Volume Title: A Disequilibrium Model of Demand for Factors of Production

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Volume Publisher: NBER

Volume ISBN: 0-870-14261-5

Volume URL: http://www.nber.org/books/nadi74-1

Publication Date: 1974

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Chapter URL: http://www.nber.org/chapters/c0575

Chapter pages in book: (p. 55 - 80)

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ESTIMATION OF THE MODEL: TOTAL MANUFACTURING

THIS chapter contains estimates of model (2.7) using total manufacturing data as described in Chapter 3. An important problem of specification of this model concerns the content of the permanent and transitory shocks that drive the system, as was noted in Chapter 2. Many alternative approximations are possible, and we have experimented with a number of them. These experiments will be summarized in the following chapter. As will be shown there, the main conclusion to be drawn from these experiments is that alternative specifications of sales forecasts do not change the results in any substantive way. Therefore, we present our preferred set of estimates of model (2.7) in which target input values are considered to be log-linear functions of actual sales, relative prices, and trend. This choice is dictated by the ease of computation in view of the insensitivity noted above.

On these assumptions the model is

$$Y_{it} - Y_{it-1} = \sum^{6} \beta_{ij}(Y^{*}_{jt} - Y_{jt-1}) + \varepsilon_{it};\ i = 1, 6;$$

and

$$Y^{*}_{jt} = a_{0j} + a_{ij}S_t + a_{2j}(w/c)_t + a_{3j}T + \varepsilon'_{jt};\ j = 1, \ldots, 6;$$

where all variables are measured in logarithms and ε and ε' are random variables. The change in each input in period t is taken to be a function of the deviations of all the inputs from their target values at the end of period $t - 1$. In addition, the target value of each input is taken to be a function of sales, relative input prices, and a trend term. T is a trend term taking integer values and with its origin in 1947. Combining these two relations, the equations to be estimated are

$$Y_{it} = m_{i0} + m_{it}S_t + m_{i2}(w/c)_t + m_{i3}T + b_{i1}Y_{1t-1} + b_{i2}Y_{2t-1} + \dots + b_{i6}Y_{6t-1} + u_{it}; \; i = 1, 2, \dots, 6; \quad (4.1)$$

where the m_{ij} terms are linear functions of β_{ij} and a_{ij} above, and the b_{ij} terms are naturally related to β_{ij}; that is, $b_{ij} = -\beta_{ij}$ for $i \neq j$ and $b_{ii} = (1 - \beta_{ii})$. The u_{it} are random error terms. As before, all the variables except for trend are measured in natural logarithms.

A. PROBLEMS OF ESTIMATION

Two issues must be considered prior to estimation. One is the question of imposing production function restrictions on the estimates of equations (4.1) a priori. The other is the selection of an appropriate estimation technique.

Although it is computationally inconvenient, imposition of production function constraints is feasible. It requires estimation of all six equations at the same time, with an appropriate adjustment of the over-all covariance matrix. At an early stage of the investigation, we decided not to follow that procedure for three reasons. First, it seems to us that imposing such restrictions makes too much of a presumption that the model is completely correct. If the model is truly "correct" then the unrestricted estimates should satisfy the a-priori restrictions. Second, data limitations already noted necessarily force us to maintain certain hypotheses concerning omitted variables. It might be appropriate to relist these factors. We have no data on "user cost" of labor inputs and hours of work of nonproduction workers. Average wage rates have been used rather than marginal wage rates; inventories consist of goods in process plus final goods rather than each component separately; we have no price data for inventories and nonproduction labor; and our utilization estimates fall short of an ideal measure. Third, the data are highly aggregated. As Fisher [1971] has shown, aggregate data in this context often conceal true underlying microrelationships. Therefore stringent tests of the restrictions undoubtedly require much better and more disaggregated data than two-digit classifications provide.

It is well known that choice of an estimation technique depends on properties of the residuals in the model to be estimated. The presence of serially dependent residuals in almost all economic time-series models is very well documented. Use of ordinary least squares procedures under such circumstances leads to biased results. To avoid this situation, the hypothesis of first-order serially correlated residuals in each equation of model (4.1) is maintained and the implied first-order serial correlation in the residuals for *each* equation is estimated by the Cochrane and Orcutt [1949] search method. This method combines ordinary least squares (OLS) and estimation of the first-order serial correlation of the disturbances. It uses an (internal) ordinary least-squares (OLS) regression to form an initial guess of the first-order correlation coefficient ρ. Then the iterative process finds the value of ρ (denoted by $\hat{\rho}$) which minimizes the sum of squares of the residuals for the particular equation.[1] The range of search for the value for ρ was chosen in the interval -0.9900 to 0.9900. The iterations were terminated either when ρ changed by less than 0.005 from one iteration to another or when twenty iterations had occurred.

We calculated an F statistic, testing the null hypothesis that each equation of the model prior to the ρ transformation did not differ statistically from its counterpart after the transformation. For testing $\rho = 0$ against $\rho = \hat{\rho}$, the approximate F statistic is

$$F(1, n - k - 1) = \frac{[SSR(\rho = 0) - SSR(\rho = \hat{\rho})]/1}{SSR(\rho = \hat{\rho})/n - k - 1},$$

where k is the number of parameters estimated (including ρ), n is the number of observations, and SSR stands for the sum of squared residuals. The calculated values of this test for $n = 80$ and $k = 10$, using SSR from ordinary least squares and generalized least squares for each equation of the model are:

	$\ln Y_1$	$\ln Y_2$	$\ln Y_3$	$\ln Y_4$	$\ln Y_5$	$\ln Y_6$
Calculated F	17.02	3.40	189.3	3.15	53.11	27.43

The critical F values are: 7.04 at 0.01 per cent and 3.99 at 0.05 per cent (1, 69) degrees of freedom. The comparison clearly suggests rejection of the null hypothesis for all equations except for the utilization rates Y_2

1. The procedure uses an internal OLS regression to form an initial guess of ρ, say ρ_0. Then all the variables are transformed by ρ_0 to form the new data set $(Y_t - \rho_0 Y_{t-1})$, and the regression is fitted to the transformed data. The regression coefficients are transformed back into the original variables for re-calculating the serially correlated errors, which provides a new estimate of ρ. The process continues until it converges to a single $\hat{\rho}$.

and Y_4 at the 0.01 level of significance. We accept the presence of first-order serial correlation in each equation.

It should be noted that the Cochrane-Orcutt method applied to our model implicitly assumes serial independence of residuals across equations. If this more complex pattern of serial dependence is present, maximum likelihood methods require estimation of a 6-by-6 matrix of correlation coefficients within and across equations in which the off-diagonal elements are not necessarily zero as has been assumed in our procedure. An investigation of cross-equation serial correlation is presented below.

B. STRUCTURAL ESTIMATES

Structural estimates of model (4.1) for total manufacturing are exhibited in Table 4.1. Judging by the high adjusted R^2 statistics and small standard errors and sums of squared residuals, shown at the bottom of the table, the fit of the model in the sample period is impressive.[2] The goodness of fit is clearly indicated by Charts 4.1 to 4.6, which show actual and predicted values of each variable over the sample period. Note, in particular, that cycle turning points are tracked extremely well, an attribute not often achieved with the same degree of success in alternative models of input demand.

Initial impact effects of sales, trend, and relative prices are indicated in the second, third, and fourth rows of Table 4.1. The sales variable is highly significant in all equations except that of capital stock ($\ln Y_3$) and inventories ($\ln Y_5$). Judging by the magnitude of the regression coefficients, the impact of sales is strongest on the generalized utilization rate ($\ln Y_4$), followed by production worker employment ($\ln Y_1$), and hours per man ($\ln Y_2$). Its effect on nonproduction workers is small, but significant. With the exception of the coefficient in the inventory equation, these results are much as expected. However, inventories serve as a buffer between production and sales and are closely related to decisions of the firm on the acquisition and utilization of the stock of inputs. For example, as inventories of finished goods are drawn down to meet the demands for output, the firm may replenish its stock of goods in process and raw materials; these in turn require higher rates of utilization of the existing stocks of capital and labor and/or additions to them. Therefore, we

2. Note that levels of inputs rather than first differences have been used as dependent variables. R^2 statistics would have been lower had first differences been used, though parameter estimates would have been identical to those reported in Table 4.1.

TABLE 4.1

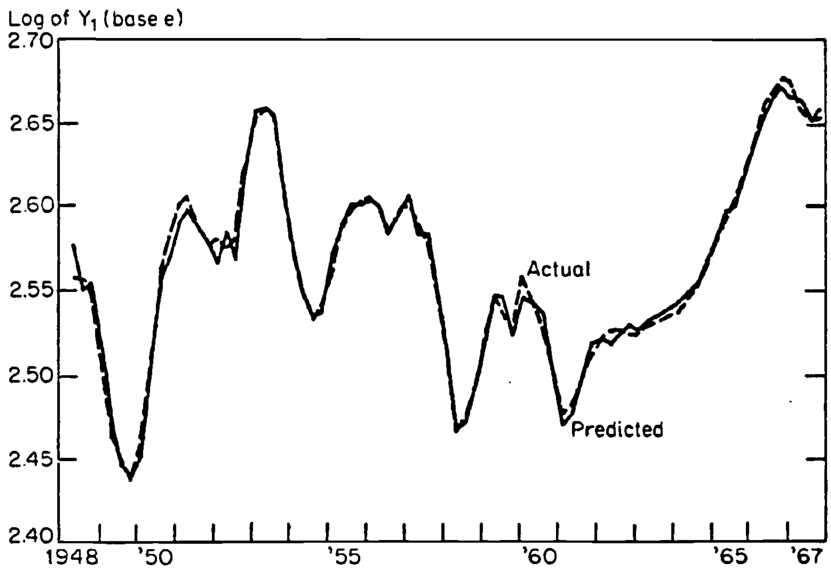
ESTIMATED STRUCTURE OF MODEL (4.1) FOR TOTAL MANUFACTURING
(sample period: 1948I–1967IV; all variables except trend are in natural logarithms)

Independent Variables	Dependent Variables: Prod. Emp. (Y_{1t})	Hours (Y_{2t})	Capital (Y_{3t})	Util. (Y_{4t})	Inven. (Y_{5t})	Nonprod. Emp. (Y_{6t})
Constant	−2.708 (3.266)	.3186 (.8357)	.1137 (.5412)	.8643 (.4129)	−1.344 (1.226)	.0911 (.1529)
Sales	.4394 (12.84)	.1554 (7.891)	−.0048 (.7009)	1.100 (10.62)	.0004 (.0096)	.0531 (2.657)
Trend	−.0004 (6.035)	−.0010 (5.595)	.0001 (.6421)	−.0085 (7.415)	.0012 (1.405)	.0015 (1.889)
Rel. prices (w/c)	−.0177 (.7669)	−.0058 (.7324)	.0017 (.3268)	−.0986 (2.127)	−.0267 (.9263)	−.0112 (.7564)
Y_{1t-1}	.4575 (7.417)	−.0992 (4.040)	.0435 (2.608)	−.2295 (1.658)	.3139 (4.000)	.0352 (.7474)
Y_{2t-1}	.4447 (2.286)	.8525 (8.709)	.0168 (.4322)	−.5874 (1.045)	.5626 (2.115)	.0057 (.2043)
Y_{3t-1}	.1784 (1.877)	−.0137 (.4671)	.9050 (27.94)	−.2794 (1.683)	−.1114 (1.043)	−.0093 (.3150)
Y_{4t-1}	−.0236 (.9932)	−.0820 (5.997)	−.0093 (1.837)	.1953 (2.686)	−.0931 (2.813)	.0408 (2.820)
Y_{5t-1}	.0053 (.1093)	−.0400 (1.982)	.0114 (.9492)	−.6623 (5.892)	.6244 (10.03)	.0153 (.4470)
Y_{6t-1}	−.0649 (.8616)	.0462 (1.814)	.0767 (2.459)	.9811 (6.889)	.3432 (4.047)	.7133 (8.180)
R^2	.9855	.9414	.9999	.9088	.9982	.9995
$\hat{\rho}$	.6665	−.2378	.9330	.0181	.5086	.9255
SEE	.0072	.0047	.0015	.0231	.0099	.0044
SSR	.0036	.0015	.0002	.0367	.0068	.0013

NOTE: Figures in parentheses are t statistics. R^2 is the coefficient of determination; *SEE*, the standard error of estimate; and *SSR*, the sum of squared residuals. For $\hat{\rho}$, see text note 1.

CHART 4.1

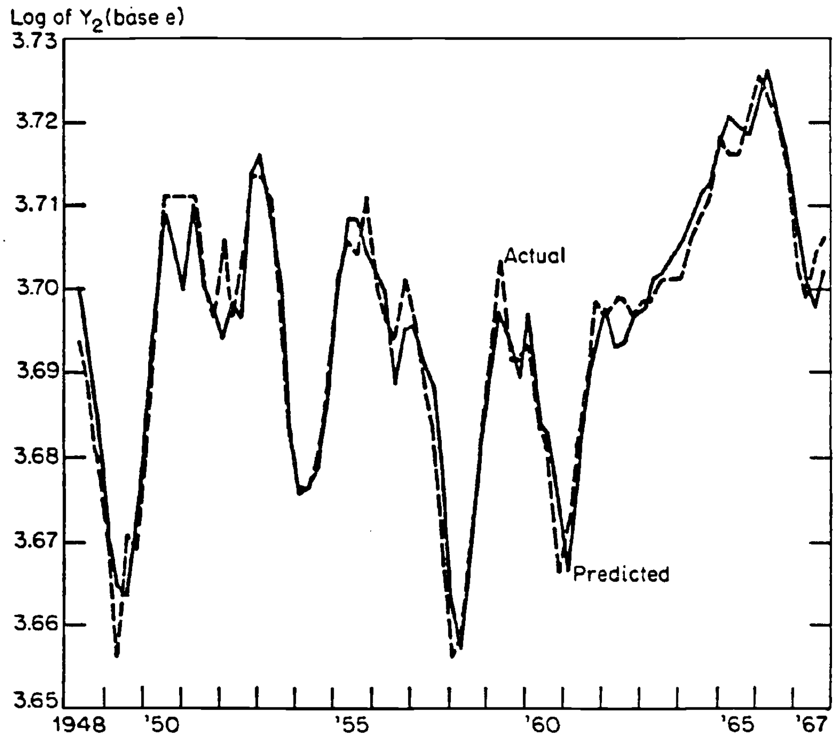
ACTUAL AND ESTIMATED VALUES OF THE STOCK OF PRODUCTION WORKERS (Y_1), 1948I–1967IV



SOURCE: Based on model (4.1.)

CHART 4.2

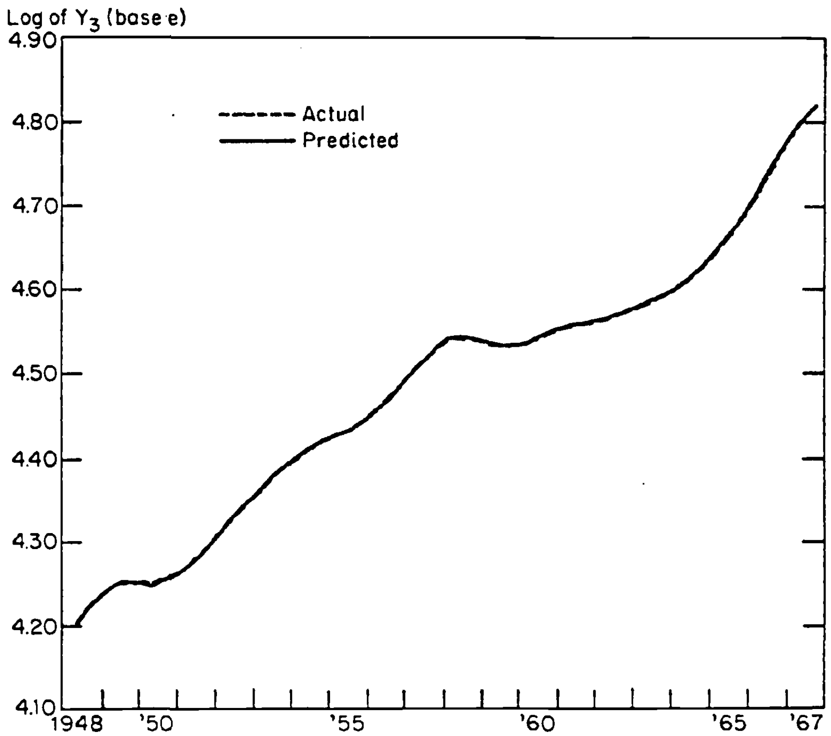
ACTUAL AND ESTIMATED VALUES OF HOURS OF WORK OF PRODUCTION WORKERS (Y_2), 1948I–1967IV



SOURCE: Based on model (4.1).

CHART 4.3

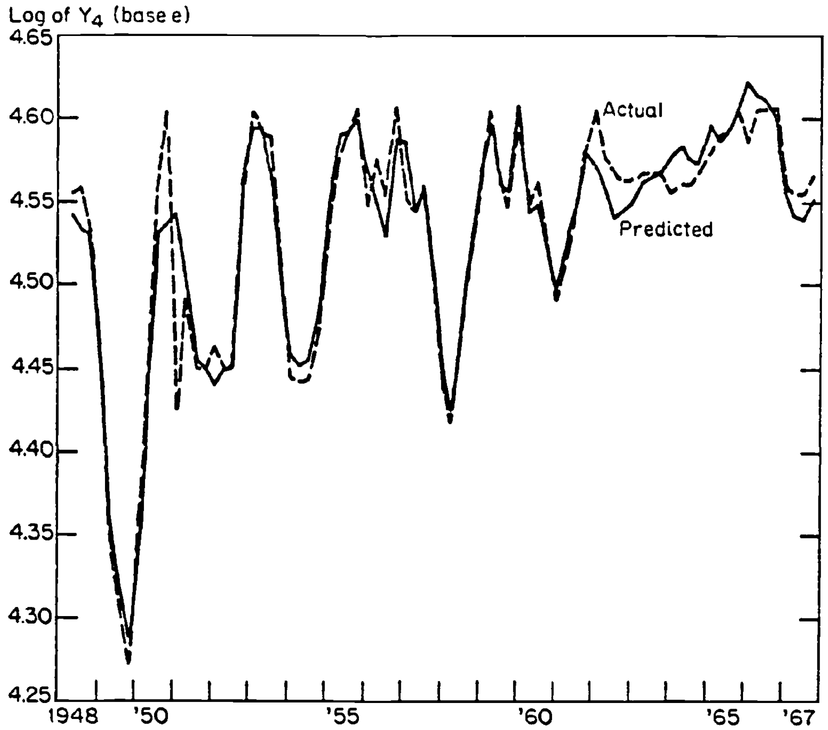
ACTUAL AND ESTIMATED VALUES OF DEFLATED CAPITAL STOCK (Y_3), 1948I–1967IV



SOURCE: Based on model (4.1).

CHART 4.4

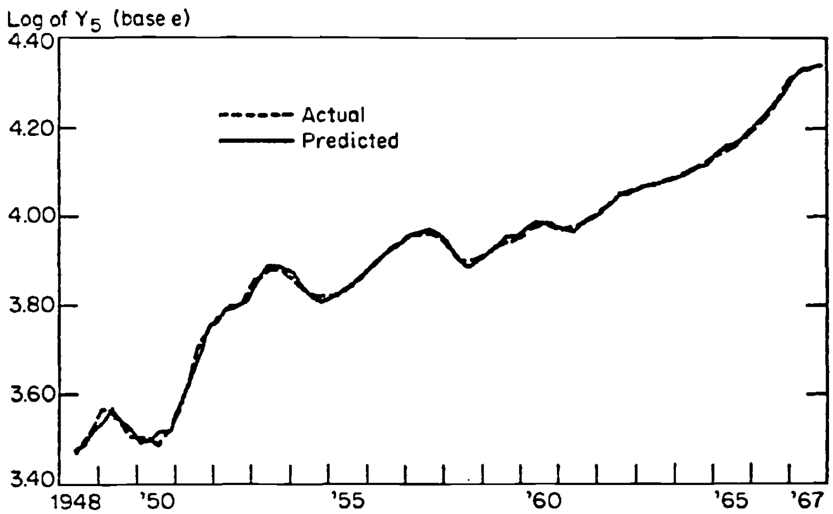
ACTUAL AND ESTIMATED VALUES OF THE UTILIZATION RATE (Y_4), 1948I–1967IV



SOURCE: Based on model (4.1).

CHART 4.5

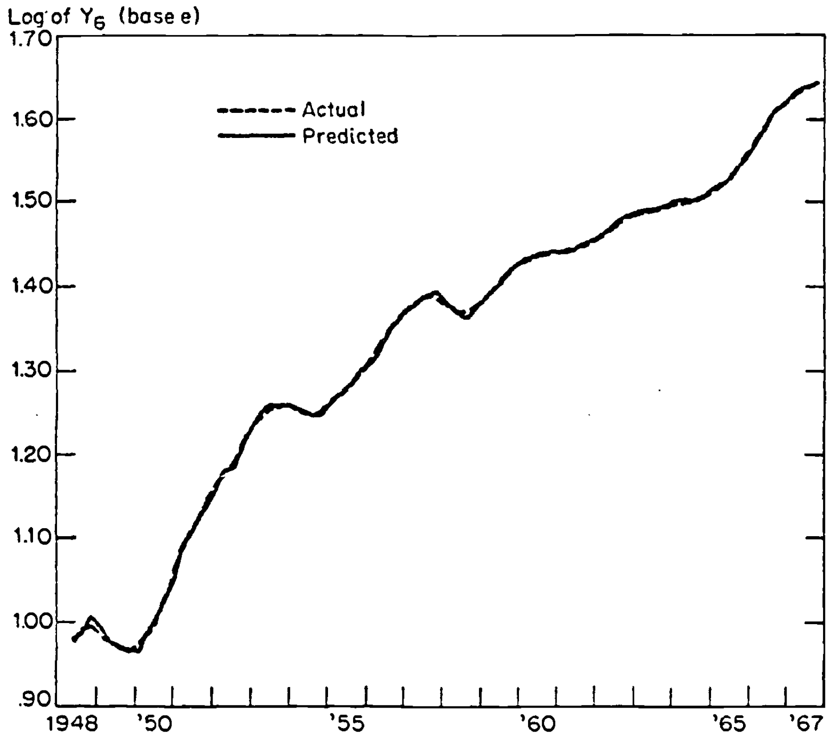
ACTUAL AND ESTIMATED VALUES OF MANUFACTURERS' TOTAL INVENTORIES IN CONSTANT DOLLARS (Y_5), 1948I–1967IV



SOURCE: Based on model (4.1).

CHART 4.6

ACTUAL AND ESTIMATED VALUES OF THE STOCK OF NONPRODUCTION WORKERS (Y_6), 1948I–1967IV



SOURCE: Based on model (4.1).

TABLE 4.2

DIRECTION EFFECT OF EXOGENOUS VARIABLES AND ADJUSTMENT COEFFICIENTS[a]

	Dependent Variables					
Independent Variables	Prod. Emp. (Y_1)	Hours (Y_2)	Capital (Y_3)	Util. (Y_4)	Inven. (Y_5)	Nonprod. Emp. (Y_6)
	(a) Direction of Impact Effects					
Sales	+	+	−	+	+	+
Trend	−	−	+	−	+	+
Rel. prices (w/c)	−	−	+	−	−	−
	(b) Cross- and Own-Adjustment Signs					
Y_{1t-1}	( )	+	−	+	−	−
Y_{2t-1}	−	( )	−	+	−	−
Y_{3t-1}	−	+	( )	+	+	+
Y_{4t-1}	+	+	+	( )	+	−
Y_{5t-1}	−	+	−	+	( )	−
Y_{6t-1}	+	−	−	−	−	( )

a. These directional effects are based on the signs of the structural coefficients reported in Table 4.1. Entries in panel (b) are estimated signs of β_{ij}, based on the relation $\beta_{ij} = -\hat{b}_{ij}$, for $i \neq j$ in model (4.1). The underlying data are in logarithms.

generally expect inventories to be inversely related to the sales variable. However, this relationship is not immutable. It depends on whether the depletion of finished goods inventories is offset or exceeded by the increase in goods in process and raw materials. This compositional effect cannot be ascertained with the aggregate data available. The estimates suggest that the effects are approximately equal in the initial period of the shock.

The time trend is significant in all equations (at 0.05) except for capital stock and inventories. It has negative signs in production worker employment and both utilization rate equations, but is positive in the equation for nonproduction worker employment. The coefficients on the relative price variable are extremely small in magnitude and are insignificant in all equations except in the equation for the general utilization rate, Y_4. This result may be due to high collinearity between trend and relative prices in the sample period, suggesting that the parameters of substitution and technological change cannot be identified in this data. The direction of impact effects is compactly presented in Table 4.2.

The cross- and own-adjustment effects in each equation are shown in Table 4.1 by the columns of regression coefficients of lagged input variables. In each column the own lag coefficient b_{ii}, (e.g., the coefficient of $\ln Y_{4t-1}$ in the Y_4 equation) is an estimate of $1 - \beta_{ii}$ in model (4.1). The other coefficients, b_{ij}, are estimates of $-\beta_{ij}$, or the cross-adjustment parameters of the model. Of course, own-adjustment coefficients are expected to be positive and less than unity, and that is the case in all equations. However, cross-adjustment coefficients can take either sign. The direction of these effects are summarized in Table 4.2. Judging from the t values in Table 4.1 and using 1.6 as the cutoff point, 18 out of 30 possible cross-adjustment coefficients are significantly different from zero. Since all cross adjustments need not be nonzero a priori, this is strong evidence in favor of an interrelated model.[3]

The largest number of nonsignificant cross effects occurs in the equation for nonproduction worker employment (Y_6), followed to a lesser extent by production worker employment (Y_1). Even so, both production and nonproduction worker employment strongly interact with all other variables in the system. Moreover, at least one variable interacts significantly with hours of work of production workers and with the generalized utilization rate of nonproduction workers. Therefore, all the input adjustments are interrelated. Furthermore, the signs of the cross adjustments in Table 4.2 indicate that there is no special tendency toward symmetry. In only 9 out of 15 possible paired comparisons (β_{ij} and β_{ji}) are the signs identical.

The signs of the cross-adjustment coefficients, β_{ij}, can be given a worthwhile interpretation in terms of "dynamic substitution." This concept differs in meaning from the conventional concepts of substitution and complementarity, which are equilibrium concepts. In a dynamic setting, firms may temporarily substitute one factor for another, even though the factors are complements in the long run, because short-run adjustment costs make it advantageous to do so. If β_{ij} is positive, excess demand for factor j increases the short-run demand for factor i, and consequently i and j can be considered as dynamic substitutes. If β_{ij} is negative, they can be considered complements.

To illustrate this point, consider the equation for production worker employment (Y_1) in Table 4.1 and also the effect of excess demand for production workers—the coefficient of Y_{1t-1}—in other equations. For all practical purposes, disequilibria in the levels of inventories, non-

3. If 1.85 is used as the cutoff, one-half of the coefficients are significant.

production workers, and generalized utilization rates have negligible effects on demand for production workers in the short run. However, demand for Y_1 is significantly affected by disequilibria in hours (Y_{2t-1}) and in capital stock (Y_{3t-1}). Excess demand for hours and capital stock decreases production worker employment, suggesting short-run complementarity relationships between these inputs. Examining the impact of disequilibrium in nonproduction worker employment on other variables of the system, note a similar complementary relation with capital stock and inventories (i.e., the coefficient of Y_{1t-1} in the Y_{3t} and Y_{5t} equations is positive). However, the coefficients suggest substitution between Y_1 and hours and the generalized utilization rate. Thus, we again note some nonsymmetry in dynamic responses among certain inputs, making it impossible to identify dynamic substitutes and complements in all cases. The feedback relationships among other variables can be interpreted in a similar manner by reference to the signs in panel (b) of Table 4.2.

C. THE GOODNESS OF FIT AND FORECASTING PERFORMANCE

We shall examine the goodness of fit of the model and its performance against an autoregressive model such as

$$Y_{it} = a_{i0} + a_{i1}Y_{it-1} + a_{i2}Y_{it-2} + a_{i3} + Y_{it-3} + \varepsilon_{it}; \; i = 1, \ldots, 6;$$

where ε_{it} is the stochastic residual, and all the variables are measured in natural logarithms. This third-order autoregressive model is essentially a generalization of the familiar naive models often used in the literature (Christ [1956] and Jorgenson-Hunter-Nadiri [1970]). Comparison with an autoregressive model is a very stringent test of quarterly models, and many analytical models often fail to pass it.[4] The sum of squared residuals for each equation of model (4.1) and its autoregressive counterpart were used to compute F statistics. These results indicate that the null hypothesis of no difference between the analytical model (4.1) and the autoregressive model is rejected. Comparison of the turning points predicted by the two models also clearly shows the superiority of model (4.1), especially for the more fluctuating series such as the utilization rate.

Using the structural estimates of model (4.1) reported in Table 4.1,

4. More complicated autoregressive models with fourth- and fifth-order lags were used, but the results were similar to those of the third-order autoregressive model.

conditional forecasts for each dependent variable for the next ten quarters can be generated. We used the following performance indexes to test forecasts of the model for the period 1968I–1970IV:

Mean forecast error:

$$m_1 = \frac{1}{n} \sum (Y_t - \hat{Y}_t);$$

Absolute mean error:

$$m_2 = \frac{1}{n} \sum | Y_t - \hat{Y}_t | ;$$

Mean square error:

$$m_3 = \left[\frac{n}{n-k} \sum (Y_t - \hat{Y}_t)^2 \right]^{1/2}.$$

The summary statistics on the forecast errors of the estimated equations of Table 4.1 are shown in Table 4.3. Several characteristics of the results

TABLE 4.3

FORECAST PERFORMANCE INDEXES FOR MODEL (4.1) FOR TOTAL MANUFACTURING

(forecast period: 1968I-1970II; all variables are measured in natural logarithms[a])

	Prod. Emp. (Y_1)	Hours (Y_2)	Capital (Y_3)	Util. (Y_4)	Inven. (Y_5)	Nonprod. Emp. (Y_6)
Mean error (m_1)	−.0030	−.0003	.0005	−.0099	−.0027	−.0026
Mean absolute error (m_2)	.0051	.0048	.0007	.0174	.0071	.0038
Mean square error (m_3)	.0059	.0061	.0012	.0219	.0085	.0049

a. The original units are: Y_1, millions of workers; Y_2, hours per man per week; Y_3, billions of 1954 dollars; Y_4, fraction of full capacity; Y_5, billions of 1954 dollars; Y_6, millions of workers.

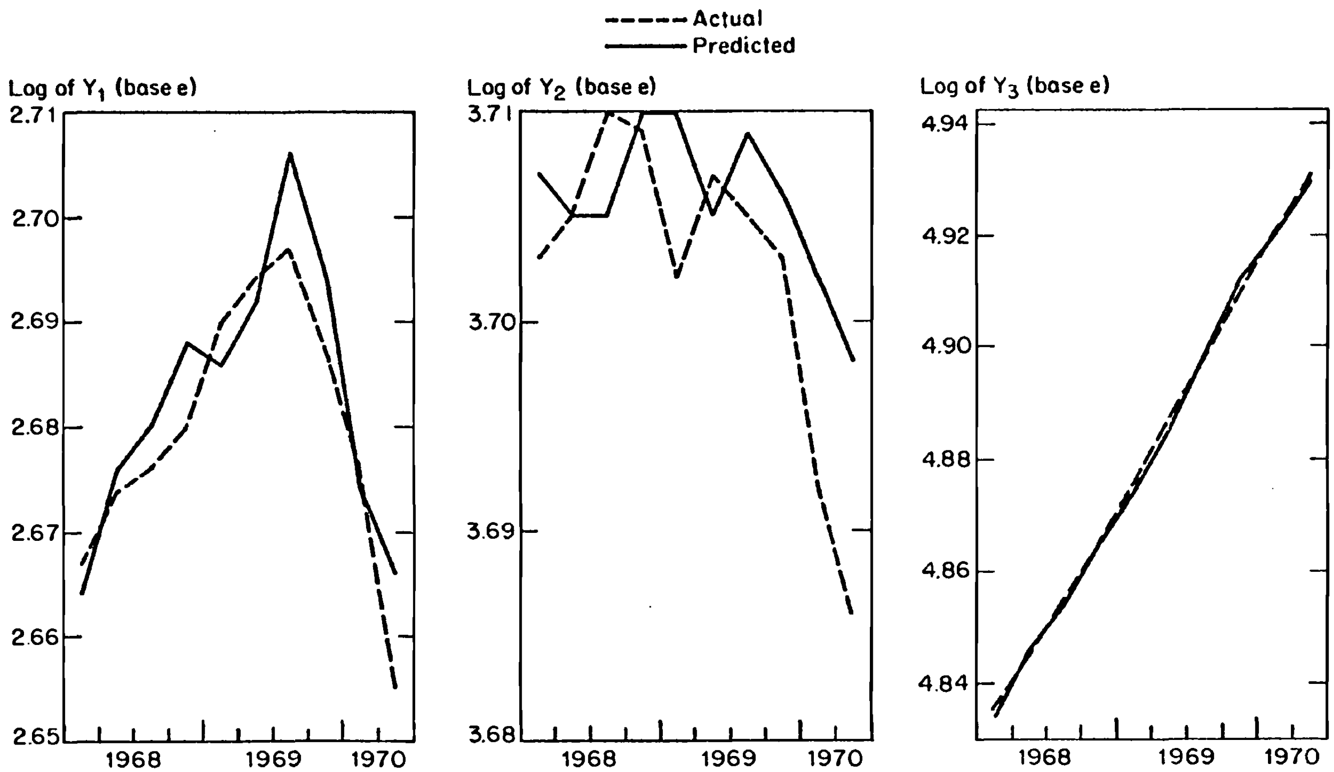
are worth noting. The mean error, m_1; the absolute mean error, m_2; and the mean square error, m_3, are all very small (as a percentage of the means) for each variable. The mean error of each equation, except for capital stock, is negative, which indicates that, on the whole, the model slightly overforecasts the value of most dependent variables. The m_2's and m_3's are all positive, of course, and naturally larger than m_1 for each equation. The size of forecast errors relative to mean values of the dependent variables in the forecast period are about 0.10 to 0.15 per cent for production worker employment, hours, capital stock, and nonproduction worker employment. It is about 0.065 per cent for inventories and about 40.0 per cent for capital utilization. The latter value results from restricting capital utilization to the interval (0, 1). Our measure uses the logarithms of such numbers, which are often close to zero.

All these summary statistics, though different in magnitude, suggest the same story. They are relatively larger for the capital utilization rate and inventory equations. Forecast errors for production workers exceed those of nonproduction workers and capital stock.

A fruitful way to examine the forecasting performance is to look at the pattern of forecast errors during the period 1968I–1970II and see how closely the level and turning points of the actual data are predicted. The forecast errors are presented in Charts 4.7 and 4.8. Actual and conditional forecasts of the dependent variables are also indicated. The sign and magnitudes of residuals in each equation vary over the period. The model on the whole overpredicts the level of the dependent variables in 1969 and the first two quarters of 1970. The forecast errors are generally negative in this range of the forecast period. The level of the stock variables is very well forecasted for production workers, capital stock, and nonproduction workers. The turning points are perfectly forecast for production workers and capital stock. The model wrongly predicts one turning point and does not predict another one for nonproduction workers in the last two quarters, 1970I and 1970II. In the inventory equation two turning points are missed, as in the hours equation. However, the model is not as successful in calling the turning points of the rates of utilization. As can be seen from Chart 4.7, forecasted hours lag behind the actual series by one quarter. If the predicted values are displaced by one quarter, very few of the turning points will be missed in this series. Once again, this result may occur because actual hours lead all other series used in the model by one or two quarters. The same picture

CHART 4.7

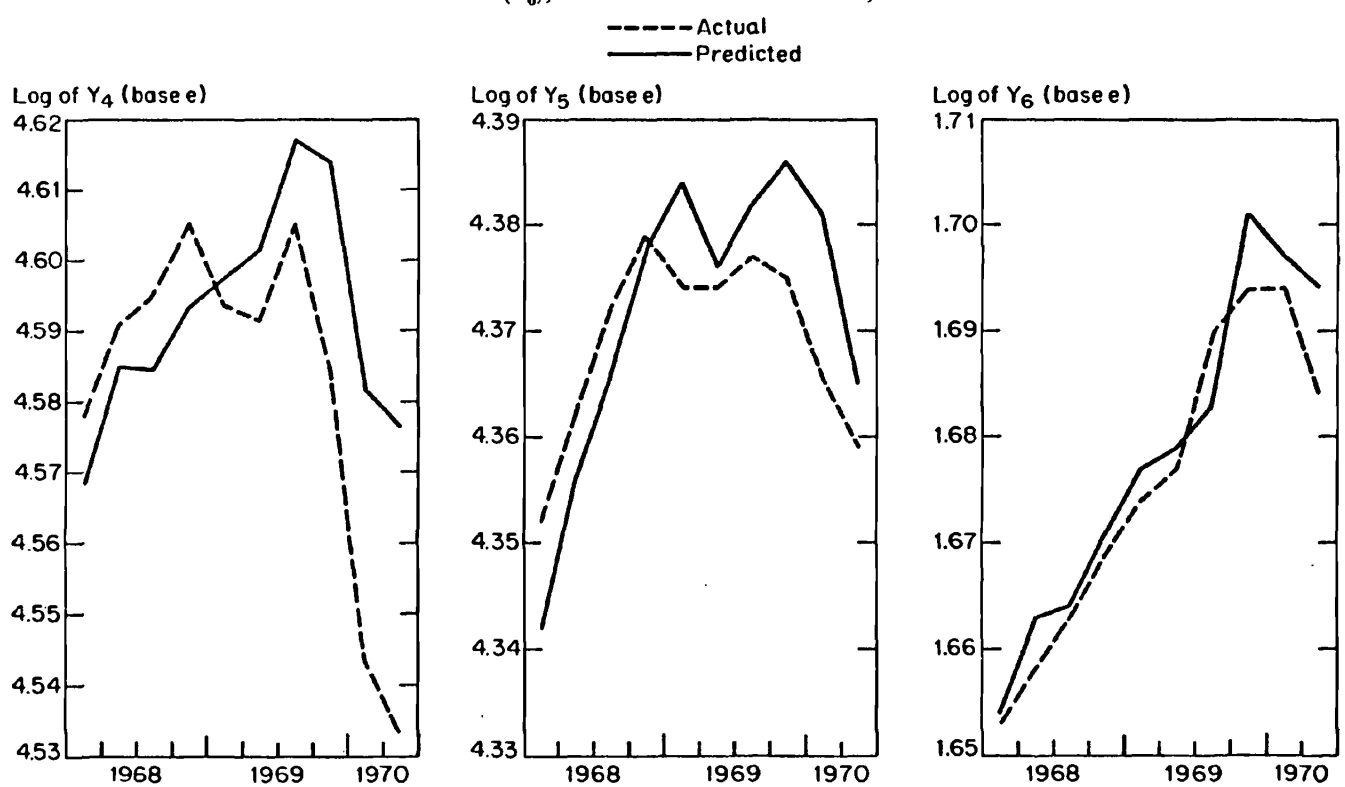
ACTUAL AND FORECAST VALUES OF THE STOCK OF PRODUCTION WORKERS (Y_1), WEEKLY HOURS (Y_2), AND CAPITAL (Y_3), FOR TOTAL MANUFACTURING, 1968I–1970II



SOURCE: Based on model (4.1).

CHART 4.8

ACTUAL AND FORECAST VALUES OF THE UTILIZATION RATE (Y_4), TOTAL INVENTORIES (Y_5), AND THE STOCK OF NON-PRODUCTION WORKERS (Y_6), FOR TOTAL MANUFACTURING, 1968I–1970II



SOURCE: Based on model (4.1).

is drawn for the general rate of utilization. Two out of three turning points are not predicted.[5]

An alternative way of testing forecast performance is based on certain test statistics on forecast errors. These are reported in Table C.1 and indicate no evidence of structural change between sample and forecast periods, confirming the stability of the estimates in Table 4.1.

D. RESPONSE CHARACTERISTICS OF THE MODEL

This section examines distributed lag patterns and estimated long-run scale and price elasticities of demand for various inputs that are implicit in the estimate of model (4.1). As discussed at length in Chapter 2, distributed lag properties of the model are found by transforming the systematic part of (4.1) into an equivalent reduced form. Long-run elasticities are merely the sums of corresponding distributed lag responses in each period.

i. Computational Methods

To summarize the earlier discussion, specifying desired inputs, ln Y_t^*, to be log-linear functions of sales, trend, and relative prices, we have estimated βA_1 and $(I - \beta)$ in

$$Y_t = \beta A_1 q_t + (I - \beta) Y_{t-1}, \tag{4.2}$$

where A_1 is a matrix of fixed coefficients, q is a vector of the exogenous variables: sales, trend, and relative prices (i.e., $Y_t^* = A_1 q_t$), and all are measured in natural logarithms. By recursion, equation (4.2) may be transformed to a reduced form

$$Y_t = \beta A_1 q_t + (I - \beta)\beta A_1 q_{t-1} + (I - \beta)^2 \beta A_1 q_{t-2} + (I - \beta)^3 \beta A_1 q_{t-3} + \dots . \tag{4.3}$$

5. When the forecasting performance of the model for each equation is compared with those of the autoregressive model, above, model (4.1) does better (in terms of the forecast statistics of Table 4.3). This is especially true of inputs such as nonproduction workers and the rate of utilization of capital. The autoregressive model underpredicts *all* the variables during the forecast period, though in the last four quarters (1969III–1970III) the magnitude of this underestimation becomes very small for Y_1 and Y_4. The turning points are more often missed by the autoregressive model in the fluctuating series than in our model. On the other hand, the autoregressive model performs as well in predicting both the level and the turning points of series for capital stock and nonproduction workers.

Equivalently, the reduced form may be expressed in terms of the lag operator L, with $LY_{it} = Y_{it-1}$, and so on. In these terms (4.2) is

$$[I - (I - \beta)L]Y_t = \beta A_1 q_t;$$

$$Y_t = [I - (I - \beta)L]^{-1}\beta A_1 q_t. \qquad (4.4)$$

Thus (4.3) and (4.4) are different ways of expressing the same thing. It was demonstrated in Chapter 2 that the matrix $[I-(I - \beta)L]^{-1}$ is a rational function of L, i.e., a ratio of two polynomials in L. The determinant is a sixth-order polynomial in L (fifth order if the restrictions hold), and each cofactor is a fifth-order polynomial. Hence, each term in the inverse is a ratio of a fifth- to sixth-order polynomial. This implies what might be termed "semireduced form" expressions, in which each equation in (4.2) can be written in terms of a *finite* number of lagged values of q and Y's *own* lagged values. Indeed, it is this particular form that is most often utilized in the literature.

At the risk of repetition, an illustration might be in order. Consider a 2×2 case, in which there are only two variables in Y. Then

$$\beta = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix}.$$

If q has three components, then A_1 is 2×3. Ignoring restrictions, we have

$$[I - (I - \beta)L]^{-1} = \frac{1}{\Theta(L)} \begin{bmatrix} 1 - (1 - \beta_{22})L & \beta_{21}L \\ \beta_{12}L & 1 - (I - \beta_{11})L \end{bmatrix}$$

$$= \frac{1}{\Theta(L)} \{\Theta_{ij}(L)\},$$

with $\Theta(L) = 1-(2 - \beta_{11} - \beta_{22})L+(1 - \beta_{11})(1 - \beta_{22})-\beta_{12}\beta_{21}L^2$. Hence, carrying through the multiplication it is seen that

$$Y_{it} = \frac{\Theta_i(L)}{\Theta(L)} q_t, \qquad (4.5)$$

where $\Theta_i(L)$ is a row vector of polynomial functions of L related to the $\Theta_{ij}(L)$ terms above (along with the appropriate elements of β and A), and all variables are measured in logarithms, as usual. Multiplying through by $\Theta(L)$ and operating with L, it is seen that $\ln Y_{it}$ can be expressed as a linear function of the natural logarithms of q_t, q_{t-1}, Y_{it-1}, and Y_{it-2}. Therefore, the ratio of $\Theta_i(L)/\Theta(L)$ is the *generating function* of the lag distribution (Griliches [1967]) of the variables in q and $\ln Y_{it}$. In the 6×6 case considered here, $\Theta_i(L)$ is a fifth-order polynomial, and $\Theta(L)$ is sixth order. Thus, expressions such as (4.5) are also equivalent to (4.2) and (4.4).

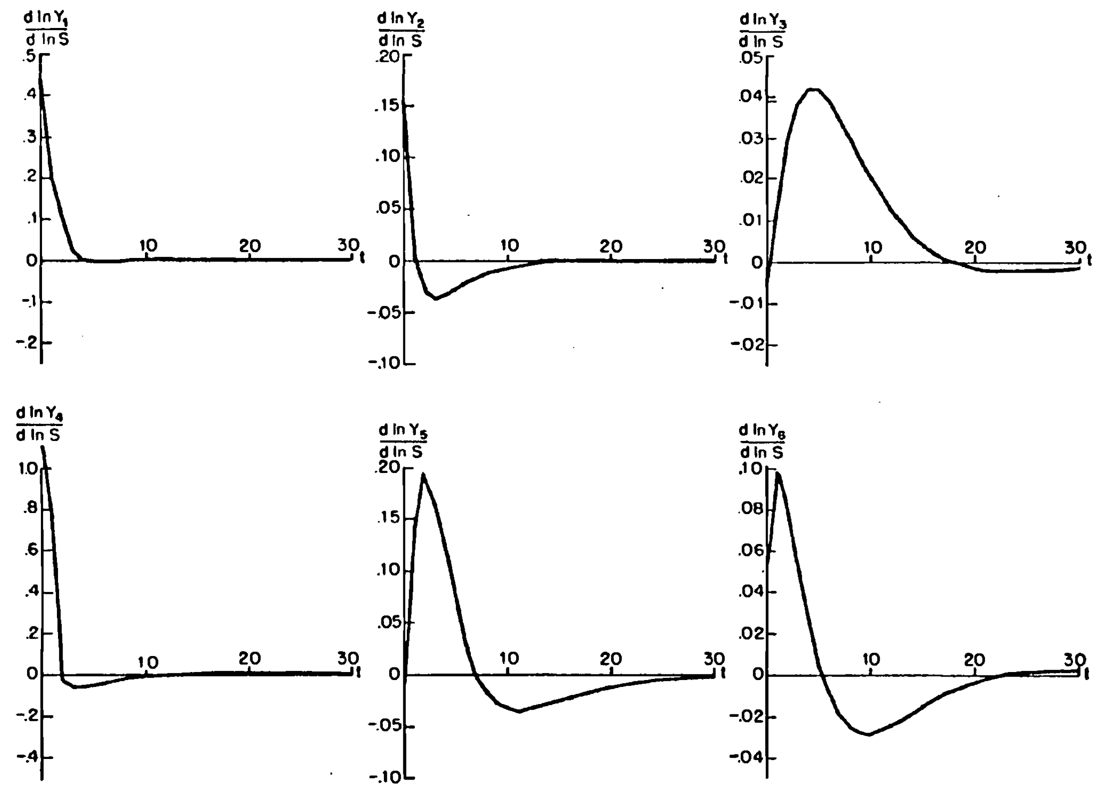
Define the matrix $B^k = (I - \beta)^k \beta A_1$, which is seen to enter the corresponding lagged value of q_{t-k} in (4.3). In our model there are six variables in Y_t. Hence β and $(I - \beta)$ are square matrices of order six. Moreover, there are three variables in q (ignoring constant terms); thus A has six rows and three columns. Hence B^k has dimension 6×3. Consider the sequence of coefficients in the ith row and jth column of B^k for $k = 1, 2, \ldots$. These coefficients are in fact the (nonnormalized) distributed lag responses of input Y_i to a unit impulse in q_{jt}. Since $(I - \beta)$ and βA_1 have been estimated and reported in Table 4.1, these responses can be calculated by picking off successive ijth elements in B^k. All variables (other than trend) are measured in logarithms; so the sums of these elements are the long-run elasticities of response of each input to the corresponding variable in q. Of course, there is no need to sum these infinite series to obtain long-run responses. Instead, set $Y_t = Y_{t-1} = \bar{Y}$, and let q_t be constant in (4.3). Then long-run responses are computed from $[I-(I - \hat{\beta})]^{-1}\hat{\beta}\hat{A}_1$, which of course yields an estimate of the matrix of "long-run" coefficients.

Figure 4.1 presents responses of distributed lag sales implicit in the estimates of Table 4.1, model (4.1). They have been computed in the manner described above. Similarly, distributed lag patterns can be obtained for relative price responses. Since impact responses of relative prices are so small and unreliable, we do not present them here.

In panel (a) of Figure 4.1, the distributed lag response of production worker employment is related to a unit impulse in sales. Most of the response of employment occurs in the first five or six quarters, and there is no evidence that production worker employment overshoots its ultimate value. The transient sales response of hours per man to sales is shown in panel (b), and that for general utilization in panel (d). These results imply that hours and general utilization are truly variable factors of production and tend to absorb shocks in the face of slowly adjusting stocks. *Both Y_2 and Y_4 overshoot their ultimate long-run values after very large initial responses in the first two quarters so that production and sales are maintained*

FIGURE 4.1

IMPLIED DISTRIBUTED LAG RESPONSES TO A UNIT SALES IMPULSE, BASED ON MODEL (4.1)



SOURCE: Table 4.1.

as other inputs are slowly adjusted. They then slowly converge to their long-run equilibrium values as other inputs steadily increase in response to the stimulus. Panel (c) contains the distributed lag response of capital stock to sales; the curve shows a characteristic bell-shaped pattern found in many other studies. It is noteworthy that these results are based entirely on a first-order *system* and that no second-order lag terms in capital stock have been used. Evidently, the mean lag is long for capital, suggesting that capital stock is the most "fixed" of all inputs.

The distributed lag response of total inventory to sales is shown in panel (f). In interpreting this result, notice that our measure of inventory includes *both* raw materials and goods in process as well as stocks of finished goods. One might anticipate that the latter component would tend to fall after an initial sales impulse and that this would show up in the lag patterns that display some initial negative values. However, the goods-in-process component would not be expected to behave in this manner but, so long as some part of the shock was anticipated, quite the reverse. The net effect of both types of change is shown in the diagram. It is quite clear that the net effect during the first ten to twelve periods after the shock is positive, and so increased holdings of goods in process and raw materials dominate the observations during that period. After about twelve periods, the distributed lag response turns negative, that is, inventories overshoot their ultimate long-run equilibrium value. By that time most of the production labor adjustments have been made, and inventories are run down. Evidently, production is speeded up after the initial shock, and goods in process and raw materials increase sufficiently to meet sales out of finished inventories long after most of the other inputs have been adjusted, and while capital stock is still being built up.

A pattern similar to that of inventories characterizes responses of nonproduction labor. The patterns of initial positive response are like those of production workers, though somewhat delayed and more dispersed over time. This suggests that nonproduction labor is subject to greater adjustment costs than production labor, as might be expected a priori. The pattern in Figure 4.1 indicates some overshooting of the long-run values for nonproduction workers, in distinction to the result for production labor. Perhaps nonproduction workers are more necessary, in the earlier periods, to supervise the great changes in utilization rates that occur at such times. After these changes have damped out, nonproduction labor is laid off. Again, such an explanation is consistent with the notion that labor stocks and utilization as well as capital utilization and inventories bear the brunt of short-run adjustments, while capital stock slowly adjusts to its ultimate long-run value.

ii. Stability

The extended discussion in Chapter 2 showed that the dynamic stability of the difference equation system (4.1) depends on the magnitude of the characteristic roots of $(I - \beta)$. The distributed lag patterns in Figure 4.1 all show that the system is in fact stable. Otherwise, the lag responses would not converge to zero. Another way of examining this behavior is to compute the characteristic roots themselves. For the estimates of Table 4.1 these are 0.9752, 0.8231, 0.8231, 0.5567, 0.5567, 0.0132. The largest root does not exceed unity in absolute value, indicating stability. However, sampling distributions of these statistics are not available, precluding a precise test of stability. The two repeating roots have complex parts that are very small, implying very small oscillations that have no perceptible effect on distributed lag patterns, as is also apparent from Figure 4.1. The convergence properties of the system depend essentially on the largest root, for it dominates response patterns as t grows larger; after the shock, the smaller roots converge at a much faster pace. The largest root is about 0.975 in absolute value. Hence, though the system does indeed converge, its rate of convergence to steady state values is *very slow*. The diagrams clearly reveal this sluggishness to be due to the unusually long lags in capital stock responses.

It is very important to note that the smallest root is very small by comparison with the others. In fact, it is only about 0.013, whereas the next largest is forty times as great, 0.56. As was argued in Chapter 3, production function restrictions depend on the singularity of the adjustment matrix $(I - \beta)$. The magnitude of the smallest root does suggest the near singularity of β, even though no a-priori restrictions were imposed on the estimates. This fact and the overshooting of the distributed lag response for utilization rates are strong verifications of the model specification.

E. LONG-RUN RESPONSE

As noted above, long-run response elasticities are computed from $[I-(I - \beta)]^{-1}\beta A_1 = \hat{A}_1$, since regression estimates in the tables above convey the necessary information about $(I - \beta)$ and βA_1. Computations based on the estimates of Table 4.1 are shown in Table 4.4. These figures have been calculated by using double-precision computer algorithms and should be as accurate as the regression coefficients on which they are based.

Later on, we present evidence of high sensitivity of estimated long-run elasticities to changes in specification. For now, note the following:

i. There is little evidence of significant long-run price or substitution effects in this data. The relative price elasticity for production workers is positive but rather small in absolute value. The relative price elasticity for capital stock is 0.045 and positive as expected. These results are consistent with those of many independent investigations, where estimated time-series elasticities of substitution are found to be very small (Nerlove [1967b]). Relative price elasticities of hours per man are also small, but positive, contrary to hypothesis. The relative price elasticity of nonproduction workers is negative, perhaps because our procedure, in which wage rates and hours per man of nonproduction workers are derived from the rates and hours of production workers, is untenable. The relative price elasticity of inventories ($\ln Y_5$) is positive, suggesting that inventories are substitutes for capital in the "sales production function." The relative price elasticity of the general utilization variable is strongly

TABLE 4.4

LONG-RUN ELASTICITIES FOR TOTAL MANUFACTURING

(all variables except trend are measured in natural logarithms)

	Dependent Variables					
Independent Variables	Prod. Emp. (Y_1)	Hours (Y_2)	Capital (Y_3)	Util. (Y_4)	Inven. (Y_5)	Nonprod. Emp. (Y_6)
Sales (S)	.7301	−.1302	.2933	1.200	.1774	.1595
Relative prices (w/c)	.1067	.1005	.0451	−0.5463	.1634	−.1393
Trend (T)	.0010	.0064	.0051	−0.0366	.0175	.0028

NOTE: Computational formula: $[I - (I - \beta)]^{-1}\beta A$. Each entry gives estimated long-run response elasticity of each input (columns) to a 1 per cent change in each exogenous variable (rows).

SOURCE: Table 4.1.

negative. However, caution is in order in interpreting this result because of the problems, mentioned earlier, of measuring the utilization rate.

Recall that these results are based on very small estimated coefficients on ln (w/c) in the structural equations. Since ln (w/c) has a substantial trend, the impact effects of the structure may only capture transitory price responses and, consequently, our "long-run" estimates of Table 4.3 do not necessarily reflect full responses of permanent changes in relative prices. Also, note again that price variables used in the estimates are far from ideal due to data limitations. Consequently, the estimates may be biased.

ii. There is some evidence of increasing returns to production worker employment, since the sales elasticity is about 0.73, suggesting returns to scale of about 1.3. Note, however, that the sales production function is overidentified, since the restrictions do not hold exactly, and that sales, rather than output, are used. Hence, lack of identification precludes any strong statement about returns to scale. No evidence of long-run scale effects on hours per man is present, consistent with our a-priori hypothesis. This implies that variations in hours per man are almost completely short run and serve a buffer function of insulating changes in demand from changes in input stocks. However, there is some evidence of significant long-run scale effects on general utilization. There is evidence of strong increasing returns to scale for capital stock, inventories, and nonproduction workers, their output elasticities falling far short of unity. Nonuniformity of these effects is not consistent with the type of multiplicative production function postulated in Chapter 2, but might be expected to be the case on the basis of more general considerations. Again, however, the production function is overidentified.

iii. The trend coefficients for production worker employment and general utilization are consistently negative, while the remaining coefficients are positive. Though these results are not consistent with Cobb-Douglas assumptions because embodied and disembodied technical change cannot be identified in that case, they might be consistent with a more general production function.