the savings ratio in the long run is independent of the absolute level of income and remains constant, but in the short run depends on the ratio of current income to highest past income.

As translated into a function of the form (1.23), this means that Y_0 remains constant on the downswing of the cycle and on that part of the upswing for which current income is less than Y_0. During this stage, (1.23) is equivalent to:

$$(1.43) \qquad \frac{S}{Y} = \alpha + \beta' Y$$

where $\beta' = \beta / Y_0$ is constant.

Once the upswing reaches the point where current income exceeds Y_0, at which time it is assumed that secular forces predominate, (1.23) may be represented as:

$$(1.44) \qquad \frac{S}{Y} = \alpha + \beta \frac{Y}{Y_{-1}}$$

because Y_0 now becomes the income of the previous year. It is in this stage of the cycle that old standards of living are presumably being raised to new, higher standards of living based on the higher level of income. In the ensuing downswing, Y_0 again reverts to a constant, though not the same constant as in the prior downswing (unless the preceding upswing had failed to carry beyond the value of Y_0 established in an earlier cycle).

[28] *Income, Saving, and the Theory of Consumer Behavior* (Harvard University Press, 1949), Ch. III, V.

A variation on this approach is the suggestion by Davis[29] that the highest previous consumption, C_0, be used to adjust for cyclical and secular influences instead of Y_0. The rationale behind this suggestion is that consumption habits are acquired only as actual consumption takes place so that people become adjusted to a certain standard of consumption, and hence it is past consumption expenditures rather than past income that influences current consumption.[30] In the prewar period, this distinction would not be of much practical importance, because years of peak income and of peak consumption coincided, but when the two variables do not fluctuate concomitantly, as in the early post-World War II years, when many goods were in short supply, such a distinction can lead to substantial differences in predictions of consumption.

[29] Davis, T. E., "The Consumption Function As A Tool for Prediction," *The Review of Economics and Statistics,* Vol. 34 (1952), pp. 270-7.

[30] Davis, *Ibid.,* p. 274.