

This PDF is a selection from a published volume  
from the National Bureau of Economic Research

Volume Title: NBER International Seminar on Macroeconomics  
2005

Volume Author/Editor: Jeffrey Frankel and Christopher  
Pissarides, editors

Volume Publisher: MIT Press

Volume ISBN: 0-262-06265-8, 978-0-262-06265-7

Volume URL: <http://www.nber.org/books/fran07-1>

Conference Date: June 17-18, 2005

Publication Date: May 2007

Title: Dual Inflation and the Real Exchange Rate  
in New Open Economy Macroeconomics

Author: Balázs Világi

URL: <http://www.nber.org/chapters/c0349>

---

# Dual Inflation and the Real Exchange Rate in New Open Economy Macroeconomics

Balázs Világi, *Magyar Nemzeti Bank (Central Bank of Hungary)*

## 1. Introduction

The traditional approach in international macroeconomics has attempted to explain real exchange rate behavior by the movements of domestic relative prices, that is, by the *internal real exchange rate*. This was a consequence of the assumptions it employed: strong homogeneity in international goods markets, where *purchasing power parity* (PPP) is dominant and the only source of heterogeneity is the distinction between *tradables* and *non-tradables*. In recent years, however, the literature has switched sides. According to the recent approach consumer markets are segmented, PPP has little explanatory power, and the main determinant of real exchange rate movements is the *external real exchange rate*, which is the relative price of domestic and foreign tradables. This new focus of research was initiated on the basis of empirical findings, see, e.g., the papers of Engel (1999) and Rogoff (1996). It appeared that, as Obstfeld (2001) put it “apparently, consumer markets for tradables are just about as segmented internationally as consumer markets for non-tradables.”

After the collapse of the Bretton Woods system, floating exchange rate regimes became widespread. This enabled scrutiny of the relationship between nominal and real exchange rate behavior: It turned out, as was first forcefully documented by Mussa (1986), that nominal and real exchange rates were strongly correlated, and moving from fixed to floating exchange rate regimes resulted in a dramatic rise in the variability of the real exchange rate. The need for a comprehensive explanation for the aforementioned empirical findings stimulated the birth of *new open economy macroeconomics* (NOEM), initiated by the seminal paper of Obstfeld and Rogoff (1995), which combines the heterogeneity of goods with *nominal rigidities* in models with micro-foundations.

Although the empirical literature related to NOEM revealed the importance of the external real exchange rate, in fast-growing and emerging market countries there are considerable movements of the internal real exchange rate. Permanent *dual inflation*, namely a significant divergence of inflation rates for tradable and non-tradable goods, is a frequent phenomenon of such markets: the inflation rate of non-tradables is permanently higher than that of tradables, which results in long-run real appreciation of the CPI-based real exchange rate. This phenomenon was documented by Ito, Isard, and Symansky (1997) for the case of Japan and some Southeast Asian countries, as well as by Coricelli and Jazbec (2001), Halpern and Wyplosz (2001), Égert (2002), Égert, Drine, Lommatzsch, and Rault (2002) and Kovács (2002) for European post-communist countries. Of course, this does not mean that in these countries the empirical phenomena emphasized in the NOEM literature are not present. For example, the required disinflation efforts, related to future EMU accession, have revealed that the connection between the consumer price index and the nominal exchange rate is weak, which, of course, violates the PPP and implies a strong comovement of nominal and real exchange rates.

The objective of this paper is to build a NOEM model which is able to replicate both sets of empirical facts observable in emerging markets: the strong correlation of the nominal and real exchange rate, and dual inflation accompanied by appreciation of the CPI-based real exchange rate.

The problem is the following. The majority of empirical studies explain the coexistence of dual inflation and the appreciation of the CPI-based real exchange rate in emerging markets by the Balassa-Samuelson (BS) effect, i.e., the relatively rapid productivity growth in the tradable sector. However, dual inflation accompanies appreciation of the CPI-based real exchange rate only if growth in tradable productivity does not result in a significant depreciation of the external real exchange rate. But the external real exchange rate does not depreciate considerably if the common currency prices of domestically produced and foreign tradables cannot deviate strongly from each other, i.e., if domestically produced and foreign tradables are *close* substitutes. On the other hand, the strong comovement of the nominal and real exchange rates stressed by the NOEM literature requires considerable deviations in the short run between domestic and foreign tradable prices (denominated in the same currency). Yet this requirement can be fulfilled only if the prod-

ucts of the aforementioned sectors are *distant* substitutes and/or *pricing to market* (PTM) is possible.

The paper demonstrates that no intermediate degree of international substitution exists that simultaneously guarantees the operation of the BS effect and strong comovement of the nominal and real exchange rate. One possible remedy is an assumption of PTM. In this case, it is possible that domestically produced export goods are close substitutes of foreign tradables, which ensures the existence of the BS effect. On the other hand, with PTM the common currency price of the exported and locally sold domestically produced goods can be substantially different over the short-run. Hence, nominal-exchange-rate movements can influence the behavior of the real exchange rate.

The paper also shows that the presence of decreasing returns to scale, which can be rationalized by a certain combination of real and nominal rigidities, has significant impact on the magnitude of the difference between sectoral inflation rates. As a consequence, the size of the effect of asymmetric sectoral productivity growth, in line with empirical observations, becomes smaller than predicted by the models of the traditional approach.

The paper is structured as follows. Section 2 surveys the empirical literature which initiated the research of this study. Section 3 presents the model and the solution technique employed. In section 4 the Balassa-Samuelson hypothesis is examined; under study is how the model can reproduce the co-existence of dual inflation and appreciation of the CPI-based real exchange rate, and the relationship between asymmetric productivity growth and the magnitude of sectoral inflation differentials is examined. Section 5 presents the conclusions.

## 2. Previous Empirical Results

This section briefly reviews the empirical literature which initiated the research of this paper. First, findings related to the internal real exchange rate are surveyed. On this issue the evidence is ambiguous. In developed economies, internal-real-exchange-rate movements are negligible, while in several emerging economies dual inflation is an important phenomenon. Second, findings on the strong relationship between the nominal and real exchange rates are considered, which are relevant in both developed and emerging economies.

## 2.1 *Dual Inflation and Real Appreciation*

As mentioned in the introduction, NOEM literature focuses on the behavior of the external real exchange rate, instead of the internal one, which was mainly studied by the previous traditional literature. This switch of interest was partly initiated by the findings of Engel (1999), who, using U.S. data, showed that the volatility of the real exchange rate can be explained nearly perfectly by the movements of the external real exchange rate.

However, the validity of this finding is not general. Even in developed countries one can observe significant movements of the internal real exchange rate, as De Gregorio and Wolf (1994), or more recently, López-Salido, Restoy, and Vallés (2005) have documented, but the real importance of this phenomenon is manifested in high growth and emerging market countries. Several empirical studies demonstrate that the BS effect plays a significant role in these countries.

Balassa (1964) and Samuelson (1964) formulated the hypothesis that the difference in productivity growth rates in tradable and non-tradable sectors results in dual inflation, and, as a consequence, appreciation of the CPI-based real exchange rate.<sup>1</sup> Ito, Isard, and Symansky (1997) showed that mainly in Japan, Korea, and Taiwan, but to some extent in other Southeast Asian countries as well, the BS effect was determinant at particular stages of their development process. It also plays an important role in the transition of European post-communist countries, as the empirical studies of Coricelli and Jazbec (2001), Halpern and Wyplosz (2001), Égert (2002), Égert, Drine, Lommatzsch, and Rault (2002), and Kovács (2002) have documented.

Coricelli and Jazbec (2001) examined the determinants of the real exchange rate in 19 transition economies between 1991 and 1998.<sup>2</sup> Halpern and Wyplosz (2001) studied the relevance of the BS effect in nine European post-communist countries by estimating a panel regression for the period 1991–1998.<sup>3</sup> Égert (2002) used time series and panel cointegration techniques to study the BS effect in five east European accession countries between 1991 and 2001.<sup>4</sup> Égert, Drine, Lommatzsch, and Rault (2002) examined the BS effect in nine European accession countries by panel cointegration techniques on a data set covering the period from 1995 to 2000.<sup>5</sup> The paper edited by Kovács (2002) summarizes the results of research on the BS effect conducted by the central banks of central European accession countries.<sup>6</sup>

The above studies demonstrate that in most European post-communist countries the coexistence of dual inflation and appreciation of the CPI-based real exchange rate can be observed in their transition period. In addition, dual inflation is related to sectoral productivity growth differentials, and appreciation of the CPI-based real exchange rate is due to the appreciation of both the external and internal real exchange rates.

Coricelli and Jazbec (2001), Halpern and Wyplosz (2001), Égert (2002), and Égert, Drine, Lommatzsch, and Rault (2002) estimated the relationship between the relative price of non-traded to traded goods and the sectoral productivity differential.<sup>7</sup> Their findings are summarized in Table 1.

According to Coricelli and Jazbec (2001, equation 19), if the productivity differential rises by 1 percent, the relative price rises by 0.87 percent. Égert (2002, Table 1-7) found a significant cointegration relationship between the relative price and productivity differential. The cointegration coefficient measuring the long-run relationship between the relative prices and productivity factors varies from 0.49 to 0.95 in individual country estimates, and 0.72 is the common estimate for the coefficient provided by the panel cointegration analysis. In Égert, Drine, Lommatzsch, and Rault (2002, Table 5) the same cointegration coefficient ranges from 0.73 to 1, depending on the applied definition of tradable and non-tradable sectors. Unlike the previous studies, Halpern and Wyplosz (2001, Table 7) estimated the effects of tradable and non-tradable productivity developments separately. They found significant coefficients with correct signs, although the estimated coefficients

**Table 1**  
Empirical long-run relationship between sectoral prices and productivity measures

	Type of regression	Estimated coefficient
Coricelli-Jazbec (2001)	price differential on productivity differential	0.87
Égert (2002)	panel, price differential on productivity differential	0.72
Égert (2002)	individual, price differential on productivity differential	0.49–0.95
Égert et al. (2002)	price differential on productivity differential	0.73–1
Halpern-Wyplosz (2001)	tradable price on tradable productivity	0.43
Halpern-Wyplosz (2001)	non-tradable price on non-tradable productivity	0.32

are quite small. If tradable productivity rises by 1 percent, the sectoral relative price rises by 0.24 percent in the short-run and by 0.43 percent in the long-run. A 1 percent rise of non-tradable productivity results in a 0.18 percent decrease of the relative price in the short-run and a 0.32 percent decrease in the long-run.

In summary, all papers found a significant relationship between sectoral prices and productivity measures. Magnitudes of estimated coefficients locate in quite a wide range. However, according to all but one estimate, productivity differentials are greater than the accompanying price differentials.

According to the original BS hypothesis, productivity induced real appreciation of the internal real exchange rate results in CPI-based real appreciation, since the external real exchange rate is fixed due to the assumed validity of PPP.

Kovács (2002, Table 1-1) documented that between 1993 and 2002 the annual average CPI-based real appreciation of the examined countries varied from 2.2 to 5.8 per cent. However, the BS effect does not fully explain the observed CPI-based real appreciations. Only 33–72 percent of it can be attributed to productivity growth induced internal real exchange rate movements; the rest can be assigned to the external real exchange rate. Égert (2002, Table 9) also reveals that productivity induced appreciation of the internal real exchange rate cannot completely explain CPI-based real appreciation. According to his panel analysis, it is responsible for 38–60 percent of CPI-based appreciation. He also stresses the importance of a trend appreciation of the external real exchange rate to explain the observed phenomena. Égert et al (2002) presented similar findings and reinforced the conclusions of the above papers.

Although in this paper I study only productivity induced dual inflation, I should mention that studies analyzing the BS effect have often detected other non-productivity factors in the determination of the sectoral relative price. Moreover, Arratibel, Rodríguez-Palenzuela, and Thiman (2002) do not simply provide alternative explanations for dual inflation, they deny the role of productivity factors in the determination of the examined countries. However, the authors admit that one should interpret this result with caution because of the poor quality of productivity data.<sup>8</sup>

## 2.2 *The Comovement of the Nominal and Real Exchange Rates*

As mentioned in the introduction, the NOEM literature was partly initiated by the empirical findings of Mussa (1986), who first documented

the strong connection between the nominal and real exchange rates. Using Monacelli (2004), I summarize some important findings. The post-1971 data from 12 developed countries reveal that the unconditional correlation of real and nominal depreciation rates is 0.98. In flexible exchange rate regimes the unconditional variance of the real depreciation rate is nearly equal to the unconditional variance of the nominal depreciation rate.

Violation of PPP is a necessary condition for the above findings. Moreover, the violation of PPP is not a transitory phenomenon, as several empirical studies have shown. Chari, Kehoe, and McGrattan (2002) studied the persistence of the real-exchange-rate shocks using HP-filtered quarterly data for the USA and 11 developed European countries for the period 1973:1–2000:1. Their estimated quarterly autocorrelation is 0.84.<sup>9</sup> Though the above empirical results are all related to developed countries, the violation of PPP can also be detected in European post-communist countries, which are the primary focus of this study,<sup>10</sup> although the supporting evidence is mainly only stylized facts.

### 3. The Model

This paper studies how to construct a model which can simultaneously guarantee the empirical regularities characterized in section 2, i.e., the comovement of the nominal and real exchange rates and generate the BS effect, i.e., the coexistence of productivity based dual inflation and appreciation of the CPI-based real exchange rate.

To guarantee the empirically observable correlation between the nominal and real exchange rates the model needs sticky prices and heterogeneous international tradable markets. Obviously, to consider the BS effect it is necessary to have two sectors with different total factor productivities (TFP).

International market heterogeneity can be captured in different ways. I therefore examine whether model versions with different descriptions of market heterogeneity can generate the BS effect. I consider a version (version *A*) without pricing to market (PTM) and with the assumption that domestic and foreign tradables are imperfect substitutes. In version *B* PTM combined with local currency pricing (LCP) is added to the model.<sup>11</sup>

The paper also considers the relationship between the magnitude of sectoral relative price and productivity differential. In frictionless,



sectorally symmetric models the two quantities are equal. Yet this is not in line with empirical results, which reveal that the relative price of non-tradables to tradables is smaller than the sectoral productivity differential. Nominal rigidities help to explain this phenomenon: if prices are sticky the adjustment of the sectoral relative price is not immediate. In addition, *decreasing returns* amplify the impact of sticky prices, making the adjustment process even slower and incomplete, which provides a better fit in terms of empirical results.

Decreasing returns are guaranteed in the model by the assumption of *fixed capital stock*. This approach makes the model simple and tractable. Besides, it can be considered as the limiting case of the *firm-specific-investments* model of Altig, Christiano, Eichenbaum, and Linde (2005) and Woodford (2005). As they show, even if technology exhibits constant returns to scale, the lack of an economy-wide rental market for physical capital and frictions in investments formation combined with sticky asynchronous price setting results in suboptimal input allocation, and decreasing returns to scale.

### 3.1 Households

The domestic economy is populated by a continuum of infinitely-lived identical households. To simplify the notation, household indices are dropped, since this does not cause confusion. The utility accrued to a given household at date  $t$  is

$$U(c_t, l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{l_t^{1+\varphi}}{1+\varphi},$$

where  $c_t$  is the consumption,  $l_t$  is the labor supply of the representative household at date  $t$ , and  $\sigma, \varphi > 0$ .

The consumption good  $c_t$  is composed of *tradable* and *non-tradable* consumption goods:

$$c_t = \left[ a_T^\eta (c_t^T)^\frac{\eta-1}{\eta} + a_N^\eta (c_t^N)^\frac{\eta-1}{\eta} \right]^\frac{\eta}{\eta-1}, \quad (1)$$

where  $c_t^T$  is the tradable,  $c_t^N$  is the non-tradable consumption good,  $\eta$  and  $a_T = 1 - a_N$  are non-negative parameters.

The intertemporal budget constraint of a given household is the following:

$$P_t^T c_t^T + P_t^N c_t^N + P_t^B B_t = \zeta_t B_{t-1} + W_t l_t + T_t,$$

where  $P_t^T$  and  $P_t^N$  are the price indices of tradables and non-tradables,  $B_t$  is the household's nominal portfolio at the beginning of date  $t$ ,  $P_t^B$  is its price, and  $\zeta_t$  is its stochastic payoff.  $W_t$  is the nominal wage, while  $T_t$  is a lump-sum tax/transfer variable.

It is well known that the linear homogeneity of function (1) implies that the households' problem can be solved in two steps. First, they maximize the intertemporal objective function

$$\sum_{t=0}^{\infty} \beta^t E_0 [U(c_t, l_t)],$$

with respect to  $c_t$  and  $l_t$  subject to the following modified budget constraint:

$$P_t c_t + P_t^B B_t = \zeta_t B_{t-1} + W_t l_t + T_t, \quad (2)$$

non-negativity constraints, and no-Ponzi schemes, where  $0 < \beta < 1$  is the discount factor of households. In the budget constraint (2) the consumer price index  $P_t$  is defined by the following expression:

$$P_t = [a_T (P_t^T)^{1-\eta} + a_N (P_t^N)^{1-\eta}]^{\frac{1}{1-\eta}}. \quad (3)$$

Second, knowing  $c_t$  it is possible to determine  $c_t^T$  and  $c_t^N$  by the demand functions

$$c_t^T = a_T \left( \frac{P_t}{P_t^T} \right)^{\eta} c_t, \quad c_t^N = a_N \left( \frac{P_t}{P_t^N} \right)^{\eta} c_t. \quad (4)$$

The assumption of complete asset markets implies that the optimal intertemporal allocation of consumption is determined by the following condition in all states of the world:

$$\beta \frac{\Lambda_{t+1} P_t}{\Lambda_t P_{t+1}} = D_{t,t+1}, \quad (5)$$

where  $\Lambda_t$  is the marginal utility of consumption,

$$\Lambda_t = c_t^{-\sigma},$$

and  $D_{t,t+1}$  is the stochastic discount factor, which satisfies the condition

$$P_t^B = E_t [D_{t,t+1} \zeta_{t+1}].$$

Since in this economy the asset markets are also complete internationally, the foreign equivalent of equation (5) is also held:

$$\beta \frac{\Lambda_{t+1}^* e_t P_t^{F*}}{\Lambda_t^* e_{t+1} P_{t+1}^{F*}} = D_{t,t+1}, \quad (6)$$

where  $\Lambda_i^*$  is the marginal utility of foreign households,  $P_i^{F^*}$  is the foreign consumer price index in foreign currency terms, and  $e_i$  is the nominal exchange rate. For simplicity  $P_i^{F^*}$  is assumed to be constant. Combining equations (5) and (6) and applying recursive substitutions yields formula

$$\frac{\Lambda_i e_i P_i^{F^*}}{\Lambda_i^* P_i} = \iota, \quad (7)$$

where  $\iota$  is a constant, which depends on initial conditions.

The solution of the households' problem implies that the real wage  $w_i$  is equal to the marginal rate of substitution between consumption and labor, i.e.,

$$w_i = c_i^\sigma l_i^\sigma,$$

which determines the labor supply decision.

### 3.2 Production

There are two stages of production in the model: in the first step import goods and labor are transformed into differentiated intermediate goods in both the tradable and non-tradable sectors,<sup>12</sup> while in the second step homogenous final goods are produced and distributed using intermediate products.

Final goods are produced in competitive markets by constant-returns-to-scale technologies from a continuum of differentiated inputs,  $y_i^s(i)$ ,  $i \in [0, 1]$ , where  $s = T, N$ , with  $T$  referring to tradable sector and  $N$  to non-tradable.

In version *A* there are two types of final goods, a tradable one, used for domestic consumption and exports, and a non-tradable one, used only for domestic consumption. The technology of final goods production is represented by the following CES production function:

$$y_i^s = \left( \int_0^1 y_i^s(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}},$$

where  $\theta > 1$ . As a consequence, the output price  $P_i^s$  is given by

$$P_i^s = \left( \int_0^1 P_i^s(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}},$$

where  $P_i^s(i)$  denotes the price of differentiated good  $i$  in sector  $s$ . The demand for differentiated goods is determined by

$$y_i^T(i) = \left( \frac{P_i^T}{P_i^T(i)} \right)^\theta (c_i^T + x_i), \quad y_i^N(i) = \left( \frac{P_i^N}{P_i^N(i)} \right)^\theta c_i^N, \tag{8}$$

where  $x_i$  denotes exports.

In version *B* intermediate goods are also manufactured in sector *T* and *N*, and the non-tradable final good is produced and distributed in the same way as in version *A*. However, the market for tradable final goods is segmented, domestic consumption and export goods are sold in different markets. This market structure is represented by the assumption that two diverse final good producers/distributors operate in these markets. As a consequence, domestic and foreign prices of tradables denominated in the same currency can diverge. Final goods distributors apply the previously described CES technologies, and export prices are set in local currency. Hence, the demand for tradable intermediate goods are given by the following functions:

$$c_i^T(i) = \left( \frac{P_i^T}{P_i^T(i)} \right)^\theta c_i^T, \quad x_i(i) = \left( \frac{P_i^{x^*}}{P_i^{x^*}(i)} \right)^\theta x_i, \tag{9}$$

where  $P_i^{x^*}(i)$  is the foreign currency price of exported tradables,  $y_i^T(i) = c_i^T(i) + x_i^T(i)$ , and

$$P_i^{x^*} = \left( \int_0^1 P_i^{x^*}(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}.$$

The continuum of goods  $y_i^s(i)$  are produced in a monopolistically competitive market in each sector ( $s = T, N$ ). Each  $y_i^s(i)$  is made by an individual firm using the following uniform technology:

$$y_i^s(i) = A_i^s (\bar{k}^s)^\alpha z_i^s(i)^{1-\alpha}, \tag{10}$$

where  $0 < \alpha < 1$ ,  $A_i^s$  is total factor productivity of sector *s*,  $\bar{k}^s$  is the stock of fixed physical capital in sector *s*, and  $z_i^s(i)$  denotes an individual firm's utilization of the composite input  $z_i^s$  defined in the following way:

$$z_i^s(i) = N_s l_i^s(i)^{n_s} m_i^s(i)^{1-n_s}, \tag{11}$$

where  $l_i^s(i)$  is an individual firms' utilization of labor  $l_\nu$  and  $m_i^s(i)$  is the utilization of imported good  $m_\nu$ ,  $n_s$  is a given non-negative parameter, and  $N_s = n_s^{-n_s} (1 - n_s)^{n_s-1}$ . The price of  $z_i^s$  is given by

$$W_i^{z,s} = W_i^{n_s} (e_i P_i^{m^*})^{1-n_s}, \tag{12}$$

where  $P_i^{m^*}$  is the foreign currency price of the imported good.

Intermediate goods producers solve the standard static cost minimization problem. The solution of the cost minimization problem determines the labor and import demand of a particular firm by

$$l_t^s(i) = n_s \frac{W_t^{z,s}}{W_t} z_t^s(i), \quad m_t^s(i) = (1 - n_s) \frac{W_t^{z,s}}{e_t P_t^{m,s}} z_t^s(i). \quad (13)$$

Intermediate goods producers follow a sticky price setting practice. As in the model of Calvo (1983), each individual firm in a given time period changes its price in a rational, optimizing, forward looking manner with probability  $1 - \gamma_s$ . Those firms which do not optimize at a given date follow a rule of thumb, as in Christiano, Eichenbaum, and Evans (2001) and Smets and Wouters (2003), and update their prices according to the past sectoral inflation rate.

In version *A* all firms in sector  $s = T, N$  which follow the simple indexation rule at date  $t$  update their prices according to formula

$$P_t^s(i) = P_{t'}^s(i) \left( \frac{P_{t-1}^s}{P_{t-1}^s} \right)^{\theta_s}. \quad (14)$$

Those which set their prices rationally at date  $t'$  take into account that  $P_t^s(i)$  will exist with probability  $\gamma_s^{t-t'}$  at date  $t$ . Thus, they maximize the expected profit function

$$\sum_{t=t'}^{\infty} E_{t'} \left[ \gamma_s^{t-t'} D_{t,t} \left\{ (1 - \tau) P_{t'}^s(i) y_t^s(i) - W_t^{z,s} \left( \frac{y_t^s(i)}{A_t^s} \right)^{\frac{1}{1-\alpha}} (\bar{k}^s)^{\frac{\alpha}{1-\alpha}} \right\} \right], \quad (15)$$

with respect to  $P_t^s(i)$  and  $y_t^s(i)$  subject to the constraints (8) and (14), where  $\tau$  is a tax/transfer variable which modifies firms' markup.<sup>13</sup> I used equation (10) to derive the marginal-cost term in the above formula.

In version *B* export prices of non-optimizing firms are given by

$$P_t^{x^*}(i) = P_{t'}^{x^*}(i) \left( \frac{P_{t-1}^{x^*}}{P_{t-1}^{x^*}} \right)^{\theta_x}. \quad (16)$$

In sector *N* optimizing firms set their prices the same way as in version *A*. In sector *T* instead of equation (15), they maximize the expected profit function

$$\sum_{t=t'}^{\infty} E_{t'} [ \gamma_s^{t-t'} D_{t,t} (1 - \tau) \{ P_{t'}^T c_t^T(i) + e_t P_{t'}^{x^*}(i) x_t(i) \} ] \\ - \sum_{t=t'}^{\infty} E_{t'} \left[ \gamma_s^{t-t'} D_{t,t} W_t^{z,T} \left( \frac{c_t^T(i) + x_t(i)}{A_t^T} \right)^{\frac{1}{1-\alpha}} (\bar{k}^T)^{\frac{\alpha}{1-\alpha}} \right], \quad (17)$$

with respect to  $P_i^T(i)$ ,  $P_i^{x^*}(i)$ ,  $c_i^T(i)$  and  $x_i(i)$ , subject to the constraints (9), (14), and (16).

### 3.3 Exports Demand

Foreign behavior is not modeled explicitly. It is assumed that the following *ad hoc* equation determines demand for exports:

$$x_i = \left( \frac{P_i^{FT^*}}{P_i^{x^*}} \right)^{\eta^*} x^*, \quad (18)$$

where  $P_i^{x^*}$  is the foreign currency price of the export goods,  $P_i^{FT^*}$  is the foreign currency price of the rival goods (which is constant by assumption),  $x^*$  is an exogenous parameter representing the volume of demand, and  $\eta^* > 0$  is an exogenous parameter.

In version *A* of the model, exported goods are produced by the tradable sector, and  $P_i^{x^*} = P^T/e_i$ . While in version *B* local tradables and export goods are different, hence their prices denominated in the same currency can be different, i.e., it is possible that  $P_i^{x^*} \neq P^T/e_i$ .

### 3.4 Real Exchange Rate Indices

In this study the following real exchange indices will be considered:

$$q_i = \frac{e_i P_i^{F^*}}{P_i}, \quad q_i^T = \frac{e_i P_i^{FT^*}}{P_i^T}, \quad P_i^R = \frac{P_i^N}{P_i^T}, \quad (19)$$

where  $q_i$  is the CPI-based real exchange rate and  $q_i^T$  is the external real exchange rate. The movements of  $P_i^R$ , the domestic relative price of non-tradables to tradables, unambiguously determine the fluctuation of the internal real exchange rate, since it is assumed that  $P^{FT^*}$  and  $P^{FN^*}$  are constant.

### 3.5 The Log-linearized Model

To solve the model its log-linear approximation around the steady state is taken. In this section, instead of the description of the complete log-linearized model, the most important equations of the system are reviewed. Variables without time indices refer to their steady-state values, and the tilde denotes the log-deviation of a variable from its steady-state value.

### 3.5.1 Domestic Price Setting

Following Woodford (2005, chapter 3) and using equations (12) and (19), one can show that the solution of the maximization of the expected profit functions (15) and (17) yields formula

$$\pi_t^s - \vartheta_s \pi_{t-1}^s = \beta E_t [\pi_{t+1}^s - \vartheta_s \pi_t^s] \quad (20)$$

$$+ \xi_s \left[ \frac{\alpha}{1-\alpha} \frac{c^s \tilde{c}_t^s + \bar{x}^s \tilde{x}_t}{c^s + \bar{x}^s} - \frac{1}{1-\alpha} \tilde{A}_t^s + n_s \tilde{w}_t + (1-n_s) \tilde{q}_t + \chi_s \tilde{P}_t^R \right]$$

for determining domestic prices, where  $s = T, N$ ,  $\pi_t^s = \bar{P}_t^s - \bar{P}_{t-1}^s$  is the sectoral inflation rate and

$$\xi_s = \frac{(1-\gamma_s)(1-\beta\gamma_s)}{\gamma_s \left( 1 + \theta \frac{\alpha}{1-\alpha} \right)}. \quad (21)$$

Furthermore,  $\bar{x}^T = x$ ,  $\bar{x}^N = 0$ ,  $\chi_T = a_N$  and  $\chi_N = -a_T$ .

### 3.5.2 Export Market

In version *A* of the model  $\tilde{q}_t^T = \bar{P}_t^{x^*}$ , hence the log-linearized version of the exports demand equation (18) becomes

$$\tilde{x}_t = \eta^* \tilde{q}_t^T. \quad (22)$$

In version *B* the log-linearized exports demand is

$$\tilde{x}_t = -\eta^* \tilde{P}_t^{x^*}. \quad (23)$$

Since in version *A* the law of one price is valid in tradable goods market, the foreign currency price of exported goods is determined by the nominal exchange rate and the domestic price of tradables. However, in version *B* the assumption of pricing to market implies that one needs an additional equation to determine export prices. The maximization of (17) yields the following log-linear formula for export prices:

$$\pi_t^{x^*} - \vartheta_T \pi_{t-1}^{x^*} = \beta E_t [\pi_{t+1}^{x^*} - \vartheta_T \pi_t^{x^*}] \quad (24)$$

$$+ \xi_T \left[ \frac{\alpha}{1-\alpha} \frac{c^T \tilde{c}_t^T + x x_t}{c + x} - \frac{1}{1-\alpha} \tilde{A}_t^T + n_T (\tilde{w}_t - \tilde{q}_t) - \tilde{P}_t^{x^*} \right],$$

where  $\pi_t^{x^*} = \bar{P}_t^{x^*} - \bar{P}_{t-1}^{x^*}$ .

### 3.5.3 Policy Rule

In this model monetary policy is represented by the following simple log-linear nominal exchange rate rule:

$$d\tilde{e}_t = -\omega(a_T \pi_{t-1}^T + a_N \pi_{t-1}^N) + S_t^{de}, \quad (25)$$

where  $d\tilde{e}_t = \tilde{e}_t - \tilde{e}_{t-1}$  is the nominal depreciation rate, and  $S_t^{de}$  is an exogenous nominal depreciation shock.

### 3.6 Model Solution and Parameterization

To solve the model, Uhlig's (1999) implementation of the *undetermined coefficients* method is used and the numerical results are generated by the aforementioned author's MATLAB algorithm.

Benchmark values of the basic parameters are found in Table 2.

The value of  $\beta$  is taken from King and Rebello (1999). The value  $\alpha$  is chosen in such a way that capital's share in GDP is 0.4.<sup>14</sup> The values of  $\sigma$ ,  $\varphi$ ,  $a_T$  and  $\eta$  are widely accepted in the literature. The value of  $\theta$  was chosen in such a way as to obtain the same degree of strategic complementarity of price setting as in Woodford (2003, 2005). I take the values of  $\gamma_s$  and  $\vartheta_s$  from the study of Galí, Gertler, and López-Salido (2001), which also contains Euro area estimates.<sup>15</sup> The value of parameter  $\eta^*$  is not fixed: in the simulation exercises of section 4 several different val-

**Table 2**  
Parameter values of the benchmark economy

	Parameter	
	Name	Value
	$\beta$	0.984
	$\sigma$	1.000
	$\varphi$	3.000
	$a_T$	0.500
	$\eta$	1.000
	$\alpha$	0.250
	$\theta$	10.80
	$\gamma_s$	0.817
	$\vartheta_s$	0.365
	$\omega$	1.000

Note:  $s = T, N$ .



ues are considered. Finally,  $\omega$  was chosen in such a way that the model fits the empirical findings of section 2.

#### 4. Examination of the Balassa-Samuelson Effect

As discussed in section 2, there is a strong relationship between the nominal and real exchange rates, and asymmetric sectoral productivity growth results in dual inflation and appreciation of the CPI-based real exchange rate in developing countries. Under study in this section is how it is possible to reproduce both sets of evidence in a NOEM model.

First, it will be demonstrated that, unlike in the models of the traditional approach, in NOEM models productivity induced dual inflation is not necessarily accompanied by CPI-based real appreciation, which contradicts the empirical findings discussed previously. It will be shown that the international substitution parameter  $\eta^*$  in equations (22) and (23) has a key role in generating appreciation of the CPI-based real exchange rate. On the other hand,  $\eta^*$  also influences the degree of comovement of the nominal and real exchange rates. According to my numerical simulations, the assumption of pricing to market (PTM) is necessary to find such a value of  $\eta^*$  which ensures both the strong comovement of the nominal and real exchange rates and the CPI-based real appreciation related to asymmetric productivity growth.

Second, it will be shown that it is difficult to reproduce the observed slow adjustment of the sectoral relative price to the sectoral productivity differential by frictionless models. However, decreasing returns to scale, which can be rationalized by the coexistence of heterogeneity in capital accumulation and sticky prices, help to explain this phenomenon.

##### 4.1 *Productivity Induced Dual Inflation and Real Appreciation*

As discussed in section 2.1, in European post-communist countries in the 1990s the fast productivity growth of the tradable sector resulted in dual inflation, i.e., appreciation of the internal real exchange rate, which accompanied the appreciation of the external and the CPI-based real exchange rate.

Usually productivity induced coexistence of dual inflation and CPI-based real appreciation, i.e., the BS effect, is analyzed with models of the traditional approach. These models can successfully explain the

coexistence of dual inflation and appreciation of the CPI-based real exchange rate, since in these models PPP is assumed, which prevents external real exchange rate movements. On the other hand, due to PPP they cannot reproduce the observable appreciation of the external real exchange rate.

It seems that with NOEM models it is even more problematic to explain the discussed empirical phenomena. It is typical in NOEM models that although a positive productivity shock in the tradable sector results in appreciation of the internal real exchange rate, at the same time, due to increasing productivity, domestic tradables become cheaper, i.e., the external real exchange rate depreciates. As Benigno and Thoenissen (2002) demonstrated, the latter effect suppresses internal appreciation, hence the CPI-based real exchange rate also depreciates.

This possibility is especially important in version A. Consider the exports demand equation (22). If the international substitution parameter  $\eta^* = +\infty$  then  $\tilde{q}_i^T = 0$ , i.e., the external real exchange rate becomes constant, and there will not be any relationship between the nominal and the real exchange rate, which contradicts empirical results. On the other hand, if  $\eta^*$  is low, and  $\tilde{P}_i^T$  is sticky, i.e., it responds to shocks slowly, then  $\tilde{q}_i^T = \tilde{e}_i - \tilde{P}_i^T$  will move together with the nominal exchange rate. However, in this case high tradable-productivity growth may cause strong external-real-exchange depreciation. The question is whether there is an intermediate value of  $\eta^*$  which can replicate both sets of empirical findings in version A of the model.

In version B even a high value of  $\eta^*$  can guarantee a strong comovement of the nominal and real exchange rates. On the other hand, in this case the foreign currency price of domestically produced export goods  $\tilde{P}_i^{x^*}$  does not deviate much from the prices of their foreign rivals. As a consequence,  $\tilde{P}_i^T - \tilde{e}_i$  remains stable, since the marginal costs of domestic tradable and export productions are the same. Thus, the conjecture is that in version B it is possible to find appropriate values for the substitution parameter, which guarantee that asymmetric sectoral productivity growth results in appreciation of the CPI-based real exchange rate.

First, it is studied which value of the substitution parameter  $\eta^*$  is consistent with the strong comovement of the nominal and real exchange rates discussed in section 2. In the simulation exercises the depreciation shock  $S_i^{dc}$  is the only source of nominal-exchange-rate movements. This approach is supported by several empirical studies. In a closed economy context Smets and Wouters (2003) and Ireland (2004) demonstrated by their estimated models that nominal shocks have a primary role while

technological shocks have only an auxiliary role in explaining business cycles. Clarida and Galí (1994) showed that in open economies 35–41 percent of real exchange rate movements can be attributed to nominal shocks. The prominent importance of the nominal-exchange-rate shocks in emerging markets is documented by Calvo and Reinhart (2002).

Instead of calculating simple contemporaneous correlations, I use statistics, which describe movements of the considered variables in a more complex way. Simple correlation coefficients can capture only a certain qualitative property of comovement. Namely, if the nominal and real exchange rates usually move to the same direction, then the value of the coefficient will be high, even if the size and time pattern of the movements are different. I therefore follow Chari, Kehoe, and McGrattan (2002), and study the autocorrelation structure of the CPI-based real exchange rate in response to nominal-exchange-rate shocks.<sup>16</sup> I also considered the relative variance of depreciation of the nominal and the CPI-based real exchange rates, which measure the relative magnitude of their movements and can capture varying magnitudes of real-exchange-rate reactions to nominal-exchange-rate shocks.

In the following simulations all parameters, except for  $\eta^*$ , are set to their benchmark values (see Table 2). Table 3 displays the results. Empirical values of the statistics in the table are taken from section 2.2.

Let us consider the autocorrelation function. If  $\eta^* = 1$  both versions of the model reproduce the 1-quarter value of empirical autocorrelation quite well. However, they undershoot the observed 1-year and 2-year autocorrelation coefficients.<sup>17</sup>

In version *A* all autocorrelation coefficients significantly diminish as  $\eta^*$  increases. In particular, the 1-year and 2-year coefficients become very small compared to the empirical values. On the other hand, in version *B* the auto-correlation coefficients are much less sensitive to the substitution parameter, moreover as  $\eta^*$  increases the fit of the model slightly improves.

Another measure indicating the strength of the comovement of nominal and real exchange rates is the relative variance of nominal and real depreciations. In version *A* this statistic decreases as  $\eta^*$  increases, and becomes definitively smaller than the empirical value. On the other hand, in version *B* the relative variance does not react to changes of the substitution parameter.

In summary, while model version *B* is quite insensitive to changes of  $\eta^*$ , version *A* is sensitive to the variation of the substitution parameter. It can approximate the empirical results only if  $\eta^*$  has low values,

**Table 3**

The relationship between the nominal and the CPI-based real exchange rate in the model economy

Statistics	Parameter values of $\eta^*$				
	Data	1	5	15	20
Version A					
Autocorrelation of the real exchange rate					
1 quarter	0.84	0.78	0.74	0.67	0.64
1 year	0.50	0.34	0.26	0.16	0.13
2 years	0.25	0.10	0.06	0.03	0.03
The relative variance of the real and nominal depreciations	1	0.93	0.91	0.87	0.86
Version B					
Autocorrelation of the real exchange rate					
1 quarter	0.84	0.80	0.81	0.82	0.82
1 year	0.50	0.37	0.40	0.42	0.43
2 years	0.25	0.13	0.16	0.18	0.19
The relative variance of the real and nominal depreciations	1	0.93	0.93	0.93	0.93

i.e., domestically produced export goods and their foreign rivals are far substitutes.

The next issue is whether dual inflation induced by asymmetric sectoral productivity growth is accompanied by CPI-based real appreciation. The role of the international substitution parameter  $\eta^*$  in equations (22) and (23) will be studied by numerical simulations.

In the simulation exercises I imitate some characteristics of productivity developments of transition countries. The model's steady state represents the state of the economy at the beginning of its transition process. Foreign productivity growth is normalized to zero, hence the productivity variables  $\tilde{A}_i^T$  and  $\tilde{A}_i^N$  represent relative productivity of the examined small open economy. In the model transition is driven by increasing productivity. The start of the process is captured by an unexpected productivity shock. It is assumed that during transition the growth rate of productivity is constant. After the transition process the growth rate of productivity in the small open economy will be equal to zero as well. The steady state belonging to the new level of productivity represents the after-transition state of the economy. However, this new

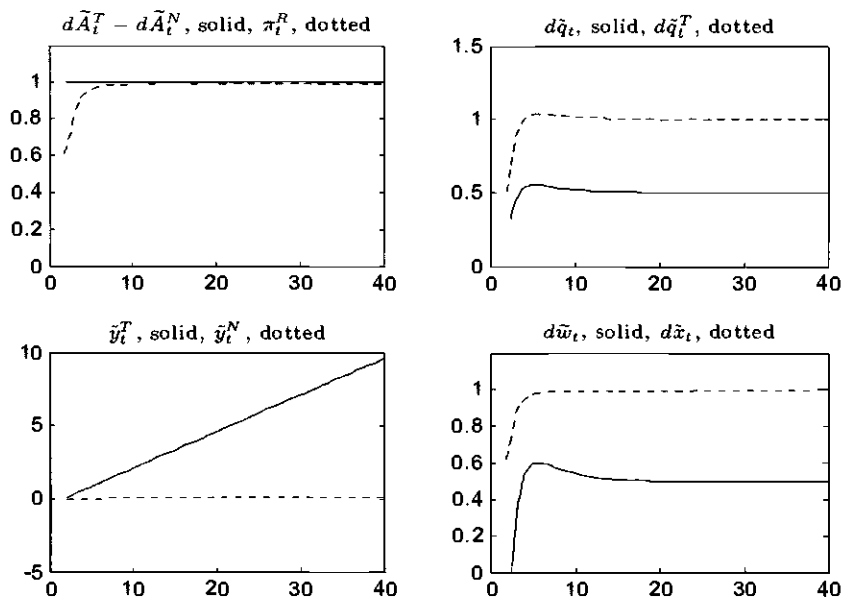
state of the economy is beyond my focus. I assume that the transition process is mainly driven by tradable productivity, hence I assume that in the examined transition period the growth rate of non-tradable productivity is equal to zero. In the simulation exercises I set the annual growth rate of the tradable TFP to 1 percent.

In the following simulation exercises differences between the responses of the two model versions are negligible, since nominal-exchange-rate movements are small. Hence, it is sufficient to report the outcomes belonging to version *B*. Figure 1 displays the simulation results for the benchmark economy with  $\eta^* = 1$ . The first panel of the figure plots the difference between the growth rates of sectoral productivity factors  $d\tilde{A}_i^T - d\tilde{A}_i^N$ , and the inflation differential  $\pi_i^R = \pi_i^N - \pi_i^T$ . The latter determines the movements of the internal real exchange rate. If  $\pi_i^R$  is positive, then the internal real exchange rate appreciates. The second panel plots the depreciation of the CPI-based real exchange rate  $d\tilde{q}_i^N$ , and the external real exchange rate  $d\tilde{q}_i^T$ . Positive values of  $d\tilde{q}_i^N$  and  $d\tilde{q}_i^T$  mean depreciation. Formulas (3) and (19) imply that the connection between the real exchange rate indices is

$$d\tilde{q}_i^N = d\tilde{q}_i^T - a_N \pi_i^R.$$

The third panel displays  $\hat{y}_i^T = (c^T \tilde{c}_i^T + x\bar{x}_i)(c^T + x)^{-1}$  and  $\hat{y}_i^N = \tilde{c}_i^N$ . As equation (20) reveals, beyond productivity factors these quantities also influence sectoral inflation rates. Finally, the fourth panel plots the growth rates of the real wage and exports. All growth rates are expressed in annualized terms.

Simulation results reveal that although the internal real exchange rate appreciates, the real exchange rate depreciates since the effect of the depreciating external rate is stronger than that of the internal rate. The reason is that productivity growth of the tradable sector is higher than those of the non-tradable sector and foreign tradable sectors. As a consequence, the relative price of domestically produced tradables to foreign tradables decreases. That is, the external real exchange rate depreciates. If domestically produced and foreign tradables were perfect substitutes, then the reduced relative price would induce a large instant increase of demand for domestic tradables. Hence, domestic real wages and tradable prices would increase and the prices of domestic and foreign tradables denominated in the same currency would equalize immediately. But in the studied case domestic and foreign tradables are far substitutes, hence increasing demand does not result in equalized prices.



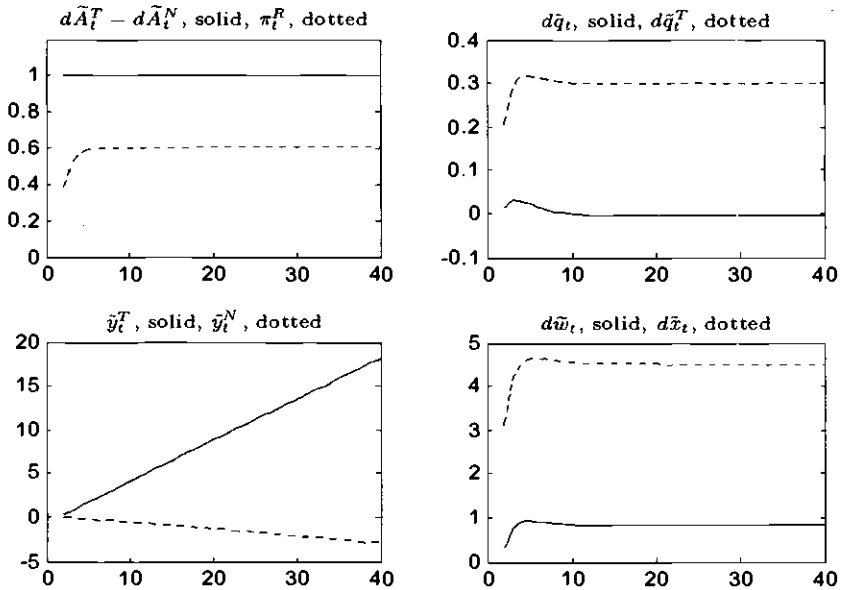
Units on a horizontal axis represent quarters, on a vertical axis percentage points. Growth rates are displayed in annualized terms.

**Figure 1**  
Balassa-Samuelson effect  
PTM - version B  
 $\eta^* = 1$

Figure 2 plots simulation results belonging to a higher value of the substitution parameter ( $\eta^* = 15$ ). The figure reveals that if domestic and foreign tradables are closer substitutes than in the previous case, then the depreciation of the external real exchange rate becomes more moderate. However, even this moderate level of depreciation prevents appreciation of the CPI-based real exchange rate. As a consequence, even this value of the international substitution parameter  $\eta^*$  is insufficient to reproduce empirical findings.

Figure 3 displays the results belonging to  $\eta^* = 20$ . Since in this case export goods are relatively close substitutes of their foreign rivals their prices cannot deviate much, hence the depreciation of the internal real exchange rate is moderate. As a consequence, the CPI-based real exchange rate appreciates in the long run.

In summary, it was demonstrated that the international substitution parameter  $\eta^*$  had a key role in reproducing empirical facts related to the BS effect. If  $\eta^*$  is low, i.e., domestic and foreign tradables are far substitutes, then the external real exchange rate depreciates too much, and

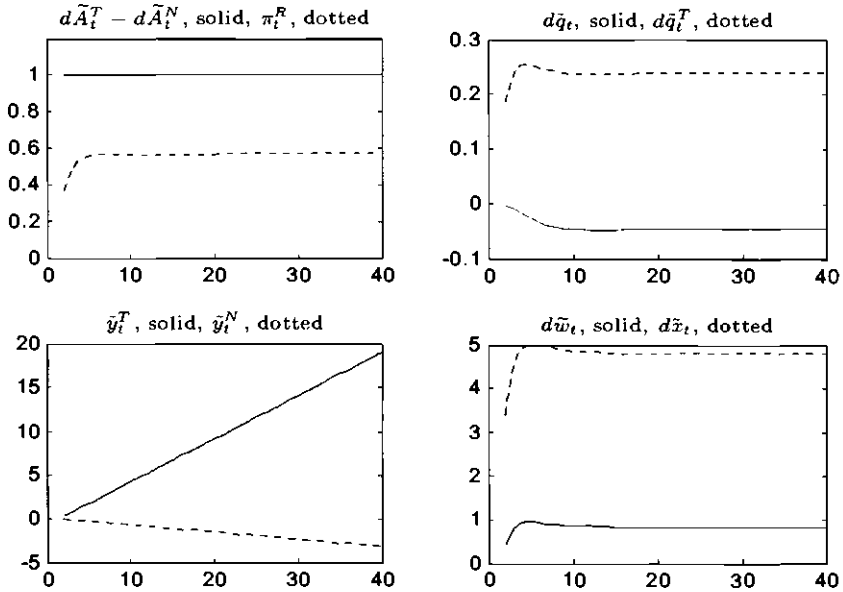


Units on a horizontal axis represent quarters, on a vertical axis percentage points. Growth rates are displayed in annualized terms.

**Figure 2**  
Balassa-Samuelson effect  
PTM – version B  
 $\eta^* = 15$

prevents the appreciation of the CPI-based real exchange rate. Hence, relatively high values of parameter  $\eta^*$  are the only possible candidates to generate results consistent with empirical findings. However, in version A, when PTM is not allowed, sufficiently high values of  $\eta^*$  result in an insufficient and weak relationship between the nominal and real exchange rates. In version A to generate CPI-based real appreciation  $\eta^* > 15$  is necessary, but these parameter values induce small autocorrelation coefficients and relative variance of the real exchange rate (recall Table 3). Hence, PTM seems necessary to appropriately describe the BS effect in NOEM models.

One may criticize the applied high values of the substitution parameter  $\eta^*$ , since estimates using macro data are usually much lower, around 1.5 to 2. However, micro data yields estimates in the range of 5 to 20; see the references in Obstfeld and Rogoff (2000b). Moreover, recent estimation of an open economy macro model by Adolfson, Laseén, Lindé, and Villani (2005) also supports high values of the elasticity of substitution. I provide some further informal arguments why it is reasonable to assume high values of  $\eta^*$  in the case of European post-communist econ-



Units on a horizontal axis represent quarters, on a vertical axis percentage points. Growth rates are displayed in annualized terms.

**Figure 3**  
Balassa-Samuelson effect  
PTM – version B  
 $\eta^* = 20$

omies. First, traditionally they exported little differentiated goods, e.g., agriculture products. Second, during the transition as a result of adoption of developed foreign technologies they started exporting highly differentiated products. However, those are manufactured by plants of foreign multinational firms, which usually produce the same product varieties in these countries as in any other countries. Hence, the majority of export products of European post-communist countries are still very similar to foreign products, and the main source of their imperfect substitutability is not variety but transportation and distribution costs.

One more remark related to market segmentation. To simplify the exposition I did not discuss the possibility of PTM with producer currency pricing (PCP), but it is possible to show that in the present framework it provides practically the same results as version B. As a consequence, I would rather not take sides in the LCP vs. PCP debate since both approaches can be consistent with the BS effect.<sup>18</sup> PCP can be applied without the assumption of price discrimination. Moreover, in most cases PCP is applied without PTM, which is equivalent to applying version A. The reason for this is that the arguments of the support-



ers of PCP remain valid without PTM. However, my results point out that if one wants to capture the particularities of emerging markets, then the PCP approach cannot be applied without the assumption of international price discrimination.

As was discussed in section 2.1, in European post-communist countries the observed long-run appreciation of the CPI-based real exchange rate is only partly caused by dual inflation; the long-run appreciation of the external real exchange rate also lies behind this phenomenon. The presented model is not able to reproduce the long-run appreciation of the external real exchange rate.<sup>19</sup>

To explain this phenomenon it seems necessary to relax the assumption of constant quality, or fixed structure of goods in the model. García Solanes, Flores, and Portero (2005) provide indirect evidence that increasing demand for tradables due to their improving quality results in appreciation of the external real exchange rate in new member states of the European Union. Broda and Weinstein (2004) demonstrate that in the U.S. increasing variety of goods is not properly captured by the statistical system, hence the rise of tradable price index is overestimated by 1.2 percent per year. This finding suggests that the appreciation of the external real exchange rate in European post-communist countries can partly be explained by measurement errors as well.

#### 4.2 *The Adjustment of the Relative Price of Non-tradables to Tradables*

As discussed in section 2.1 and displayed in Table 1, according to most of the estimations of Coricelli and Jazbec (2001), Halpern and Wyplosz (2001), Égert (2002), and Égert, Drine, Lommatzsch, and Rault (2002), in the long-run the magnitude of the relative price of non-tradables to tradables ( $\tilde{P}_t^R$ ) is significantly smaller than that of the sectoral productivity differential  $\tilde{A}_t^T - \tilde{A}_t^N$ . In addition, Halpern and Wyplosz found that the short-run adjustment of the relative price was very slow.

It is difficult to explain these facts by models of the traditional approach. Applying classical assumptions to the present model,<sup>20</sup> it is easy to show that the relative price is determined by

$$\tilde{P}_t^R = \frac{n_N}{n_T} \tilde{A}_t^T - \tilde{A}_t^N, \quad (26)$$

where  $n_T$  and  $n_N$  are the labor utilization parameters in the technological equation (11). If the tradable productivity process  $\tilde{A}_t^T$  is dominant, then the only way to reproduce the aforementioned empirical long-run relationship is to assume that the tradable sector is more labor intensive

than the non-tradable one. But this is counterfactual. In addition, the above formula implies instant adjustment of the relative price to the productivity differential.

In this section I show how the presence of decreasing returns, which can be rationalized as the limiting case of the firm-specific-investments model of Altig, Christiano, Eichenbaum, and Linde (2005) and Woodford (2005), helps to explain the above empirical findings, even if  $n_N \geq n_T$ . For expositional simplicity, I assume that  $n_N = n_T$ . Combine sectoral sticky price equations represented by formula (20), and for expositional simplicity assume that  $\xi_T = \xi_N = \xi$  and  $\vartheta_T = \vartheta_N = \vartheta$ . Then the inflation differential  $\pi_t^R = \pi_t^T - \pi_t^N$  is determined by

$$\pi_t^R - \vartheta\pi_{t-1}^R = \beta E_t[\pi_{t+1}^R - \vartheta\pi_t^R] + \frac{\xi}{1-\alpha}(\tilde{A}_t^T - \tilde{A}_t^N) \quad (27)$$

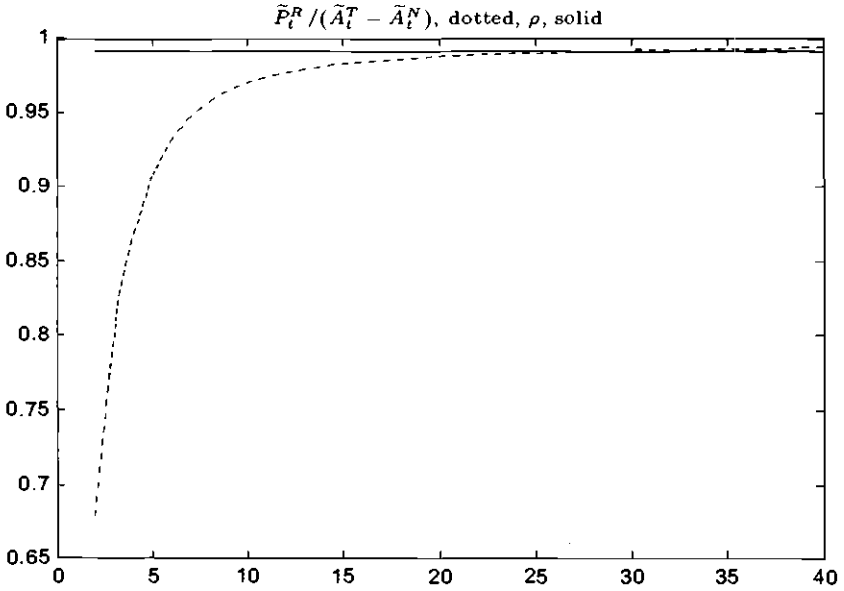
$$+ \frac{\xi\alpha}{1-\alpha} \left( \tilde{c}_t^N - \frac{c^T \tilde{c}_t^T + x\tilde{x}_t}{c^T + x} \right) - \xi \tilde{P}_t^R.$$

Terms  $\tilde{c}_t^N$ ,  $\tilde{c}_t^T$ , and  $\tilde{x}_t$  appear in the above equation, due to decreasing returns to scale. In the constant-returns-to-scale version of the present model, i.e., when  $\alpha = 0$ , only the productivity factors  $\tilde{A}_t^T$ ,  $\tilde{A}_t^N$  and the relative price  $\tilde{P}_t^R$  would influence the evolution of the inflation differential.

Relative price adjustment in the presence of sticky prices is definitely slower than in flexible price models of the traditional approach represented by formula (26). Obviously, speed of adjustment of  $\tilde{P}_t^R$  depends on the magnitude of parameter  $\xi$ . The smaller  $\xi$  is, the slower is the adjustment process. However, nominal rigidities without decreasing returns are not sufficient to reproduce the empirical estimates, as the simulation exercise belonging to Figure 4 demonstrates. The figure plots the adjustment process of the relative price to the sectoral productivity differential: it displays the fraction of the relative price to the productivity differential, i.e.,  $\tilde{P}_t^R / (\tilde{A}_t^T - \tilde{A}_t^N)$ . In the simulation exercise I apply the same productivity process as previously, and use version B with  $\eta^* = 20$ , but I assume that  $\alpha = 0$ , i.e., technology exhibits constant returns to scale. Hence, terms  $\tilde{c}_t^N$ ,  $\tilde{c}_t^T$  and  $\tilde{x}_t$  are missing from formula (27). To compare simulation results with empirical estimates I calculated the OLS regression

$$\tilde{P}_t^R = \rho(\tilde{A}_t^T - \tilde{A}_t^N) + u_t$$

using the simulated ten-year-long time series. The obtained OLS coefficient  $\rho$  represents the empirical "long-run" estimates of the studied relationship. The magnitude of the OLS coefficient  $\rho$  is also displayed



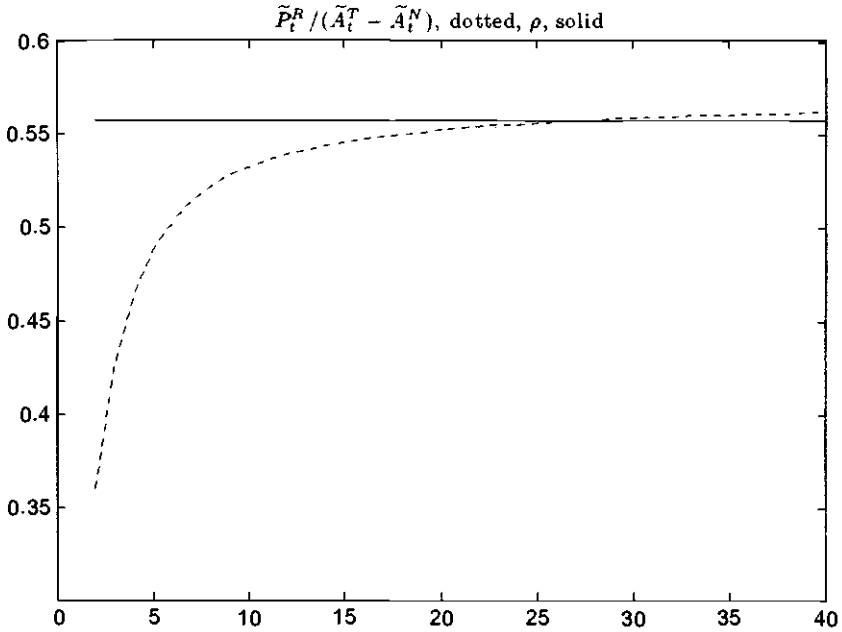
Units on a horizontal axis represent quarters, on a vertical axis percentage points.

**Figure 4**  
Adjustment of the relative price of non-tradables to tradables  
PTM – version B, constant returns to scale,  $\eta^* = 20$

in the figure. Figure 4 reveals that although the adjustment of  $\tilde{P}_t^R$  is not instant,  $\rho$  is nearly equal to 1. However, with one exception the empirical estimates are significantly smaller than this.

In the presence of decreasing returns to scale the adjustment process becomes slower and incomplete. First, as formula (21) reveals, if  $\alpha > 0$ , then  $\xi$  becomes smaller than in the constant-returns-to-scale case, which slows down the adjustment process. Second, terms  $\tilde{c}_t^N$ ,  $\tilde{c}_t^T$ , and  $\tilde{x}_t$  in the real marginal cost function triggers a feedback effect. As  $\tilde{A}_t^T$  increases  $\pi_t^R$  and  $\tilde{P}_t^R$  start increasing as well. As a consequence, the demand for  $\tilde{c}_t^T$  and  $\tilde{x}_t$  will rise and for  $\tilde{c}_t^N$  will decrease. But according to formula (27) this change of demand will decrease the rise of  $\pi_t^R$  and  $\tilde{P}_t^R$ , hence the adjustment process will be slower. Third, the sectoral consumption and export terms make the adjustment incomplete, since the long-run rise of productivity in the tradable sector results in a long-run rise of tradable consumption and exports, see Figures 1–3. Hence, formula (27) implies that sectoral price differential will not converge to productivity differential.

Figure 5 illustrates this. In this simulation exercise I used the original decreasing-returns-to-scale ( $\alpha > 0$ ) form of version B with  $\eta^* = 20$ . The



Units on a horizontal axis represent quarters, on a vertical axis percentage points.

**Figure 5**

Adjustment of the relative price of non-tradables to tradables  
PTM – version B, benchmark economy,  $\eta^* = 20$

figure reveals that now the adjustment is slower and  $\rho = 0.56$ , which is in line with empirical estimates.

In summary, although both flexible price models and sticky price models with constant returns to scale can roughly capture the relationship between sectoral price and productivity differentials, they fail to reproduce the exact empirical magnitudes. The presence of decreasing returns to scale, which can be rationalized by the coexistence of frictions in capital accumulation and nominal rigidities, helps to explain the observed phenomena.

## 5. Conclusions

This paper has reviewed how models of the new open economy macroeconomics (NOEM) can explain the permanent dual inflation and the accompanying appreciation of the CPI-based real exchange rate often observed in emerging markets.

The coexistence of dual inflation and CPI-based real appreciation is usually explained by the BS effect, i.e., by the faster productivity growth in the tradable sector. Traditionally, the BS effect is derived from models with flexible prices and internationally homogenous tradable goods markets. On the other hand, NOEM models assume sticky prices and/or wages and heterogeneous goods markets. The traditional approach focuses on the determinants of the internal real exchange rate, while NOEM emphasizes the importance of the external real exchange rate.

It was shown that a NOEM model can simultaneously guarantee the strong comovement of the nominal and real exchange rates and can generate the BS effect only if there is pricing to market in the model.

The study also investigates how the presence of decreasing returns to scale, which can be rationalized by the coexistence of nominal rigidities and frictions in capital accumulation, modifies the effects of asymmetric productivity growth on dual inflation and the external real exchange rate. The paper demonstrated that decreasing-returns-to-scale features help to explain the slow and incomplete adjustment of the relative price of non-tradables to tradables observable in post-communist European countries.

Although it was not studied in this paper, it is worth mentioning here that decreasing returns to scale can also explain the role of demand factors in generating dual inflation as documented in Arratibel, Rodríguez-Palenzuela, and Thiman (2002) and López-Salido, Restoy, and Vallés (2005).

## Acknowledgements

This paper was presented at the *NBER International Seminar on Macroeconomics* held 17–18 June, 2005 at Magyar Nemzeti Bank (Central Bank of Hungary). I would like to thank my discussants, Richard Clarida and Refet S. Gürkaynak. I am also grateful to Péter Benczúr, Giancarlo Corsetti, Chris Pissarides, Philip Lane, David López-Salido, Hugo Rodríguez Mendizábal, and János Vincze. All errors are, of course, my own.

## Notes

1. On the Balassa-Samuelson effect see Obstfeld and Rogoff (1996, chapter 4).
2. The examined countries were Armenia, Azerbaijan, Belarus, Bulgaria, Croatia, Czech Republic, Estonia, Hungary, Kazakhstan, Kyrgyzstan, Latvia, Lithuania, Poland, Romania, Russia, Slovakia, Slovenia, Ukraine, and Uzbekistan.

3. The countries in the sample were the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Russia, Romania, and Slovenia.
4. The examined countries are the Czech Republic, Hungary, Poland, Slovakia, and Slovenia.
5. The studied countries are Croatia, the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Slovakia, and Slovenia.
6. The examined countries and the length of the data set: the Czech Republic (1994–2001), Hungary (1992–2001), Poland (1990–2001), Slovakia (1995–2000), and Slovenia (1992–2001).
7. Since reliable estimates of total factor productivity were not available, due to the lack of capital stock data, they used labor productivity measures.
8. In their paper they studied the inflation processes in ten European post-communist countries. Their results support the existence of dual inflation in these countries. However, according to their estimations a positive productivity shock negatively influences the inflation rate in the non-tradable sector.
9. Diebold, Husted, and Rush (1991) and Lothian and Taylor (1996) using long annual time series of different currencies found much more persistent real-exchange-rate shocks than Chari, Kehoe, and McGrattan (2002). It is difficult to explain their findings purely by nominal rigidities. Rogoff (1996) refers to this phenomenon as the "PPP puzzle." Engel and Morley (2001) built an empirical model, which may help to resolve this puzzle.
10. Hornok, Jakab, Reppa, and Villányi (2002) tried to perform econometric estimations on very short time series and the half-time they found is approximately 2.8 years. On the other hand, Darvas (2001) using the data of the Czech Republic, Hungary, Poland, and Slovenia found very short, less than one year, half-lives. But in the studied time periods narrow-band crawling peg regimes were typical in these countries, which may explain his results.
11. Although it is rarely studied in the literature, there is a third logical possibility, namely PTM with producer currency pricing. For the sake of clear presentation, I omit discussion of this case.
12. Thus, I apply the approach of McCallum and Nelson (2001), Smets and Wouters (2002), and Laxton and Pesenti (2003), who consider imports as a production input.
13. Since the government's budget is balanced, the tax/transfer represented by  $\tau$  is compensated by  $T_t$  lump-sum tax/transfer variable in equation (2). In the present model the only role of  $\tau$  is to simplify steady-state calculations, see the Appendix.
14. In this model  $\alpha$  is not equal to capital's share in GDP since one has to subtract the value of imports from the value of total output to obtain GDP.
15. In their study they interpret inflation persistence differently from the approach I use. They use the model of Galí and Gertler (2000) and assume that each firm updates its price in a given period by probability  $1 - \gamma$ . Hence, according to the law of large numbers in a given period  $1 - \gamma$  fraction of the firms change their prices. But only  $1 - \vartheta$  fraction of the price setters choose their prices in an optimal forward-looking manner, the rest update their prices according to the past inflation rate. If  $\beta = 1$ , then the approach I use and the one used by Galí and Gertler coincides, if  $\vartheta_s = \vartheta/\gamma$  and  $(1 - \gamma)^2 \gamma_s^{-1} = (1 - \vartheta)(1 - \gamma)^2 \gamma^{-1}$ ,  $s = T, N$ . Although in our case  $\beta \neq 1$ , as an approximation I used the above mentioned formula to determine the values of  $\gamma_s$  and  $\vartheta_s$ .

16. The speed of the pass-through of the nominal exchange rate to domestic CPI, a key issue both in academics and policy applications, is also related to the autocorrelation of the CPI-based real exchange rate.
17. This contradicts the simulation results of Chari, Kehoe, and McGrattan (2002), who found weaker simulated autocorrelations. However, Benigno (2004) demonstrated that if monetary policy is described by a rule with inertia, and the foreign and home country are asymmetric in such a way that monetary shocks result in terms of trade changes, then the required persistence can be attained by the model. These conditions are fulfilled in my model.
18. LCP vs. PCP is one of the most important undecided debates in the NOEM literature, since the choice of the optimal exchange rate is not independent of this problem. One can read pro LCP arguments in Engel (2002a, 2002b). Obstfeld (2001, 2002) and Obstfeld and Rogoff (2000a) present arguments supporting the PCP approach. Two recent studies on this topic are Bergin (2004), which provides evidence supporting LCP, and Koren, Szeidl, and Vincze (2004) with findings reinforcing PCP.
19. As it is shown in an extended version of the present study, see Világi (2005), applying Woodford's (2005) firms-specific-investments model can explain initial appreciation of the external real exchange rate due to initial bias of investments demand for tradables. However, it cannot account for its long run appreciation.
20. Flexible price setting, internationally homogeneous goods and capital markets.

## References

- Adolfson, M., S. Laseén, J. Lindé, and M. Villani. 2005. "Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through." Sveriges Riksbank Working Paper no. 179.
- Altig D., L.J. Christiano, M. Eichenbaum, and J. Linde. 2005. "Firm-Specific Capital, Nominal Rigidities and the Business Cycle." NBER Working Paper no. 11034. Cambridge, MA: National Bureau of Economic Research.
- Arratibel, O., D. Rodríguez-Palenzuela, and C. Thiman. 2002. "Inflation Dynamics and Dual Inflation in Accession Countries: A "New Keynesian Perspective." European Central Bank Working Paper no. 132.
- Balassa, B. 1964. "The Purchasing Power Doctrine: a Reappraisal" *Journal of Political Economy* 72: 584–596.
- Benigno, G. 2004. "Real Exchange Rate Persistence and Monetary Policy Rules." *Journal of Monetary Economics* 51: 473–502.
- Benigno, G., and C. Thoenissen. 2002. "Equilibrium Exchange Rates and Supply-Side Performance." Bank of England Working Paper no. 156.
- Bergin, P.R. 2004. "How Well Can the New Open Economy Macroeconomics Explain the Exchange Rate and Current Account?" NBER Working Paper no. 10356. Cambridge, MA: National Bureau of Economic Research.
- Betts, C., and M. Devereux. 1998. "Exchange Rate Dynamics in a Model of Pricing to Market." *Journal of International Economics* 47: 569–598.

- Broda, C., and D.E. Weinstein. 2004. "Globalization and the Gains from Variety." NBER Working Paper no. 10314. Cambridge, MA: National Bureau of Economic Research.
- Calvo, G. 1983. "Staggered Price Setting in a Utility Maximizing Framework." *Journal of Monetary Economics* 12: 383–398.
- Calvo, G., and C. Reinhart. 2002. "Fear of Floating." *Quarterly Journal of Economics* 117(2): 379–408.
- Chari, V.V., P.J. Kehoe, and E.R. McGrattan. 2002. "Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?" Federal Reserve Bank of Minneapolis Research Department Staff Report 277.
- Christiano, L.J., M. Eichenbaum, and C.L. Evans. 2001. "Nominal Rigidities and the Effects of a Shock to Monetary Policy." NBER Working Paper no. 8403. Cambridge, MA: National Bureau of Economic Research.
- Clarida, R., and J. Galí. 1994. "Sources of Real Exchange rate Fluctuations: How Important are Nominal Shocks?" *Carnegie Rochester Conference Series on Public Policy* 41: 1–56.
- Coricelli, F., and B. Jazbec. 2001. "Real Exchange Rate Dynamics in Transition Economies." CEPR Discussion Paper no. 2869.
- Darvas, Z. 2001. "Exchange Rate Pass Through and Real Exchange Rate in the EU Candidate Countries." Discussion Paper of the Economic Research Centre of the Deutsche Bundesbank 10/01.
- De Gregorio, J., and H.C. Wolf. 1994. "Terms of Trade, Productivity and the Real Exchange Rate." NBER Working Paper no. 4807. Cambridge, MA: National Bureau of Economic Research.
- Diebold, F.X., S. Husted, and M. Rush. 1991. "Real Exchange Rate under the Gold Standard." *Journal of Political Economy* 99: 1252–1271.
- Égert, B. 2002. "Investigating the Balassa-Samuelson Hypothesis in Transition: Do We Understand What We See?" Bank of Finland Discussion Papers 2002/6.
- Égert, B., I. Drine, K. Lommatzsch, and C. Rault. 2002. "The Balassa-Samuelson Effect in Central and Eastern-Europe: Myth or Reality?" William Davidson Working Paper no. 483.
- Engel, C. 1999. "Accounting for U.S. Real Exchange Rate Changes." *Journal of Political Economy* 107: 507–538.
- Engel, C. 2002a. "The Responsiveness of Consumer Prices to Exchange Rates and Implications for Exchange-Rate Policy: A Survey of a Few Recent New Open-Economy Macro Models." NBER Working Paper no. 8725. Cambridge, MA: National Bureau of Economic Research.
- Engel, C. 2002b. "Expenditure Switching and Exchange Rate Policy." NBER Working Paper no. 9016. Cambridge, MA: National Bureau of Economic Research.
- Engel, C., and J.C. Morley. 2001. "The Adjustment of Prices and the Adjustment of the Exchange Rate." NBER Working Paper no. 8550. Cambridge, MA: National Bureau of Economic Research.
- Galí, J., and M. Gertler. 2000. "Inflation Dynamics: A Structural Econometric Analysis." NBER Working Paper no. 7551. Cambridge, MA: National Bureau of Economic Research.



- Galí, J., M. Gertler, and J.D. López-Salido. 2001. "European Inflation Dynamics." NBER Working Paper no. 8218. Cambridge, MA: National Bureau of Economic Research.
- García Solanes, J., F. Torrejón Flores, and I. Sancho Portero. 2005. "Más allá de la Hipótesis de Balassa-Samuelson en los Países de la Ampliación Europea: El Sesgo de Calidad y la Segmentación de Mercados." Mimeo, Universidad de Murcia.
- Halpern, L., and C. Wyplosz. 2001. "Economic Transformation and Real Exchange Rates in the 2000s: The Balassa-Samuelson Connection." In *Economic Survey of Europe 2001*, No. 1, Chapter 6. Geneva: United Nations Economic Commission for Europe.
- Hornok, C., M.Z. Jakab, Z. Reppa, and K. Villányi. 2002. "Inflation Forecasting at the Magyar Nemzeti Bank." Mimeo, Magyar Nemzeti Bank (Central Bank of Hungary).
- Ito, T., P. Isard, and S. Symansky. 1997. "Economic Growth and Real Exchange Rate: An Overview of the Balassa-Samuelson Hypothesis in Asia." NBER Working Paper no. 5979. Cambridge, MA: National Bureau of Economic Research.
- Ireland, P. 2004. "Technology Shocks in the New Keynesian Model." NBER Working Paper no. 10309. Cambridge, MA: National Bureau of Economic Research.
- King, R.G., and S.T. Rebello. 1999. "Resuscitating Real Business Cycles." In J.B. Taylor and M. Woodford, eds., *Handbook of Macroeconomics* Vol. 1. Amsterdam: Elsevier Science B.V.
- Koren, M., Á. Szeidl, and Vincze J. 2004. "Export Pricing in New Open Economy Macroeconomics: An Empirical Investigation." Mimeo, Harvard University and Corvinus University of Budapest.
- Kovács M.A., ed. 2002. "On the Estimated Size of the Balassa-Samuelson Effect in Five Central and Eastern European Countries." MNB (Central Bank of Hungary) Working Paper no. 2002/5.
- Laxton, D., and P. Pesenti. 2003. "Monetary Rules for Small, Open, Emerging Economies." NBER Working Paper no. 9568. Cambridge, MA: National Bureau of Economic Research.
- López-Salido, D., F. Restoy, and J. Vallés. 2005. "Inflation Differentials in EMU: the Spanish Case." Banco de España, paper presented at the European Central Bank Workshop on *Monetary Policy Implications of Heterogeneity in a Currency Area*, 13 and 14 December, 2004, Frankfurt am Main.
- Lothian, J.R., and M.P. Taylor. 1996. "Real Exchange Rate Behavior: The Recent Float from the Perspective of the Past Two Centuries." *Journal of Political Economy* 107: 507-538.
- McCallum, B.T., and E. Nelson. 2001. "Monetary Policy for an Open Economy: An Alternative Framework with Optimizing Agents and Sticky Prices." NBER Working Paper no. 8175. Cambridge, MA: National Bureau of Economic Research.
- Monacelli, T. 2004. "Into the Mussa Puzzle: Monetary Policy Regimes and the Real Exchange Rate in a Small Open Economy." *Journal of International Economics* 62: 191-217.
- Mussa, M. 1986. "Nominal Exchange Regimes and the Behavior of Real Exchange Rates: Evidence and Implications." *Carnegie-Rochester Conference Series on Public Policy* 25:117-214.
- Obstfeld, M. 2001. "International Macroeconomics: Beyond the Mundell-Fleming Model." NBER Working Paper no. 8369. Cambridge, MA: National Bureau of Economic Research.

Obstfeld, M. 2002. "Exchange Rate and Adjustment: Perspectives from the New Open Economy Macroeconomics." NBER Working Paper no. 9118. Cambridge, MA: National Bureau of Economic Research.

Obstfeld, M., and K. Rogoff. 1995. "Exchange Rate Dynamics Redux." *Journal of Political Economy* 103: 624–660.

Obstfeld, M., and K. Rogoff. 1996. *Foundations of International Macroeconomics*. Cambridge, MA: MIT Press.

Obstfeld, M., and K. Rogoff. 2000a. "New Directions for Stochastic Open Economy Models." *Journal of International Economics* 50(1): 117–153.

Obstfeld, M., and K. Rogoff. 2000b. "The Six Major Puzzles in International Macroeconomics: is There a Common Cause?" In B. Bernanke and K. Rogoff, eds., *NBER Macroeconomics Annual*. Cambridge, MA: MIT Press.

Rogoff, K. 1996. "The Purchasing Power Parity Puzzle." *Journal of Economic Literature* 34(2): 647–668.

Samuelson, P.A. 1964. "Theoretical Notes on Trade Problems." *Review of Economics and Statistics* 46: 145–54.

Smets, F., and R. Wouters. 2002. "Openness, Imperfect Exchange Rate Pass-Through and Monetary Policy." *Journal of Monetary Economics* 49: 947–981.

Smets, F., and R. Wouters. 2003. "An Estimated Stochastic Dynamic General Equilibrium Model of the Euro Area." *Journal of the European Economic Association* 1: 1123–1175.

Uhlig, H. 1999. "A Toolkit for Analyzing Dynamic Stochastic Models Easily." In R. Marimon and A. Scott, eds., *Computational Methods for the Study of Dynamic Economies*. Oxford: Oxford University Press.

Világi, B. 2005. "Dual Inflation and the Real Exchange Rate in New Open Economy Macroeconomics. Available at <<http://www.ecb.int/events/conferences/html/mpimphet.en.html>>.

Woodford, M. 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton, NJ: Princeton University Press.

Woodford, M. 2005. "Firm-Specific Capital and the New Keynesian Phillips Curve." NBER Working Paper no. 11149. Cambridge, MA: National Bureau of Economic Research.

## Appendix

### *The Steady State*

In this section the non-stochastic steady state of the benchmark model is described. Variables without time indices refer to their steady-state values.

In the steady state there is no difference between the two model versions, and there is no intra-household and intra-sector heterogeneity. Therefore the index  $i$  of firms are omitted to simplify the notations. The level of fixed capital stock used in the model is set to be equal to the steady state capital stock of the vari-

able capital version of the model with zero depreciation rate and steady state level of investments.

It is assumed that  $P = P^T = P^N = 1$ . Then the demand equations of formula (4) imply that

$$c^T = a_T c, \quad c^N = a_N c. \quad (28)$$

Furthermore, it is assumed that  $P^T x = e^{P^{m^*}} m$ . Hence,

$$GDP = a^T c^T + a^N c^N = c.$$

The real interest rate  $r$  is determined by

$$r = \frac{1}{\beta} - 1.$$

The values of  $\tau$  is set in such a way that the markup is equal to 1,

$$1 - \tau = \frac{\theta}{\theta - 1}.$$

Then it is true for all sectors that the marginal product of capital is equal to  $r$ . Thus, equation (10) implies that

$$\kappa = \left( \frac{r}{\alpha} \right)^{\frac{1}{1-\alpha}},$$

where  $\kappa = z^T/k^T = z^N/k^N$ . Furthermore, equation (10) implies that

$$c^T + x = k^T \kappa^{1-\alpha}, \quad c^N = k^N \kappa^{1-\alpha}. \quad (29)$$

It is assumed that  $w = W = e^{P^{m^*}}$ , then equation (12) implies that  $w^s = w$ . Since in each sector  $w^z$  is equal to the marginal product of  $z^s$

$$w = (1 - \alpha) \kappa^{-\alpha}.$$

In the benchmark economy  $w = 1.865$ . Let us denote the exogenous exports/GDP ratio by  $s_x$ , and I set  $s_x = 0.6$ . Since  $x = e^{P^{m^*}} m$ ,

$$s_x = \frac{x}{c} = \frac{e^{P^{m^*}} m}{c}. \quad (30)$$

It is assumed that in the benchmark economy  $n_N = n_T = n$ . Then the imports demand equation in formula (13) implies that

$$m = (1 - n)(z^T + z^N).$$

Then one can show that

$$m = (1 - n) \kappa (k^T + k^N) = (1 - n) \kappa k. \quad (31)$$

Using the previous expression for  $m$  and equation (30) yields

$$c = K \kappa, \quad (32)$$

where

$$K = eP^{m^*}(1-n)\kappa s_x^{-1}.$$

By equation (29) one can similarly show that

$$k\kappa^{1-\alpha} = c + \delta k + x = eP^{m^*}(1-n)\kappa s_x^{-1}k + eP^{m^*}(1-n)\kappa k.$$

This implies that

$$n = 1 - \frac{\kappa^{1-\alpha}}{eP^{m^*}\kappa(1+s_x^{-1})}.$$

In the benchmark economy  $n = 0.5$ .

In the steady state the labor supply function of households takes the form

$$w = c^\sigma l^\varphi. \quad (33)$$

As for imports, one can derive a similar expression for labor:

$$l = n\kappa k. \quad (34)$$

Substituting equations (32) and (34) into equation (33) yields an expression for the capital stock:

$$k = [wK^{-\alpha}(n\kappa)^{-\varphi}]^{\frac{1}{\sigma+\varphi}}.$$

Using this expression one can calculate the steady-state value of the capital stock and investments. In the benchmark economy  $k = 67.296$ . Then using formula (32) yields the value of consumption,  $c = 2.736$ , and equation (34) provides the value of labor,  $l = 0.88$ .