PRICE INDEX AGGREGATION: PLUTOCRATIC WEIGHTS, DEMOCRATIC
WEIGHTS, AND VALUE JUDGMENTS

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Abstract

The question of how to aggregate individual cost-of-living indexes is not a simple one. The recent National Research Council Panel's report (Schultze *et al.* (2002). *At What Price? Conceptualizing and Measuring Cost-of-Living and Price Indexes*. Washington: National Academy Press) discusses two methods. The first, the "plutocratic" method uses expenditure weights in the aggregation; the second, the "democratic" method weights each household equally. This paper argues that the choice of weights depends on value judgments about income distribution. It is shown that, save in very special circumstances, neither the plutocratic nor the democratic method is satisfactory.

1. Introduction

The theory of the cost-of-living index is well developed so far as concerns a single household. When it comes to aggregation over households, however, there is far less agreement as to how to proceed. Yet such aggregation is inevitable. What then is to be done?

The recent report of the National Research Council's Panel on Conceptual Measurement and Other Statistical Issues in Developing Cost-of-Living Indexes (Schultze, *et al.*, 2002)¹ examines this issue (among many others) and offers two alternative aggregation methods – neither of which seems fully satisfactory.² In this paper, I evaluate those methods by stating the aggregation problem in a form in which value judgments about income distribution can be made explicit. I show that while the so-called "plutocratic" method, as the panel notes, will usually be undesirable, the so-

* I am grateful to E. R. Berndt for comments but retain responsibility for error.

called “democratic” method, which the panel appears to prefer, is difficult or impossible to rationalize under *any* set of circumstances and reasonable value judgments about income distribution. While I have unfortunately been unable to offer any practical recommendation, I hope that setting out the explicit theory will eventually prove helpful.

I begin by defining terms. There are H households, indexed by $h = 1, \dots, H$ and n goods. The h th household’s consumption vector is denoted by x^h , and the vector of total consumption over all households is denoted by X . Subscripts denote consumption of a particular good.

The reference (or base) period is denoted by $t = 0$ and the comparison (or current) period by $t = 1$.

The income of the h th household in the t th period is denoted by y^{th} , and total income of all households by Y^t .

It is assumed that all households face the same prices.³ For convenience, the price vector in the comparison period is denoted simply by p , and that in the reference period by p^0 . Again, the price of a particular good is denoted by a subscript.

The utility of the h th household (in the base period) is given by u^{0h} , and, at that utility level, its expenditure function in the comparison period is denoted by $E^h(p, u^{0h})$. Of course, $E^h(p^0, u^{0h}) = y^{0h}$, so that h ’s cost-of-living index can be written as:

$$(1.1) \quad I^h \bullet E^h(p, u^{0h}) / E^h(p^0, u^{0h}) = E^h(p, u^{0h}) / y^{0h}, \quad h = 1, \dots, H.$$

The Laspeyres index corresponding to this is:

¹ All references are to this unless otherwise stated.

² See pp. 51-52 and Chapter 8, especially pp. 237-40.

$$(1.2) \quad L^h \bullet px^{oh}/p^0 x^{oh} = px^{oh}/y^{oh} ,$$

where the obvious inner products are involved.

The aggregation problem is that of combining the indexes given in (1.1) and (1.2) into single aggregate indexes for the entire group of H households.

As discussed by the panel (pp. 51-52), following Pollak (1980, 1981), one way to proceed is to define what he calls the “social cost-of-living index” and ask for the minimum income that would be required in the comparison period to give each household the same utility it achieved in the reference period. That income, summed across households, is then divided by the total income required to achieve those utilities in the reference period. This yields an index:

$$(1.3) \quad J \bullet \sum_h E^h(p, u^{oh})/Y^0 = \sum_h \{E^h(p, u^h)/y^{oh}\} \{ \{ y^{oh}/Y^0 \} = \sum_h I^h \{ \{ y^{oh}/Y^0 \} ,$$

an average of the individual indexes weighted by the respective expenditures of the households. The corresponding Laspeyres index is then:

$$(1.4) \quad L \bullet \sum_h L^h \{ y^{oh}/Y^0 \} = pX/p^0 X ,$$

the standard form of the consumer price index.

³ The problem is quite hard enough with this assumption.

Because of the expenditure weights, these indexes are called ‘plutocratic’. They give more weight to the indexes of the rich than to the indexes of the poor. The panel finds this unappealing for some purposes⁴ (and I agree), and wonders whether unweighted averages – called ‘democratic’ indexes – would not be better.

The democratic index is defined as:

$$(1.5) \quad D \bullet (1/H) \sum_h \{E^h(p, u^h)/y^{oh}\}$$

with the corresponding Laspeyres index:

$$(1.6) \quad F \bullet (1/H) \sum_h L^h \quad .$$

2. Formally Modeling the Problem

It is clear that the issue here necessarily involves value judgments, especially value judgments about income distributions. It follows that the analysis should be framed in a way in which such value judgments can be made explicit. I do so in the following way.⁵

Let $W(u^I, \dots, u^H) \bullet W(u)$ be a social welfare function defined over the utilities of the households. Parallel to the definition of the expenditure function in the theory of the individual household, we can define a social welfare expenditure function (WEF). In its unrestricted form, that function is defined as:

⁴ But not for all. The plutocratic index is consistent with accounting identities in the national accounts.

⁵ The formal set-up is similar to that in Fisher (2003), although the problem is different.

$$(2.1) \quad E^* = E^*(p, W) \bullet \text{Min } pX, \text{ subject to } W(u) = W.$$

In other words, E^* is the minimum total money income needed by the household sector to attain social welfare W when prices are p , *provided that such total income can be allocated by the social planner whose welfare function is $W(\cdot)$.*

But, of course (and, perhaps, fortunately), social planners – or, more generally, people presenting their views as to the relative desirability of different allocations – typically do not have such powers, and this will increase the cost of obtaining welfare level, W .⁶ Thus, while E^* measures the total cost of achieving a given welfare level if the distribution of income can be altered by the planner, we need to examine the (greater or equal) total cost of achieving that level with the income distribution held constant. This is a second-best solution from the planner's viewpoint.

Formally, let s^h denote h 's share of total income in the fixed income distribution. Define the restricted welfare expenditure function by:

$$(2.2) \quad E = E(p, W) \bullet \text{Min } pX, \text{ subject to } W(u) = W \text{ and } y^h / Y = s^h \ (h = 1, \dots, H).$$

It is evident that $E \bullet E^*$, since the latter is calculated as a minimum without restrictions.

There is another way to look at the restricted problem (2.2), and that way brings us back to the price-index aggregation problem. Let the fixed income distribution be that of the reference period. Suppose that a single, aggregate price index is to be published and that such aggregate is to be used to adjust each household's income by the same

⁶ Pollak (1981, pp. 333-34) considers this but in a different way than is done in the present paper.

percentage. Then the restricted problem is seen to be that of minimizing the total amount of expenditure required under such conditions to achieve a given level of welfare.

Of course, in both the restricted and unrestricted cases, the minimization problem is to be solved at comparison period prices and the result divided by Y^0 (the solution at reference period prices) to obtain the desired index.⁷

Now consider the Pollak social cost-of-living index given in (1.3). The denominator is, of course, Y^0 , while the numerator is the total expenditure required to give each household the same utility at prices p^1 that it achieved in the reference period at prices p^0 .⁸ Evidently, this is one way of achieving the reference level of social welfare as in the unconstrained problem. It will generally not be the efficient way, however, for, isomorphic to the reasons that a Laspeyres index overstates the corresponding cost-of-living index, the plutocratic index, J , fails to allow for substitution among individuals in the social welfare function and therefore overstates the unrestricted first-best solution. In other words, $J \bullet E^*/Y^0$. Further, the plutocratic Laspeyres, L , will certainly overstate J , so that:

$$(2.3) \quad L \bullet J \bullet E^*/Y^0 \quad .$$

⁷ A reader has commented that he sees no reason to think that individual product prices would be the same in the current and optimal income distribution worlds. I agree. That issue, however, is irrelevant to the discussion in this paper which takes the current prices as given and constructs indices comparing them to those of the reference period.

⁸ This is a loose way of stating what is involved. The phrase “the same utility” has no operational meaning, and one should say ‘making the household indifferent between facing the comparison period prices with the calculated income and facing the reference period prices with reference period income’. See Fisher and Shell (1972, pp. 2-4). In the present context, there is no need for that precision, but I would not like it to be thought that my views on the subject have changed.

But, of course, this means that the plutocratic indexes (including the standard consumer price index) overstate an index which is itself lower than the index of interest – the restricted index, E/Y^0 . This does not mean that the standard indexes are likely to be right, however. To examine that question, we must look at the restricted problem in more detail.

It will be convenient to do this in terms of Laspeyres indexes. Formally, a Laspeyres consumer price index is a linear approximation to the cost-of-living index taken at the reference year's purchases. We shall compare the weight given to a particular price (p_i), in the plutocratic Laspeyres index, L (the actual consumer price index), with that which would be given in a Laspeyres index approximating E/Y^0 . We do that by evaluating the derivatives of E at the original prices.⁹

The LaGrangian for the restricted minimization problem is:

$$(2.4) \quad \tilde{E} = pX - \tilde{\lambda} \{ W(u) - W \} - \sum_h \delta^h \{ px^h - s^h pX \},$$

where $\tilde{\lambda}$ and the δ^h are LaGrange multipliers.

One (relatively minor) point before proceeding.¹⁰ The constraints in (2.2) are not independent, since if every household but one has its current-period income share equal to its reference-period income share, so will the remaining household. This means that we should omit that constraint for one of the household, say the H th. To avoid complicating the notation in (2.4) and later equations, I simply define $\tilde{\alpha}^H = 0$. It may help to fix ideas to take that household as one with the median income in the reference-period income distribution, and I do so in the following discussion.

⁹ I am indebted to Eduardo Ley for calling to my attention the need to make this explicit.

Note that, by the Envelope Theorem:

$$(2.5) \quad \ddot{C}^h = \frac{\partial E / \partial s^h}{pX} \quad (h = 1, \dots, H).$$

Hence, if, according to the value judgments of the social planner, h has too high a share of income in the fixed distribution relative to the median, then $\ddot{C}^h > 0$, since a lower share for h would reduce the cost of achieving the given welfare level. Similarly, $\ddot{C}^h < 0$ corresponds to too low a share for h relative to the median. If all $\ddot{C}^h = 0$, then the given income distribution is the same as would be imposed by the social planner in the first-best solution, and $E = E^*$ when both are minimized.

Again using the Envelope Theorem,

$$(2.6) \quad \bullet E / \bullet p_i = \bullet \ddot{E} / \bullet p_i = X_i - \sum_h \delta^h \{ x_i^h - s^h X_i \} \quad (i = 1, \dots, n),$$

so that the weight given to p_i in a Laspeyres index would be:

$$(2.7) \quad w_i \bullet (1/Y^0)(X_i - \sum_h \delta^h \{ x_i^h - s^h X_i \}),$$

whereas the weight given to p_i in the plutocratic index is simply $(1/Y^0)(X_i)$. These will be the same for all i under any of the following conditions¹¹:

Case 1: $x_{i1}^h - s^h X_{i1} = 0$, all i . In this case, each household consumes each good in the same proportion to its income as does every other household. If this holds globally, then the household's all have the same homothetic indifference map, and their behavior

¹⁰ Through an oversight, this paragraph was omitted from Fisher (2003).

¹¹ And, except for cases of measure zero, only then.

can be represented as that of an aggregate household.¹² In this case, the plutocratic and democratic indexes are the same. Indeed, *any* weighted average of individual cost-of-living indexes would give the same result.

Case 2: $\bar{C}^h = 0$, $h = 1, \dots, H$. Here there are two possibilities.

- a. We are indifferent to income distribution questions;
- b. We regard the existing income distribution as optimal.

Note that, in all three of these cases, the restricted minimum problem has the same solution as the unrestricted one; that is, $E = E^*$. Note further, that, in Case 2b, the ‘P’ in ‘Plutocratic Index’ might just as well stand for ‘Panglossian’ or for the ‘Popian’ as in ‘Alexander Pope’, since the view taken can be described either as ‘All’s for the best in this best of all possible worlds,’ or ‘Whatever is, is right,’ at least when it comes to income distribution. These are not attractive views.

Given this, the reader may now think that I shall come down in favor of the democratic index, but that is not the case.

For the democratic index, D , defined in (1.5) to coincide with E/Y^0 , it must be the case that the two corresponding Laspeyres indices also coincide. This requires that, for all i , the weight given to p_i in F , defined in (1.6) equal w_i , defined in (2.7). This means:

$$(2.8) \quad (1/H) \sum_h \{x_i^h / y^{0h}\} = (1/Y^0)(X_i - \sum_h \delta^h \{x_i^{1h} - s^h X_i\}), \quad i = 1, \dots, n.$$

Except for cases of measure zero, this can only happen under one of the following conditions:

¹² If the condition holds locally, then aggregation is possible in some neighborhood.

Case 1: $x_{i1}^h - s^h X_{i1} = 0$, all i . In this case, each household consumes each good in the same proportion to its income as does every other household. We have already seen that this is the case of perfect aggregation over households, and that, if it holds, the plutocratic and democratic indexes are the same.

Case 2: For every $h = 1, \dots, H$, $\ddot{C}^h = 0$, and $y^{0h} = Y^0 / H$. In this case, the income distribution is egalitarian, and (as we should expect, given the “democratic” nature of the index to be used), such egalitarian distribution is also considered to be the optimal one. Note that, in this case also, the democratic and plutocratic indexes coincide.

The interesting feature of Case 2 is that the democratic index is optimal only if the income distribution is already egalitarian and considered optimal. The reason for this is fairly easy to see.

Assume that the aggregate price index is to be used to provide the same adjustment for price changes to each household’s income. The use of the democratic index provides that equal weight is to be given to each household. But this provides equal treatment for each household only going forward – equal treatment as regards changes from the reference to the comparison period. The Case 2 result shows that the value judgment as to income distribution that this relies on is that the optimal distribution involves equal treatment for each household. But, if the income distribution is not already egalitarian, and if particular price changes have different (proportional) effects on the costs of living of different households (the absence of Case 1), then different households should be weighted differently, so as to correct for the lack of optimality of the income distribution. An increase in the price of goods heavily bought by the poor should get a higher weight than an increase in the price of goods heavily bought by the

rich, even if the number of households in each group is the same. This would alter the distribution of *real* income in the direction of equality while leaving the distribution of money income unchanged.

Hence, the same value judgment on income distribution that leads to the use of the democratic index also implies that such use is suboptimal unless the income distribution is already egalitarian.

3. Summary and Conclusions

We have shown that, when embedded in a context permitting value judgments on income distribution to be made explicit, neither the plutocratic nor the democratic index are really satisfactory – apart from the obvious case in which aggregation of preferences is possible – a case in which the two indexes coincide and also coincide with any other method of averaging individual indexes.

The plutocratic index is optimal only if we either do not care about income distribution at all or else believe that the existing income distribution is optimal.

The democratic index is optimal only if we already have an egalitarian income distribution and believe that distribution to be optimal – formally a more restrictive set of circumstances than that required for the plutocratic index (even though they would coincide in that case).

Having said this, I wish I had a truly constructive suggestion to offer, but I do not. I sympathize with the view that the democratic index is to be preferred to the plutocratic one, but note that it only provides equal treatment going forward from a given position that is likely to be unequal. Since the use of aggregation methods to affect the

distribution of real income will doubtless be controversial, perhaps the democratic index is the best we can do.

Of course, all this suggests that the use of a single aggregate index for large, disparate groups of households is likely to be unsatisfactory. The obvious conclusion is that different indexes should be constructed for different groups wherever possible.

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