# THE "ARCHITECTURE" OF CAPITAL ACCOUNTING: BASIC DESIGN PRINCIPLES<sup>1</sup>

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"... it is not reasonable for us to expect the government to produce statistics in areas where concepts are mushy and where there is little professional agreement on what is to be measured and how (Griliches (1994), page 14)."

National income accounting would be a relatively simple matter were it not for "capital." All flows of output would then be for immediate consumption, and labor would be the sole factor of production (and relatively undifferentiated labor at that, since there would be no investments in health and education to complicate matters). How to define the boundary between market and non-market activity would be one of the main issues of contention, how to measure the real output associated with intangible services would be another. However, both problems are largely issues of implementation rather than of basic theory, since there is no conceptual reason to exclude the non-market use of economic resources from a complete set of national accounts, nor is there any controversy about the need to express inputs and output in both current and constant prices.

When it comes to capital, however, it is more a question of "what" to do rather than "how" to do it. No issue has given economic theory more trouble, from Karl Marx and the

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Austrian capital theorists, to Keynes and the Cambridge Controversies, and the ambiguity has only gotten worse with the increased theoretical focus on Schumpeterian uncertainty, partial information, and imperfect competition, and with the emerging literature on the importance of intangible capital as sets. This unsettled state of affairs is obviously a problem for the design of national income accounts, since, as Griliches observed, it is hard to measure something when there is fundamental disagreement about what exactly 'it'' is. Indeed, this ambiguity is reflected in the current design of national accounting systems, as well as in the structure of financial accounting systems. No system currently in place achieves a complete account of capital in its many facets and dimensions.

These observations are the starting point for this paper on the architecture of the capital accounts. To use the architectural analogy, the paper is about the abstract design principles and not a blueprint for a particular building; it is about the logically prior question of 'what" should be done, rather than a discussion of how do 'tt." It is inspired by Koopmans' famous injunction about the need to avoid 'measurement without theory." This injunction argues that theory should guide measurement practice in order to guide the selection and definition of the variables included in the accounts and to define the boundaries, insure internal consistency among these variables, and facilitate their interpretation and subsequent use. The Koopmans injunction is especially important for defining the role of capital in the national accounts, not only because of the tendentious history of capital theory, but also because of the many practical debates over issues like the treatment of depreciation, obsolescence, and quality change, as well as net versus gross measures of capital and output. However, it does not specify any particular theory and, of course, there are many candidates for this role: capital accounts can be built along Marxist or

Keynesian lines (as with the traditional structure of the U.S. National Income and Product Accounts (NIPAs), or can be broadly defined to include environmental, social, and quality-of-life indicator variables. This paper opts for the neoclassical theory prevalent in most textbook treatments of "standard" economics and describes an architecture for capital similar to the structure outlined in the work of Christensen and Jorgenson (1969,1970), with special emphasis on the neglected role of the Hall-Jorgenson user cost of capital in current accounting structures.

While neoclassical theory provides a Koopmanian underpinning for the capital accounts described in this paper, the actual architecture is based on the circular flow model of payments and commodities (or 'CFM'). The structure of the CFM is particularly well-suited to the task of accounting for the different dimensions of 'capital,' which is both a stock and a flow, an output and an input of the production process, is both tangible and intangible, and can be given different valuations by consumer and producers. One point of this paper is to show how these various characteristics can be expressed in the structure of a wealth-augmented version of CFM. Moreover, the CFM also has the advantage of being a fairly general framework that can accommodate more than one underlying theoretical structure -- indeed, there are at least three ways of describing capital within the neoclassical model alone.

Because of the complex nature of capital, the paper starts with a minimalist description of the CFM in which there is no capital of any kind. Using this as the baseline case, the paper then explores the role of time in a variant of the CFM in which "capital" arises only from a temporal mismatch between the production and use of consumption goods, and in which there are no capital goods like machinery and structures. These sections establish the following principle: all capital ultimately is derived from the decision to defer current consumption in order to

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enhance or maintain expected future consumption. The capital-as-deferred-consumption aspect is then extended to the situation in which explicit capital goods are introduced into the CFM, and the capital-as-a-goods-producing-good aspect is linked to the deferred consumption aspect using the neoclassical framework of standard theory. Special attention is paid here to the role of the Hall-Jorgenson user cost of capital in the circular flow, one of the most problematic areas of contemporary national income accounts, and its link to financial intermediation. A final section explores issues beyond the strict neoclassical formulation, including problems associated with imperfect competition, externalities, and uncertainty and imperfect information.

# II. The Basic Circular Flow Model

The CFM provides a minimalist view of economic activity that distinguishes two basic functions, the production of goods and the consumption of these goods, and two basic types of goods, inputs and output. It describes how the flow of payments and quantities associated with these functions and goods are related in a four-part diagram in which the production activities are located on one side of a diagram and consumption on the other side, and in which inputs consigned to the lower half of the diagram and outputs are put in the upper part. As shown in Figure 1, inputs and outputs flow through 'markets' where the goods are transferred from one sector to the other. When formal markets exist, they provide an explicit valuation of the flows of goods at the prevailing prices. These exchanges are the essential feature of CFM, and establish the equivalence between the various value flows: revenue  $= \cos t = income = expenditure$ . These are portrayed around the outer edge of the Figure 1, and they give rise to the fundamental accounting identity relating the value of output to the value of input:  $P^{C}C=P^{L}L$  (the consumption good is here denoted by C and its price by  $P^{C}$ ; L is labor input, L, and  $P^{L}$  the wage rate). This basic national income and product accounting identity exists independently of the CFM, but the CFM gives it the added dimension that both the output side and the input side of the fundamental identify can be measured in two ways (revenue and expenditure for goods, cost and income for inputs).

As it currently stands, there is no explicit time dimension in Figure 1. However, C and L are flows and there must therefore be an implicit duration over which the flow is measured (C and L per some unit of time). The duration of this period is of little economic significance per se in the non-capital case, but the time dimension does enter the problem when accounts are constructed for successive time periods and bundled into a time series. The ability to track input and output over time is, indeed, one of the main reasons that the accounts are constructed in the first place. The construction of time series does, however, open up the issue of comparability over time, particularly with regard to prices. The price of labor and consumption can be normalized to an index of one in the base time period 0, , i.e.  $P^{C}(0) = P^{L}(0) = 1$ , by the appropriate choice of the units in which C and L are measured. If these prices remain constant over time, there is no problem of comparability and, in fact, no need for considering prices at all. However, there is no reason to expect  $P^{C}(t)$  and  $P^{L}(t)$  to remain constant, since both productivity change (C/L) and monetary inflation (an autonomous increase in the money supply per unit of goods), will cause prices to change over time, both relatively and absolutely. When this happens, the accounting identity (1) must be modified to

$$P_{C}(t)C(t) = P_{L}(t)L(t), \qquad (1)$$

and supplemented by a constant price account which allows a times series on C and L to be constructed:

$$P^{C}(0)C(t) = A(t)P^{L}(0)L(t).$$
(2)

The factor A(t) must be included in the constant price account in order to allow for autonomous changes in productivity over time.

The implementation of (2) usually requires an independent estimate of prices or quantities, since the accountant typically observes only the product of the two. This is, unfortunately, often easier said than done. First, the separation is relatively easy when C is a tangible good whose units are readily observable, but determining the units of measurement is often rather hard when C is intangible (e.g., education services), and the units may be hard even to define. Secondly, the circular flows of Figure 1 are shown flowing through a market in which the prices are determined. Indeed, one function of a system of national accounts is to track the use of available resources, L in this case, and there are clearly many non-market uses of time. Leisure is one such use, household production is another, and education yet another, and a complete description of a nation's production and consumption possibilities would regard L and an endowment of time to be distributed among competing uses, market and non-market.<sup>2</sup>

The discussion of the all consumption-labor version of the CFM could be left at this point

<sup>&</sup>lt;sup>2</sup> The CFM does have the feature that market and non-market circular flows are financially separable, in the sense that a balanced flow of payments occurs within the market subsector (each dollar paid is a dollar received by someone else). The problem with ignoring the non-market subsector is that the underlying economic structure is inherently non-separable.

without burdening the exposition with formal theory. However, the theory of value shows the value of theory in defining the boundaries of the accounts. While there is no explicit reference to formal economic theory, there is a rudimentary theory embedded in the architecture of Figure 1 simply by virtue of its organization into sectors and markets. The flow of L into the producers' sector and the flow of C out of the sector implies a transformation of input into output, which is formalized in standard theory as the production function

$$C(t) = F(L(t),t).$$
(3)

The 't', here, allows for costless advances in the efficiency of production. In a multi-product version of the CFM, each of the N types of consumer goods has its own production function, and in principle use some of each of the M types of labor input:  $C_i(t) = F^i(L_{i1}(t), ..., L_{iM}(t), t)$ . Similarly, the flow of C into the consumers' sector and the outflow of L implies consumer choice among competing alternatives, which is modeled in standard theory by the ordinal utility function<sup>3</sup>

$$U(C_1(t), ..., C_N(t); L_1(t), ..., L_M(t)).$$
 (4)

These last two equations have the helpful feature that they establish natural boundaries for the flow accounts in the CFM of Figure 1, or, indeed, for any set of accounts whose purposes is to provide a complete description of how available resources are used to satisfy economic wants (and, as a secondary objective, to provide insights into the economic structure driving the

<sup>&</sup>lt;sup>3</sup> Since the focus is ultimately on the definition of capital, all consumers here are assumed to have identical utility functions.

economic system). This line of argument suggests that any <u>produced</u> good that yields utility, and any input that is necessary for production should be located within the boundaries of a complete economic account, and other items excluded. The principle also implies that goods which are distributed outside of formal markets should be included in the accounts.<sup>4</sup>

One final remark about intermediate goods will be useful for the subsequent discussion of capital accounting. Intermediate goods are produced for immediate use in other industries (steel to make autos, for example), and the production functions shown above must be expanded to included the interindustry flow of such goods. This can be illustrated by the example of a two-industry economy in which one industry produces the intermediate good  $M(t) = F^M(L_M)$  and the second produces the consumption good  $C(t) = F^C(L_C)$ , with the constraint that  $L = L_M + L_C$ . The total revenue in this economy is  $P^C(t)C(t)+P^M(t)M(t)$  and is equal to total cost  $P^L(t)L(t)+P^M(t)M(t)$ . This would seem to modify the fundamental accounting identity shown in

(1), but since the intermediate good term  $P^{M}(t)M(t)$  appears on both sides of the gross accounting identity, they cancel out leaving the net identity (1). The fundamental accounting identity is still the correct measure of aggregate economy activity on both the input and output sides of the production sector, but the flows must be given a different interpretation:  $P^{C}(t)C(t)$  is the value of deliveries to final demand in the consumer sector, and  $P^{L}(t)L(t)$  is the value added by labor (and

<sup>&</sup>lt;sup>4</sup> Theory can also be helpful in valuing commodity flows which occur outside of the formal market. The shadow prices established by the optimization of utility subject to the production constraint can in principle be used in place of market prices, though this is often difficult in practical applications. The valuation of leisure time using the wage as an opportunity cost is one successful application (e.g., Jorgenson and Fraumeni (1989,1992)), the Hall-Jorgenson user-cost imputation for capital services is another. National accounting practice has generally shied away from imputations, preferring to limit the boundary of the accounts to sectors and goods for which valuation is a matter of data acquisition, rather than construction.

income to the consumer sector). Intermediate goods also disappear from the aggregate production possibility frontier,

$$\Phi[C_1(t),...,C_n(t);L_1(t),...,L_n(t));A_1((t),...,A_n(t)] = 0$$
<sup>(5)</sup>

which indicates efficient combinations of the consumption goods  $[C_1(t), ..., C_n(t)]$  that can be produced given the available amount of the different types of labor input and the level of technical efficiency prevailing in each industry.

#### III. Capital as Deferred Consumption

Once the length of the accounting period has been chosen for the CFM in Figure 1 (e.g., a year, a quarter, a month), the only remaining time-related problem is to insure that the methods used in any one time period are consistent with those of the other periods as the sequence of accounts unfolds over time. Beyond this intertemporal consistency issue, the data flows in the Figure 1 account of any one year are not connected to the accounts of previous or subsequent years, i.e., there is no economic connection between C(t-1) and C(t), or between L(t-1) and L(t), or between the corresponding prices. This temporal independence is intentional, since the goal of Figure 1 was to describe an economy utterly devoid of capital, and therefore of a meaningful time dimension, precisely in order to show how capital and time might be introduced into the CFM.

A first step in this direction is to retain the assumption that consumption and labor are the only goods, but allow consumers to lend some of their current allotment of consumption goods to others in return for a repayment in a subsequent period of their life. The total amount of consumption in each year is not affected (aggregate  $P^{C}(t)C(t)$  must still equal  $P^{L}(t)L(t)$  in every year), so the saving of lenders must just balance the dissaving of the borrowers. Therefore, no net wealth is created at the economy-wide level of aggregation, but a balance sheet does exist for each individual agent, which records in each year the net consequences of all past saving and dissaving, and constitutes the individual's net worth (new wealth). This lead to the conclusion that the possibility of shifting consumption across time by individuals is sufficient to introduce a wealth variable into the CFM even though there are no explicit capital goods and no consumption goods are actually shifted between years. Because this consumption-shifting mechanism is the starting point of all types of capital formation, and because the formal analytics of the accounting model in this simple case persists in some form throughout capital accounting, a further elaboration of this case is useful.

The life-cycle model of intertemporal choice is the basic consumption-shifting model of standard theory. This model is a variant of the atemporal framework in which the variables in the utility function are consumption of the same type of good at different points in time:

$$U^{j}(C_{j}(1), ..., C_{j}(T); L_{j}(0), ..., L(T)).$$
 (6)

This intertemporal utility is maximized subject to the constraint that the optimal path  $[C^*_{j}(1), ..., C^*_{j}(T)]$  can be purchased with income available to the j individuals over their lifetime, T. When expressed in present value terms to make the income of different years comparable, the lifetime budget constraint takes the form:

$$W_{j}(0) = \sum_{t=1}^{T} \frac{P^{C}(t)C_{j}(t)}{(1+r)^{t}} = \sum_{t=1}^{T} \frac{P^{L}(t)L_{j}(t)}{(1+r)^{t}} , \qquad (7)$$

The time discount factor (1+r) is assumed to be constant over time for simplicity of exposition. The optimal consumption path satisfies this intertemporal constraint, but not necessarily the annual income identity  $P_C(t)C_j(t) = P_L(t)L_j(t)$ , since the optimal path may imply a mismatch of consumption and income in any one year. When a mismatch occurs, the individual must either saving or dissave, leading to an annual income identity of the form,

$$P^{C}(t)C_{j}(t)+S_{j}(t) = P^{L}(t)L_{j}(t).$$
(8)

 $S_j(t)$  has a positive or negative value depending on whether there is net saving or dissaving in that year. Total saving across individuals,  $E_jS_j(t)$ , must be zero in each year, since all goods produced within a given year must be consumed within that year. Individual saving must also have a zero balance over the lifetime of each person, since loans must ultimately be repaid:

$$NW_{j}(0) = \sum_{t=1}^{T} \frac{P^{C}(t)C_{j}(t)}{(1+r)^{t}} - \sum_{t=1}^{T} \frac{P^{L}(t)L_{j}(t)}{(1+r)^{t}} = \sum_{t=1}^{T} \frac{S_{j}(t)}{(1+r)^{t}} = 0$$
(9)

The term  $NW_i(0)$  is the net worth of the individual at the beginning of the time interval.

A specific solution to this problem is shown in Figure 2 for two individuals and two years. Both have the same income in both years,  $I_j(t) = P^L(t)L_j(t)$ , denoted by the point <u>a</u> in the figure, and the same gross wealth  $W_j$ . However, they have different preferences for current and future consumption, and therefore choose different points on the wealth constraint, <u>b</u> and <u>c</u> respectively. At these point the first individual is a borrower in year 1 whose dissaving is exactly matched by the saving of the second person  $(-S_1(1) = S_2(1))$ . This is no accident, of course, since total consumption is fixed, but it is worth noting that the interest rate <u>r</u> adjusts to bring the borrowing and lending into a net balance of zero. The roles are reversed in the second year, when the second person's consumption exceeds income by the amount of year one saving plus interest,  $S_2(2)=(1+r)S_2(1)$ , while the second individual faces a corresponding deficit in consumption.

'Capital" in this context is limited to contemporaneous transfers between people, and it is therefore of no direct consequence for a set of aggregate accounts, at least for an economy that is closed to international flows of goods. However, when a economy is open to international flows, a net asset balance is possible. If, for example, the two agents in Figure 2 are nations rather than individual people, the aggregate wealth constraint of each 'country" is

$$W_{j}(0) = \sum_{t=1}^{T} \frac{P^{C}(t)C_{j}(t)}{(1+r)^{t}} = K_{j}(0) + \sum_{t=1}^{T} \frac{P^{L}(t)L_{j}(t)}{(1+r)^{t}}$$
(10)

 $K_j(0)$  is the cumulative balance of past loans up to the beginning of the decision interval (the 'present'). For any two adjacent years, it is the balance carried forward from the preceding year, with interest, plus current saving:

$$K_j(t) = S_j(t) + (1+r) K_j(t-1).$$
 (11)

 $K_j(t)$  can be thought of as the stock of debt associated with a given flow of past annual saving; it plays the role in accounting practice of the balance sheet associated with the income statement

represented in equation (8). The balance in any year may be positive or negative, and a times series of the  $K_j(t)$  may switch from one to the other. In any event,  $K_j(t)$  is an important statistic of the open economy.

This line of analysis leads to the conclusion that a form of "capital" is implicit in economic activity even when there are no explicit capital goods and, indeed, even when all the goods that are produced within a time period are also consumed (by someone) during that period. This highlights a defining feature of all capital assets: they involve the shifting of consumption across time. Indeed, it will be apparent in the following sections that the equations worked out for this exact-consumption-loan model apply quite generally. Moreover, it will be apparent that the length of the accounting period is critically important for determining what is and is not capital -- a shift in consumption from month to month would generate a capital balance, which might disappear if the accounting period is extended to one year.

# IV. Capital as an Inventory of Goods

As small 'tweak" to the analysis of the preceding section gives further insights to the capital problem. The discussion of Section III was based on a consumption good that had to be consumed in the period it was produced, and time shifting occurred through the issuance of 'paper" debt agreements. An important variant on this theme arises when the consumption good can be stored, and therefore shifted directly from one period to the next. While this 'tweak" is small, the implications are not. There is now a transfer of real goods over time, and it is thus possible to speak of a 'stock of capital" in this case, albeit a stock composed entirely of consumption goods.

The amount of the consumption stock inherited from the past, K(0), augments the stream of consumption produced in future years, and is therefore part of the feasible intertemporal consumption possibility set,

$$\Phi[C(1),...,C(T);K(0);L(1),...,L(T));A(1),...,A(T)] = 0$$
(12)

The macroeconomic problem is to find the feasible consumption path  $[C^*(1), ..., C^*(T)]$  that maximizes the intertemporal utility function (11) subject to this constraint (again with fixed labor input to simplify matters). This problems differs somewhat from that of the preceding section, where the constraint was on individual wealth (11), but it is in fact more general in that the previous wealth constraint is an endogenous feature of the optimal solution.

This result is shown for two periods in Figure 3 for the case of identical consumers (the heterogenous utility functions of Figure 2 could also be used in this figure, but are unnecessary since the point is to generalize the definition of "capital" from the transfer of goods among individuals at the same point in time, to the transfer of goods over time). The optimal solution in this case is the point of tangency, <u>a</u>, where optimal consumption is the vector  $C^*(t)$ , whereas income  $I(t)=P^L(t)L(t)$  is shown at the point <u>b</u>. Optimal saving is the difference between the two, with saving in the first year balanced by dissaving in the second. The capital accounting equations developed in the all-consumption loan case of the preceding section carry over pretty much intact to the current situation, with a change in interpretation of the capital variable. The aggregate income identity is  $P^C(t)C(t) + S(t) = P^L(t)L(t)$ , as before, but now the saving variable is the change in the inventory of consumption goods, permitting the distinction between the act of saving and the means of saving through investment in real goods. This distinction is evident in the accumulation equation,

$$P^{C}(t)K(t) = S(t) + P^{C}(t)K(t-1),$$
(13)

which indicates that the amount of consumption differed in year t, S(t), is equal to the change in the inventory of consumption goods valued in current prices,  $P^{C}(t)[K(t)-K(t-1)]$ .<sup>5</sup>

The distinction between saving and investment carries over to the difference between wealth and the value of capital stock. The gross wealth of the economy represented in Figure 3 is the present value of the maximal consumption stream, represented in Figure 3 by the point W and expressed analytically as

$$W(0) = \sum_{t=1}^{\infty} \frac{P^{C}(t)C(t)}{(1+r)^{t}} = P^{C}(0)K(0) + \sum_{t=1}^{\infty} \frac{P^{L}(t)L(t)}{(1+r)^{t}} .$$
(14)

Total consumption over time (in present value terms) is the sum of the consumption goods inherited from previous periods plus those produced with labor input after year zero. The net worth of this economy is then the present value of consumption less the present value of the input costs, which leads to the balance sheet equation of net wealth (in the sense of net deferred consumption) and the value of the stock of capital *cum* inventory of consumption goods,  $P^{C}(0)K(0)$ . This may seem like a distinction without much of a difference in the context of this class of all-consumption models, but it is the fundamental balance sheet identity of the economy and its importance will be more apparent

<sup>&</sup>lt;sup>5</sup> Some mention might be made, here, of the price of consumption goods in successive time periods. In standard intertemporal optimization theory, the slope of the wealth line WW' in Figures 2 and 3 defines the relative price of consumption in successive years,  $P^{C}(t)/P^{C}(t-1)$ . This ratio is equal to (1+r), leading to  $P^{C}(t) = (1+r)P^{C}(t-1)$ . This expression indicates that the accumulation equation of this section is equivalent to the debt equation (X) of the preceding section ( $P^{C}(t)K(t-1) = (1+r)P^{C}(t-1)K(t-1)$ ).

when tradition capital goods are introduced.

## V. Capital as a Produced Means of Production

The capital accounts described up to this point are missing one essential thing: the entity that most people intuitively regard as 'capital.' The common conception of capital is of something rather solid and durable like machines and buildings, something that is used to produce output and is itself produced. Much capital is like this, and while there are some obvious differences between this type of capital and the all-consumption conception of capital in the preceding sections, there is also an underlying unity: both types originate from the decision by consumers to defer consumption by saving out of current income, and both represent wealth that yields a future return to the consumer. Because of this underlying unity, the accounting framework of the preceding sections can be extended to handle the case of fixed capital investment with little difficulty. Three extensions are needed. First, the production possibility frontier must now include both types of output, consumption goods C(t) and investment goods I(t), and second, it must include the stock of capital, K(t), as a productive input. Thus,

$$\Phi[C(t),I(t);L(t),K(t);A_{C}((t),A_{I}(t)] = 0 , \qquad (5')$$

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(here we have shown only one type of consumption good, one type of capital, and one type of labor for simplicity). Third, the flow of investment goods must be linked to the stock of capital, which is the sum all surviving past investments weighted by their remaining productive efficiency,  $N_i$ :

$$K(t) = N_0 I(t) + N_1 I(t-1) + \dots + N_T I(t-T).$$
(15)

More will be said about the  $N_i$  in the next section; for now, we will assume that they all equal one.

There is also a question about how capital goods should be valued when they are both a stock and a flow. We will start by assuming, in this section, that all productive capital can be rented in active markets, so the cost of using one unit of K(t) for one year is  $P^{K}(t)$ . The price of the new investment good,  $P^{I}(t)$ , is also established in the product market, but the two price concepts are not independent, since the willingness to pay for a unit of new capital stock must be related to the future stream of rents generated by that good,  $P^{K}(t+\vartheta)$ . Under the assumption that investment will continue in any year up to the point at which the cost of the last unit just equals the present value of the rental income it generates:

$$P^{I}(t,0) = \sum_{\tau=0}^{T} \frac{P^{K}(t+\tau,\tau)}{(1+r)^{\tau+1}} \quad .$$
(16)

When intertemporal utility is maximized subject to the constraint (5') the rental price  $P^{K}(t)$  is related to the value of the marginal product of capital, the asset price  $P^{I}(t)$  to the marginal rate of substitution between current and future consumption, and the discount rate <u>r</u> to the rate of time preference.

The income and wealth accounts in this world are an extension of the accounts described in the preceding sections. The income accounting identity must now include the value of capital inputs and output, and becomes

$$P^{C}(t)C(t) + P^{I}(t)I(t) = P^{L}(t)L(t) + P^{K}(t)K(t).$$
(17)

The analogue to the gross wealth equation (14) is

$$W(0) = \sum_{t=1}^{\infty} \frac{P^{C}(t)C(t)}{(1+r)^{t}} = \sum_{t=1}^{\infty} \frac{P^{L}(t)L(t)}{(1+r)^{t}} + \sum_{t=1}^{\infty} \frac{P^{K}(t)K(t)}{(1+r)^{t}} - \sum_{t=1}^{\infty} \frac{P^{I}(t)I(t)}{(1+r)^{t}}$$

Equations (15) and (16) insure that the terms involving K(t) and I(t) cancel out except for the value of the initial endowment of capital carried forward from the past, leaving  $P^{I}(0)K(0)$ . When the present value of the primary input, labor, is netted out of gross wealth as before, the result is again the basic balance sheet identity between the net worth of the economy's deferred consumption and the net value of its capital stock.

Figure 4 is a modified version of Figure 1 in which the flows refer to movement of produced capital goods rather than of labor. Labor input is suppressed, here, in order to emphasize that the flows of capital and labor input are perfectly symmetric when capital input is rented. Indeed, a comparison of Figures 1 and 4 reveals that the architecture of the accounts is not fundamentally changed when the concept of capital is extended to include capital as a produced means of production.

### VI. Owner-Operated Fixed Capital

Accounting for fixed assets would be relatively easy if only they were rented in active markets. The capital flows in Figure 4 would correspond to observed data, with prices based on actual transactions. Unfortunately, this is not the way capital markets normally operate. Capital stock does not come home from work each night as workers do, because most fixed capital is

purchased and operated by businesses. One consequence is that only a small fraction of the stock flows through a formal rental market, so there is thus no explicit rental price  $P^{\kappa}(t)$ , nor a rental flow  $P^{\kappa}(t)K(t)$ , for the national accountant to observe. The lower left 'cost' branch in Figure 4 is therefore blank, and the accountant cannot complete the circular flow description of the economy. Moreover, the lower 'income' branch does not contain the rental payments generated by the consumers' ownership of the capital stock, but mainly consists of a flow of payments generated by financial instruments like stocks and bonds issued by the businesses that own the actual capital goods. These financial instruments generate a flow of dividends, interest, non-corporate profits, some rents, and retained earnings *cum* capital gains, which ultimately accrue to consumers through the direct ownership of stock and bonds, or other assets. To complicate matters, much of the financial flow reaches consumers indirectly through financial intermediaries like mutual funds, pensions funds, and life insurance companies.

A full accounting of these indirect financial flows requires a flow of funds account to track the various stages and channels of the transfer of income from the business to the consumer sector. This account would reveal the complexity of the lower right-hand income side of the capital flow model, but measuring these flows would not help with the problem of estimating the capital costs on the lower left-hand side. The lower left-hand side cost of capital is the rent,  $P^{K}(t)$ , when formal rental markets exist, but it is <u>not</u> the flow of interest, dividends, etc., when those markets do not exist. A firm that finances all its capital formation with retained earnings and pays no dividends or interest will still have an implicit, or shadow, rent on the capital it 'leases'' to itself as owner-user. This shadow rent is equal, in equilibrium, to the value of the marginal product of capital, and is therefore an important magnitude to include in a complete set of economic accounts. Fortunately, economic theory provides a way of using the aggregate financial data to impute the unobserved rental price. The solution is based on the Hall-Jorgenson user cost of capital approach, in which the asset pricing equation (16) is solved to yield an expression for the implicit rental price:

$$P^{K}(t) = [r(t) - \rho(t) + (1 + \rho(t))\delta] P^{I}(t) .$$
(19)

This is the Hall-Jorgenson user cost for a new asset without provision for taxes or other complications. It is the opportunity cost that the owner-user of the capital must recover in each year, and it is equivalent to the rental price of capital. Estimates of the user cost can be constructed for each type of asset because the elements on the right hand side of (19) are measurable: the rate asset price revaluation,  $\Delta$ , the rate of economic depreciation, \*, the rate of return, *I*, and the acquisition price of the asset, P<sup>I</sup>. The result is an imputed value of the rental price for each type of capital.

This formulation, combined with the capital accumulation equation (15), permits the circular flows in the both halves of Figure 4 to be reconstructed. The one provides an estimate of the rental price of each type of capital,  $P_{i}^{K}$ , while the other gives an estimate of the stock itself,  $K_{i}$ . The total cost of capital in Figure 4 is the sum of these elements:

$$\Pi(t) = \sum_{i=1}^{N} P^{K}_{i}(t) K_{i}(t) . \qquad (20)$$

This expression defines total capital cost A(t) from the point of view of the business sector. It is equal in the CFM to the A(t) received by consumers as interest payments, dividends, retained earnings, etc. In practice, however, the two estimates of A(t) will differ because they are based on different data sources and assumptions. But, here again, theory comes to the rescue. Jorgenson and Griliches (1967), and Christensen and Jorgenson (1969, 1970), show that the two independent estimates of A(t) will be brought into equality if the user cost (19) is based on an *ex post* rate of return to capital, *r*, rather than on an exogenously imposed *ex ante* rate of interest as was assumed above. Indeed, the *ex post* rate of return is <u>defined</u> as the rate which brings the financial income flows into equality with (20).<sup>6</sup>

This Jorgenson-Griliches adding-up procedure forces the capital-cost and income branches of the circular flow model to come into balance. It has the additional advantage that it thereby brings the wealth account into balance as well. The total value of the capital stock is simply the sum of the individual values of each type the stock,  $P_i^{I}(t)K_i(t)$ ; the value recognized by the owner/consumers is the value of the financial claims against this stock, the value of equity E(t) plus the value of debt B(t). The income stream generated from the 'real side,''i.e., from (X), is the source of the financial income stream accruing to debt and equity, and in view of (20), the corresponding stock must therefore balance:

$$\sum_{i=1}^{N} P_{i}^{I}(t) K_{i}(t) = E(t) + B(t).$$
(21)

This forms the basis for the accounting balance sheet in this case of indirect ownership of the productive capital stock. To complete the picture, new investment in capital stock in any year,  $P_i^{I}(t)I_i(t)$ , must be balanced by corresponding changes in either debt or equity, ) E(t)+) B(t), implying a balance between investment and saving.

<sup>&</sup>lt;sup>6</sup> The Jorgenson-Griliches solution is not just an expedient forced by the absence of rental markets. The *ex post* has an interesting economic interpretation as a Marshallian quasi-rent (Berndt and Fuss (1986)). The algebra and economic interpretation of the *ex post* user cost model is developed in some detail in the exposition by Hulten (1990).

## VII. Owner-Constructed Fixed Capital

The preceding section examined the implications for national income and wealth accounting of indirect ownership, where the problem was the absence of an explicit rental price or rental income flow. A further problem arises when capital stock is not only owner-operated, but also ownerconstructed. When capital assets are constructed by the owner-operator, there is no observable market transaction on which to base estimates of the investment price,  $P_i^{I}(t)$ , or the size of the nominal investment flow,  $P_i^{i}(t)I_i(t)$ . Instead, accounting practice mainly treats such expenditures as a current expense that is lumped together with current operating costs. When construction expenses are separately broken out, as with some R&D expenditures, it is the cost of constructing the asset in nominal prices that is recorded. Construction costs may or may not be equal to the value of the asset,  $P_i^{i}(t)I_i(t)$ , but even when they are, a problem still arises because there is no price deflator,  $P_i^{i}(t)$ , with which to isolate the real quantity of the investment,  $I_i(t)$ .

For business <u>tangible</u> capital, the problem of self-construction arises mainly in the case of maintenance and repair. However, it is the dominant situation for investment in business <u>intangibles</u> like research and development, computer software developed within firms, human competencies, and product marketing. Most items in these categories tend to be firm specific, both in the sense that they are of value only (or mainly) to the firm that makes them, and in the sense that firm-specific expertise is required for their production. Other items, like R&D, may also have a value outside the firm, but are closely held because of appropriability problems. All share the feature that it is almost impossible to define the quantity units in which they might be measured in principle, much less observed in practice, and, indeed, their very presence may be a matter of some dispute because they

have no tangible embodiment (and are thus potentially subject to manipulation by firms seeking to "improve" balance sheets and income statements). As a result, accountants at both micro and macro levels are reluctant to treat them as capital expenditures.

While the case for expensing intangible inputs may have a certain procedural rationale, the real issue for accounting theory is whether this practice accurately represents the evolving production and consumption possibilities of an economy (or firm) over time. Here, again, economic theory comes to the rescue. The consumption-based conception of capital developed in Sections III and IV suggests the following test to determine what is and is not capital: if a current expenditure is made in order to increase future consumption (or to prevent a decrease in future consumption), it should be regarded as saving and treated as adding to capital. In terms of Figure 3, capital formation involves a movement along the frontier AA'.

When applied to specific cases, this rule suggests that maintenance and repair expenditures should largely be treated as capital, since they are made with the expectation that they will prevent a reduction of consumption in the future. R&D spending is also capital formation under this rule, as are many other intangible business expenditures made with the intention of increasing future output and thus potential consumption. As a general matter, investments in intangible capital should be treated symmetrically with investment in tangible assets. Outside the business sector, the opportunity cost of delaying entry into the labor force in order to acquire additional education should be treated as an investment in human capital, as should many health-related expenditures.

The amount of intangible investments currently excluded from capital formation in conventional accounting practice is potentially quite large. Corrado, Hulten, and Sichel (forthcoming) find that more than one trillion dollars of business intangibles are currently excluded

from U.S. investment spending each year, a sum approximately equal to the amount business spends annually on tangible fixed capital. Others have arrived at similar estimates. Jorgenson and Fraumeni (1989, 1992) also argue that large amounts of human capital spending are ignored by current practice. Another line of analysis approaches the problem of measuring intangibles from the financial side of the accounts (e.g., Hall (2001)). Investment in intangible capital should raise the financial value of a firm just as tangible investments do in (21), implying ) E(t)+) B(t) should respond positively to both types of investment. This hypothesis is examined in Figure 5, which compares the financial value of the U.S. private business sector to the sector's value of fixed capital (the right-hand side of (21) versus the left-hand side). The disconnect is apparent: financial markets assigned a greater value to U.S. businesses than the value of their fixed asset during the 1990s, suggesting that the omission of intangibles, which are heavily weighed to investments in knowledge capital, may have understated the true dynamism of the U.S. economy over this period. However, the experience of the prior decades call this explanation into question (more will be said about this below). It would be interesting to know what Figure 5 would look like if national accounting practice had treated tangible and intangible capital symmetrically as theory dictates.

### VIII. Depreciation and Obsolescence

Special mention must be made of that portion of the fixed capital stock that is 'used up' in the process of production, through wear, tear, and obsolescence. Wear, tear, and use can cause the productive efficiency of the asset to erode as it ages; specifically, the relative efficiency terms,  $N_i$ , of the stock accumulation equation (15) decline over time until they become zero at the time, T,

when the asset is finally retired from service.<sup>7</sup> The decline in the  $N_i$  weights is associated with a corresponding decline in the value of the asset, in part because of the loss in productivity before retirement, and in part because each year that passes moves the asset closer to the end of its productive life, thus shortening the remaining stream of income. The decline in value is termed 'depreciation,' and is separate and distinct from deterioration ()  $N_i$ ) in all but the case of geometric depreciation/deterioration. It is an important economic variable, and an equally important part of the capital accounting story, albeit one with a long and tendentious history. An full account of this history is beyond the scope of this paper, as is the algebra of the link between asset deterioration, depreciation, and asset valuation, so only a few observations will be offered here.<sup>8</sup>

When assets depreciate, some portion of the annual gross return must be considered to be a return of the principal amount of the original investment, and not as income to the owner. This depreciation is the loss in value that must be recovered (or 'put back') in order to maintain the original investment intact. This principle carries over to macro accounting, with the recognition that GDP net of depreciation, not GDP itself, is the appropriate indicator of annual output's contribution to intertemporal utility. This idea is rigorously developed in Weitzman (1976), who shows that the optimal solution  $[C^*(1), ..., C^*(T)]$  to the problem of maximizing the intertemporal utility function (equation (5) above) subject to the production constraint (12) and the accumulation equation (15), was equally  $[C^*(1)+p(1))$  K\*(1), ..., C\*(T)p(T)) K\*(T)], where p(t) is the investment good price relative to the price of consumption (which is normalized to one). This expression C\*(t)p(t)) K\*(t)

 $<sup>^{7}</sup>$  An asset for which the N<sub>i</sub> weights remain at full capacity -- an index value of one -- and then falls abruptly to zero at retirement is termed "one-hoss shay."

<sup>&</sup>lt;sup>8</sup> A more complete account can be found in Hulten and Wykoff (1996) and Triplett (1996). A mathematical formulation of the issues is provided in Hulten (1990).

is Haig-Simon income, consumption plus change in net worth, but it is also equal to factor income (17) less depreciation as well as to net domestic product (Hulten (1992)). The Weitzman result has been interpreted as implying that NDP is a superior measure of aggregate economic activity than GDP (but not by Weitzman). However, the CFM makes clear that GDP belongs on the upper left-hand branch of the circular flow, because it is a measure of the actual output of the business sector, while NDP refers to the net implication of GDP for the consumers on the right-hand side of the CFM. GDP is thus a necessary concept for studying the structure of production while, as Weitzman puts it, NDP 'is a proxy for the present discounted value of future consumption" (page 156).

The conceptual problem of defining economic depreciation has also troubled the field of national accounting. There has been a confusion between the loss of productive capital from wear, tear, and retirement from service, on the one hand, and the corresponding loss in asset value (the true definition of depreciation (Jorgenson (1973), Hulten (1990), Triplett (1996)). This confusion has led to such erroneous accounting concepts as real NDP, conceived of as a measure of real output net of the capital used up in production. There has also been some confusion about the time dimension of depreciation. The loss in asset value of given asset does indeed unfold over time, but it is the age of the asset and not time *per se* that is the right dimension of depreciation. The equilibrium value of the asset is the discounted present value of the rents generated by the asset over its remaining life (a straight-forward extension of the asset pricing equation (16) above). The dimensionality of the asset's price as it evolves over time is therefore P<sup>i</sup>(t,s); depreciation is conceptually the partial derivative of the asset's price with respect to age, s, while asset revaluation (the term  $\Delta(t)$  in the Hall-Jorgenson user cost expression (19)) is the partial derivative of price with respect to time. An estimate of depreciation can be obtained by measuring the price differential between two similar

assets of different ages at the same point in time. This is the basis for the Hulten-Wykoff measures of economic depreciation embodied in the U.S. National Accounts.<sup>9</sup>

Obsolescence is another aspect of asset valuation, and it greatly complicates the asset valuation model. This phenomenon occurs when technological improvements are embodied in the design of new capital goods. Older vintages of capital are put at a competitive disadvantage, even though their own productivity has not changed, and this disadvantage is capitalized in the price of older assets. The key insight by Hall (1968) was that the capital-embodied technical change entered the asset valuation model in a way that made it impossible to disentangle its average effect, 8, from the average wear and tear effect, \*, and the average revaluation effect,  $\Delta$ , using price data on used assets, P<sup>i</sup>(t,s). In this case, the partial derivative of P<sup>i</sup>(t,s) with respect to s is \*+8, and the partial derivative with respect to t is  $\Delta$ +8. Estimates of the age effect thus contain both the effects or wear and tear, plus the obsolescence factor. This is not necessarily a problem for the measurement of wealth on the right-hand consumer side of the CFM, where consumer wealth is measured, but it is a problem for the left-hand side measurement of productive capital. The obsolescence of term 8 must be stripped out of the cross-sectional depreciation estimate \*+8 and used to increase the

<sup>&</sup>lt;sup>9</sup> Another confusion sometimes arises between the definition of depreciation and the empirical implementation of the definition, which tends to produce results that display a rapid decline in asset value in the early years of asset life. This pattern seems to contradict the intuition that a physical asset like a chair retains most of its productive value up the point that it is retired from service (the one-hoss shay model). This intuition should not discredit the definition of depreciation as the partial derivative with respect to age even if it were correct, since the case of the one-hoss shay is included as a special case. However, the intuition is itself flawed by the fallacy of composition. The depreciation experience of a single asset is not necessarily the same as the average experience of the whole cohort of similar assets that were put in place in the same year. The members of the cohort will generally be retired at different ages, and if the retirement pattern is normally distributed, the average rate of depreciation for the cohort will be close to the geometric rate (Hulten and Wykoff (1981)). It is investment cohorts that matter for aggregate income accounting.

estimated quantity of investment, I(t).<sup>10</sup>

# IX. The 'Nonzero-Rent Economy"

The accounting architecture described in the preceding sections has two levels: a foundation based on the circular flow of goods and payments, and a superstructure based on the application of economic theory. The first is rather general, but the latter employs specific assumptions about the valuation of the stocks and flows, assumptions that Hall (2001) terms the "zero-rent economy." That economy is characterized by competitive markets, constant returns to scale, and the possibility that all factors can be freely adjusted in the long run. There are thus no economic rents in this economy, leading to the income identity (20) connecting the total gross return to capital assets,  $\Gamma$ P<sup>K</sup><sub>i</sub> K<sub>i</sub>, and the flow of income to consumers generated by the assets. This is one of the foundations of the accounting models of Jorgenson and Griliches (1967), and Christensen and Jorgenson (1969, 1970). The identity between the value of the capital stock,  $\Gamma$  P<sup>I</sup><sub>i</sub> K<sub>i</sub>, and the stock of consumer wealth, E+B in our notation, also hold in this world.

These identities have a different interpretation in an economy with economic rents generated

<sup>&</sup>lt;sup>10</sup> This solution is part of the larger issue of how to treat changes in the quality of products over time. When quality improvements occur, there is effectively 'more" of the new variety of the good embodied in one physical unit of the item than before. This can be captured by grossing-up the quantity of the good to reflect the improvement, in effect measuring the good in efficiency units. In the case of capital goods, 8 is the appropriate factor to apply. This factor is applied 'up front," and then adjustments for wear and tear are made as the assets ages. Hall (1968) advocates the hedonic price model as one way to isolate 8; the Hulten-Wykoff depreciation studies are actually an application of the hedonic model in which age and time variables are the main characteristics, with other factors included to standardize the asset. A full hedonic treatment would use these other factors to isolate the quality improvement component.

by monopoly power, intramarginal efficiency rents, persistent disequilibrium, imperfect information, or uncertainty. In this more realistic world, some of the income thought to accrue to the collection of capital assets,  $\Gamma P^{K}_{i} K_{i}$ , is in reality a return to entrepreneurship or to the owners of the firm. Any attempt to impose the zero-rent-economy rules in this world results in an over-estimate of the return to the specific capital assets included in the analysis. Moreover, there is a potential disconnect between the value of capital stock and the amount of wealth (indeed, this may be another explanation for the discrepancy evident in Figure 5). However, this does not mean that the use of theory, *per se*, is at fault, but rather that it is important to use the right theory. Nor does it mean that the zero-rent model is irrelevant. Given the difficulty of adapting models of imperfect competition, Schumpetarian entrepreneurship, and uncertainty to national income accounting problems, the zero-rent model is a logical and important step along the way toward Koopmans' vision of measurement with theory.

## X. Summary and Conclusion

This paper has discussed the design principles of a set of income and product accounts based on the circular flow model of payments and goods. The CFM is basically a schematic representation of an economy which portrays the various flows associated with the goods produced and consumed by the economy. It is more general than the zero-rent theory of basic neoclassical economics, but it is consistent with that theory (indeed, it can be derived from the assumptions of that theory). It is also more general than the accounts of contemporary practice, since it renders an account of the sources and uses of all goods, market and non-market, though the two sets of implied flows are separable. Moreover, its leads to a quadruple-entry accounting framework, since the flows in each quadrant of the CFM must be equal, and thus provides an opportunity for four statistical discrepancies (three if the Jorgenson-Griliches procedure is used to impute user cost). This fact alone may cool enthusiasm for its full implementation, but, then, some of the best architectural designs are hard to build, just as some of the most brilliant classical music is difficult to perform.

The CFM also provides important insights about the definition and treatment of capital. First, and foremost, the clear division between the consumer and business sectors applies to capital as well as to current account expenditures. This structure calls attention to the fact that capital originates with the decision by consumers to defer consumption, and leads to a design principle for determining what is and is not capital: if an expenditure is made with the intention of increasing future rather than current consumption, then it should be treated as being capital. This rule clearly applies to conventional business fixed capital, which can also be identified by its physical durability. It also applies to items like maintenance and repairs, which are undertaken to prevent a reduction in future consumption potential, and R&D and other intangible expenditures which are not treated as capital under prevailing durability notions but which are undertaken with the express intention of increasing future output. Carried to its logical extreme and applied to human capital, that is, to health and educational expenditures, the amount of capital formation missed by the failure to apply the deferred consumption rule dwarfs the amount of conventionally defined capital.

The structure of the CFM also calls attention to two areas where conventional accounting practice is particularly problematic. When capital is owner-operated, there are no explicit rental prices with which to value the lower left-hand branch of the CFM, and imputations must be made

for specific types of capital. Hall-Jorgenson procedures can be applied, here, in a "zero-rent" economy, but otherwise a problem persists. When capital is owner-constructed, there are no explicit investment good prices with which to value the upper left-hand branch of the CFM, but imputations here are almost impossible. However, the alternative of ignoring this kind of capital formation by treating it as a current expenditure runs afoul of the old dictum that is better to be imprecisely right than precisely wrong.

Finally, the division between the consumer and business sectors in the CFM points to another problem: with non-zero rents and, in particular, with uncertainty and imperfect information, there is a potential disconnect between the sectoral valuations of the same underlying flows of input and output. These are difficult problems, which cannot be solved without a further elaboration of theory. The quote by Griliches cited at the beginning of this paper applies at the end as well: "... it is not reasonable for us to expect the government to produce statistics in areas where concepts are mushy and where there is little professional agreement on what is to be measured and how." Using the CFM as the paradigm for an integrated national income and wealth account at least reduces the degree of mushiness and disagreement (e.g., in the treatment of intangibles), and provides a framework for future theoretical development.

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Figure 1 Circular Flow Model without Capital



Figure 2





Figure 4 Circular Flow Model with Capital





Figure 5 Source: Corrado, Hulten, and Sichel (forthcoming)